On the Use of the Lossy Transmission Line Theory for the SRF Characterization

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Outline

The Lossy Transmission Line Theory

- Introduction
- Basis
- Analysis

RF and SRF cavities: a different approach

- Introduction
- The high-losses case
- The Lossy TLT connection

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Limitations. Proposals. Conclusions.

The Lossy Transmission Line Theory : *Introduction*

Transmission Line Theory (TLT):

Objective: Complete characterization of **propagative waves** in **PEC surrounded waveguides** by means of **TL parameters**.

$\updownarrow$

Mathematically: characterization of the **closed-form homogeneous solutions to $\square^2$** when "**ideal**" **BCs** (Dirichlet & Neumann) and $\boldsymbol{x_i}$ **-invariance** are imposed.

Lossy TLT (LTLT):

Generalization: The lossless case is only a particular case, among others.

$\updownarrow$

Mathematically: nonhomogeneous solutions to $\square^2$ with general BCs.

The Lossy Transmission Line Theory : *Introduction*

LTLT:

- TEM modes, i.e. plane waves ↔ analytic solutions in multiple connected planar (e.g. z-invariant) regions.

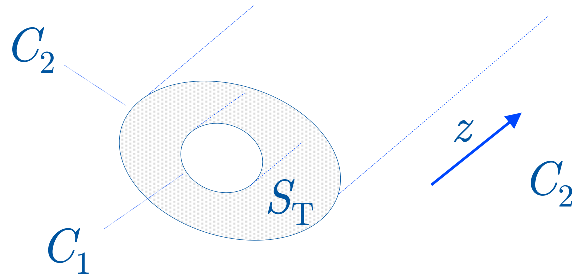


Fig. 1. Two-conductor waveguide that supports the TEM mode represented in some cylindrical coordinate system.

- Easily described in the frequency domain ↔ time (harmonic) $\mathcal{FT}$ (coordinates on $e^{j\omega t}$ basis).
- Complex functions and parameters → Complex Analysis.

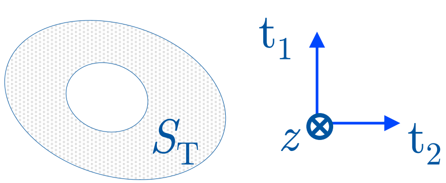
The Lossy Transmission Line Theory : *Basis*

From the analyticity of TEM solutions in S_T, the integration of the *Maxwell equations* unequivocally leads to the *Telegrapher's equations* (written in the frequency domain):

Line parameters

$$\begin{cases} \dfrac{dV(z)}{dz} = -\,(R+j\omega L)\cdot I(z)\ ; & R = \omega\mu'' \cdot \mathrm{H}_{12}/\ell_2\ ,\ L = \mu' \cdot \mathrm{H}_{12}/\ell_2 \in \mathbb{R}^+ \\ \dfrac{dI(z)}{dz} = -\,(G+j\omega C)\cdot V(z)\ ; & G = \omega\varepsilon'' \cdot \ell_2/\mathrm{H}_{12}\ ,\ C = \varepsilon' \cdot \ell_2/\mathrm{H}_{12} \in \mathbb{R}^+ \end{cases}\ ;$$

where, using generalized orthogonal coordinates on S_T: $(\mathrm{t}_1,\mathrm{t}_2)$;
$\mathrm{H}_{12} = \int_{\langle \mathrm{t}_1\rangle} h_1/h_2 h\ dt_1$, $\ell_2 = \int_{\langle \mathrm{t}_2\rangle} dt_2$, (e.g. polar coord.: $\mathrm{t}_1=r$, $\mathrm{t}_2=\varphi$).



◀ Fig. 2. General orthogonal coordinate system.

