On Beam Break Up (BBU) *estimations* for Pulsed Beams

W. Weingarten

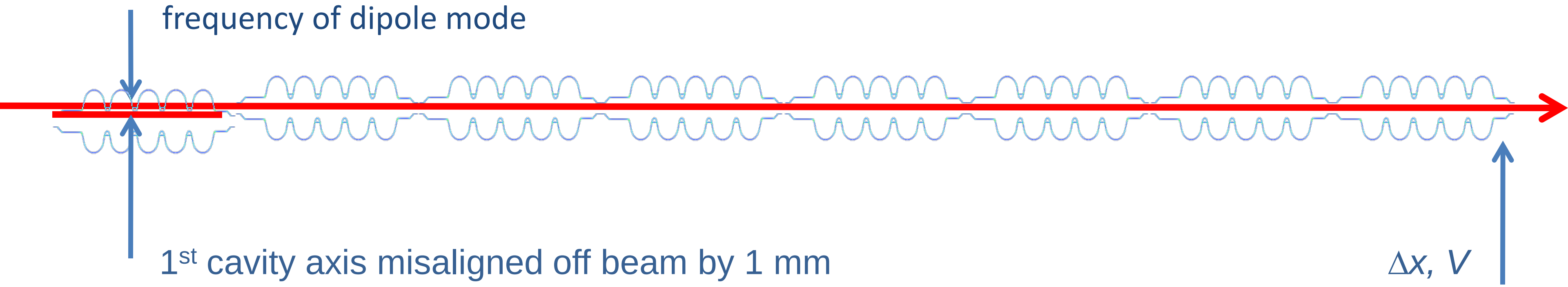
Simulating the interaction of dipole cavity modes with pulsed particle beams 1/5

This is what is simulated with Mathematica

- Input:
  - String of 8 cavities
  - 1st cavity misaligned by 1 mm
  - Cavities are excited in dipole mode at 2.1 GHz

I	40 mA
$(R/Q)_x$	100 Ω
Q_0	10^{10}

- Output:
  - Voltage V of dipole mode and deviation Δx of beam in last (8th) cavity
  - Study of dependence of V and Δx on Q_{ext} and frequency of dipole mode

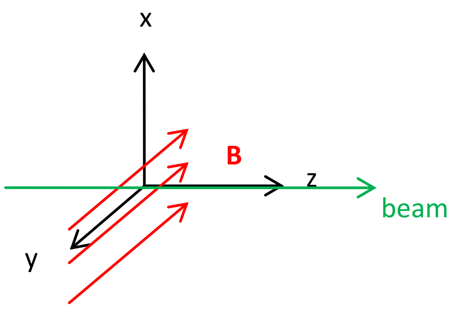


Simulating the interaction of dipole cavity modes with pulsed particle beams 2/5

Definitions

- coordination system: z in direction of beam, x in transversal direction, *x'*=dx/dz.
- *m* is the mass, *p* is the momentum of the particle, β and γ are the usual relativistic factors):

$$\Delta x' = \frac{\Delta v_x}{\beta \cdot c} = \frac{\Delta p_x}{p_z} .$$



From the Panofsky-Wenzel theorem follows in all generality

$$\Delta p_x = -\frac{ie}{\omega}\frac{\partial V_z}{\partial x} .$$

This theorem can be made plausible, in the approximation of a short pillbox cavity excited in a mode with only longitudinal electric fields:

From the Lorentz-force and Maxwell equations

$$F_x = e\left(v_y B_z - v_z B_y\right) \qquad \mathrm{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

follows

$$\Delta p_x = F_x \Delta t = -e v_z B_y \Delta t = -e v_z \frac{i}{\omega} \cdot \frac{\partial E_z}{\partial x} \Delta t = -\frac{ie}{\omega}\frac{\partial V_z}{\partial x} .$$

