# Rational Terms of Two-Loop All-Plus Amplitudes

Sebastian Pögel

3rd SAGEX Workshop, Zoom

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- Last piece missing in all-*n* understanding at leading color

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**0**
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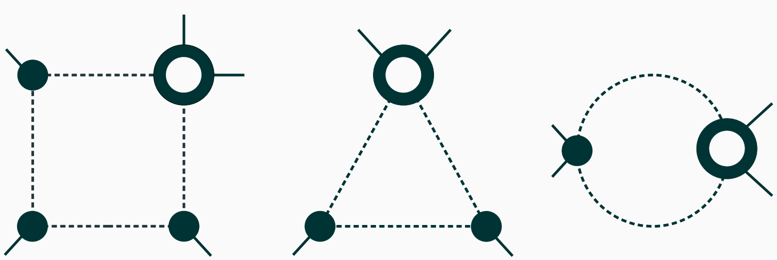

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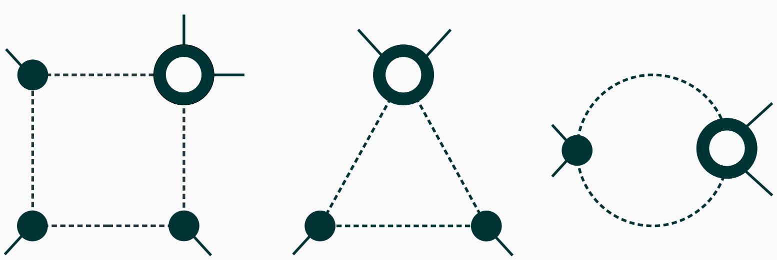

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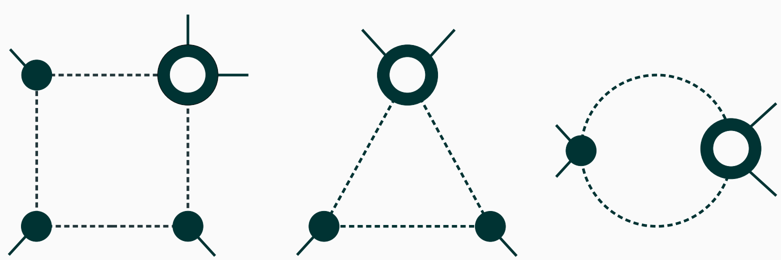

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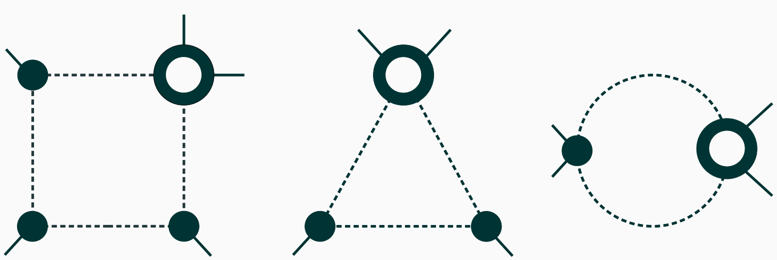

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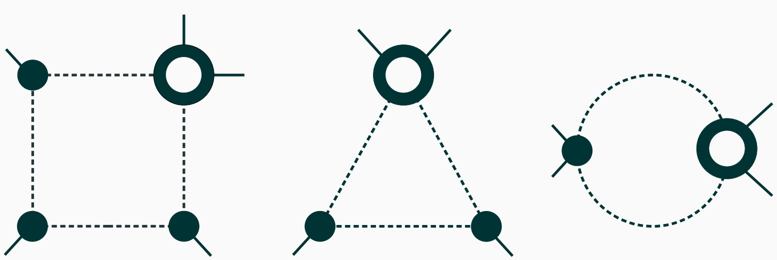

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4-, 5-, 6- and 7-point known
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Sufficient to compute rational terms of $A^{2s}_{6\mathrm{D}}$

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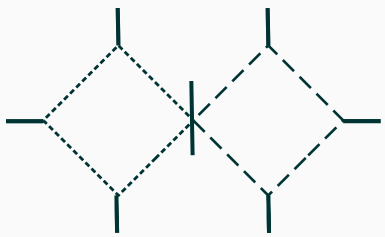

