SAGEX

Scattering Amplitudes: from Geometry to Experiment

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The Double Copy for Heavy Particles

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3RD SAGEX WORKSHOP

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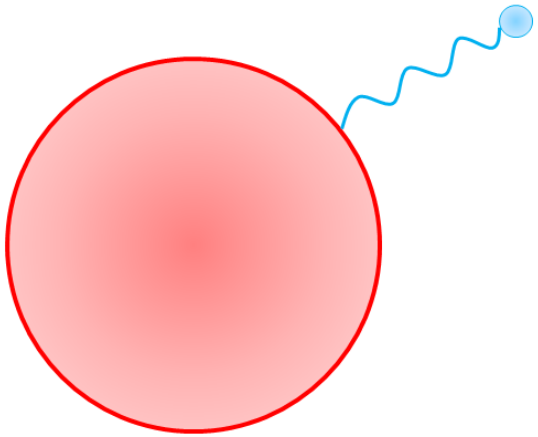
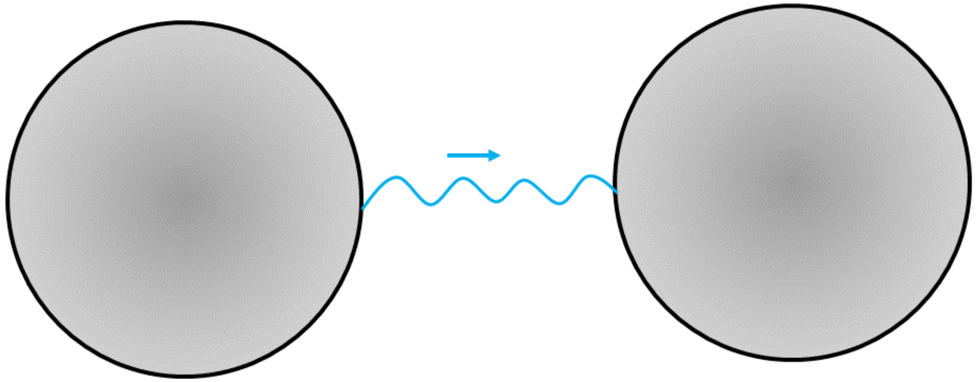
BASED ON WORK WITH ANDREAS HELSET [ARXIV:2005:13897]

Review: HQET, HBET

- EFTs describing very heavy particles interacting via Yang-Mills (Heavy Quark Effective Theory, HQET) or GR (Heavy Black Hole Effective Theory, HBET)
- Heavy particles treated as point sources of mediating bosons, with approximately constant velocity v^μ
- $\mathcal{O}(\hbar)$ corrections to velocity due to interactions (residual momentum, k^μ): total momentum $p^\mu = mv^\mu + k^\mu$