Simulating the interaction of dipole cavity modes with pulsed particle beams 3/5

For highly relativistic beams, in analogy to

$$\Delta p_z \cdot c = \Delta E = eV_z \Rightarrow V_z = \Delta p_z \cdot c/e,$$

we obtain

$$\Delta p_x = e \cdot V_x/c.$$

With the approximation for dipole modes,

$$V_z = const \cdot x \Rightarrow \frac{\partial V_z}{\partial x} = \frac{V_z}{x},$$

we obtain

$$\Delta p_x = e \cdot V_x/c = -\frac{ie}{\omega}\frac{\partial V_z}{\partial x} = -\frac{ie}{\omega} \cdot \frac{V_z}{x} \Rightarrow V_z(x) = i \cdot \left(\frac{\omega}{c}\right) \cdot x \cdot V_x,$$

and

$$\Delta x' = \frac{\Delta p_x}{p_z} = \frac{e \cdot V_x/c}{p_z} = \frac{e}{\underbrace{c \cdot p_z}_{a}} \cdot V_x.$$

Simulating the interaction of dipole cavity modes with pulsed particle beams 4/5

More definitions:

Decay time	$T_d = 2 \cdot \frac{Q_L}{\omega}$	Complex frequency	$p = i\omega + \frac{1}{T_d}$
Loaded Q-value	$Q_L = \frac{1}{1/Q_0 + 1/Q_{ext}}$	R/Q value	$\left(\frac{R}{Q}\right)_z = \frac{V_z(x)^2}{\omega U} = \underbrace{\frac{V_x^2}{\omega U}}_{\left(\frac{R}{Q}\right)_x} \cdot x^2 \cdot \left(\frac{\omega}{c}\right)^2$

Similarly to longitudinal case we get for the voltage increment ΔV_z (q is the bunch charge):

$$\Delta V_z(x) = \omega \cdot \frac{\left(\frac{R}{Q}\right)_z}{2} \cdot q = \omega \cdot \frac{\left(\frac{R}{Q}\right)_x}{2} \cdot \left(\frac{\omega}{c}\right)^2 \cdot q \cdot x^2 = i\frac{\omega}{c} \cdot x \cdot \Delta V_x$$

$$\Rightarrow \Delta V_x = \underbrace{-i \cdot c \cdot \frac{\left(\frac{R}{Q}\right)_x}{2} \cdot \left(\frac{\omega}{c}\right)^2}_{b} \cdot q \cdot x \quad .$$

Simulating the interaction of dipole cavity modes with pulsed particle beams 5/5

Recursive definition of induced **transversal** voltage $V = V_x$ and transversal deviation x, Δz being the distance between cavities:

$$x'_{m,n-1} + \mathrm{Re}\left(a \cdot V_{m-1,n-1} \cdot e^{-pT_b}\right) \rightarrow x'_{mn};\; n = 2,\ldots,n_0;\; m = 1,\ldots,m_0$$

$$x'_{mn} \cdot \Delta z + x_{m,n-1} \rightarrow x_{mn}$$

$$V_{m-1,n} \cdot e^{-pT_b} + b \cdot x_{mn} \rightarrow V_{mn}$$

$$x_{1,1} = 0;\; x'_{1,1} = 0;\; V_{1,1} = b \cdot x_0$$

$$x_{m,1} = 0;\; x'_{m,1} = \mathrm{Re}\left(a \cdot V_{m-1,1} \cdot e^{-pT_b}\right);\; V_{m,1} = b \cdot x_0 + V_{m-1,1} \cdot e^{-pT_b};\; m = 2$$

m: bunch number, *n*: cavity, x_0: misalignment of the 1st cavity, n_0 : number of cavities, m_0: number of bunches per pulse.

Some formula were taken from this reference: J. Tuckmantel, Crab Cavities: Speed of voltage change, presentation in CCIS Working Group, 27 Nov. 2009.

