Higgs Basis Reloaded

Summary of SMEFT in Higgs basis

(last compiled July 8, 2020)

Contents

1 Pep talk

We consider the extension of the Standard Model (SM) by dimension-6 operators Q_i invariant under the SM gauge symmetries [1, 2]:

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i C_i\, Q_i\,. \tag{1.1}$$

The Q_i set is assumed to be complete and non-redundant, that is to form a *basis*. Such sets are known to consist of 2499 distinct operators; explicit constructions include the Warsaw basis [2] and the SILH basis [3]. The parameters C_i are called the *Wilson coefficients*. Together with the parameters of the SM Lagrangian, they make the parameter space of the the SMEFT. In our conventions the Wilson coefficients C_i have dimensions $[\text{mass}]^{-2}$ and they count as $\mathcal{O}(\Lambda^{-2})$ in the EFT expansion. Operators with dimensions higher than six, as well as dimension-5 operators are ignored in this discussion.

The idea behind the Higgs basis [4] was to create a new parameterization of the space of dimension-6 SMEFT operators satisfying the following properties:

1. Each Wilson coefficient has a simple physical interpretation;
2. Higgs observables at leading order are affected by a minimal set of Wilson coefficients;
3. Large correlations between the Higgs and electroweak constraints are avoided.

These features are very helpful for global analyses targeting the multi-parameter space of the SMEFT, see e.g. [5, 6, 7, 8]. The Higgs basis is a realization of the idea first laid out in [9], and its Wilson coefficients are what is called *primary effects* in that reference.

2 Definition of the Higgs basis

In Ref. [4] the Higgs basis was introduced as follows. One started with the SMEFT Lagrangian including dimension-6 operators in the SILH basis [3]. From it, a Lagrangian of mass eigenstates after electroweak symmetry breaking was derived. That Lagrangian was brought to a more convenient form by a series of fields and couplings redefinitions. Finally, the Higgs basis was introduced by identifying a set of independent linear combinations of SILH Wilson coefficients that fully characterizes the mass eigenstate Lagrangian. This algorithm permitted to construct a 1-to-1 linear map connecting the Higgs basis and SILH basis Wilson coefficients.

In this note we follow a different route. We define the Higgs basis via a map $M_{W\to H}$:[1]

$$\vec{c}_{\rm HB} = M_{W\to H}\vec{C}_{\rm WB}. \tag{2.1}$$

Here, $\vec{c}_{\rm HB}$ is a 2499-dimensional vector of Wilson coefficients in the Higgs basis to be defined shortly. By convention, all components of $\vec{c}_{\rm HB}$ are dimensionless. On the other

[1]Alternatively, rather than defining the Higgs basis via a rotation of Wilson coefficients we could define it as linear combinations of gauge-invariant dimension-6 operators. At the operator level, a definition equivalent to the one in Eq. (2.1) would be $Q_{\rm HB} = M^{-1\,T}_{W\to H} Q_{\rm WB}$, where $Q_{\rm WB}$ is the complete set of dimension-6 operators in the Warsaw basis.

hand, $\vec{C}_{\rm WB}$ is a 2499-dimensional vector of Wilson coefficients in the Warsaw basis of dimension-6 operators [2]. If one uses SMEFT beyond tree level, one needs to specify at which scale the map in Eq. (2.1) is defined; in our conventions, Eq. (2.1) holds at the scale m_Z. Finally, $M_{W\to H}$ is a 2499×2499-dimensional invertible matrix. It was obtained in Ref. [11] and is quote below.[2] As we shall see shortly, it depends on the parameters g_L, g_Y, and g_s, which are the SM gauge couplings at the scale m_Z, and on v and λ, which are the Higgs VEV and self-coupling. Their central values are

$$g_s = 1.2172, \quad g_L = 0.6485, \quad g_Y = 0.3580, \quad v = 246.22 \text{ GeV}, \quad \lambda = 0.1291. \tag{2.2}$$

and their errors can be ignored from the present purpose.

We are ready to write down the map $M_{W\to H}$. The vector $\vec{c}_{\rm HB}$ in Eq. (2.1) contains the following Wilson coefficients:

- The Higgs couplings

$$\delta c_z, \; c_{z\Box}, \; c_{zz}, \; c_{\gamma\gamma}, \; c_{z\gamma}, \; c_{gg}, \; \delta\lambda_3, \; \tilde{c}_{zz}, \; \tilde{c}_{\gamma\gamma}, \; \tilde{c}_{z\gamma}, \; \tilde{c}_{gg}, \; \delta y_u, \; \delta y_d, \; \delta y_e, \tag{2.3}$$

  where the blue parameters are real, and δy are 3×3 complex matrices in the flavor

[2] With respect to that reference, we adapted sign, naming, and flavor conventions to match those of WCxf [10]. See Appendix A for more details.

space. They are related to the Wilson coefficients in the Warsaw basis as

$$\begin{aligned}
v^{-2}\delta c_z &= C_{\varphi\Box} - \frac{1}{4}C_{\varphi D} - \frac{3}{2}\Delta_{G_F}, \\
v^{-2}c_{z\Box} &= \frac{1}{2g_L^2}\left(C_{\varphi D} + 2\Delta_{G_F}\right), \\
v^{-2}c_{gg} &= \frac{4}{g_s^2}C_{\varphi G}, \\
v^{-2}c_{\gamma\gamma} &= 4\left(\frac{1}{g_L^2}C_{\varphi W} + \frac{1}{g_Y^2}C_{\varphi B} - \frac{1}{g_L g_Y}C_{\varphi WB}\right), \\
v^{-2}c_{zz} &= 4\left(\frac{g_L^2 C_{\varphi W} + g_Y^2 C_{\varphi B} + g_L g_Y C_{\varphi WB}}{(g_L^2+g_Y^2)^2}\right), \\
v^{-2}c_{z\gamma} &= 4\left(\frac{C_{\varphi W} - C_{\varphi B} - \frac{g_L^2-g_Y^2}{2g_L g_Y}C_{\varphi WB}}{g_L^2+g_Y^2}\right), \\
v^{-2}\tilde{c}_{gg} &= \frac{4}{g_s^2}C_{\varphi\tilde{G}}, \\
v^{-2}\tilde{c}_{\gamma\gamma} &= 4\left(\frac{1}{g_L^2}C_{\varphi\tilde{W}} + \frac{1}{g_Y^2}C_{\varphi\tilde{B}} - \frac{1}{g_L g_Y}C_{\varphi W\tilde{B}}\right), \\
v^{-2}\tilde{c}_{zz} &= 4\left(\frac{g_L^2 C_{\varphi\tilde{W}} + g_Y^2 C_{\varphi\tilde{B}} + g_L g_Y C_{\varphi W\tilde{B}}}{(g_L^2+g_Y^2)^2}\right), \\
v^{-2}\tilde{c}_{z\gamma} &= 4\left(\frac{C_{\varphi\tilde{W}} - C_{\varphi\tilde{B}} - \frac{g_L^2-g_Y^2}{2g_L g_Y}C_{\varphi W\tilde{B}}}{g_L^2+g_Y^2}\right), \\
v^{-2}\delta\lambda_3 &= -\frac{1}{\lambda}C_{\varphi} + 3C_{\varphi\Box} - \frac{3}{4}C_{\varphi D} - \frac{1}{2}\Delta_{G_F}, \\
v^{-2}[\delta y_f]_{JK} &= -\frac{v}{\sqrt{2m_{f_J}m_{f_K}}}[C^\dagger_{f\varphi}]_{JK} + \delta_{JK}\left(c_{\varphi\Box} - \frac{1}{4}C_{\varphi D} - \frac{1}{2}\Delta_{G_F}\right),
\end{aligned} \tag{2.4}$$

where $\Delta_{G_F} = [C^{(3)}_{\varphi l}]_{11} + [C^{(3)}_{\varphi l}]_{22} - \frac{1}{2}[C_{ll}]_{1221}$. The interpretation of the Higgs basis Wilson coefficients on the left-hand-side of Eq. (2.4) as certain Higgs boson couplings will become clear in Section 4.

