Towards light-cone conformal blocks for N=4 amplitudes

27/07/2020

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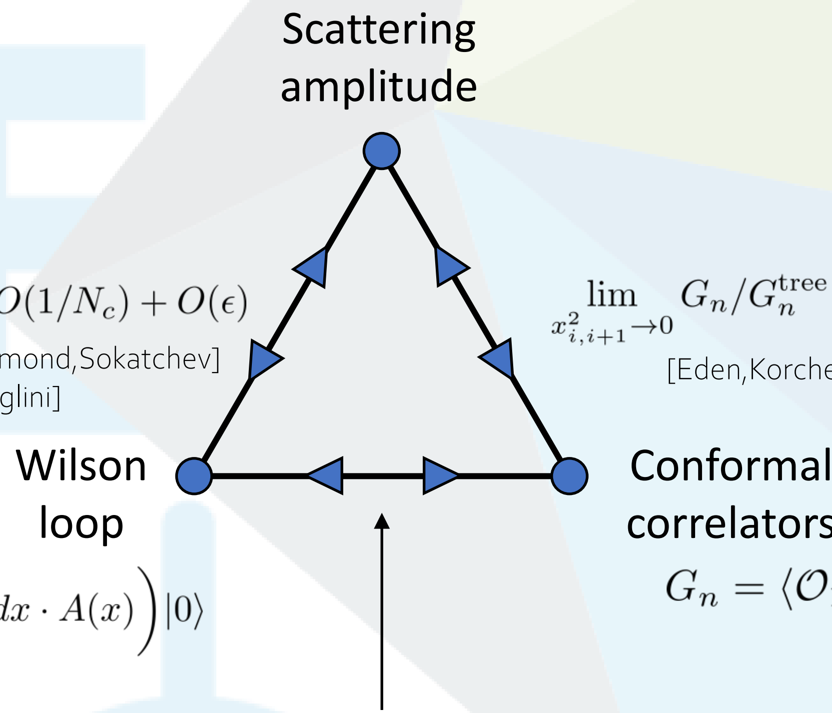
This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 764850 (SAGEX).

MHV Amplitudes in N=4 SYM

$$A_n = A_n^{\text{tree}} \times \mathcal{M}_n^{\text{BDS}} \times \mathcal{R}_n$$

$$\ln\left(A_n/A_n^{\text{tree}}\right) = \ln\left(W[C_n]\right) + O(1/N_c) + O(\epsilon)$$

[Alday,Maldacena][Korchemsky,Drummond,Sokatchev]
[Brandhuber,Heslop,Travaglini]



$$\lim_{x_{i,i+1}^2 \to 0} G_n/G_n^{\text{tree}} = \left(A_n/A_n^{\text{tree}}\right)^2$$

[Eden,Korchemsky,Sokatchev]

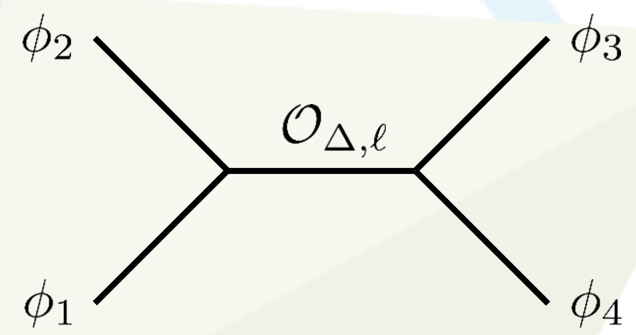
$$W[C_n] = \frac{1}{N_c} \langle 0|\text{tr}\, P \exp\left(ig \oint_{C_n} dx \cdot A(x)\right)|0\rangle$$

$$G_n = \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\ldots\mathcal{O}_n(x_n)\rangle$$

$$\lim_{x_{i,i+1}^2 \to 0} G_n/G_n^{\text{tree}} = (W[C_n])^2$$

[Alday,Eden,Korchemsky,Maldacena ,Sokatchev]

