Coherent Diffraction Radiation as a Tool for Non-invasive Bunch Length Diagnostics, theory

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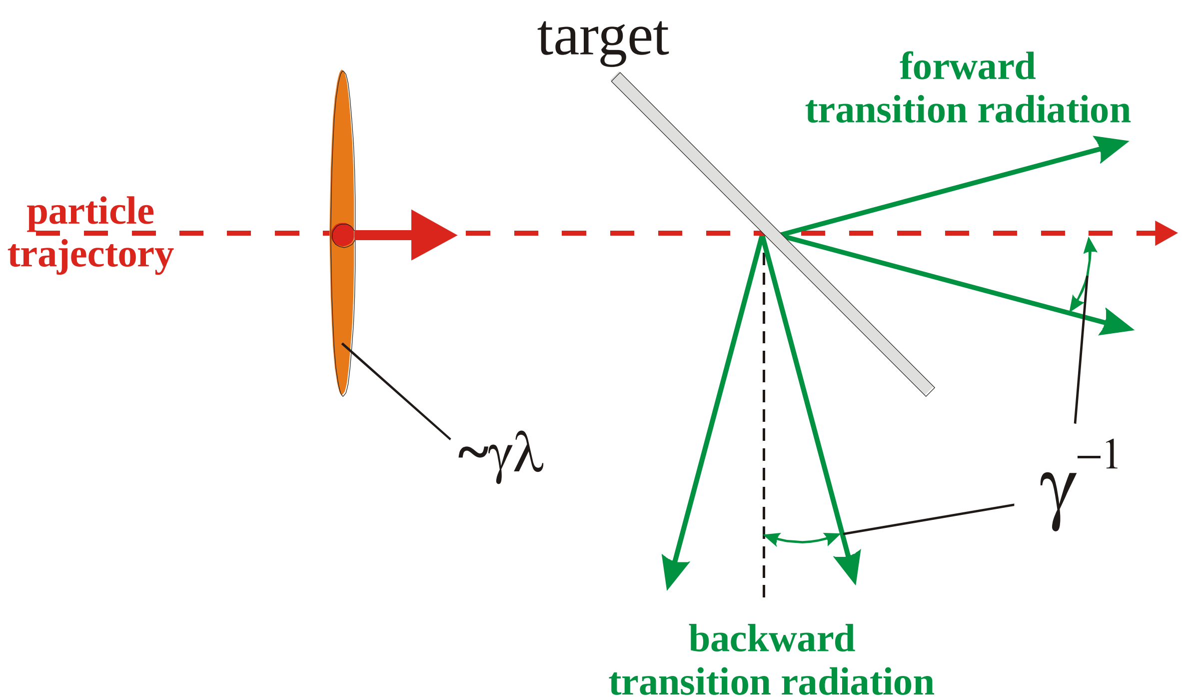
1.1Transition and diffraction radiation

Transition radiation appears when a charged particle crosses a boundary between two media with different dielectric properties.



- Invasive mechanism
- High brightness beam might destroy a target
- A target can change a beam parameters

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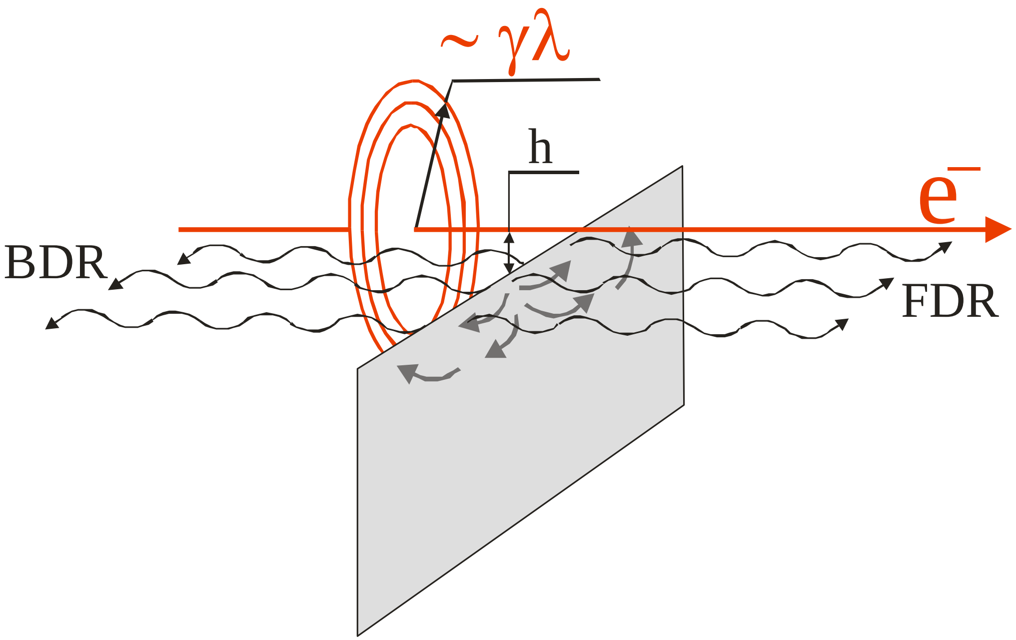
1.1Transition and diffraction radiation