- The vertex corrections

$$\delta g_L^{Ze},\ \delta g_R^{Ze},\ \delta g_L^{Zu},\ \delta g_R^{Zu},\ \delta g_L^{Zd},\ \delta g_R^{Zd},\ \delta g_L^{W\ell},\ \delta g_R^{Wq}, \tag{2.5}$$

which are all 3×3 Hermitian matrices in the flavor space, except for δg_R^{Wq} which is a general complex matrix. They are related to the Wilson coefficients in the

Warsaw basis as

$$
\begin{aligned}
v^{-2}\delta g_L^{W\ell} &= C_{\varphi l}^{(3)} + f(1/2,0) - f(-1/2,-1), \\
v^{-2}\delta g_L^{Z\ell} &= -\frac{1}{2}C_{\varphi l}^{(3)} - \frac{1}{2}C_{\varphi l}^{(1)} + f(-1/2,-1), \\
v^{-2}\delta g_R^{Z\ell} &= -\frac{1}{2}C_{\varphi e}^{(1)} + f(0,-1), \\
v^{-2}\delta g_L^{Zu} &= \frac{1}{2}C_{\varphi q}^{(3)} - \frac{1}{2}C_{\varphi q}^{(1)} + f(1/2,2/3), \\
v^{-2}\delta g_L^{Zd} &= -\frac{1}{2}C_{\varphi q}^{(3)} - \frac{1}{2}C_{\varphi q}^{(1)} + f(-1/2,-1/3), \\
v^{-2}\delta g_R^{Zu} &= -\frac{1}{2}C_{\varphi u} + f(0,2/3), \\
v^{-2}\delta g_R^{Zd} &= -\frac{1}{2}C_{\varphi d} + f(0,-1/3), \\
v^{-2}\delta g_R^{Wq} &= \frac{1}{2}C_{\varphi ud},
\end{aligned} \tag{2.6}
$$

where

$$
f(T^3,Q) \equiv \left\{-Q\frac{g_L g_Y}{g_L^2 - g_Y^2}C_{\varphi WB} - \left(\frac{1}{4}C_{\varphi D} + \frac{1}{2}\Delta_{G_F}\right)\left(T^3 + Q\frac{g_Y^2}{g_L^2 - g_Y^2}\right)\right\}\mathbf{1}. \tag{2.7}
$$

- The F^3 couplings

$$
\lambda_z,\ \tilde{\lambda}_z,\ \lambda_g,\ \tilde{\lambda}_g, \tag{2.8}
$$

which are all real. They are related to the Wilson coefficients in the Warsaw basis as

$$
\begin{aligned}
v^{-2}\lambda_z &= \frac{3}{2}g_L C_W, \qquad v^{-2}\tilde{\lambda}_z = \frac{3}{2}g_L C_{\tilde{W}}, \\
v^{-2}\lambda_g &= \frac{C_G}{g_s^3}, \qquad v^{-2}\tilde{\lambda}_g = \frac{C_{\tilde{G}}}{g_s^3}.
\end{aligned} \tag{2.9}
$$

- The dipole couplings

$$
d_{Gu},\ d_{Gd},\ d_{Ae},\ d_{Au},\ d_{Ad},\ d_{Ze},\ d_{Zu},\ d_{Zd}, \tag{2.10}
$$

which are all complex 3×3 matrices. They are related to the Wilson coefficients in the Warsaw basis as

$$
\begin{aligned}
v^{-2}d_{Gf} &= -\frac{16}{g_s^2}C_{fG}^*, \\
v^{-2}d_{Af} &= -\frac{16}{g_L^2}\left(\eta_f C_{fW}^* + C_{fB}^*\right), \\
v^{-2}d_{Zf} &= -16\left(\eta_f\frac{1}{g_L^2+g_Y^2}C_{fW}^* - \frac{g_Y^2}{g_L^2(g_L^2+g_Y^2)}C_{fB}^*\right), \\
v^{-2}d_{Wf} &= -\frac{16}{g_L^2}C_{fW}^*,
\end{aligned} \tag{2.11}
$$

where $\eta_u = +1$, $\eta_{d,e} = -1$.

- Four-fermion couplings

$$c_{ll},\ c_{qq}^{(1)},\ c_{qq}^{(3)},\ c_{lq}^{(1)},\ c_{lq}^{(3)},\ c_{ee},\ c_{uu},\ c_{dd},\ c_{eu},\ c_{ed},\ c_{ud}^{(1)},\ c_{ud}^{(3)}$$
$$c_{le},\ c_{lu},\ c_{ld},\ c_{qe},\ c_{qu}^{(1)},\ c_{qu}^{(8)},\ c_{qd}^{(1)},\ c_{qd}^{(8)},\ c_{ledq},\ c_{quqd}^{(1)},\ c_{quqd}^{(8)},\ c_{lequ}^{(1)},\ c_{lequ}^{(3)}. \tag{2.12}$$

which are all 4-index tensors in the flavor space. They are trivially related to the Wilson coefficients in the Warsaw basis as

$$v^{-2}c_i = C_i. \tag{2.13}$$

The full set of Higgs basis Wilson coefficients is displayed in Eqs. (2.3), (2.5), (2.8), (2.10), and (2.12). The map $M_{W\to H}$ is completely specified by Eqs. (2.4), (2.6), (2.9), (2.11), and (2.13). The physical interpretation of the Higgs basis Wilson coefficients will be clarified in Section 4.

3 Lagrangian for mass eigenstates

In order to derive physical predictions of the SMEFT the first step is to recast its Lagrangian in Eq. (1.1) in terms of the mass eigenstates after electroweak symmetry breaking. In the Warsaw basis this exercise was completed in [12], where all the interactions vertices with the corresponding Feynman rule were given. To derive the mass eigenstate Lagrangian in the Higgs basis one could for example borrow the interaction terms from that reference and translate the couplings to the Higgs basis using the map in Eqs. (2.4), (2.6), (2.9), (2.11), and (2.13).[3] Below I quote the part of the mass eigenstate Lagrangian most relevant for the LHC and Higgs phenomenology.

By definition of the mass eigenstate basis, the kinetic terms for the Higgs, W, Z bosons, photons, gluons, and fermions are diagonal and canonically normalized:

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu \hat{h}\partial_\mu \hat{h} - \frac{1}{2}W^+_{\mu\nu}W^-_{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z_{\mu\nu} - \frac{1}{4}A_{\mu\nu}A_{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \sum_{f\in u,d,e,\nu} i\bar{f}\gamma_\mu\partial_\mu f. \tag{3.1}$$

Above, we mark the Higgs boson field $\hat{h}$ because later we will switch to a more convenient variable to describe this particle. The mass terms for the Higgs, W, Z bosons, and fermions are also diagonal:

$$\mathcal{L} \supset -\frac{1}{2}m_h^2 h^2 + m_W^2 W^+_\mu W^-_\mu + \frac{1}{2}m_Z^2 Z_\mu Z_\mu - \sum_{f\in u,d,e} m_f \bar{f}f, \tag{3.2}$$

[3] In practice, we rederive all interactions using a custom-made computer code.

where in the Higgs basis

$$\begin{aligned}
m_h^2 &= 2\hat{\lambda}\hat{v}^2\Bigg[1 - \frac{3\hat{g}_L^4}{\hat{g}_L^2-\hat{g}_Y^2}c_{z\Box} - \frac{3\hat{g}_L^2\hat{g}_Y^2}{\hat{g}_L^2-\hat{g}_Y^2}c_{zz} + \frac{3\hat{g}_L^4\hat{g}_Y^4}{(\hat{g}_L^2-\hat{g}_Y^2)(\hat{g}_L^2+\hat{g}_Y^2)^2}c_{\gamma\gamma} + \frac{3\hat{g}_L^2\hat{g}_Y^2}{\hat{g}_L^2+\hat{g}_Y^2}c_{z\gamma} \\
&\quad -\frac{5}{2}\delta c_z + \frac{3}{2}\delta\lambda_3 - 3\Delta\Bigg], \\
m_W^2 &= \frac{\hat{g}_L^2\hat{v}^2}{4}\left[\frac{\hat{g}_L^2}{2}c_{zz} + \frac{\hat{g}_L^2\hat{g}_Y^4}{2(\hat{g}_L^2+\hat{g}_Y^2)^2}c_{\gamma\gamma} + \frac{\hat{g}_L^2\hat{g}_Y^2}{\hat{g}_L^2+\hat{g}_Y^2}c_{z\gamma}\right] \\
m_Z^2 &= \frac{(\hat{g}_L^2+\hat{g}_Y^2)\hat{v}^2}{4}\Bigg[\hat{g}_Y^2 c_{z\gamma} - \frac{\hat{g}_Y^2(\hat{g}_L^2+\hat{g}_Y^2)}{\hat{g}_L^2-\hat{g}_Y^2}c_{z\Box} + \frac{\hat{g}_L^2\hat{g}_Y^4}{\hat{g}_L^4-\hat{g}_Y^4}c_{\gamma\gamma} \\
&\quad + \frac{\hat{g}_L^6 - \hat{g}_L^4\hat{g}_Y^2 - 3\hat{g}_L^2\hat{g}_Y^4 - \hat{g}_Y^6}{2\hat{g}_L^2(\hat{g}_L^2-\hat{g}_Y^2)}c_{zz} - \left(1+\frac{\hat{g}_Y^2}{\hat{g}_L^2}\right)\Delta\Bigg]
\end{aligned} \tag{3.3}$$

and $\Delta \equiv \delta g_L^{We} + \delta g_L^{W\mu} - \frac{1}{2}[c_{ll}]_{1221}$. Above $\hat{g}_L$ and $\hat{g}_Y$ are the $SU(2)\times U(1)$ gauge couplings in the SM Lagrangian $\mathcal{L}_{\rm SM}$ in Eq. (1.1), $\hat{v}$ is the vacuum expectation value of the Higgs field, $\langle H^\dagger H\rangle = \hat{v}^2/2$, and $\hat{\lambda}$ is the quartic Higgs coupling in the SM Lagrangian. Since dimension-6 operators affect the input observables from which these couplings are determined in the SM context, in SMEFT one *cannot* assume the hatted coupling have the numerical values in Eq. (2.2). In fact, their numerical value vary as a function of dimension-6 Wilson coefficients.