The Lossy Transmission Line Theory : *Basis*

Solutions:

$$V(z) = V_0^+ e^{-kz} + V_0^- e^{+kz} = V_0^+ e^{-kz} (1 + \Gamma_0 e^{-2kz}) \,,$$
$$I(z) = I_0^+ e^{-kz} - I_0^- e^{+kz} = (V_0^+ / Z_0)\, e^{-kz} (1 - \Gamma_0 e^{-2kz}) \,;$$

Basic parameters

$$Z_0 = V_0^+ / I_0^+ = V_0^- / I_0^- = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \,,\; k = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta \in \mathbb{C}$$

Wave parameters

$$V(z)/I(z) = Z(z) = 1/Y(z) = Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)} \;;\; \Gamma(z) = \Gamma_0 e^{2kz}$$

The Lossy Transmission Line Theory : *Analysis*

Basic parameters

(fixed frequency analysis, ω_0)

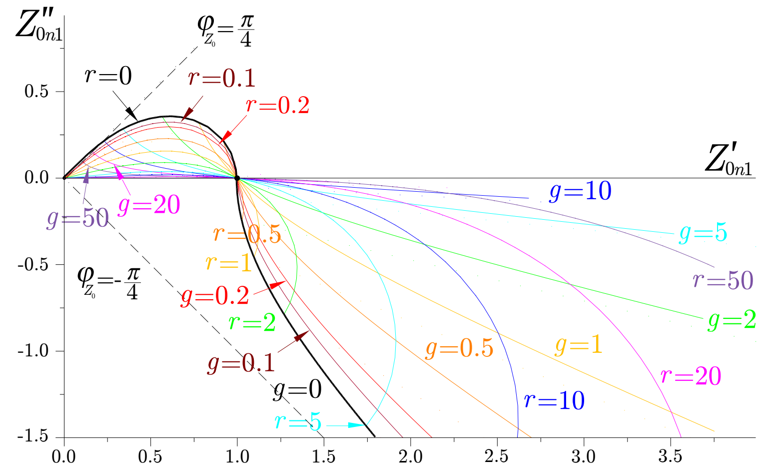
Parameterizations:

$$\begin{cases} r = R/(\omega_0 L) \\ g = G/(\omega_0 C) \end{cases}$$

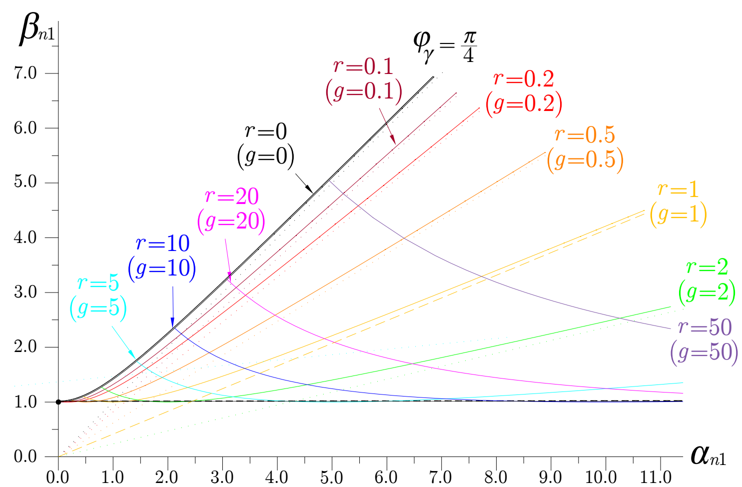
Normalizations :

$$Z_{0n1} = Z_0 / Z_{0,lossless} = \sqrt{\frac{1-jr}{1-jg}}$$

$$k_{n1} = k/\beta_{lossless} = j\sqrt{(1-jr)(1-jg)}$$



(a)



(b)

Fig. 3. Normalized (a) characteristic impedance and (b) propagation constant. ▶

The Lossy Transmission Line Theory : *Analysis*

Wave parameters

Normalizations :

$Z_{n0} = Z/|Z_0|;\ Y_{n0} = Y|Z_0|;\ \Gamma$

$Z_{0n} = Z_0/|Z_0| = e^{j\varphi_{Z0}}$

Parameterizations:

$$\begin{cases} \varphi_{Z0} \\ (Z'_{n0}, Z''_{n0}) \text{ or } (|Z_{n0}|, \varphi_Z) \end{cases};$$

$$\begin{cases} \varphi_{Z0} \\ (Y'_{n0}, Y''_{n0}) \text{ or } (|Y_{n0}|, \varphi_Y) \end{cases};$$

$$\begin{cases} \varphi_{Z0} \\ (\Gamma', \Gamma'') \text{ or } (|\Gamma|, \varphi_\Gamma) \end{cases}.$$

Usual Smith Chart: $\begin{cases} \varphi_{Z0} = 0 \\ (Z'_{n0}, Z''_{n0}) \end{cases}$.