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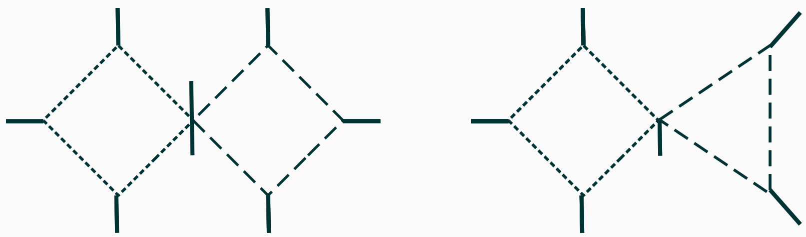

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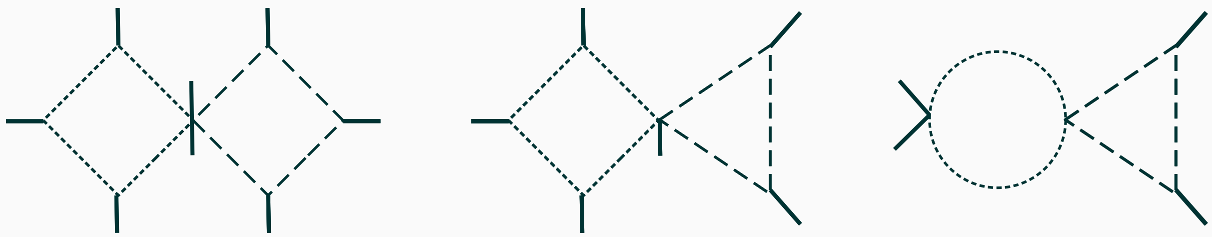

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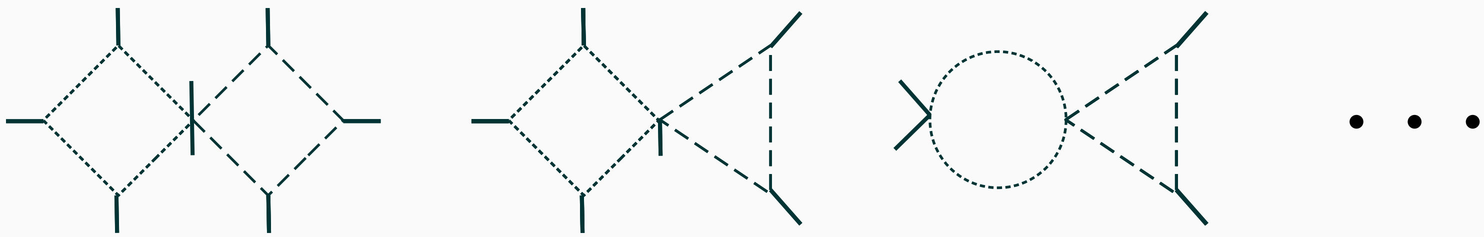

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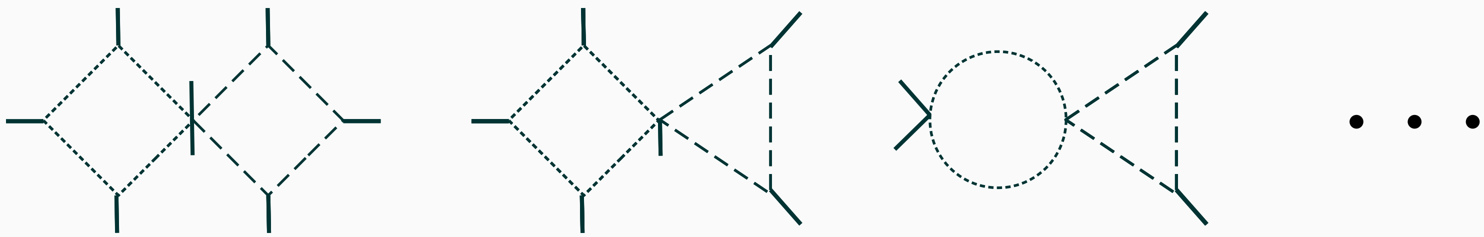

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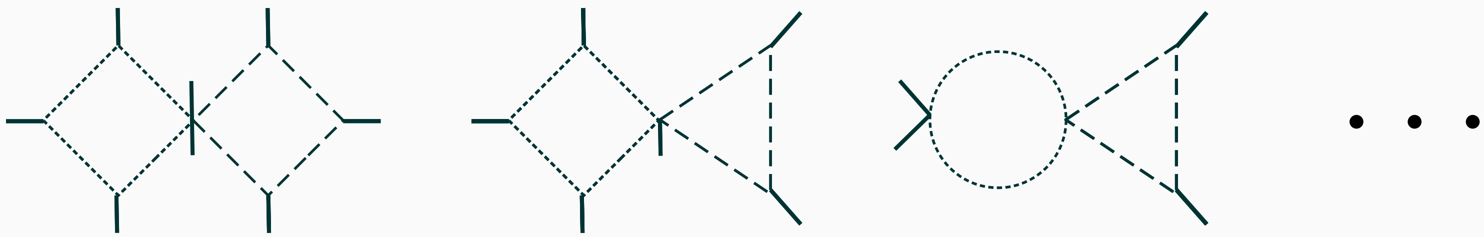