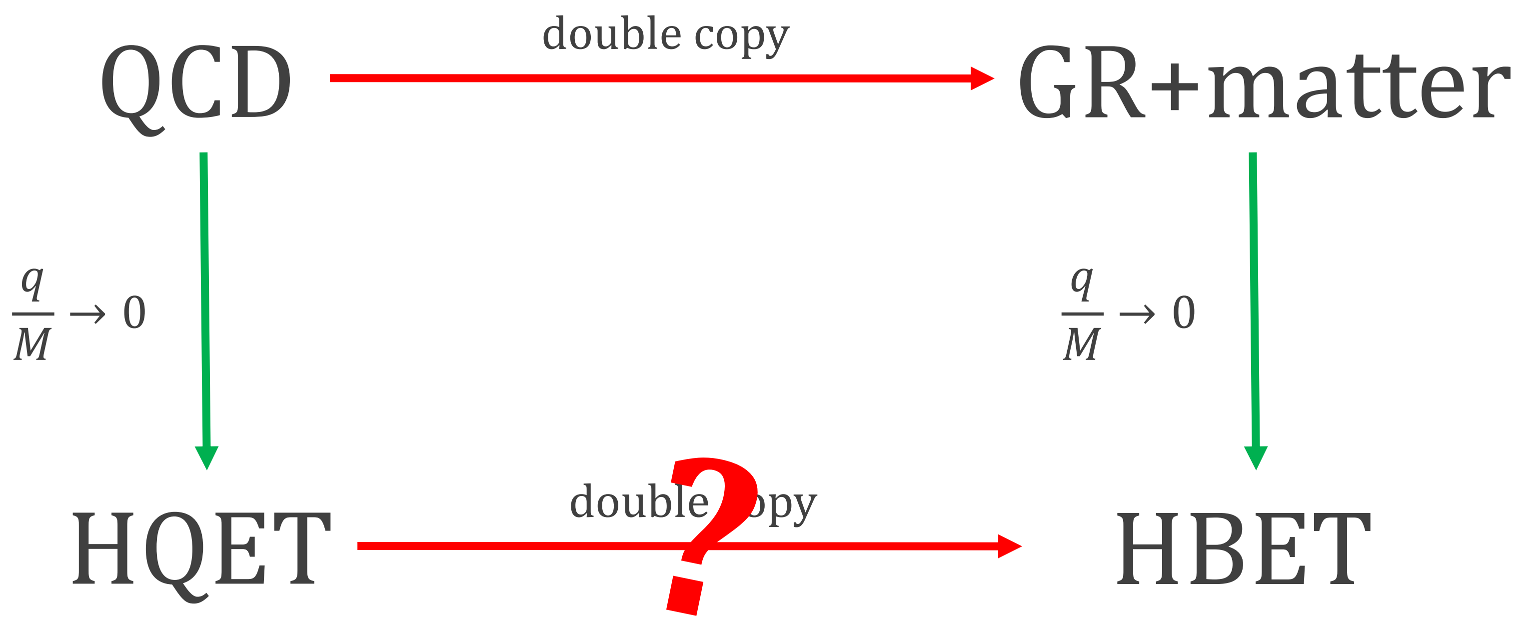


$p = mv + k$



$p_1 = m_1 v_1 - k$ $\qquad$ $p_2 = m_2 v_2 + k$

$\mathrm{HQET}^2 = \mathrm{HBET}\,?$



Question is how, not if

$$\text{HQET} = \text{QCD} \xrightarrow{\text{double copy}} \text{GR} + \text{matter} = \text{HBET}$$

LSZ reduction ⇒ invariance of S-matrix

Color-kinematics duality and the double-copy

- Gauge theory amplitude:

$$A_n = \sum_{i\in\Gamma} \frac{c_i n_i}{d_i}.$$

- Color-kinematics duality:

$$c_i + c_j + c_k = 0 \Leftrightarrow n_i + n_j + n_k = 0$$

- The double copy

$$M_n = \sum_{i\in\Gamma} \frac{\tilde{n}_i n_i}{d_i}$$

  is a gravitational amplitude.*
- **Note:** $\tilde{n}_i$ and n_i need not be from same gauge theory, only one needs to satisfy CK.

*sometimes [Johnson, Jones, Paranjape].

Double copying in effective theories

- $n_{i,\mathrm{EFT}}$ not purely process dynamics in EFTs
  - Contain kinematic effects arising from field redefinitions ⇒ kinematic effects which define the external state.
- Factorize $n_{i,\mathrm{EFT}}$:

$$n_{i,\mathrm{EFT}} = n_{i,\mathrm{d}} \mathcal{R}_s^{-1/2}.$$

- $\mathcal{R}_s^{-1/2}$ is the wavefunction normalization factor (WNF).
  - Process dependent, diagram independent (same for all numerators of a certain process).
  - Field redefinition dependent:

$$\varepsilon_{\mathrm{canonical}} \to \mathcal{R}_s^{-1/2} \varepsilon_{\mathrm{EFT}}.$$

- WNF affects external state after double copy; must be controlled for.