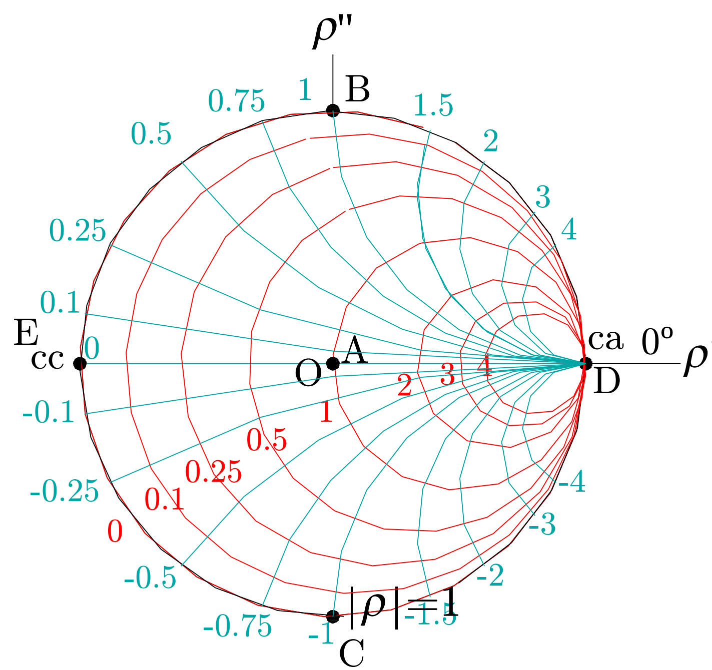


Fig. 4. Transformations of the real and imaginary parts of the Z_{n0}-plane into the Γ-plane.

The Lossy Transmission Line Theory : *Analysis*

Wave parameters

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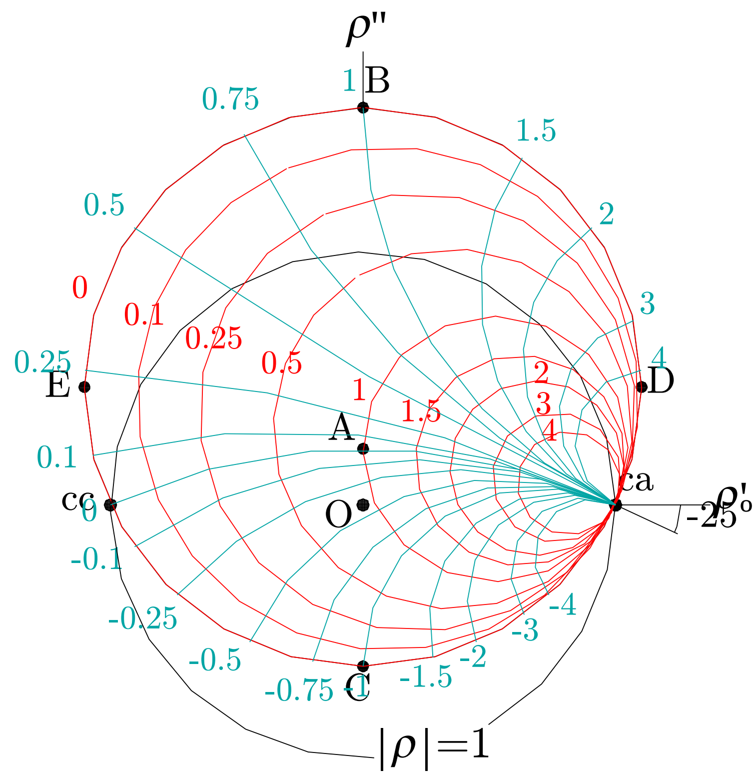
$$\begin{cases} \varphi_{Z0} \\ (\Gamma', \Gamma'') \text{ or } (|\Gamma|, \varphi_\Gamma) \end{cases}.$$

Generalized Smith Chart: $\begin{cases} \varphi_{Z0} = -25^o \\ (Z'_{n0}, Z''_{n0}) \end{cases}.$



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The Lossy Transmission Line Theory : *Analysis*