Diffraction radiation appears when a charged particle moves in the vicinity of a medium.

Impact parameter h – the shortest distance between a particle and a target.

Advantages

- Non-invasive method
- Instantaneous emission
- Large emission angles
- Single electron spectrum is predictable

λ – observation wavelength,

$$\gamma = \frac{E}{mc^2} \text{ – Lorentz factor.}$$

$$h \leq \gamma\lambda$$

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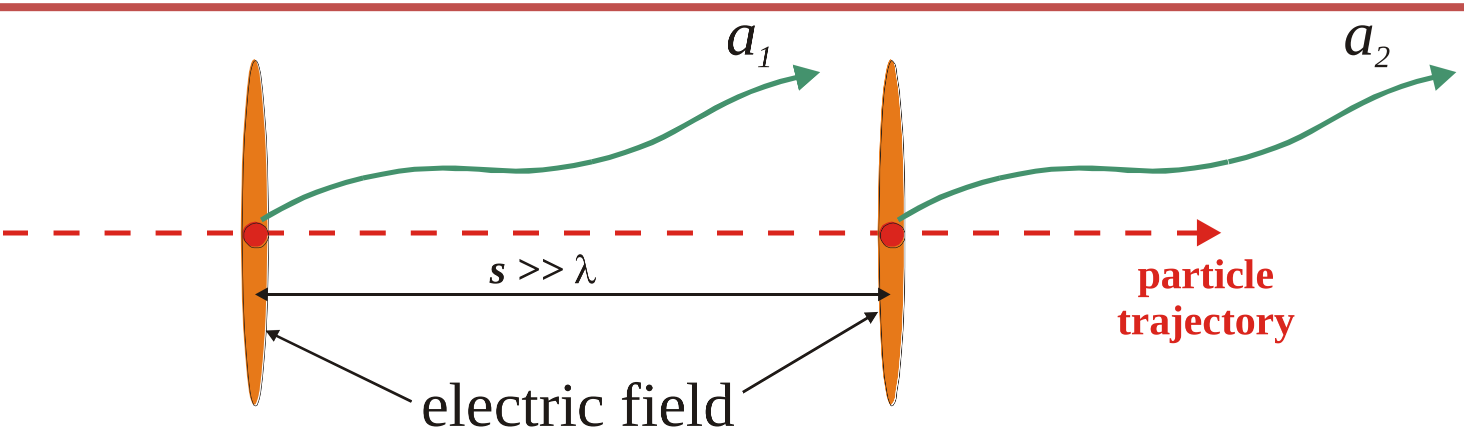
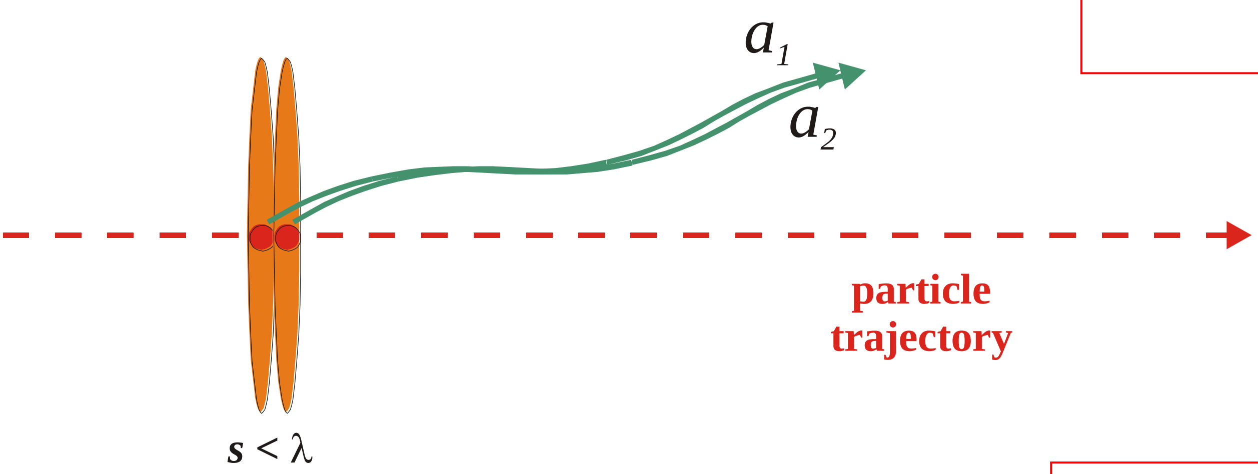
1.2 Incoherent and coherent radiation