Double copying in effective theories

- Control for WNF in one of three ways:
  1. Double copy with $n_{i,\mathrm{EFT}}$ and combine WNFs in spin-dependent manner to identify new asymptotic matter states,
  2. Identify $n_{i,\mathrm{d}}$ and double copy with it,
  3. Double copy with n_i from theory with trivial WNF.
- Will choose option 3 $\Rightarrow$ asymptotic matter states identical before and after double copy.
- Theory with trivial WNF?

$$\int d^4x\, e^{ip\cdot x}\langle\Omega|T\phi(x)\phi(0)|\Omega\rangle = \frac{i\mathcal{R}_s}{p^2-m^2} = \frac{i}{p^2-m^2}$$

- Scalar QCD!

Matching to HBET

- Option 3 ⇒ asymptotic matter states identical before and after double copy.
- Define asymptotic states and write HQET/HBET in terms of these states.

$$s=0: \quad \varphi(x)=\frac{e^{-imv\cdot x}}{\sqrt{2m}}\left[1-\frac{1}{2m+iv\cdot\partial+\frac{\partial_\perp^2}{2m}}\frac{\partial_\perp^2}{2m}\right]\phi_v(x), \quad \partial_\perp^\mu\equiv\partial^\mu-v^\mu v\cdot\partial,$$

$$s=\frac{1}{2}: \quad \psi(x)=e^{-imv\cdot x}\left[1+\frac{i}{2m+iv\cdot\partial}\left(\gamma_\mu\partial^\mu-v\cdot\partial\right)\right]Q_v(x),$$

$$s=1: \quad A^\mu(x)=\frac{e^{-imv\cdot x}}{\sqrt{2m}}\left[\delta_\nu^\mu-\frac{iv^\mu\partial_\nu-\frac{\partial^\mu\partial_\nu}{2m}}{m+\frac{i}{2}v\cdot\partial}\right]B_v^\nu(x).$$

- With these asymptotic states,

$$(\mathrm{QCD})_{s=0}\times(\mathrm{HQET})_s=(\mathrm{HBET})_s,$$
$$(\mathrm{HQET})_{s=1/2}\times(\mathrm{HQET})_{s=1/2}=(\mathrm{HBET})_{s=1}.$$

Example: Three-points

- Spin-0 QCD amplitude:

$$\mathcal{A}_3^{s=0} = -\mathrm{T}_{ij}^a \epsilon_q^{*\mu}(2mv_\mu + k_{1\mu} + k_{2\mu}).$$

- Spin-0 HQET amplitude:

$$\mathcal{A}_3^{\mathrm{H},s=0} = -\mathrm{T}_{ij}^a \epsilon_q^{*\mu} \phi_v^* \left(1 + \frac{k_1^2 + k_2^2}{4m^2}\right) \phi_v \frac{2m}{2m}\left[v_\mu + \frac{(k_1+k_2)_\mu}{2m}\right] + \mathcal{O}(m^{-4}).$$

- How does LSZ produce equality? Heavy scalar external state non-trivial, equal to

$$\phi_v = \mathcal{R}_{s=0}^{1/2}.$$

Cancels factor in red.

- KLT double-copy (stripping color generators):

$$\mathcal{A}_3^{\mathrm{H},s=0} \mathcal{A}_3^{s=0} = \epsilon_q^{*\mu\nu} \phi_v^* \left(1 + \frac{k_1^2 + k_2^2}{4m^2}\right) \phi_v \frac{4m^2}{2m}\left[v_\mu v_\nu + \frac{v_\nu (k_1+k_2)_\mu}{m} + \frac{(k_1+k_2)_\mu (k_1+k_2)_\nu}{4m^2}\right].$$

✓

Example: Compton scattering

- For brevity, write amplitudes in gauge where $q_i \cdot \epsilon_j = \epsilon_i \cdot \epsilon_j = 0$. Results hold in general gauge too.
- Spin-0 QCD amplitude:

$$\mathcal{A}_4^{s=0} = \frac{c_s n_s}{d_s} + \frac{c_u n_u}{d_u} + \frac{c_t n_t}{d_t},$$

$$c_s = \mathrm{T}^a_{ik}\mathrm{T}^b_{kj}, \qquad c_u = \mathrm{T}^b_{ik}\mathrm{T}^a_{kj}, \qquad c_t = i f^{abc}\mathrm{T}^c_{ij},$$

$$n_s = -4m^2 \epsilon^{*\mu}_{q_1} \epsilon^{*\nu}_{q_2} v_\mu v_\nu, \qquad n_u = n_s\Big|_{q_1 \leftrightarrow q_2}, \qquad n_t = 0.$$