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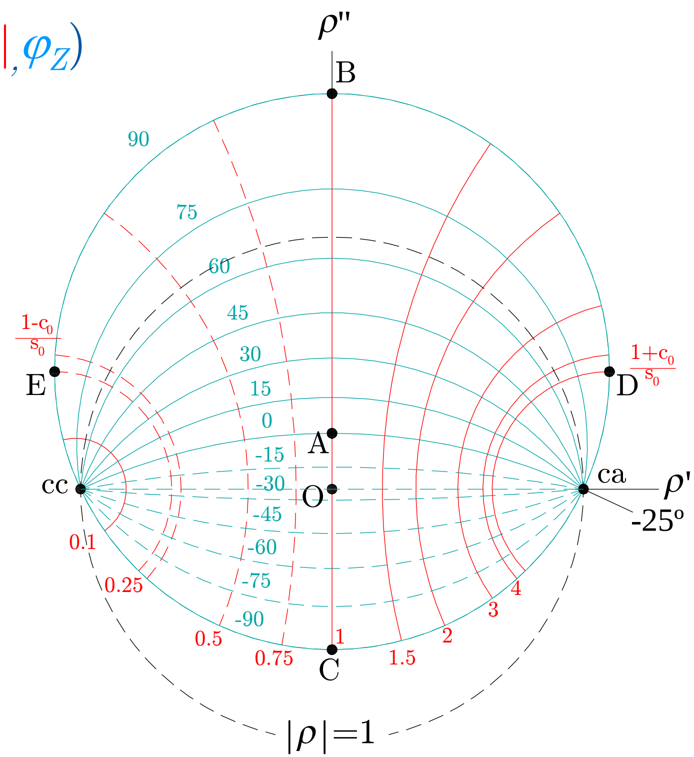


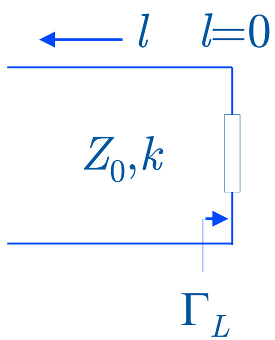
Fig. 5. Transformations of the modulus and phase of the Z_{n0}-plane into the Γ-plane.

The Lossy Transmission Line Theory : *Analysis*

Example of analysis: Wave parameters along the TL

$\Gamma = \Gamma_L e^{-2kl}$ (*logarithmic spiral* in the Γ-plane)

$\Gamma_L = 0.756\angle -4.118^o$, $\varphi_{Z_0} = 25.526^o$



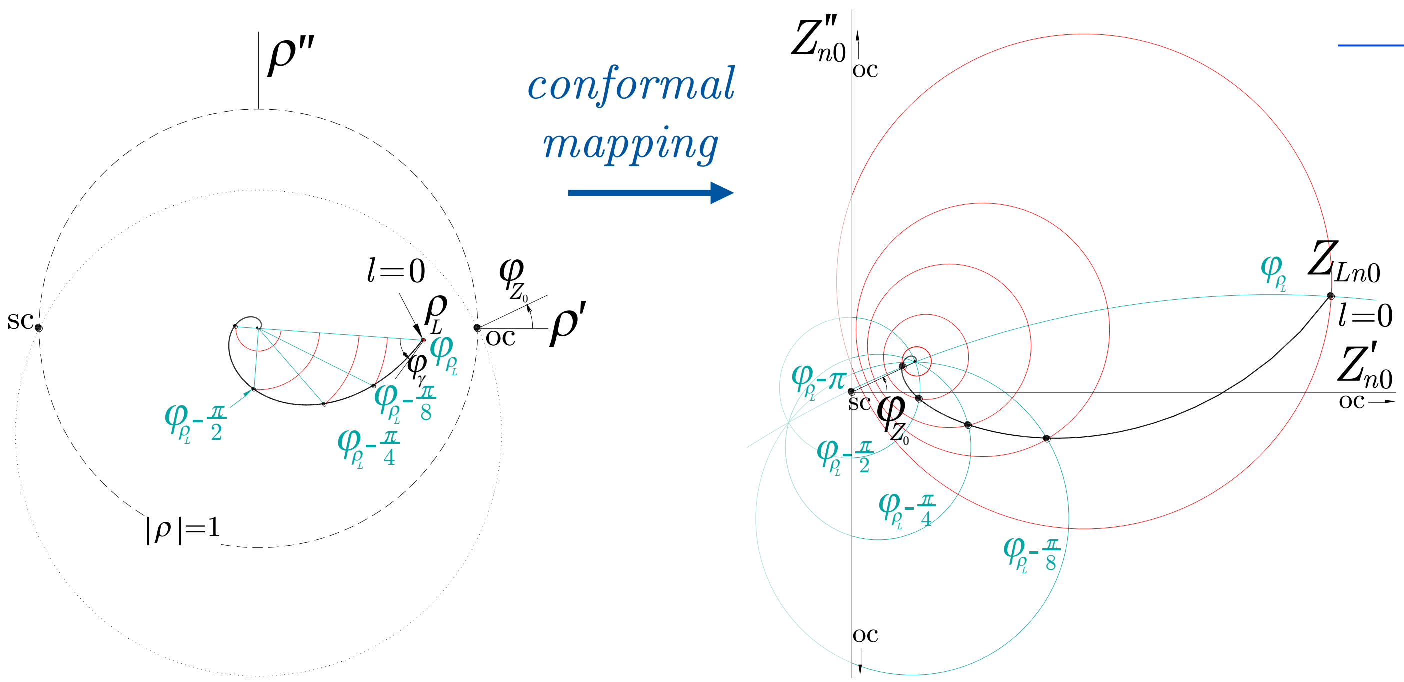
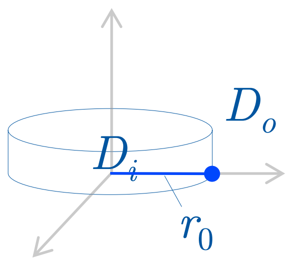