$$I = |a_1|^2 + |a_2|^2 = 2|a|^2 \rightarrow N|a|^2$$



$$I = |a_1 + a_2|^2 = |2a|^2 = 4|a|^2 \rightarrow N^2|a|^2$$

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1.3 Diffraction radiation spectrum

$$S(\omega) = S_e(\omega)\left[N + N(N-1)F(\omega)\right]^* \qquad (1.3.1)$$

$$S_{coh}(\omega) = N^2 S_e(\omega) F(\omega) \qquad (1.3.2)$$

$S(\omega)$ - radiation spectrum (can be measured in the experiment)

$S_{coh}(\omega)$ - coherent radiation spectrum

N - number of electrons in a bunch (known from the experiment)

$S_e(\omega)$ - single electron spectrum (has to be known)

$F(\omega)$ - longitudinal bunch form factor (purpose of the measurements)

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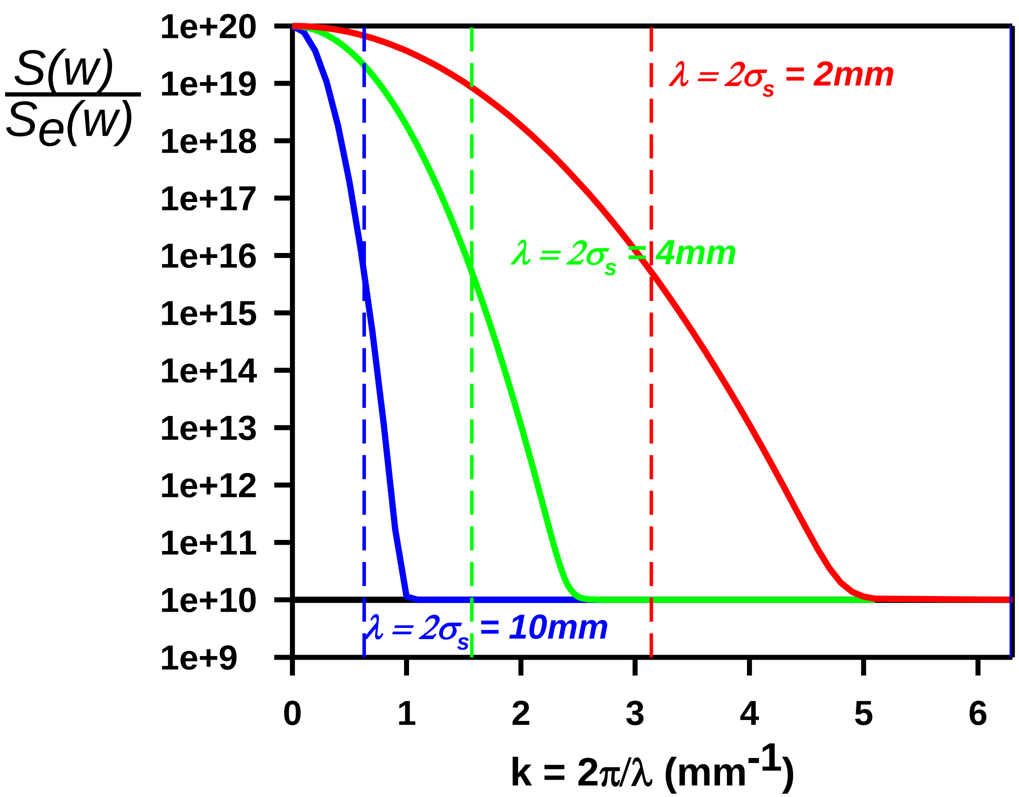
* J.S. Nodvick and D.S. Saxon, Phys. Rev. 96, 180 (1954)