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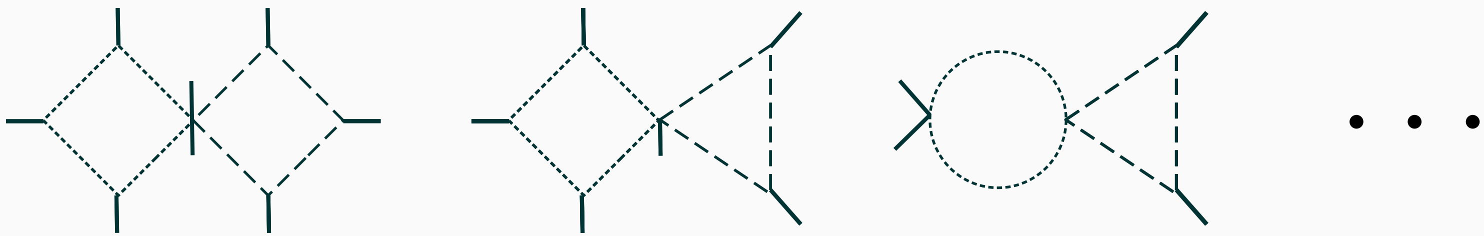

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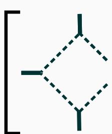
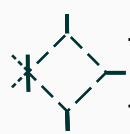
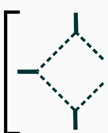
$\mathcal{R}^{(2)} = \mathcal{R}^{(1)} \Big[$  $\times \mathcal{R}^{(1)} \big[$  $\big] \Big] + \mathcal{R}^{(1)} \Big[$  $\times \mathcal{R}^{(1)} \big[$  $\big] \Big] + \dots$

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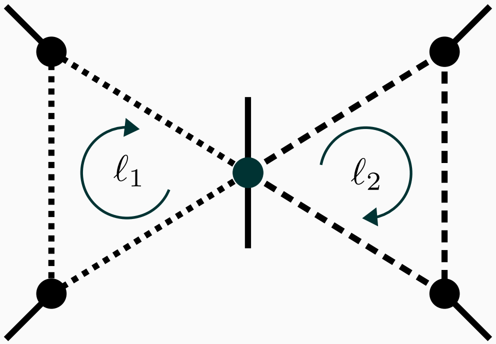

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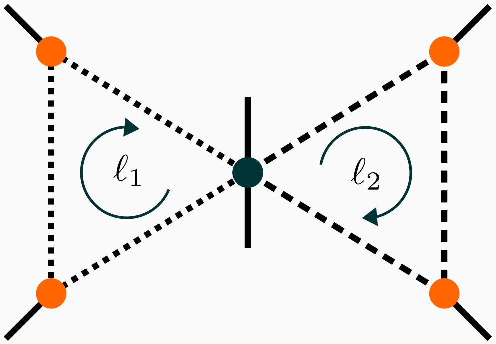


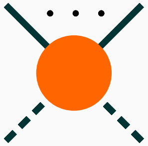

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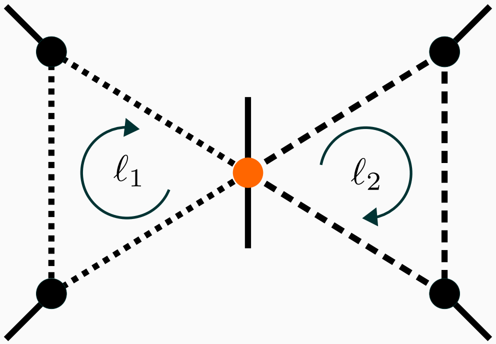


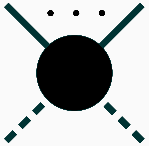

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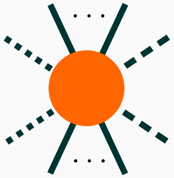