Fig. 6. Parameterization of $\Gamma(l)$ and its transformation to the Z_{n0}-plane.

RF and SRF cavities: a different approach
Introduction

Example: TM_{010}-like mode on pill-box cavity.



◀ Fig. 7. Geometry and domains of the canonical pill-box problem.

Solve : $\nabla^2 E_z + k^2 E_z = 0$, which particularizes to :
$r^2\partial_r{}^2[\mathrm{R}(r)] + r\partial_r[\mathrm{R}(r)] + r^2k^2\,\mathrm{R}(r)=0,\ \mathrm{R}(r)\in\mathcal{C}^2$
where $k_i=\omega/\mathrm{c} \in \mathbb{R}^+$ on $D_i=[0,r_0)\times[-\pi,\pi)\times[0,d]$,
and $k_o=\omega\sqrt{\varepsilon_c\mu_0} \in \mathbb{C}$ on $D_o=(r_0,\infty)\times[-\pi,\pi)\times[0,d]$;
subject to $|\mathrm{R}(0)|<\infty$ (*finiteness*)
($|\mathrm{R}(\infty)|<\infty$ automatically verifies since $k_o \in \mathbb{C}$).

General Sol : $\begin{cases} A\mathrm{J}_0(ar) \text{ on } D_i \\ B_1\mathrm{H}^{(1)}{}_0(br)+B_2\mathrm{H}^{(2)}{}_0(br) \text{ on } D_o \end{cases}$;

RF and SRF cavities: a different approach
The high-losses case

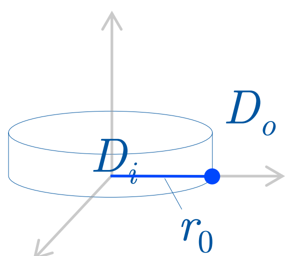
In D_o: $|\sigma|/(\omega\varepsilon_0) >> 1 \Rightarrow$ *asymptotic analysis* of solutions:

$$B_1\mathtt{H}^{(1)}{}_0(br)+B_2\mathtt{H}^{(2)}{}_0(br) \sim (B_1/\sqrt{r})\mathrm{e}^{-k_o r} + (B_2/\sqrt{r})\mathrm{e}^{+k_o r} \sim$$

$$\equiv B_1(\mathrm{e}^{-\alpha_o r}/\sqrt{r})\mathrm{e}^{-j\beta_o r} + B_2(\mathrm{e}^{+\alpha_o r}/\sqrt{r})\mathrm{e}^{+j\beta_o r} \sim$$

$$\sim \underbrace{B_1\mathrm{e}^{-k_o r} + B_2\mathrm{e}^{+k_o r}}$$

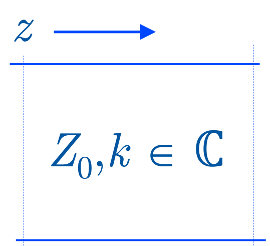
TL-like propagative waves!



◀ Fig. 7. Geometry and domains of the canonical pill-box problem.