- Spin-$s > 0$ HQET amplitude violates CK at $\mathcal{O}(m^{-2})$. Need to isolate WNF effects to remedy this.
- Fortunately, CK duality is satisfied in the form $c_s - c_u = c_t \Leftrightarrow n_s - n_u = n_t$. Therefore, don't need heavy amplitude to satisfy CK. We find

$$\mathcal{M}_4^{\mathrm{H},s} = \frac{n_s^{\mathrm{H},s} n_s}{d_s} + \frac{n_u^{\mathrm{H},s} n_u}{d_u} + \frac{n_t^{\mathrm{H},s} n_t}{d_t}.$$

$\frac{1}{2} \times \frac{1}{2}$ Double-Copy

- Two ways to double-copy to gravity with spin-1 matter [Bautista, Guevara; Johansson, Ochirov]:

$$\mathcal{M}^{s=1} = \mathcal{A}^{s=0} \otimes \mathcal{A}^{s=1} = \mathcal{A}^{s=1/2} \otimes \mathcal{A}^{s=1/2}.$$

- How to combine spinors in $\frac{1}{2} \times \frac{1}{2}$ case? For single matter line [Bautista, Guevara]

$$\mathcal{M}^{\frac{1}{2}\otimes\frac{1}{2}} = \frac{1}{2}\sum_{\alpha\beta} K_{\alpha\beta}\,\mathrm{Tr}\left[\mathcal{A}_{\alpha}^{s=1/2}(p_{1\mu}\gamma^{\mu} - m)\varepsilon_{1\nu}\gamma^{\nu}\bar{\mathcal{A}}_{\beta}^{s=1/2}(p_{2\rho}\gamma^{\rho} - m)\varepsilon_{2\tau}\gamma^{\tau}\right].$$

- Following their derivation of this, but instead using on-shell HPET variables [Aoude, KH, Helset]

$$\mathcal{M}^{\mathrm{H},\frac{1}{2}\otimes\frac{1}{2}} = \frac{m_{k_1}m_{k_2}}{4m}\sum_{\alpha\beta} K_{\alpha\beta}\,\mathrm{Tr}\left[\mathcal{A}_{\alpha}^{\mathrm{H},s=1/2}(1 + v_{\mu}\gamma^{\mu})\varepsilon_{v\nu}\gamma^{\nu}\bar{\mathcal{A}}_{\beta}^{\mathrm{H},s=1/2}(1 - v_{\rho}\gamma^{\rho})\varepsilon_{v\tau}^{*}\gamma^{\tau}\right].$$

- $\mathcal{A}$ and $\bar{\mathcal{A}}$ are color ordered and stripped of external states. $\bar{\mathcal{A}} = -\gamma_5\mathcal{A}^{\dagger}\gamma_5,\ m_k = m(1 - \frac{k^2}{4m^2})$.

Conclusion

- At tree-level three-points and Compton scattering:

$$(\text{QCD})_{s=0} \times (\text{HQET})_{s} = (\text{HBET})_{s},$$
$$(\text{HQET})_{s=1/2} \times (\text{HQET})_{s=1/2} = (\text{HBET})_{s=1},$$

  for $s \leq 1$, and the specified heavy asymptotic states.
- Not unique double-copy relation between HQET and HBET; prescription should exist that is independent of WNFs,
  - Identify and compensate for WNFs through comparison with biadjoint scalar theory.
- Expands double-copy in powers of $\hbar$,
  - Maps classical QCD amplitudes to classical GR amplitudes,
  - Can HPETs provide insight into classical double copy?