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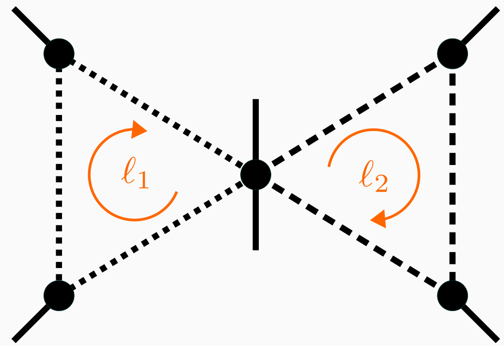


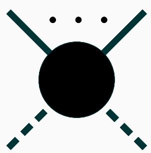

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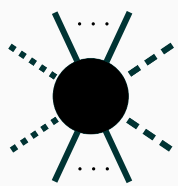

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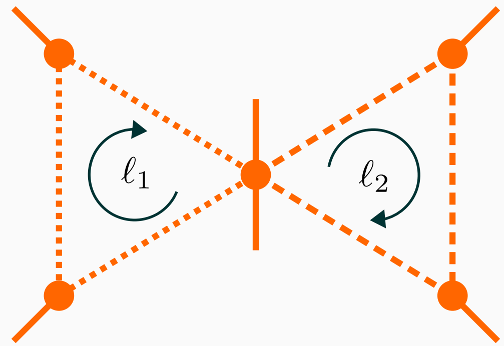


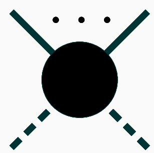

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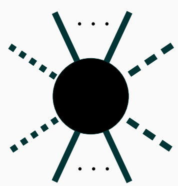

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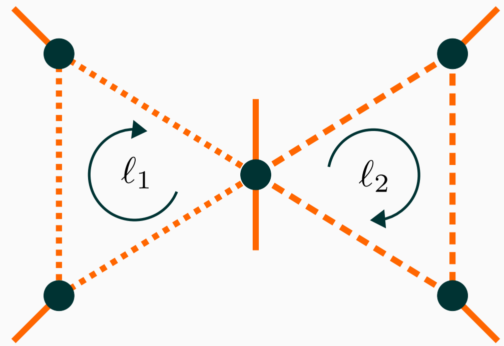


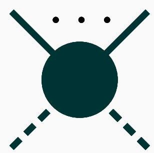

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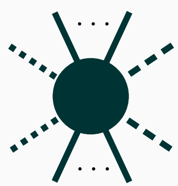

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- Using momentum twistors, rederived 4-gluon all-plus

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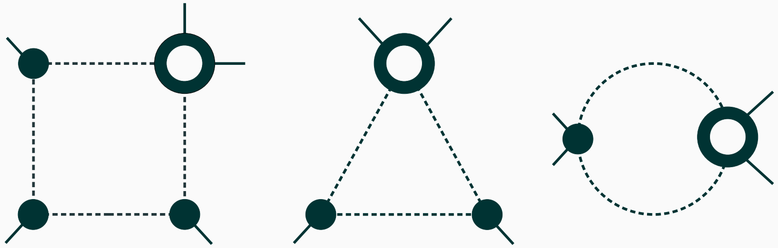

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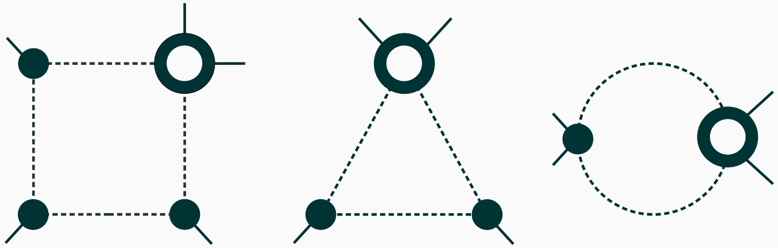

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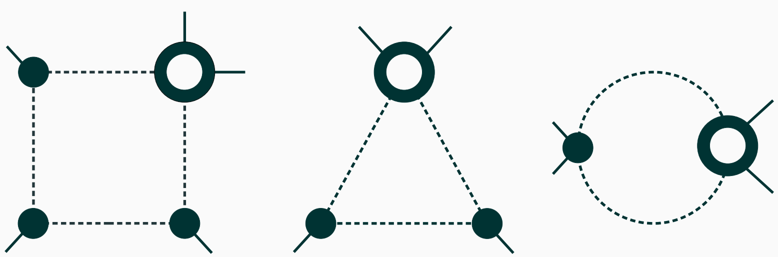