The gluon couplings to matter are given by

$$\mathcal{L} \supset -\hat{g}_s\left(1+\frac{\hat{g}_s^2}{4}c_{gg}\right)G_\mu^a\sum_{f\in u,d}\bar{f}\gamma_\mu T^a f. \tag{3.4}$$

The photon couplings to matter are given by

$$\mathcal{L} \supset -\frac{\hat{g}_L\hat{g}_Y}{\sqrt{\hat{g}_L^2+\hat{g}_Y^2}}\left(1+\frac{\hat{g}_L^2\hat{g}_Y^2}{4(\hat{g}_L^2+\hat{g}_Y^2)}c_{\gamma\gamma}\right)A_\mu\sum_{f\in u,d,e}Q_f\bar{f}\gamma_\mu f. \tag{3.5}$$

The Z boson couplings to charged leptons and quarks are given by

$$\mathcal{L} \supset -\sqrt{\hat{g}_L^2+\hat{g}_Y^2}Z_\mu\sum_{f\in u,d,e}\bar{f}\gamma_\mu\left(T_f^3 - \hat{s}_\theta^2 Q_f + \hat{\delta}g^{Zf}\right)f \tag{3.6}$$

where $\hat{s}_\theta = \hat{g}_Y/\sqrt{\hat{g}_L^2+\hat{g}_Y^2}$ and

$$\hat{\delta}g^{Zf} = \delta g^{Zf} + \left(T_f^3 + \frac{\hat{g}_Y^2}{\hat{g}_L^2-\hat{g}_Y^2}Q_f\right)\left(\frac{\hat{g}_L^2}{2}c_{z\Box} + \frac{\hat{g}_L^2+\hat{g}_Y^2}{4}c_{zz}\right) - Q_f\frac{\hat{g}_L^2\hat{g}_Y^2}{2(\hat{g}_L^2-\hat{g}_Y^2)(\hat{g}_L^2+\hat{g}_Y^2)^2}c_{\gamma\gamma}. \tag{3.7}$$

The contact interactions between fermions and the Higgs and Z bosons are given by

$$\mathcal{L} \supset -\sqrt{\hat{g}_L^2+\hat{g}_Y^2}\left(\frac{2\hat{h}}{\hat{v}}+\frac{\hat{h}^2}{\hat{v}^2}\right)Z_\mu\sum_{f\in u,d,e,\nu}\bar{f}\gamma_\mu\hat{\delta}g^{hZf}f \tag{3.8}$$

where

$$\hat{\delta}g^{hZf} = \delta g^{Zf} + \frac{\hat{g}_L^2}{2}\left(T_f^3 + \frac{\hat{g}_Y^2}{\hat{g}_L^2-\hat{g}_Y^2}Q_f\right)c_{z\Box} - Q_f\frac{\hat{g}_L^2\hat{g}_Y^2}{2(\hat{g}_L^2-\hat{g}_Y^2)}\left(c_{zz} + \frac{\hat{g}_L^2-\hat{g}_Y^2}{\hat{g}_L^2+\hat{g}_Y^2}c_{z\gamma} + \frac{\hat{g}_L^2\hat{g}_Y^2}{(\hat{g}_L^2+\hat{g}_Y^2)^2}c_{\gamma\gamma}\right). \tag{3.9}$$

The cubic Higgs boson self-interactions are given by

$$\mathcal{L} \supset -\hat{\lambda}\hat{v}(1+\hat{\delta}\lambda_3)\hat{h}^3 + \frac{\hat{\lambda}_3^{(2)}}{\hat{v}}\hat{h}(\partial_\mu \hat{h})^2, \tag{3.10}$$

where

$$\begin{aligned}
\hat{\delta}\lambda_3 &= -\frac{11\hat{g}_L^4}{2(\hat{g}_L^2-\hat{g}_Y^2)}c_{z\Box} - \frac{11\hat{g}_L^2\hat{g}_Y^2}{2(\hat{g}_L^2-\hat{g}_Y^2)}c_{zz} + \frac{11\hat{g}_L^4\hat{g}_Y^4}{2(\hat{g}_L^2-\hat{g}_Y^2)(\hat{g}_L^2+\hat{g}_Y^2)^2}c_{\gamma\gamma} + \frac{11\hat{g}_L^2\hat{g}_Y^2}{2(\hat{g}_L^2+\hat{g}_Y^2)}c_{z\gamma} \\
&\quad -\frac{9}{2}\delta c_z + \frac{5}{2}\delta\lambda_3 - \frac{11}{2}\Delta, \\
\hat{\lambda}_3^{(2)} &= -\frac{3\hat{g}_L^4}{\hat{g}_L^2-\hat{g}_Y^2}c_{z\Box} - \frac{3\hat{g}_L^2\hat{g}_Y^2}{\hat{g}_L^2-\hat{g}_Y^2}c_{zz} + \frac{3\hat{g}_L^4\hat{g}_Y^4}{(\hat{g}_L^2-\hat{g}_Y^2)(\hat{g}_L^2+\hat{g}_Y^2)^2}c_{\gamma\gamma} + \frac{3\hat{g}_L^2\hat{g}_Y^2}{\hat{g}_L^2+\hat{g}_Y^2}c_{z\gamma} \\
&\quad -2\delta c_z - 3\Delta.
\end{aligned} \tag{3.11}$$

The SM-like Higgs boson couplings to massive gauge bosons are given by

$$\mathcal{L} \supset \frac{\hat{h}}{\hat{v}}\left[(1+\hat{\delta}c_w)\frac{\hat{g}_L^2\hat{v}^2}{2}W_\mu^+W_\mu^- + (1+\hat{\delta}c_z)\frac{(\hat{g}_L^2+\hat{g}_Y^2)\hat{v}^2}{4}Z_\mu Z_\mu\right], \tag{3.12}$$

where

$$\begin{aligned}
\hat{\delta}c_w &= \delta c_z + \frac{3\hat{g}_L^4}{2(\hat{g}_L^2-\hat{g}_Y^2)}c_{z\Box} + \frac{\hat{g}_L^2(\hat{g}_L^2+2\hat{g}_Y^2)}{2(\hat{g}_L^2-\hat{g}_Y^2)}c_{zz} - \frac{\hat{g}_L^2\hat{g}_Y^4(2\hat{g}_L^2+\hat{g}_Y^2)}{2(\hat{g}_L^2-\hat{g}_Y^2)(\hat{g}_L^2+\hat{g}_Y^2)^2}c_{\gamma\gamma} - \frac{\hat{g}_L^2\hat{g}_Y^2}{2(\hat{g}_L^2+\hat{g}_Y^2)}c_{z\gamma} \\
&\quad +\frac{3}{2}\Delta, \\
\hat{\delta}c_z &= \delta c_z + \frac{(3\hat{g}_L^4-4\hat{g}_L^2\hat{g}_Y^2)}{2(\hat{g}_L^2-\hat{g}_Y^2)}c_{z\Box} + \frac{\hat{g}_L^4-\hat{g}_L^2\hat{g}_Y^2-\hat{g}_Y^4}{2(\hat{g}_L^2-\hat{g}_Y^2)}c_{zz} + \frac{\hat{g}_L^4\hat{g}_Y^4}{2(\hat{g}_L^2-\hat{g}_Y^2)(\hat{g}_L^2+\hat{g}_Y^2)^2}c_{\gamma\gamma} + \frac{\hat{g}_L^2\hat{g}_Y^2}{2(\hat{g}_L^2+\hat{g}_Y^2)}c_{z\gamma} \\
&\quad -\frac{1}{2}\Delta
\end{aligned} \tag{3.13}$$