Conformal blocks



- Basis of functions for conformally invariant objects

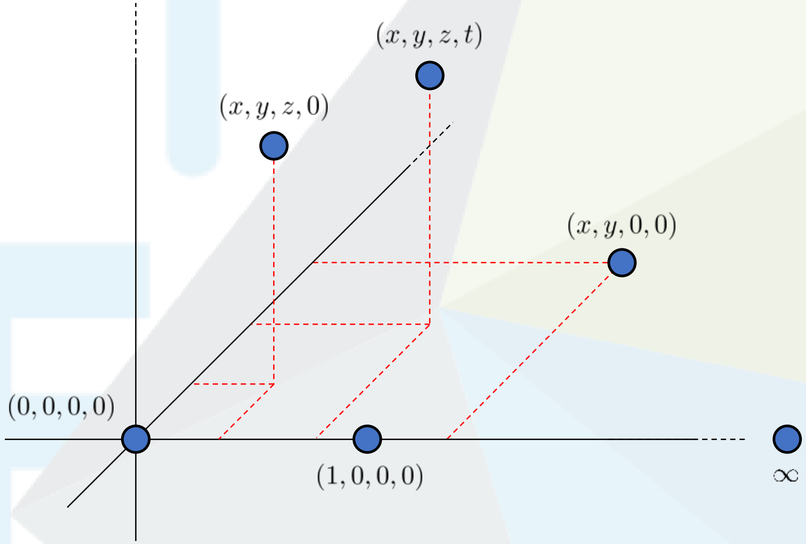
$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\rangle = \frac{1}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2}{2}}(x_{34}^2)^{\frac{\Delta_3+\Delta_4}{2}}}\left(\frac{x_{24}^2}{x_{14}^2}\right)^{\frac{\Delta_{12}}{2}}\left(\frac{x_{14}^2}{x_{13}^2}\right)^{\frac{\Delta_{34}}{2}} F(u,v)$$

cross ratios: $u = \dfrac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad v = \dfrac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$

$$F(u,v) = \sum_{\Delta,\ell} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}}\, g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(u,v)$$

- Can we do the same with N=4 amplitudes?

Cross ratios

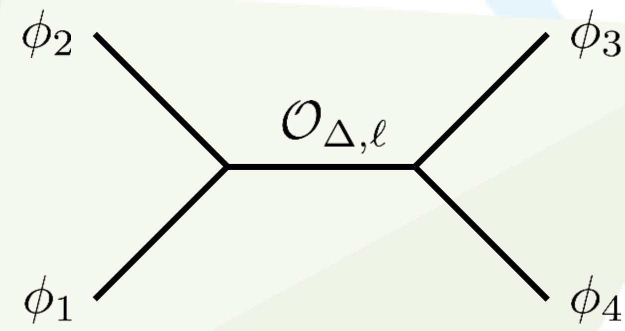


$$\#(\text{cross ratios}) = 4N - 15 \underline{- N}$$

- For amplitudes also need lightlike condition $x_{i,i+1}^2 = 0$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \to 0 \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} \to 0$$

Four-points blocks



Solution of differential equations associated to Casimir operators [Dolan,Osborn]

$$\frac{1}{2} M^{AB}_{(12)} M^{(12)}_{BA} \mathcal{O}_{\Delta,\ell} = c^{(2)}_{\Delta,\ell} \mathcal{O}_{\Delta,\ell} \Longrightarrow \mathcal{D}\, g^{\Delta_{12},\Delta_{34}}_{\Delta,\ell}(u,v) = \frac{1}{2} c^{(2)}_{\Delta,\ell}\, g^{\Delta_{12},\Delta_{34}}_{\Delta,\ell}$$