1.4 Gaussian beam

$$F(\omega)=\left|\frac{1}{\sigma_s\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{s^2}{2\sigma_s^2}} e^{-i\frac{\omega}{c}s} ds\right|^2 = e^{-\frac{\omega^2\sigma_s^2}{c^2}} = e^{-k^2\sigma_s^2}$$



Assuming that

$$N = 10^{10}\,[e/bunch]$$

Coherent radiation appears when a bunch length is comparable to or shorter then the emitted radiation wavelength.

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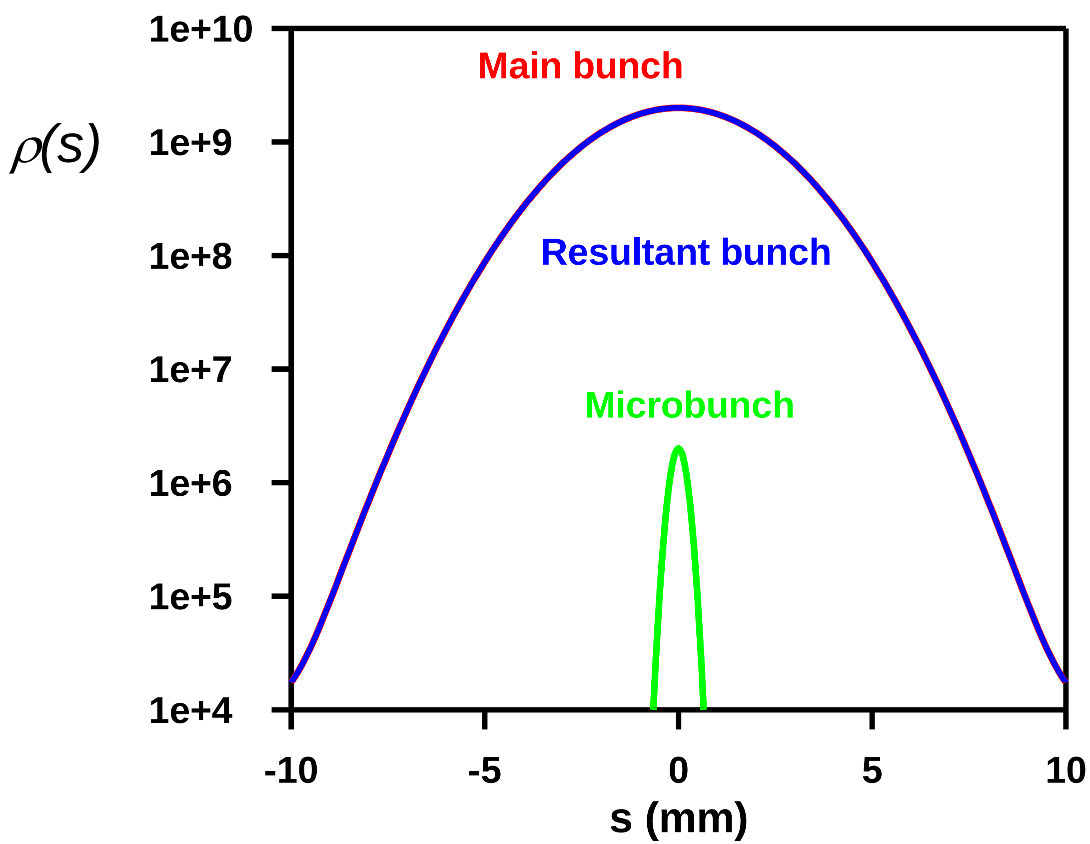
1.4 Two Gaussian beams