The 2-derivative Higgs boson couplings to gauge bosons are given by

$$\begin{aligned}
\mathcal{L} \supset\; & \frac{\hat{h}}{\hat{v}}\Big\{c_{gg}\frac{\hat{g}_s^2}{4}G_{\mu\nu}^aG_{\mu\nu}^a + \tilde{c}_{gg}\frac{\hat{g}_s^2}{4}G_{\mu\nu}^a\tilde{G}_{\mu\nu}^a + c_{ww}\frac{\hat{g}_L^2}{2}W_{\mu\nu}^+W_{\mu\nu}^- + \tilde{c}_{ww}\frac{\hat{g}_L^2}{2}W_{\mu\nu}^+\tilde{W}_{\mu\nu}^- \\
& +c_{\gamma\gamma}\frac{\hat{g}_L^2\hat{g}_Y^2}{4(\hat{g}_L^2+\hat{g}_Y^2)}A_{\mu\nu}A_{\mu\nu} + \tilde{c}_{\gamma\gamma}\frac{\hat{g}_L^2\hat{g}_Y^2}{4(\hat{g}_L^2+\hat{g}_Y^2)}A_{\mu\nu}\tilde{A}_{\mu\nu} + c_{z\gamma}\frac{\hat{g}_L\hat{g}_Y}{2}Z_{\mu\nu}A_{\mu\nu} + \tilde{c}_{z\gamma}\frac{\hat{g}_L\hat{g}_Y}{2}Z_{\mu\nu}\tilde{A}_{\mu\nu} \\
& +c_{zz}\frac{\hat{g}_L^2+\hat{g}_Y^2}{4}Z_{\mu\nu}Z_{\mu\nu} + \tilde{c}_{zz}\frac{\hat{g}_L^2+\hat{g}_Y^2}{4}Z_{\mu\nu}\tilde{Z}_{\mu\nu},\Big\},
\end{aligned} \tag{3.14}$$

where

$$\begin{aligned}
c_{ww} &= c_{zz} + \frac{2\hat{g}_Y^2}{\hat{g}_L^2+\hat{g}_Y^2}c_{z\gamma} + \frac{\hat{g}_Y^4}{(\hat{g}_L^2+\hat{g}_Y^2)^2}c_{\gamma\gamma}, \\
\tilde{c}_{ww} &= \tilde{c}_{zz} + \frac{2\hat{g}_Y^2}{\hat{g}_L^2+\hat{g}_Y^2}\tilde{c}_{z\gamma} + \frac{\hat{g}_Y^4}{(\hat{g}_L^2+\hat{g}_Y^2)^2}\tilde{c}_{\gamma\gamma}.
\end{aligned} \tag{3.15}$$

4 New variables

The Lagrangian displayed in Section 3 is perfectly legal, and come used to calculate physical predictions of SMEFT. However, it has a number of inconvenient features. In this section, we use equations of motion and field and couplings redefinitions to bring it into a more convenient form, where the physical interpretation of the Higgs basis Wilson coefficients is more transparent. We stress that this is purely cosmetic: the physical observables up to $\mathcal{O}(1/\Lambda^2)$ in the EFT expansion are exactly the same, whether calculated with the fields and couplings in Section 3 or the ones in Section 4.

Below we enumerate the redefinitions and briefly discuss motivations for each of them.

#1 In the SM, the hatted couplings $\hat{g}_s$, $\hat{g}_L$, $\hat{g}_Y$, $\hat{\lambda}$, and the VEV $\hat{v}$ can be directly related to input observables. On that basis they can be assigned well-defined numerical values with error intervals. That is no longer true in the SMEFT. As can be seen in Eq. (3.3) and Eq. (3.5), the presence of dimension-6 operators complicates the relation between the SM couplings and traditional input observables such as m_Z or α. For this reason it is convenient to introduce a new (unhatted) set of of couplings, related to the original couplings by

$$\begin{aligned}\hat{g}_s = g_s\,(1+\delta g_s)\,,\quad \hat{g}_L = g_L\,(1+\delta g_L)\,,\quad \hat{g}_Y = g_Y\,(1+\delta g_Y)\,,\\ \hat{v} = v\,(1+\delta v)\,,\quad \hat{\lambda} = \lambda\,(1+\delta\lambda)\,.\end{aligned} \tag{4.1}$$

We choose the shifts as

$$\begin{aligned}
\delta g_s &= -\frac{g_s^2}{4}c_{gg},\\
\delta g_L &= -\frac{g_L^4}{2(g_L^2-g_Y^2)}c_{z\Box} - \frac{g_L^2(g_L^2+g_Y^2)}{4(g_L^2-g_Y^2)}c_{zz} + \frac{g_L^2 g_Y^4}{4(g_L^4-g_Y^4)}c_{\gamma\gamma},\\
\delta g_Y &= \frac{g_L^2 g_Y^2}{2(g_L^2-g_Y^2)}c_{z\Box} + \frac{g_Y^2(g_L^2+g_Y^2)}{4(g_L^2-g_Y^2)}c_{zz} - \frac{g_L^4 g_Y^2}{4(g_L^4-g_Y^4)}c_{\gamma\gamma},\\
\delta v &= \frac{g_L^4}{2(g_L^2-g_Y^2)}c_{z\Box} + \frac{g_L^2 g_Y^2}{2(g_L^2-g_Y^2)}c_{zz} - \frac{g_L^4 g_Y^4}{2(g_L^2-g_Y^2)(g_L^2+g_Y^2)^2}c_{\gamma\gamma} - \frac{g_L^2 g_Y^2}{2(g_L^2+g_Y^2)}c_{z\gamma} + \frac{1}{2}\Delta,\\
\delta\lambda &= \frac{2g_L^4}{g_L^2-g_Y^2}c_{z\Box} + \frac{2g_L^2 g_Y^2}{g_L^2-g_Y^2}c_{zz} - \frac{2g_L^4 g_Y^4}{(g_L^2-g_Y^2)(g_L^2+g_Y^2)^2}c_{\gamma\gamma} - \frac{2g_L^2 g_Y^2}{g_L^2+g_Y^2}c_{z\gamma}\\
&\quad + \frac{5}{2}\delta c_z - \frac{3}{2}\delta\lambda_3 + 2\Delta.
\end{aligned} \tag{4.2}$$

Recall that $\Delta \equiv \delta g_L^{We} + \delta g_L^{W\mu} - \frac{1}{2}[c_{ll}]_{1221}$. In these new variables, the mass terms in Eq. (3.3) simplify

$$m_h^2 = 2\lambda v^2, \qquad m_Z^2 = \frac{(g_L^2+g_Y^2)v^2}{4}, \qquad m_W^2 = \frac{g_L^2 v^2}{4}\,(1+\Delta)\,. \tag{4.3}$$

Furthermore, the gluon and photon couplings to matter in Eq. (3.4) and Eq. (3.5) simplify as

$$\mathcal{L} \supset -g_s G_\mu^a \sum_{f\in u,d} \bar{f}\gamma_\mu T^a f - \frac{g_L g_Y}{\sqrt{g_L^2+g_Y^2}} A_\mu \sum_{f\in u,d,e} Q_f \bar{f}\gamma_\mu f. \tag{4.4}$$

Finally, one can show that the Fermi constant measured in muon decay is related via $G_F = \frac{1}{\sqrt{2}v^2}$ to the parameter v in Eq. (4.1). All in all, the new parameter set g_s, g_L, g_Y, v, λ introduced in Eq. (4.1), at tree level, is related to the input observables α_s, α, m_Z, G_F, m_h in the same way as the corresponding parameters in the SM. With these new parameters the mass eigenstate Lagrangian simplifies considerably, and moreover they can be assigned the numerical values displayed in Eq. (2.2), which are independent of the dimension-6 Wilson coefficients up to $\mathcal{O}(\frac{1}{16\pi^2\Lambda^2})$ corrections.

#2 The cubic Higgs boson interactions in Eq. (3.10) come in two forms: one the familiar SM-like cubic coupling, and the other a 2-derivative interaction. However, the latter can be eliminated by a suitable choice of variables, after which its effect is absorbed into the former, and into other couplings involving two Higgs bosons and gauge bosons and/or fermions. To this end one can perform the following non-linear redefinition of the Higgs field:

$$\hat{h} = h + \delta_h \left(h^2 + \frac{1}{3} h^3 \right), \tag{4.5}$$

where

$$\begin{aligned} \delta_h \;=\; & \frac{3g_L^4}{2(g_L^2 - g_Y^2)} c_{z\Box} + \frac{3g_L^2 g_Y^2}{2(g_L^2 - g_Y^2)} c_{zz} - \frac{3g_L^4 g_Y^4}{2(g_L^2 - g_Y^2)(g_L^2 + g_Y^2)^2} c_{\gamma\gamma} - \frac{3g_L^2 g_Y^2}{2(g_L^2 + g_Y^2)} c_{z\gamma} \\ & + \delta c_z + \frac{3}{2}\Delta. \end{aligned} \tag{4.6}$$

This eliminates *all* derivative Higgs boson self-interactions from the Lagrangian. In the new variable h the self-interactions take the form

$$\mathcal{L} \supset -\lambda v \left(1 + \delta\lambda_3\right) h^3 - \frac{\lambda}{4}\left(1 + \delta\lambda_4\right) h^4 - \lambda_5 \frac{\lambda}{v} h^5 - \lambda_6 \frac{\lambda}{v^2} h^6, \tag{4.7}$$

where

$$\delta\lambda_4 = 6\delta\lambda_3 - \frac{4}{3}\delta c_z, \quad \lambda_5 = \frac{3}{4}\delta\lambda_3 - \frac{1}{4}\delta c_z, \quad \lambda_6 = \frac{1}{8}\delta\lambda_3 - \frac{1}{24}\delta c_z. \tag{4.8}$$

We stress that, although the field redefinition in Eq. (4.5) changes the Lagrangian, it does *not* change on-shell S-matrix elements. More generally, on-shell S-matrix elements, whether tree- or loop-level, are not affected by general field redefinitions, even non-linear ones or non-gauge-invariant ones, as long as they satisfy certain minimal conditions [13]. Therefore, amplitudes for all Higgs boson production processes will be the same whether calculated with the Lagrangian in Section 3 using the field $\hat{h}$, or with the Lagrangian in this section using the field h.