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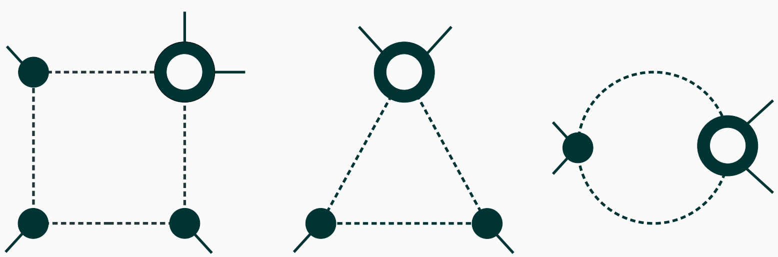
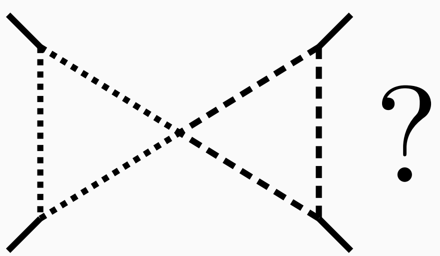
Decouples



Computed for 4 and 5 gravitons

[Dunbar, Jehu, Perkins, 1701.02934]

Decouples?



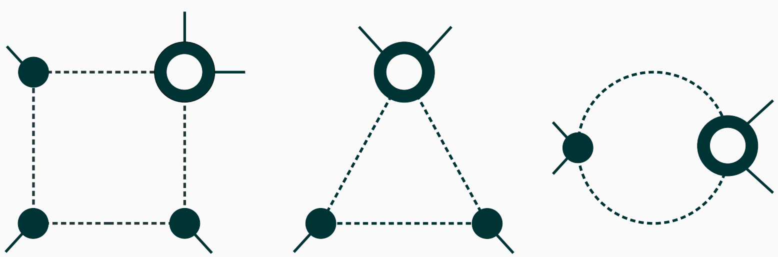
# Graviton All-Plus

For positive helicity gravitons

Loops seem to decouple as well

$$A_n^{(2)} \simeq \left( A_n^{(1)} \mathcal{I}_n^{(1)} \right) + \mathcal{P}_n^{(2)} + \mathcal{R}_n^{(2)}$$

Decouples



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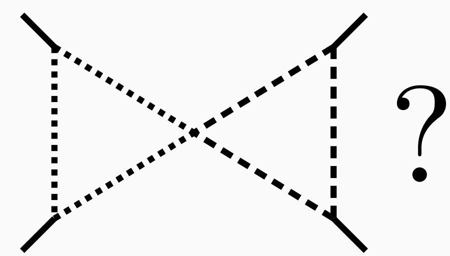

Recently computed for 4 gravitons
using numerical unitarity

[Abreu, Cordero, Ita, Jacquier, Page, Ruf, Sotnikov, 2002.12374]

# Two-Loop Gravity Ingredients

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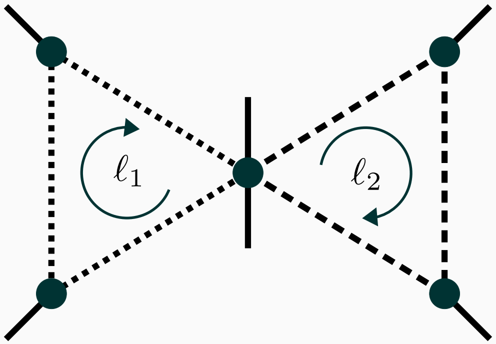

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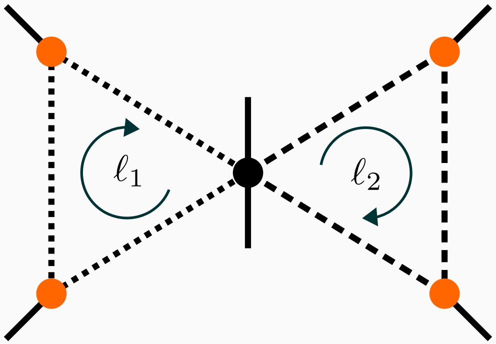


Computable from YM amplitudes via KLT

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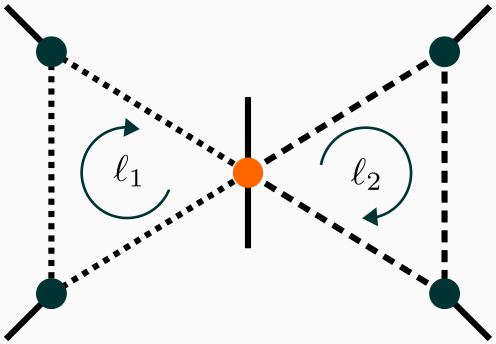


