CLASSICAL OBSERVABLES AND EFTs OF GRAVITY

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[2006.02375: Manuel A.H., Andreas Brandhuber, SDA, Gabriele Travaglini]

THE SYSTEM AND THE INTERACTIONS

$$S = \int d^4x \sqrt{-g} \left(-\frac{2}{\kappa^2}\right)\left[R + R^2 \text{ terms} + \frac{\alpha}{\Lambda^4} R^3 \text{ terms} + \frac{\beta}{\Lambda^6} R^4 \text{ terms} + \dots \right]$$

talk in Berlin

$$\text{BHs} \to + \frac{1}{2} \sum_{i=1}^{2} \left(g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + m_i^2 \phi_i^2 \right) \nrightarrow$$

binary system dynamics << cosmological scales

$$\text{photons} \to -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{\gamma}{\Lambda^2} F^{\mu\nu} F^{\rho\sigma} R_{\mu\nu\rho\sigma}$$

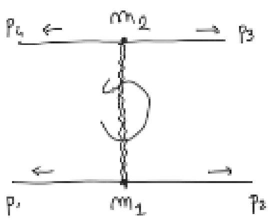
[1704.01590, 1912.09917] : about tests of these EFTs with GW.

Which classical observables?

- DEFLECTION ANGLE & TIME DELAY ← EIKONAL APPROXIMATION

for (massless) gravitons / photons deflected by the gravitational field generated by an extremely massive object (~ BH)

THE EIKONAL APPROXIMATION (a very schematic introduction) [1105.2036 : Giddings]



$$\to S = 1 + i\, T(s,t)\, (2\pi)^D \delta^D \left(\sum p_i \right)$$

$$T_0 \simeq \left(\frac{\kappa}{2}\right)^2 \frac{(t - m_1^2 - m_2^2)^2 - 2 m_1^2 m_2^2}{s}$$

$E \ggg M_P$ BUT! The impact parameter is very large.

(s small)

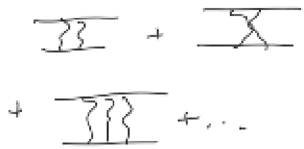
We define the EIKONAL PHASE $\chi(b,t) = \dfrac{1}{2(t-m_1^2-m_2^2)} \displaystyle\int \frac{d^{D-2}q}{(2\pi)^{D-2}}\, e^{i\vec{q}\cdot\vec{b}}\, T_{\text{tree}}(-\vec{q}^{\,2}, t)$

As far as $\boxed{\chi \ll 1}$ and b is large, the tree level is a good approximation of the actual amplitude.

BORN APPROXIMATION

In the case $\boxed{\chi \simeq 1}$ it can be noticed that the amplitude EXPONENTIATE :

$$i T_{\text{eikonal}}(s,t) = 2(t-m_1^2-m_2^2)\int d^{D-2}b\; e^{-i\vec{q}\cdot\vec{b}}\left[e^{i\chi(b,t)} - 1\right]$$



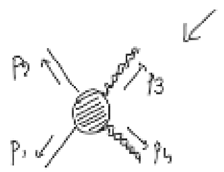
and the exponentiation comes from LOOP (CROSSED) LADDER DIAGRAMS.

Which is a good approximation for this integral?

- the expansion in powers of χ is increasingly badly divergent. NOT NICE !
- SADDLEPOINT APPROXIMATION :
  - DEFLECTION ANGLE
  - TIME DELAY

THE KINEMATIC SETUP (IN 4D)

$m_2 = 0$!



CENTRE OF MASS (CoM) FRAME : $\vec{p}\cdot\vec{q} = 0$

$p_1^\mu = -\left(E_1, \vec{p} - \frac{\vec{q}}{2}\right) \qquad p_2^\mu = \left(E_1, \vec{p} + \frac{\vec{q}}{2}\right)$

$p_4^\mu = -\left(E_3, -\vec{p} + \frac{\vec{q}}{2}\right) \qquad p_3^\mu = \left(E_3, -\vec{p} - \frac{\vec{q}}{2}\right)$

$E_1 = \sqrt{m^2 + \vec{p}^{\,2} + \frac{\vec{q}^{\,2}}{4}}$

$E_3 = \sqrt{\vec{p}^{\,2} + \frac{\vec{q}^{\,2}}{4}} \equiv \omega$

$s = -\vec{q}^{\,2} \quad , \quad t \simeq m^2 + 2m\omega$

EIKONAL LIMIT : $m \gg \omega \gg |\vec{q}|$

Gravitons and photons as external states : we need a parametrisation of the massless SPINOR HELICITY VARIABLES in the eikonal limit.