#3 In the new variables introduced in this section, the Z boson couplings to charged leptons and quarks also simplify:

$$\mathcal{L} \supset -\sqrt{g_L^2 + g_Y^2} Z_\mu \sum_{f \in u,d,e} \bar{f}\gamma_\mu \left(T_f^3 - s_\theta^2 Q_f + \delta g^{Zf}\right) f, \tag{4.9}$$

where $s_\theta = g_Y/\sqrt{g_L^2 + g_Y^2}$. That is to say, the Wilson coefficients δg^{Zf} in the Higgs basis, cf. Eq. (2.5), are interpreted as *vertex corrections* to the Z boson couplings as compared to the SM prediction. There is a similar kind of interaction in Eq. (3.8) which differs from Eq. (4.9) by the presence of additional Higgs boson fields. Since it is already $\mathcal{O}(\Lambda^{-2})$, it remains the same in the new variables, except for the trivial relabeling $\hat{X} \to X$. It is however possible to combine the vertex correction in Eq. (4.9) and the Higgs interactions in Eq. (4.9) into one compact expression:

$$\mathcal{L} \supset -\sqrt{g_L^2 + g_Y^2} Z_\mu \left(1 + \frac{h}{v}\right)^2 \sum_{f \in u,d,e} \bar{f} \gamma_\mu \delta g^{Zf} f, \tag{4.10}$$

The motivation to do so is the following. The interaction in Eq. (3.8) are certainly relevant for the LHC Higgs phenomenology (in particular in the $H \to ZZ^*$ channel) and must be taken into account to correctly assess the parameter space. On the other hand, there are strong model independent constraints on the vertex corrections δg^{Zf} [7], at the level of $\mathcal{O}(10^{-3})$ for the leptonic vertex correction. Such strongly suppressed vertex corrections will not be relevant for LHC Higgs phenomenology, where typical accuracy is $\mathcal{O}(10^{-1})$. Eliminating from Eq. (3.8) all terms *not* proportional to the vertex correction δg offers users an option to ignore this class of interactions in the LHC context. In practice, the elimination can be achieved by adding to the Lagrangian the terms

$$\begin{aligned}
\mathcal{L}_{\rm eom} \;=\;& \left(\frac{2h}{v} + \frac{h^2}{v^2}\right) \Bigg\{ x_{ZB} Z_\mu \left[\partial_\nu B_{\nu\mu} + \frac{i g_Y}{2} H^\dagger \overleftrightarrow{D_\mu} H + g_Y j_\mu^Y\right] \\
& + x_{ZW} Z_\mu \left[D_\nu W^3_{\nu\mu} + \frac{i}{2} g_L H^\dagger \sigma^3 \overleftrightarrow{D_\mu} H + g_L j^3_\mu\right] \\
& + \sum_{i=1}^{2} x_W W^i_\mu \left[D_\nu W^i_{\nu\mu} + \frac{i}{2} g_L H^\dagger \sigma^i \overleftrightarrow{D_\mu} H + g_L j^i_\mu\right] \Bigg\}.
\end{aligned} \tag{4.11}$$

where j_μ^a and j_μ^Y are the fermionic currents coupled to the $SU(2) \times U(1)$ gauge bosons in the SM, and

$$\begin{aligned}
x_{ZB} &= -\frac{g_L^2 g_Y \sqrt{g_L^2+g_Y^2}}{2(g_L^2-g_Y^2)} c_{z\Box} - \frac{g_L^2 g_Y \sqrt{g_L^2+g_Y^2}}{2(g_L^2-g_Y^2)} c_{zz} + \frac{g_L^4 g_Y^3}{2(g_L^2-g_Y^2)(g_L^2+g_Y^2)^{3/2}} c_{\gamma\gamma} + \frac{g_L^2 g_Y}{2\sqrt{g_L^2+g_Y^2}} c_{z\gamma}, \\
x_{ZW} &= -\frac{g_L^3 \sqrt{g_L^2+g_Y^2}}{2(g_L^2-g_Y^2)} c_{z\Box} - \frac{g_L g_Y^2 \sqrt{g_L^2+g_Y^2}}{2(g_L^2-g_Y^2)} c_{zz} + \frac{g_L^3 g_Y^4}{2(g_L^2-g_Y^2)(g_L^2+g_Y^2)^{3/2}} c_{\gamma\gamma} + \frac{g_L g_Y^2}{2\sqrt{g_L^2+g_Y^2}} c_{z\gamma}, \\
x_W &= -\frac{g_L^4}{2(g_L^2-g_Y^2)} c_{z\Box} - \frac{g_L^2 g_Y^2}{2(g_L^2-g_Y^2)} c_{zz} + \frac{g_L^4 g_Y^4}{2(g_L^2-g_Y^2)(g_L^2+g_Y^2)^2} c_{\gamma\gamma} + \frac{g_L^2 g_Y^2}{2(g_L^2+g_Y^2)} c_{z\gamma}.
\end{aligned} \tag{4.12}$$

Note that each term in square brackets in Eq. (4.11) vanishes by the SM equations of motion. Therefore adding it to the Lagrangian does not change S-matrix elements, whether at tree or at loop level. The role of Eq. (4.11) is to eliminate all $h^n V \bar{f} f$ contact interactions that are not proportional to the vertex corrections δg. Of course, the eliminated interaction do not vanish, but re-emerge in a different (more transparent) form. In this case, their effect on single Higgs processes is taken over by 2-derivative Higgs boson interactions with electroweak gauge bosons:

$$\mathcal{L} \supset c_{z\Box} g_L^2 Z_\mu \partial_\nu Z_{\mu\nu} + c_{\gamma\Box} g_L g_Y Z_\mu \partial_\nu A_{\mu\nu} + c_{w\Box} g_L^2 \left(W_\mu^- \partial_\nu W_{\mu\nu}^+ + \text{h.c.}\right), \tag{4.13}$$

where $c_{z\Box}$ is already one of the Higgs basis Wilson coefficients, and the other two parameters can be expressed by the Wilson coefficients as

$$\begin{aligned}
c_{w\Box} &= \frac{1}{g_L^2 - g_Y^2}\left[g_L^2 c_{z\Box} + g_Y^2 c_{zz} - \frac{g_Y^2(g_L^2 - g_Y^2)}{g_L^2 + g_Y^2} c_{z\gamma} - \frac{g_L^2 g_Y^4}{(g_L^2 + g_Y^2)^2} c_{\gamma\gamma}\right] \\
c_{\gamma\Box} &= \frac{1}{g_L^2 - g_Y^2}\left[2 g_L^2 c_{z\Box} + (g_L^2 + g_Y^2) c_{zz} - (g_L^2 - g_Y^2) c_{z\gamma} - \frac{g_L^2 g_Y^2}{g_L^2 + g_Y^2} c_{\gamma\gamma}\right]. \quad (4.14)
\end{aligned}$$

The change of variables and transformations described in **#1**, **#2**, **#3** lead to a more convenient form of the mass eigenstate Lagrangian, in which the interpretation of various Higgs basis Wilson coefficients is more transparent. The Lagrangian in these variables is the one introduced in Ref. [4]. However, we stress again that applying these transformations is a question of taste. Using the original variables and mass eigenstate Lagrangian from Section 3 would lead to the same amplitudes for all physical processes up to $\mathcal{O}(\Lambda^{-2})$, that is up to the maximum order for which our EFT is defined.

5 Final Lagrangian

In this section we summarize the SMEFT Lagrangian in the Higgs basis, rewritten in the variables introduced in Section 4. This is the same Lagrangian as the one in Ref. [4], up to change in sign and CKM conventions to match those in WCxF. The idea is to list here all terms that may be relevant for the current Higgs analyses at the LHC. The complete Lagrangian is available in a custom-made computer code, and any additional terms can be listed on request.

5.1 Kinetic and mass terms

The kinetic terms are diagonal and canonically normalized:

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu h \partial_\mu h - \frac{1}{2} W^+_{\mu\nu} W^-_{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z_{\mu\nu} - \frac{1}{4} A_{\mu\nu} A_{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_{f \in u,d,e,\nu} i \bar{f} \gamma_\mu \partial_\mu f. \quad (5.1)$$

Here h, $W^\pm_\mu$, Z_μ, A_μ, G^a_μ and f are respectively Higgs boson, W boson, Z boson, photon, gluon, and fermion fields.