Proposition 1: Asymptotically, the behavior of any kind of waves in high-lossy media may be studied by means of LTLT.

Fig. 8. Equivalent TL representing the high-lossy media. ▶



$|\sigma|/(\omega\varepsilon_0) >> 1 >> \mu_0$

$|Z_0| \rightarrow 0$

$|k| \rightarrow$? (in general: $|k| >> 1$)

RF and SRF cavities: a different approach
The high-losses case

R=0 ⇒ r=0 ; $g = \sigma'/(-\sigma'')$ $(<0!)$ ($\sigma \in \mathbb{C}$ from the two-fluid model)

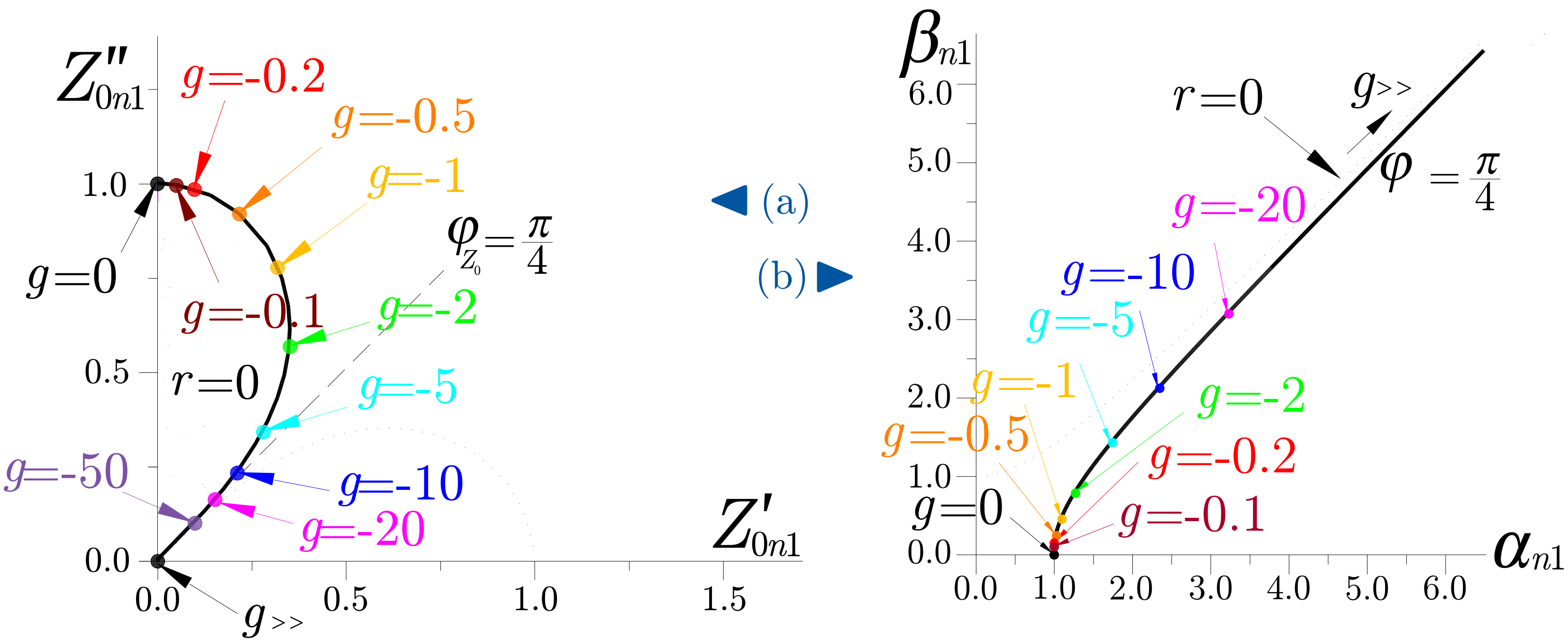


Fig. 9. Normalized (a) characteristic impedance and (b) propagation constant for good conductors.