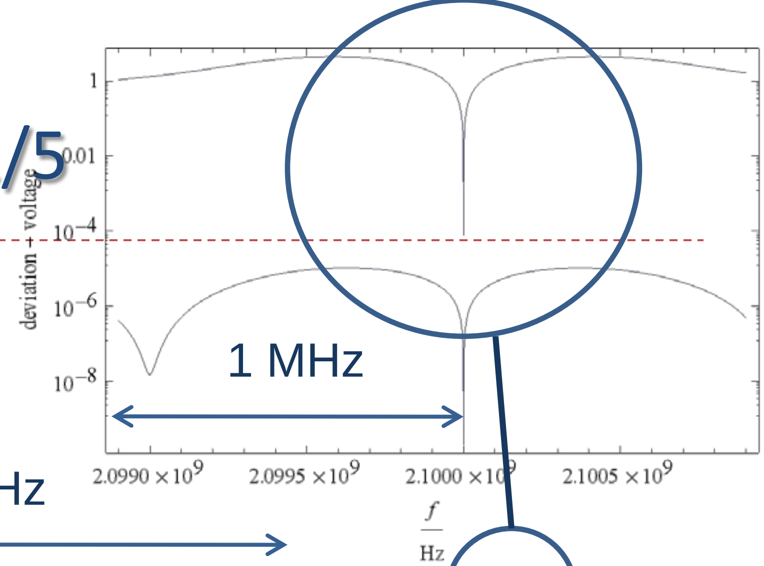
On BBU for pulsed beams - Results 1/5

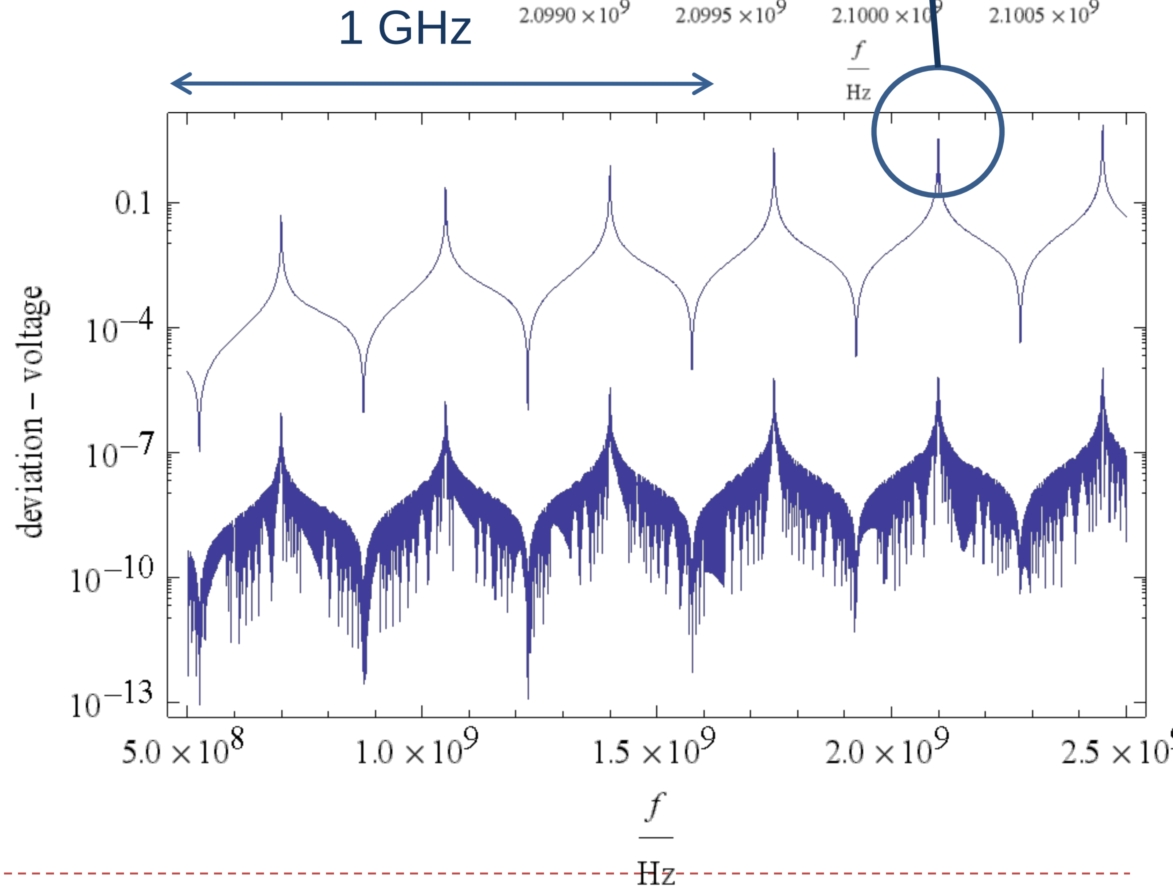
Deviation and Voltage in last (8th) cavity of cryomodule