The mass terms are given by

$$\mathcal{L} \supset -\frac{1}{2} m_h^2 h^2 + m_W^2 W^+_\mu W^-_\mu + \frac{1}{2} m_Z^2 Z_\mu Z_\mu - \sum_{f \in u,d,e} m_f \bar{f} f, \quad (5.2)$$

where

$$\begin{aligned}
m_h &= \sqrt{2\lambda} v, \\
m_Z &= \frac{\sqrt{g_L^2 + g_Y^2}\, v}{2}, \\
m_W &= \frac{g_L v}{2}\left(1 + \delta m_w\right), \qquad \delta m_w = \frac{1}{2}\delta g_L^{We} + \frac{1}{2}\delta g_L^{W\mu} - \frac{1}{4}[c_{ll}]_{1221}, \\
m_f &= \frac{Y_f v}{\sqrt{2}}\left(1 + \delta m_f\right), \qquad \delta m_{f_J} = \frac{1}{2}[\delta y_f]_{JJ} - \frac{1}{2}\delta c_z. \quad (5.3)
\end{aligned}$$

Note that the neutrinos are treated as massless here.

5.2 Single Higgs couplings

The single Higgs couplings to matter are given by

$$\begin{aligned}
\mathcal{L} \supset\ & \frac{h}{v}\Big[(1+\delta c_w)\frac{g_L^2 v^2}{2}W_\mu^+ W_\mu^- + (1+\delta c_z)\frac{(g_L^2+g_Y^2)v^2}{4}Z_\mu Z_\mu \\
& - \sum_{f\in u,d,e}\sum_{IJ}\sqrt{m_{f_I}m_{f_J}}\left[(\delta_{IJ}+[\delta y_f]_{IJ})\,\bar{f}_L f_R + \text{h.c.}\right] \\
& + c_{ww}\frac{g_L^2}{2}W_{\mu\nu}^+ W_{\mu\nu}^- + \tilde{c}_{ww}\frac{g_L^2}{2}W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- + c_{w\Box}g_L^2\left(W_\mu^-\partial_\nu W_{\mu\nu}^+ + \text{h.c.}\right) \\
& + c_{gg}\frac{g_s^2}{4}G_{\mu\nu}^a G_{\mu\nu}^a + c_{\gamma\gamma}\frac{g_L^2 g_Y^2}{4(g_L^2+g_Y^2)}A_{\mu\nu}A_{\mu\nu} + c_{z\gamma}\frac{g_L g_Y}{2}Z_{\mu\nu}A_{\mu\nu} + c_{zz}\frac{g_L^2+g_Y^2}{4}Z_{\mu\nu}Z_{\mu\nu} \\
& + c_{z\Box}g_L^2 Z_\mu\partial_\nu Z_{\mu\nu} + c_{\gamma\Box}g_L g_Y Z_\mu \partial_\nu A_{\mu\nu} \\
& + \tilde{c}_{gg}\frac{g_s^2}{4}G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + \tilde{c}_{\gamma\gamma}\frac{g_L^2 g_Y^2}{4(g_L^2+g_Y^2)}A_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{z\gamma}\frac{g_L g_Y}{2}Z_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{zz}\frac{g_L^2+g_Y^2}{4}Z_{\mu\nu}\tilde{Z}_{\mu\nu}\Big],
\end{aligned} \tag{5.4}$$

Most of the EFT parameters above are identical with the Higgs basis Wilson coefficients, as defined in Section 2. The remaining EFT parameters can be expressed by the Wilson coefficients as

$$\begin{aligned}
\delta c_w &= \delta c_z + 4\delta m_w, \qquad \delta m_w \equiv \frac{1}{2}\delta g_L^{We} + \frac{1}{2}\delta g_L^{W\mu} - \frac{1}{4}[c_{ll}]_{1221}, \\
c_{ww} &= c_{zz} + \frac{2g_Y^2}{g_L^2+g_Y^2}c_{z\gamma} + \frac{g_Y^4}{(g_L^2+g_Y^2)^2}c_{\gamma\gamma}, \\
\tilde{c}_{ww} &= \tilde{c}_{zz} + \frac{2g_Y^2}{g_L^2+g_Y^2}\tilde{c}_{z\gamma} + \frac{g_Y^4}{(g_L^2+g_Y^2)^2}\tilde{c}_{\gamma\gamma}, \\
c_{w\Box} &= \frac{1}{g_L^2-g_Y^2}\left[g_L^2 c_{z\Box} + g_Y^2 c_{zz} - \frac{g_Y^2(g_L^2-g_Y^2)}{g_L^2+g_Y^2}c_{z\gamma} - \frac{g_L^2 g_Y^4}{(g_L^2+g_Y^2)^2}c_{\gamma\gamma}\right], \\
c_{\gamma\Box} &= \frac{1}{g_L^2-g_Y^2}\left[2g_L^2 c_{z\Box} + (g_L^2+g_Y^2)c_{zz} - (g_L^2-g_Y^2)c_{z\gamma} - \frac{g_L^2 g_Y^2}{g_L^2+g_Y^2}c_{\gamma\gamma}\right].
\end{aligned} \tag{5.5}$$

5.3 Higgs self-interactions

The Higgs boson self-interactions take the form

$$\mathcal{L} \supset -\lambda v\,(1+\delta\lambda_3)\,h^3 - \frac{\lambda}{4}\,(1+\delta\lambda_4)\,h^4 - \lambda_5\frac{\lambda}{v}h^5 - \lambda_6\frac{\lambda}{v^2}h^6, \tag{5.6}$$

where $\delta\lambda_3$ is one of the Wilson coefficients in the Higgs basis, and the remaining EFT parameters can be expressed by the Wilson coefficients as

$$\delta\lambda_4 = 6\delta\lambda_3 - \frac{4}{3}\delta c_z, \quad \lambda_5 = \frac{3}{4}\delta\lambda_3 - \frac{1}{4}\delta c_z, \quad \lambda_6 = \frac{1}{8}\delta\lambda_3 - \frac{1}{24}\delta c_z. \tag{5.7}$$

5.4 Gauge interactions

The gauge interactions have the form

$$\begin{aligned}
\mathcal{L} \supset & -\frac{g_L g_Y}{\sqrt{g_L^2+g_Y^2}} A_\mu \sum_{f\in u,d,e} Q_f \bar{f}\gamma_\mu f - g_s G^a_\mu \sum_{f\in u,d} \bar{f}\gamma_\mu T^a f, \\
& - \frac{g_L}{\sqrt{2}}\left(W^+_\mu \bar{\nu}_L \gamma_\mu (\mathbf{I}+\delta g_L^{W\ell}) e_L + W^+_\mu \bar{u}_L\gamma_\mu (V_{\rm CKM}+\delta g_L^{Wq}) d_L + W^+_\mu \bar{u}_R \gamma_\mu \delta g_R^{Wq} d_R + \text{h.c.}\right) \\
& - \sqrt{g_L^2+g_Y^2} Z_\mu \left[\sum_{f\in u,d,e,\nu} \bar{f}_L \gamma_\mu (T^3_f - s_\theta^2 Q_f + \delta g_L^{Zf}) f_L + \sum_{f\in u,d,e} \bar{f}_R\gamma_\mu(-s^2_\theta Q_f + \delta g_R^{Zf}) f_R\right],
\end{aligned} \tag{5.8}$$

where all the departures from the SM couplings are parametrized by the vertex corrections δg^{Vf}. Note that, by construction, there are no vertex corrections to photon and gluon couplings. Most of the vertex corrections are Wilson coefficients of the Higgs basis, as defined in Section 2. The remaining EFT parameters can be expressed by the Wilson coefficients as

$$\begin{aligned}
\delta g_L^{Z\nu} &= \delta g_L^{W\ell} + \delta g_L^{Ze}, \\
\delta g_L^{Wq} &= V^\dagger_{\rm CKM} \delta g_L^{Zu} V_{\rm CKM} - \delta g_L^{Zd}.
\end{aligned} \tag{5.9}$$