Particular cases :

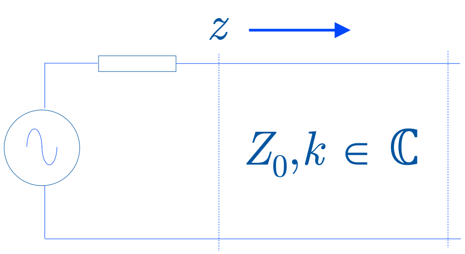
Normal conductor $(\varphi_{Z0} \to \pi/4,\ \varphi_k \to \pi/4)$: $Z_0 \to R_s(1+j)$, $k \to (1/\delta)(1+j)$

Superconductor $(\varphi_{Z0} \to \pi/2,\ \varphi_k \to 0) \to Z_0 \to R_o e^{j\pi/2}$, $k \to 1/\lambda_L$

RF and SRF cavities: a different approach
The Lossy TLT connection

(i) The surface impedance of the cavity walls is the characteristic impedance of the asymptotic approximation of the waves which propagate along lossy media.
(ii) The cavity itself behaves as a generator for these waves (as long as β_e>>1, regarding the "antenna" which keeps the fields inside static).
(iii) The impedance of the generator would play the role of the impedance of that "antenna". Then, the maximum transfer of power (*Jacobi's law*) would represent the matching between the antenna and the cavity.

Fig. 10. Proposed equivalent circuit for the cavity characterization.



Example of application
Optimization of thin films in terms of R_s

Problem: To analyze the R_s at the input (cavity-film interface) in terms of film thickness (d) and choose the minimum (optimum).

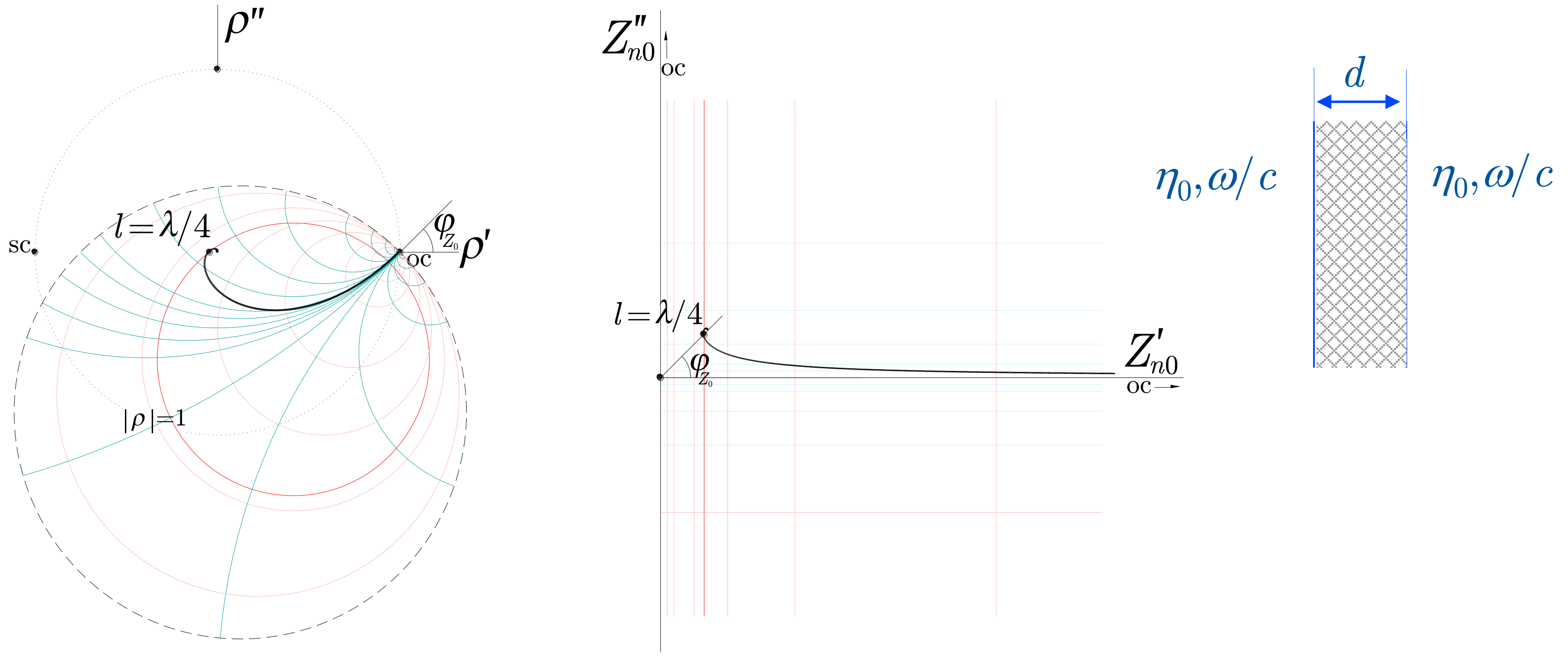
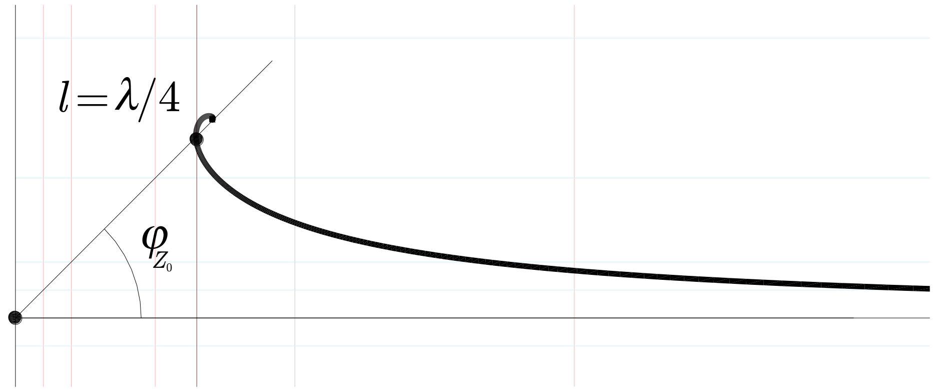