Simulation parameters:

Initial beam displacement for 1st cavity: 1 mm

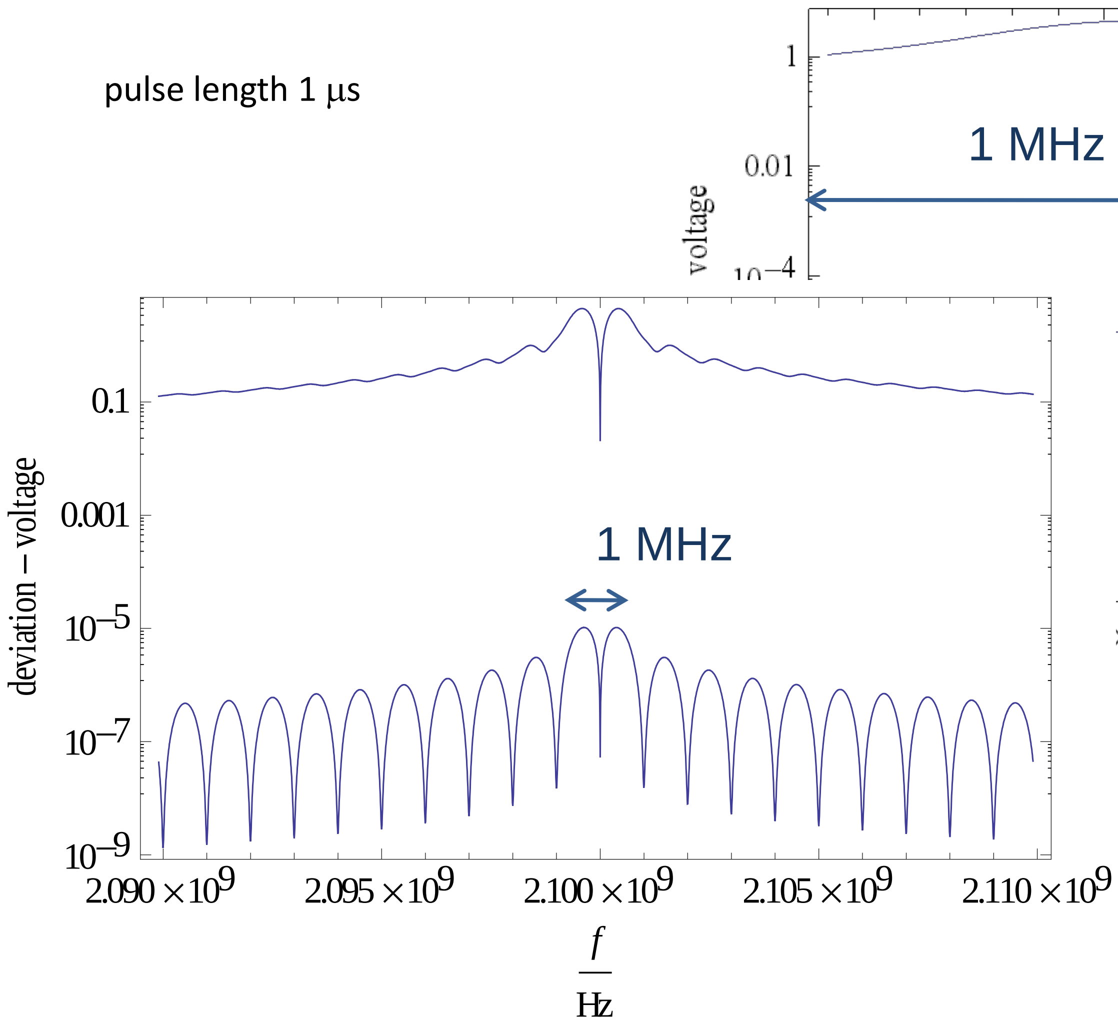
I	40 mA
$(R/Q)_x$	100 Ω
pulse length	1 μs
Number of midi-pulses	2
repetition rate	50 Hz
bunches per pulse	350
Number of cavities	8
Q_{ext}	10^6
Q_0	10^{10}