We choose $\vec{p} = |\vec{p}|\,\hat{z} \quad \Rightarrow \quad \vec{q} = q_1\hat{x} + q_2\hat{y}$

$q = q_1 + i q_2 \;,\; \bar{q} = q_1 - i q_2$

$$p_{3\,\alpha\dot{\alpha}} = \begin{pmatrix} \frac{\vec{q}^{\,2}}{8|\vec{p}|} & -\frac{\bar{q}}{2} \\ \frac{q}{2} & 2|\vec{p}| \end{pmatrix}$$

Helicity structures :

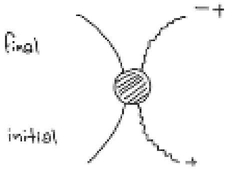
$$\lambda_3 = \sqrt{2|\vec{p}|}\left(-\frac{\bar{q}}{4|\vec{p}|}, 1\right) \qquad \tilde{\lambda}_3 = \sqrt{2|\vec{p}|}\left(-\frac{q}{4|\vec{p}|}, 1\right)$$

$$\lambda_4 = i\sqrt{2|\vec{p}|}\left(\frac{\bar{q}}{4|\vec{p}|}, 1\right) \qquad \tilde{\lambda}_4 = i\sqrt{2|\vec{p}|}\left(\frac{q}{4|\vec{p}|}, 1\right)$$

$$\Rightarrow \quad [34]^2 \simeq q^2 \quad (++) \qquad \mathcal{N} \simeq \left(\frac{\langle 3\, p_1\, 4]}{2m\omega}\right)^2 \simeq -1 \quad (-+)$$

THE EIKONAL PHASE $\longrightarrow$ THE EIKONAL MATRIX

Gravitons and photons are massless states which can have helicity ± 2 and ± 1.



$$\mathcal{A}(1_\phi, 2_\phi, 3^{\mp}, 4^{+}) = \mathcal{A}(\mp, +) \neq 0$$

$\mathcal{A}(-,+)$ / $\mathcal{A}(+,+)$

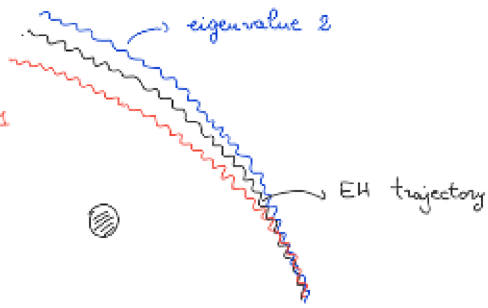
HELICITY PRESERVING / FLIPPING AMPLITUDE (counterintuitive notation: all momenta are outgoing)

$$\chi \sim \int d^2q\, e^{i\vec{q}\cdot\vec{b}} \mathcal{A} \longrightarrow \chi \sim \int d^2q\, e^{i\vec{q}\cdot\vec{b}} \begin{pmatrix} \mathcal{A}(-,+) & \mathcal{A}(+,+) \\ \mathcal{A}(-,-) & \mathcal{A}(+,-) \end{pmatrix}$$

EQUIVALENCE PRINCIPLE

This is usually hidden in EH gravity because $\mathcal{A}(\pm,\pm) \simeq 0$ in the (classical) eikonal limit.

- DIAGONALISE the EIKONAL MATRIX $\qquad \begin{pmatrix} a & b \\ b^* & a \end{pmatrix} \longrightarrow a \pm |b|$
- deflection angle: $\theta^{(i)} \simeq \frac{1}{\omega}\frac{\partial}{\partial b}\chi^{(i)}$
- time delay $\quad t^{(i)} = \frac{\partial}{\partial \omega}\chi^{(i)}$



DEFLECTION ANGLE & TIME DELAY (R^3, FFR)

$$\chi_{EH} = -\left(\frac{\kappa}{2}\right)^2 \frac{m\omega}{2\pi}\left[\frac{1}{4-D} + \log b - \left(\frac{\kappa}{2}\right)^2 \frac{15}{256\pi}\frac{m}{b} + \dots\right] \mathbb{1}_2$$

$$\rightarrow \theta_{EH} \simeq -\frac{4Gm}{b}\left(1 + G\,\frac{15\pi}{16}\frac{m}{b} + \dots\right)$$

Einstein, "On the influence of gravitation on the propagation of light"

$$\rightarrow t_{EH} = 4Gm\left(\log\frac{b_0}{b} + G\,\frac{15\pi}{16}\frac{m}{b} + \dots\right)$$