Fig. 11. Wave parameter characterization of the vacuum-thin film-vacuum structure.

Example of application
Optimization of thin films in terms of R_s

Example (good conductor): σ=5.8e7 [S·m^{-1}]



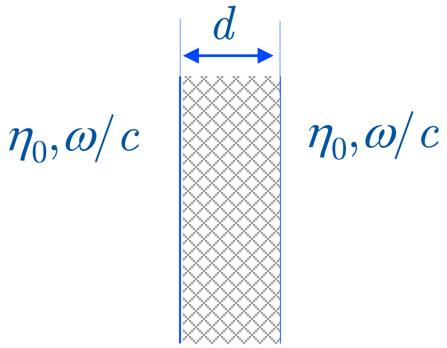


Fig. 11. Wave parameter characterization of the vacuum-thin film-vacuum structure.

A thin film of $d=\lambda/4$ (~4.83μm at 400MHz) leads to an input resistance of $0.917 \cdot R_s$!!

Proposition 2 : The structure vacuum-(super)conductor-vacuum optimizes the R_s when a thin film of $d=\lambda/4$ is used in the middle.

Limitations. Proposals. Conclusions

Limitations

(i) Depending on the SC state (given by $|g|=\sigma'/\sigma''$), it may be difficult to synthesize a thin-film ~$\lambda/4$. (from μm to nm).

(ii) However, the more SC the thin-film is, the more accurate is the asymptotic approximation.

(iii) The actual equivalent R_s is not the R_s at the surface $(E(0)/H(0))$ anymore but it is given by the definite integral of the ratio of the fields in the range $[0,d]$, that is, the wave impedance along the TL.

Proposals

(i) To use this analysis to study multilayer structures that lead to minimize the integrated R_s.

(ii) To study the viability of using more complex TL structures (e.g. parallel stubs) and use the asymptotic analysis to study more designs (e.g. (and especially) periodic structures).

Limitations. Proposals. Conclusions

Conclusions

(i) The LTLT is connected to the characterization of propagative waves in high lossy media where the cavity -inevitably- "radiates" the inner fields. The link is by means of the asymptotic analysis of the solutions, which is possible thanks to the high losses. In this sense, a complete equivalence between the cavity and the TL has been given.

(ii) The usual definition of R_s at the cavity-wall interface is only valid as limit when $d>>\lambda$. As explained and exemplified, thin-films behave differently.

(iii) The geometrical analysis of the curves represented in the planes associated to each TL parameters becomes very helpful.

(iv) A simple but illustrative example with practical application on thin-films deposition has been presented. From the example, and using the angle conservation property of conformal mappings, any multilayer structure may be analyzed.

References

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[2] E. Gago-Ribas, *Complex Transmission Line Analysis Handbook*, Vol. GW-I, "Electromagnetics & Signal Theory Notebooks" series, GREditores, 2001.

Thank you for your attention!

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