$$\mathcal{D} = (1-u-v)\frac{\partial}{\partial v}\left(v\frac{\partial}{\partial v} - \frac{\Delta_{12}}{2} + \frac{\Delta_{34}}{2}\right) + u\frac{\partial}{\partial u}\left(2u\frac{\partial}{\partial u} - d\right) - (1+u-v)\left(u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v} - \frac{\Delta_{12}}{2}\right)\left(u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v} + \frac{\Delta_{34}}{2}\right)$$

$$\frac{1}{2} M^{AB}_{(12)} M^{(12)}_{BC} M^{CD}_{(12)} M^{(12)}_{DA} \mathcal{O}_{\Delta,\ell} = c^{(4)}_{\Delta,\ell} \mathcal{O}_{\Delta,\ell}$$

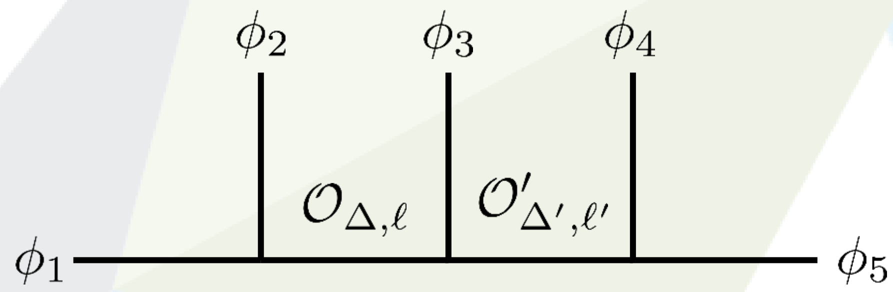
2 cross ratios, 2 diagonalized operators ⟹ integrable!

Higher-point blocks not known in literature

→ Need many commuting operators, but which ones?

Higher point case: Gaudin models

- Starting from $n = 5$,
  Casimirs are not enough



$$\#(\text{cross ratios}) \overset{N=5}{=} 5 \longrightarrow \left\{ C_2^{(12)},\, C_4^{(12)},\, C_2^{(45)},\, C_4^{(45)},\, (?) \right\}$$

Gaudin models allow construction of the whole set of conformally invariant commuting operators from invariant polynomials!

$$\mathcal{L}(z) = \sum_{r=1}^{N} \frac{T^{(r)}}{z - z_r}$$

$$\mathcal{H}_1 = \kappa^{ab} \left(T_a^{(1)} + T_a^{(2)}\right) \left(T_b^{(1)} + T_b^{(2)}\right) \qquad \mathcal{H}_2 = \kappa^{ab} \left(T_a^{(4)} + T_a^{(5)}\right) \left(T_b^{(4)} + T_b^{(5)}\right)$$

$$\mathcal{H}_3 = \tau^{abcd} \left(T_a^{(1)} + T_a^{(2)}\right) \left(T_b^{(1)} + T_b^{(2)}\right) \left(T_c^{(1)} + T_c^{(2)}\right) \left(T_d^{(1)} + T_d^{(2)}\right)$$

$$\mathcal{H}_4 = \tau^{abcd} \left(T_a^{(4)} + T_a^{(5)}\right) \left(T_b^{(4)} + T_b^{(5)}\right) \left(T_c^{(4)} + T_c^{(5)}\right) \left(T_d^{(4)} + T_d^{(5)}\right)$$

$$\boxed{\mathcal{H}_5 = \tau^{abcd} \left(T_a^{(1)} + T_a^{(2)}\right) \left(3T_b^{(1)} + 3T_b^{(2)} - 2T_b^{(3)}\right) T_c^{(3)} T_d^{(3)}}$$

Summary

- Light-cone conformal blocks could give a basis of functions for MHV amplitudes at any loop order
- We plan to obtain them by null limits of ordinary conformal blocks
- Differential equation theory for their construction in the $n > 4$ case is still missing
- We are gathering evidence that $n \geq 5$ conformal blocks are eigenfunctions of special limits of Gaudin Hamiltonians
- Next step will be to work this out for $n = 6$ and take the light-cone limits