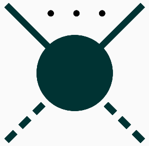

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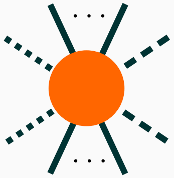

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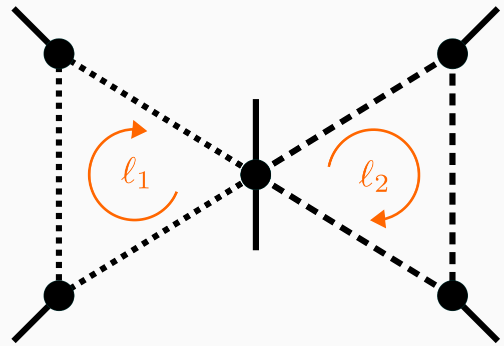


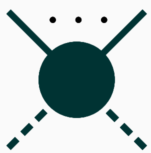

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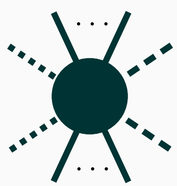

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$\ell_\square, \ell_\triangle, \ell_\circ$ Parametrization as in YM

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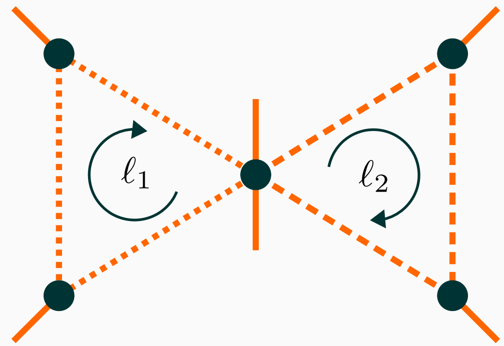


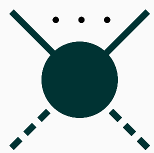

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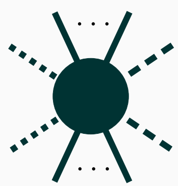

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$\ell_\square, \ell_\triangle, \ell_\circ$ Parametrization as in YM

$C_\square[\mu^8], C_\square[\mu^6], C_\square[\mu^4],$
$C_\triangle[\mu^6], C_\triangle[\mu^5], C_\triangle[\mu^2],$
$C_\circ[\mu^4], C_\circ[\mu^2]$

As in YM, but higher orders in parameters
Parameter integrals mostly known [Kilgore, 0711.5015]

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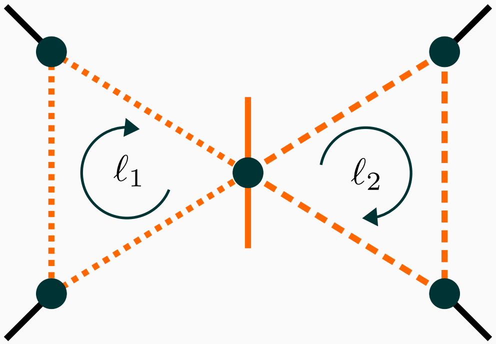


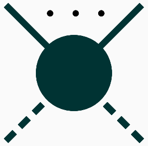

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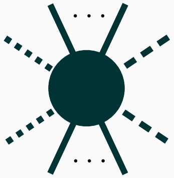

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$\ell_\Box, \ell_\triangle, \ell_\circ$ — Parametrization as in YM

$\mathrm{C}_\Box[\mu^8], C_\Box[\mu^6], C_\Box[\mu^4],$
$\mathrm{C}_\triangle[\mu^6], C_\triangle[\mu^5], C_\triangle[\mu^2],$
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As in YM, but higher orders in parameters
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$\mathrm{I}_\Box[\mu^8], I_\Box[\mu^6], I_\Box[\mu^4],$
$\mathrm{I}_\triangle[\mu^6], I_\triangle[\mu^4], I_\triangle[\mu^2],$
$\mathrm{I}_\circ[\mu^4], I_\circ[\mu^2]$

Integrals can be obtained from dimension shifting

# Conclusion

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**Cut-constructible part of two-loop all-plus amplitudes:**

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**All-Plus Graviton Amplitudes**

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**All-Plus Graviton Amplitudes**

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**All-Plus Graviton Amplitudes**

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  - ➔ Decoupling property of rationals is likely also present