5.5 Dipole interactions

The dipole-type interactions take the form

$$\begin{aligned}
\mathcal{L} \supset & -\frac{1+h/v}{4v}\left[g_s \sum_{f\in u,d} \frac{\sqrt{m_{f_I} m_{f_J}}}{v} \bar{f}_I \sigma_{\mu\nu} T^a [d_{Gf}]_{IJ} P_L f_J G^a_{\mu\nu} \right. \\
& + \frac{g_L g_Y}{\sqrt{g_L^2+g_Y^2}} \sum_{f\in u,d,e} \frac{\sqrt{m_{f_I} m_{f_J}}}{v} \bar{f}_I \sigma_{\mu\nu} [d_{Af}]_{IJ} P_L f_J A_{\mu\nu} \\
& + \sqrt{g_L^2+g_Y^2} \sum_{f\in u,d,e} \frac{\sqrt{m_{f_I} m_{f_J}}}{v} \bar{f}_I \sigma_{\mu\nu} [d_{Zf}]_{IJ} P_L f_J Z_{\mu\nu} \\
& + \sqrt{2} g_L \frac{\sqrt{m_{u_I} m_{u_J}}}{v} \bar{u}_I \sigma_{\mu\nu} [d_{Wu}]_{IJ} P_L d_J W^+_{\mu\nu} + \sqrt{2} g_L \frac{\sqrt{m_{d_I} m_{d_J}}}{v} \bar{d}_I \sigma_{\mu\nu} [d_{Wd}]_{IJ} P_L u_J W^-_{\mu\nu} \\
& \left. + \sqrt{2} g_L \frac{\sqrt{m_{e_I} m_{e_J}}}{v} \bar{e}_I \sigma_{\mu\nu} [d_{We}]_{IJ} P_L \nu_J W^-_{\mu\nu} + \text{h.c.}\right],
\end{aligned} \tag{5.10}$$

where $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, and d_{Gf}, d_{Af}, d_{Zf}, and d_{Wf} are complex 3×3 matrices. Out of these, d_{Gd}, d_{Af}, and d_{Zf} are already Wilson coefficients of the Higgs basis, as defined in Section 2. The remaining dipole parameters can be expressed by the Wilson coefficients as

$$\eta_f d_{Wf} = d_{Zf} + \frac{g_Y^2}{g_L^2+g_Y^2} d_{Af}, \tag{5.11}$$

where $\eta_u = +1$, $\eta_{d,e} = -1$.

5.6 Contact Higgs-gauge-fermion interactions

The contact Higgs-gauge-fermion interactions have the form

$$
\begin{aligned}
\mathcal{L} \supset & -\sqrt{2} g_L \left(\frac{h}{v} + \frac{h^2}{2v^2}\right) \left(W_\mu^+ \bar{\nu}_L \gamma_\mu \delta g_L^{W\ell} e_L + W_\mu^+ \bar{u}_R \gamma_\mu \delta g_L^{Wq} d_R + W_\mu^+ \bar{u}_R \gamma_\mu \delta g_R^{Wq} d_R + \text{h.c.}\right) \\
& - 2\sqrt{g_L^2 + g_Y^2} \left(\frac{h}{v} + \frac{h^2}{2v^2}\right) Z_\mu \left[\sum_{f \in u,d,e,\nu} \bar{f}_L \gamma_\mu \delta g_L^{Zf} f_L + \sum_{f \in u,d,e} \bar{f}_R \gamma_\mu \delta g_R^{Zf} f_R\right].
\end{aligned}
\tag{5.12}
$$

The EFT parameters describing these interaction are completely fixed by the vertex corrections discussed in the previous subsection.

5.7 Triple gauge couplings

The triple gauge couplings of electroweak gauge bosons are customarily parametrized as [14]

$$
\begin{aligned}
\mathcal{L} \supset & -i \frac{g_L}{\sqrt{g_L^2 + g_Y^2}} \Big\{ g_Y \left(W_{\mu\nu}^+ W_\mu^- - W_{\mu\nu}^- W_\mu^+\right) A_\nu + g_L g_{1,z} \left(W_{\mu\nu}^+ W_\mu^- - W_{\mu\nu}^- W_\mu^+\right) Z_\nu \\
& + g_Y \kappa_\gamma W_\mu^+ W_\nu^- A_{\mu\nu} + g_L \kappa_z W_\mu^+ W_\nu^- Z_{\mu\nu} + g_Y \tilde{\kappa}_\gamma W_\mu^+ W_\nu^- \tilde{A}_{\mu\nu} + g_L \tilde{\kappa}_z W_\mu^+ W_\nu^- \tilde{Z}_{\mu\nu} \\
& + \frac{\lambda_\gamma g_Y}{m_W^2} W_{\mu\nu}^+ W_{\nu\rho}^- A_{\rho\mu} + \frac{\lambda_z g_L}{m_W^2} W_{\mu\nu}^+ W_{\nu\rho}^- Z_{\rho\mu} + \frac{\tilde{\lambda}_\gamma g_Y}{m_W^2} W_{\mu\nu}^+ W_{\nu\rho}^- \tilde{A}_{\rho\mu} + \frac{\tilde{\lambda}_z g_L}{m_W^2} W_{\mu\nu}^+ W_{\nu\rho}^- \tilde{Z}_{\rho\mu} \Big\}.
\end{aligned}
\tag{5.13}
$$

Above, only λ_z and $\tilde{\lambda}_z$ are Wilson coefficients in the Higgs basis, as defined in Section 2. The remaining EFT parameters can be expressed by the Wilson coefficients as

$$
\begin{aligned}
g_{1,z} &= 1 + \frac{1}{2(g_L^2 - g_Y^2)} \left[-g_L^2(g_L^2 + g_Y^2) c_{z\Box} - g_Y^2(g_L^2 + g_Y^2) c_{zz} + g_Y^2(g_L^2 - g_Y^2) c_{z\gamma} + \frac{g_L^2 g_Y^2}{g_L^2 + g_Y^2} c_{\gamma\gamma}\right], \\
\kappa_z &= 1 + \frac{1}{g_L^2 - g_Y^2} \left[-\frac{g_L^2(g_L^2 + g_Y^2)}{2} c_{z\Box} - g_L^2 g_Y^2 c_{zz} + \frac{g_L^2 g_Y^2 (g_L^2 - g_Y^2)}{g_L^2 + g_Y^2} c_{z\gamma} + \frac{g_L^4 g_Y^4}{(g_L^2 + g_Y^2)^2} c_{\gamma\gamma}\right], \\
\kappa_\gamma &= 1 + \frac{g_L^2}{2} \left[c_{zz} - \frac{g_L^2 - g_Y^2}{g_L^2 + g_Y^2} c_{z\gamma} - \frac{g_L^2 g_Y^2}{(g_L^2 + g_Y^2)^2} c_{\gamma\gamma}\right], \\
\tilde{\kappa}_\gamma &= \frac{g_L^2}{2} \left[\tilde{c}_{zz} - \frac{g_L^2 - g_Y^2}{g_L^2 + g_Y^2} \tilde{c}_{z\gamma} - \frac{g_L^2 g_Y^2}{(g_L^2 + g_Y^2)^2} \tilde{c}_{\gamma\gamma}\right], \\
\tilde{\kappa}_z &= -\frac{g_Y^2}{2} \left[\tilde{c}_{zz} - \frac{g_L^2 - g_Y^2}{g_L^2 + g_Y^2} \tilde{c}_{z\gamma} - \frac{g_L^2 g_Y^2}{(g_L^2 + g_Y^2)^2} \tilde{c}_{\gamma\gamma}\right], \\
\lambda_\gamma &= \lambda_z, \qquad \tilde{\lambda}_\gamma = \tilde{\lambda}_z.
\end{aligned}
\tag{5.14}
$$

One can verify that these expression lead to the usual dimension-6 SMEFT relations between the triple gauge couplings: $\kappa_z = g_{1,z} - \frac{g_Y^2}{g_L^2}(\kappa_\gamma - 1)$, $\tilde{\kappa}_z = -\frac{g_Y^2}{g_L^2}\tilde{\kappa}_\gamma$.

5.8 Double Higgs couplings to matter

The interactions between two Higgs bosons and two other SM fields are given by

$$
\begin{aligned}
\mathcal{L} \supset \; & h^2 \Big\{ \left(1 + \delta c_z^{(2)}\right) \frac{g_L^2 + g_Y^2}{8} Z_\mu Z_\mu + \left(1 + \delta c_w^{(2)}\right) \frac{g_L^2}{4} W_\mu^+ W_\mu^- \\
- \; & \frac{1}{2v^2} \sum_f \sqrt{m_{f_J} m_{f_K}} \left[\bar{f}_J [y_f^{(2)}]_{JK} P_L f_K + \text{h.}c. \right] \\
+ \; & \frac{1}{8v^2} \left[c_{gg}^{(2)} g_s^2 G_{\mu\nu}^a G_{\mu\nu}^a + \tilde{c}_{gg}^{(2)} g_s^2 G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a \right] \\
+ \; & \frac{1}{8v^2} \left(2 c_{ww}^{(2)} g_L^2 W_{\mu\nu}^+ W_{\mu\nu}^- + c_{zz}^{(2)} (g_L^2 + g_Y^2) Z_{\mu\nu} Z_{\mu\nu} + 2 c_{z\gamma}^{(2)} g_L g_Y Z_{\mu\nu} A_{\mu\nu} + c_{\gamma\gamma}^{(2)} \frac{g_L^2 g_Y^2}{g_L^2 + g_Y^2} A_{\mu\nu} A_{\mu\nu} \right) \\
+ \; & \frac{1}{8v^2} \left(2 \tilde{c}_{ww}^{(2)} g_L^2 W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- + \tilde{c}_{zz}^{(2)} (g_L^2 + g_Y^2) Z_{\mu\nu} \tilde{Z}_{\mu\nu} + 2 \tilde{c}_{z\gamma}^{(2)} g_L g_Y Z_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{\gamma\gamma}^{(2)} \frac{g_L^2 g_Y^2}{g_L^2 + g_Y^2} A_{\mu\nu} \tilde{A}_{\mu\nu} \right) \\
+ \; & \frac{1}{2v^2} \left(g_L^2 c_{w\Box}^{(2)} (W_\mu^+ \partial_\nu W_{\mu\nu}^- + W_\mu^- \partial_\nu W_{\mu\nu}^+) + g_L^2 c_{z\Box}^{(2)} Z_\mu \partial_\nu Z_{\mu\nu} + g_L g_Y c_{\gamma\Box}^{(2)} Z_\mu \partial_\nu A_{\mu\nu} \right) \Big\}. \qquad (5.15)
\end{aligned}
$$