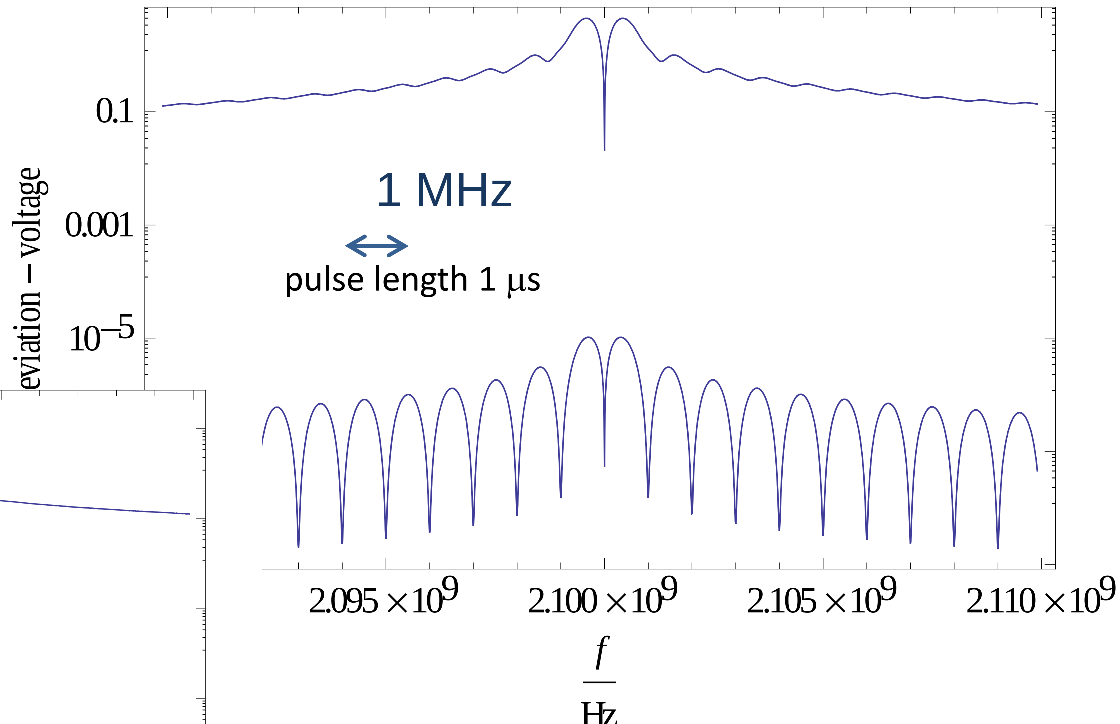


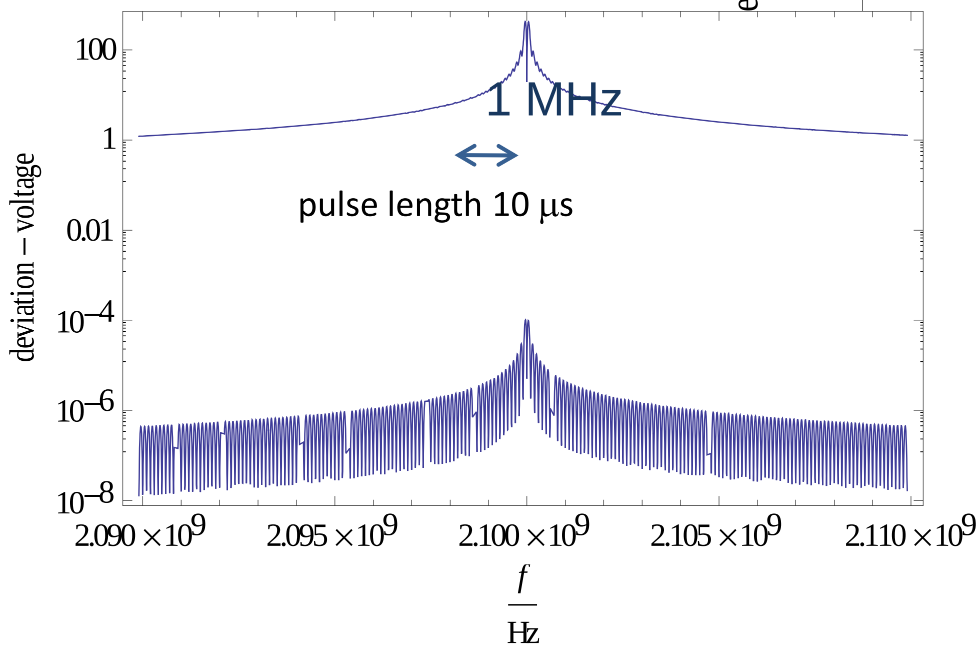
On BBU for pulsed beams - Results 2/5

pulse length 1 μs



On BBU for pulsed beams - Results 3/5