Shapiro, "Fourth Test of GR"

$$\chi_{R^3} = \chi^{(0)}_{R^3} + \chi^{(1)}_{R^3} + \dots, \qquad \chi^{(0)}_{R^3} = \left(\frac{\kappa}{2}\right)\left(\frac{\alpha'}{4}\right)^2 \frac{m\omega}{2\pi}\begin{pmatrix} 0 & \frac{3}{\bar{\mathfrak{b}}^4} \\ \frac{3}{\mathfrak{b}^4} & 0 \end{pmatrix} \qquad \chi^{(1)}_{R^3} = \left(\frac{\kappa}{2}\right)^4\left(\frac{\alpha'}{4}\right)^2 \frac{m^2\omega}{256\pi}\begin{pmatrix} -\frac{9}{b^4} & \frac{1365}{16}\frac{1}{\bar{\mathfrak{b}}^4} \\ \frac{1365}{16}\frac{1}{\mathfrak{b}^4} & -\frac{9}{b^4} \end{pmatrix}$$

$$\mathfrak{b} = \frac{b_1 + i b_2}{2}$$

$$\Delta\theta_{R^3} = -\frac{4Gm}{b}\left(\frac{\alpha'}{4}\right)^2\left[\pm\frac{132}{b^4} + \frac{5\pi}{16}\left(-9 \pm 1365\right)\frac{Gm}{b^5}\right]$$

$$\Delta t_{R^3} = 4Gm\left(\frac{\alpha'}{4}\right)^2\left[\pm 48\,\frac{1}{b^4} + \frac{\pi}{16}\left(-9 \pm 1365\right)\frac{Gm}{b^5}\right]$$

POSSIBLE NEGATIVE TIME DELAY for small enough impact parameter

$\rightarrow$ [1407.5597: Camanho, Edelstein, Maldacena, Zhiboedov]

$\rightsquigarrow$ no real problem for us (EFT point of view): $b_c < \frac{1}{\Lambda}$

Very similar story for FFR interaction with photons as external states.

I [illegible] $^{(\text{graviton})}$ [illegible] for gravitons in FFR background $\chi_{FFR} \propto \mathbb{1}$.

THE CURIOUS CASE OF R^4 INTERACTIONS

One could write 26 independent couplings involving four Riemann tensors.

Only 7 of these do not involve any Ricci tensors / scalars: these cannot be eliminated via FIELD REDEFINITION, in arbitrary D dimensions.

In FOUR DIMENSIONS only two R^4 couplings are independent:

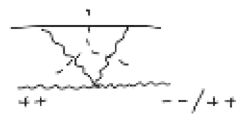
$$\mathcal{C} \equiv R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \qquad\qquad \tilde{\mathcal{C}} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta}$$

$$S_{R^4} = \frac{2}{k^2} \int d^4x \sqrt{-g} \cdot \left(\frac{\beta_1}{\Lambda^6} \mathcal{C}^2 + \frac{\beta_2}{\Lambda^6} \tilde{\mathcal{C}}^2 \right)$$

No 3-points amplitudes. (gravitons)

$$\mathcal{A}(1^{++}, 2^{++}, 3^{++}, 4^{++}) = i \frac{\beta}{\Lambda^6} \left(\frac{k}{2} \right)^2 \left([12]^4 [34]^4 + [13]^4 [24]^4 + [14]^4 [23]^4 \right) \qquad \beta = \frac{1}{2}(\beta_1 - \beta_2)$$

$$\mathcal{A}(1^{++}, 2^{++}, 3^{--}, 4^{--}) = i \frac{\tilde{\beta}}{\Lambda^6} \left(\frac{k}{2} \right)^2 [12]^4 \langle 34 \rangle^4 \qquad \tilde{\beta} = \frac{1}{2}(\beta_1 + \beta_2)$$



$$\chi^{(0)}_{R^4} = 0$$

$\chi^{(1)} \sim \omega$ for EH, R^3, FFR, STRINGS:

does R^4 exponentiate?

$$\chi^{(1)}_{R^4} = \frac{1}{\Lambda^6} \left(\frac{k}{2} \right)^4 \frac{315}{512} \frac{m^2 \omega^3}{2\pi} \frac{1}{b} \begin{pmatrix} \tilde{\beta}/b^4 & \frac{\beta}{16} \frac{1}{b^4} \\ \frac{\beta}{16} \frac{1}{b^4} & \tilde{\beta}/b^4 \end{pmatrix}$$

THANK YOU!

$$\Delta\vartheta_{R^4} = - \frac{(\tilde{\beta} \pm \beta)}{\Lambda^6} \frac{1575\pi}{16} \omega^2 \frac{(Gm)^2}{b^6}$$

$$\Delta t_{R^4} = \frac{(\tilde{\beta} \pm \beta)}{\Lambda^6} \frac{945\pi}{16} \omega^2 \frac{(Gm)^2}{b^5}$$

$\tilde{\beta} \pm \beta > 0 \Rightarrow \begin{cases} \beta_1 > 0 \\ \beta_2 > 0 \end{cases}$ to avoid causality violation.

Analytic properties of the S-matrix: POSITIVITY BOUNDS $\beta_1, \beta_2 > 0$.

[1509.00851: Bellazzini, Cheung, Remmen]