The parameters above are related to the Wilson coefficient in the Higgs basis as

$$
\begin{aligned}
\delta c_z^{(2)} &= 4\delta c_z, \qquad \delta c_w^{(2)} = 4\delta c_z + 6\Delta, \\
[y_f^{(2)}]_{JK} &= 3[\delta y_f]_{JK} - \delta c_z \, \delta_{JK}, \\
c_{vv}^{(2)} &= c_{vv}, \qquad \tilde{c}_{vv}^{(2)} = \tilde{c}_{vv}, \qquad v \in \{g, w, z, \gamma\}, \\
c_{v\Box}^{(2)} &= c_{v\Box}, \qquad v \in \{w, z, \gamma\}, \qquad (5.16)
\end{aligned}
$$

where the expression of c_{ww}, $\tilde{c}_{ww}$, $c_{z\Box}$ and $c_{\gamma\Box}$ in terms of the Higgs basis Wilson coefficients are given in Eq. (5.5).

6 Discussion

We close this note with a number of scattered comments.

- Using Warsaw or Higgs basis is totally a matter of convenience, and leads to fully equivalent results at $\mathcal{O}(1/\Lambda^2)$ in the EFT expansion. One can verify these statements for any particular process, by calculating it in both bases, and comparing the results using the map in Eq. (2.1). Moreover, using the Higgs basis with the Lagrangian in Section 3, or the one with redefined field and couplings in Section 5 leads to the same results at $\mathcal{O}(1/\Lambda^2)$.

- The Higgs basis is designed to be convenient for the characterization of Higgs processes at the LHC. It does not mean it is convenient for *any* application. One counterexample is the diboson production. In the Higgs basis, the cubic CP-even electroweak gauge couplings are described by five parameters c_{zz}, $c_{z\gamma}$, $c_{\gamma\gamma}$, $c_{z\Box}$, and λ_z, see Eq. (5.14). In diboson analyses it is more convenient to use the standard TGC parametrization in terms of $g_{1,z}$, κ_γ and λ_z, and only a-posteriori translate the results to the Higgs basis using Eq. (5.14). One could in fact construct another basis, call it *Higgs-TGC basis*, where $\delta g_{1,z} \equiv g_{1,z} - 1$ and $\delta\kappa_\gamma \equiv \kappa_\gamma - 1$ are defined as

Wilson coefficients, at the expense of two Wilson coefficients from the original Higgs basis, e.g. c_{zz} and $c_{z\Box}$. Similarly, for the analysis of $h \to WW^*$ analysis alone, one would rather use the δc_w, c_{ww}, $\tilde{c}_{ww}$, $c_{w\Box}$ variables, and only later translate to the Higgs basis using Eq. (5.5). Again, for this purpose one could also construct a new basis, call it *Higgs-WW basis*, where δc_w, c_{ww}, $\tilde{c}_{ww}$, $c_{w\Box}$ are the Wilson coefficients, at the expense of e.g. δc_z, c_{zz}, $\tilde{c}_{zz}$, $c_{z\Box}$.

- To reduce the number of free parameters in a SMEFT analysis of Higgs observables one may take advantage of the fact that some combinations of Wilson coefficients are strongly constrained by other precision measurements, notably by the electroweak data from LEP-1. A nice feature of the Higgs basis is that it separates the Wilson coefficients affecting only the Higgs observables at tree level (the ones in Eq. (2.3)) from those affecting also electroweak precision measurements and thus being strongly constrained (the ones in Eq. (2.5)). In particular, all leptonic, bottom and charm vertex corrections δg are constrained at a level of 10^{-2} or better [15]. The current experimental sensitivity at the LHC not sufficient to probe the effect of these vertex corrections on the Higgs observables, thus for all practical purpose one can simply set these δg to zero when analyzing the LHC Higgs data. Similarly, δm_w is very strongly constrained by W mass measurements, and thus can be set to zero. This greatly reduces the number of Wilson coefficients that a typical Higgs analysis has to deal with.

- Some caution regarding the point above has to be exercised, however. First, not all vertex corrections and dipole couplings have been strongly constrained by prior non-Higgs measurements. For example, the vertex corrections to the $Zt\bar{t}$ couplings, $\delta g^{Zt}_{L,R}$, for obvious reasons are not constrained by LEP-1, and thus they should not be neglected whenever they contribute to Higgs observables. Moreover, for some of the observables the effect of strongly constrained parameters may be amplified at the LHC. One such case was identified in [16]. In the case of diboson production at the LHC, the effect of the light quark vertex correction is enhanced by the factor s/v^2 at high invariant diboson mass $\sqrt{s}$, as is not negligible.

A Notation and conventions

This appendix discusses notation and conventions used in this note. Note that some of the conventions are changed as compared to Refs. [4, 11] in order to match those of WCxf [10].

The gauge couplings of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ group are denoted by g_s, g_L, g_Y, and the corresponding gauge fields by G^a_μ, W^i_μ, B_μ, $a = 1 \ldots 8$, $i = 1 \ldots 3$. The covariant derivatives read [4]

$$D_\mu f = \left(\partial_\mu + i g_s G^a_\mu T^a_f + i g_L W^i_\mu T^i_f + i g_Y Y_f B_\mu\right) f. \tag{A.1}$$

Consequently, the covariant field strength tensors are expressed by the corresponding

[4] Note the sign difference with respect to [4, 11].

gauge fields as

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_L \epsilon^{ijk} W^j_\mu W^k_\nu, \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu. \end{aligned} \tag{A.2}$$

where ϵ^{ijk} and f^{abc} are the totally anti-symmetric structure tensors of $SU(2)$ and $SU(3)$.

The fermions and Wilson coefficients of fermionic operators in SMEFT Lagrangian are given in a particular flavor basis . Namely, the basis is chosen such the fields $P_R u_J$, $P_R d_J$, $P_R e_J$ and $P_L L_J = (P_L \nu_J, P_L e_J)$ are mass eigenstates after electroweak symmetry breaking. Furthermore, the left-handed quark doublets are given by $q_J = ([V^\dagger_{\rm CKM}]_{JK} P_L u_K, P_L d_J)$, where $P_L u_J$, $P_L d_J$ mass eigenstate after electroweak symmetry breaking.[5] Note that the definition mass eigenstates may not be RG invariant in the presence of higher-dimensional operators. By conventions, we choose that we work with mass eigenstates defined by the Lagrangian at the scale $\mu = m_Z$.

Repeated Lorentz indices $\mu, \nu, \ldots$ are implicitly contracted using the Lorentz tensor $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. Similarly, repeated generation indices I, J, K, as well as repeated group indices i, j, k, a, b, c are implicitly summed over.

The components of the Higgs double field φ are parametrized as

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} i\sqrt{2}\hat{G}_+ \\ \hat{v} + \hat{h} + i\hat{G}_z \end{pmatrix}, \tag{A.3}$$

where $\hat{v}$ is the Higgs VEV, $\hat{h}$ is the Higgs boson field, and $\hat{G}$ are the unphysical Goldstone boson fields.

The SMEFT Lagrangian is given by

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i C_i \, Q_i, \tag{A.4}$$

where $\mathcal{L}_{\rm SM}$ is the SM Lagrangian, and Q_i form a basis of dimension-6 operators. The Wilson coefficients C_i have dimensions $[\mathrm{mass}]^{-2}$ and they count as $\mathcal{O}(\Lambda^{-2})$ in the EFT expansion. We ignore dimension-5 operators, as well as any effects subleading to $\mathcal{O}(\Lambda^{-2})$ (thus in particular, order C_i^2 effects are ignored). We work with the dimension-6 operators in the so-called Warsaw basis [2]. In the original reference the flavor structure of the operators was not specified. For that, we follow the notation and conventions established by the *Wilson coefficient exchange format* (WCxf) [10]. We also apply the WCxf convention that all components of $\vec{C}_{\rm WB}$ have dimension $1/\mathrm{mass}^2$ (thus, the SMEFT scale Λ, often displayed explicitly in the literature, is absorbed into C_i here).

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