On BBU for pulsed beams - Results 4/5
- cavity res. frequency on beam spectral line

Deviation [m] and Voltage [V] in last (8th) cavity of cryomodule

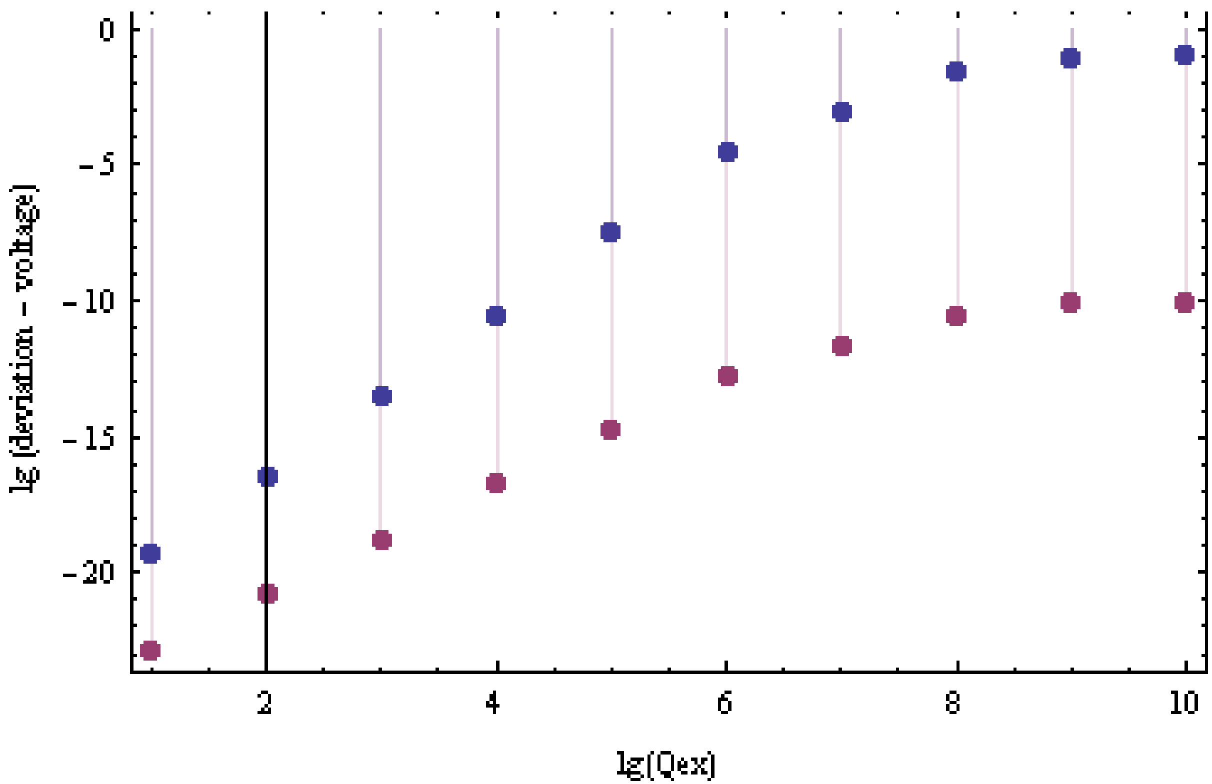
Simulation parameters:

Initial beam displacement for 1st cavity: 1 mm

I	40 mA
$(R/Q)_x$	100 Ω
pulse length	1 ms
Number of midi-pulses	2 [1]
repetition rate	50 Hz
bunches per pulse	350000
Number of cavities	8
f	2.1 GHz
Q_0	10^{10}

[1] same result as with 4 midi-pulses

f_0 = 2.1 GHz (on spectral line)



On BBU for pulsed beams - Results 5/5
- cavity res. frequency on/off beam spectral line

Deviation [m] and Voltage [V] in last (8^{th}) cavity of cryomodule

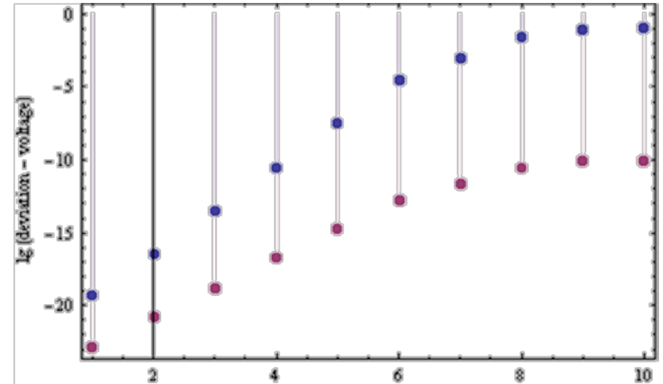
Simulation parameters:

Initial beam displacement for 1^{st} cavity: 1 mm

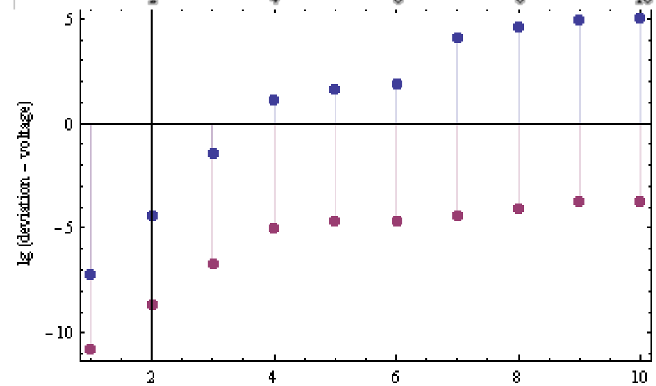
I	40 mA
$(R/Q)_x$	100 Ω
pulse length	1 ms
Number of midi-pulses	2[1]
repetition rate	50 Hz
bunches per pulse	350000
Number of cavities	8
f	2.1 GHz
Q_0	10^{10}

[1] same result as with 4 midi-pulses

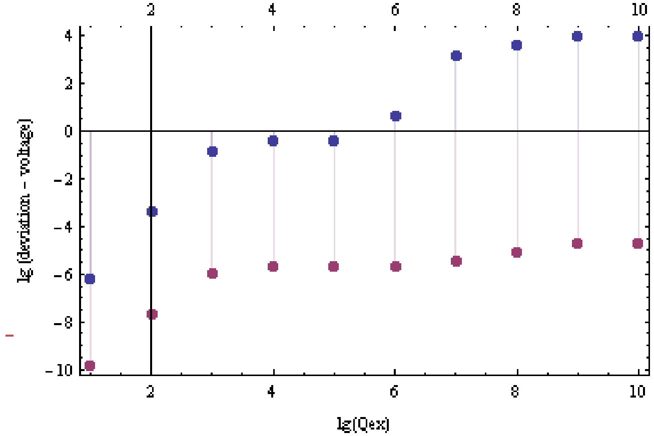
f_0 = 2.1 GHz (on spectral line)



$f = (f_0 - 0.1)$ MHz



$f = (f_0 - 1.0)$ MHz



Conclusion

- The results as presented by M. Schuh in previous meetings, based on much more detailed simulations, are confirmed:
  - For the nominal beam parameters, there is no risk of beam break up by dipole HOMs.
- Instead, the recommendation for HOM damping of $Q_{ext} < 10^5$ is based on the much denser spectrum of beam spectral lines that may lead to an excessive power deposited by the beam into the monopole HOMs under various chopping conditions.