Assume we have a main bunch with $\sigma_s = 2mm\left(6.7\,ps\right);\ \mathrm{N} = 10^{10}$

and a micro bunch in it with $\sigma_{sm} = 0.2mm\left(0.67\,ps\right);\ \mathrm{N}_m = 10^{6}$



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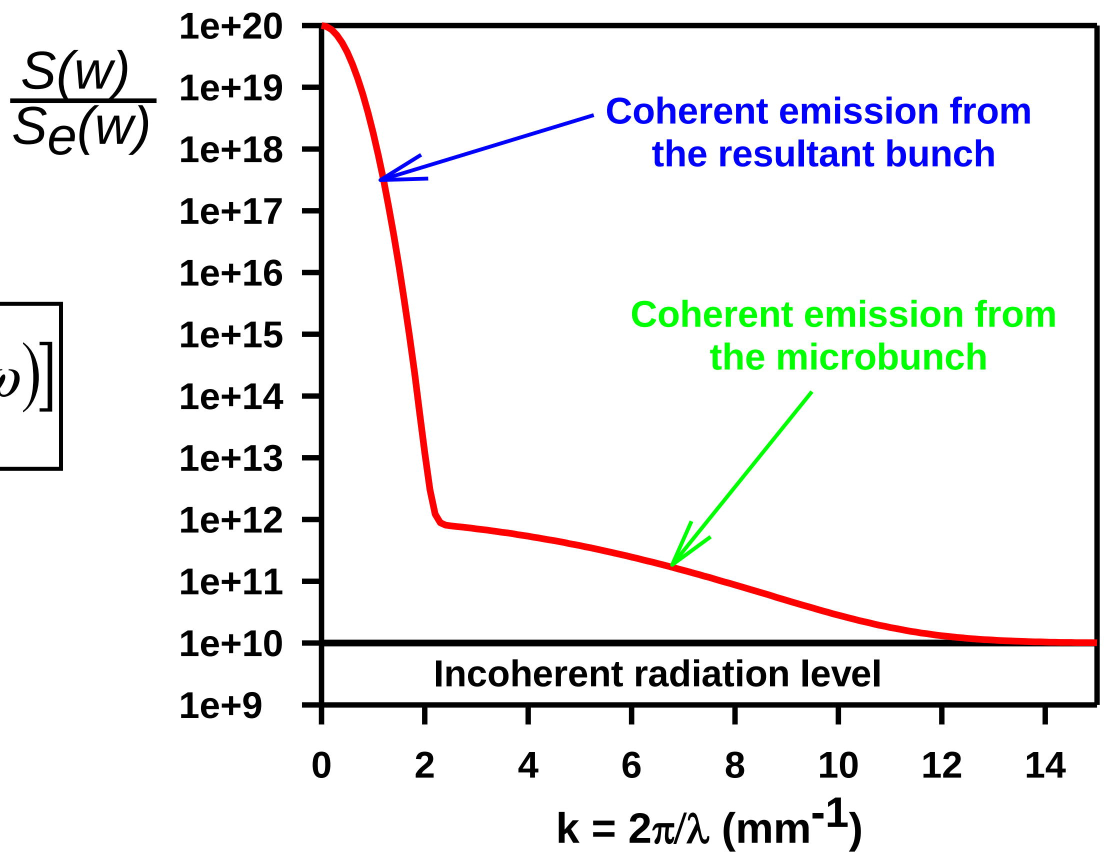
1.4 Two Gaussian beams

Assume we have a main bunch with $\sigma_s = 2mm\left(6.7\,ps\right);\ \mathrm{N} = 10^{10}$

and a micro bunch in it with $\sigma_{sm} = 0.2mm\left(0.67\,ps\right);\ \mathrm{N}_m = 10^{6}$

$$\frac{S(\omega)}{S_e(\omega)} = \left[N + N(N-1)F(\omega)\right]$$



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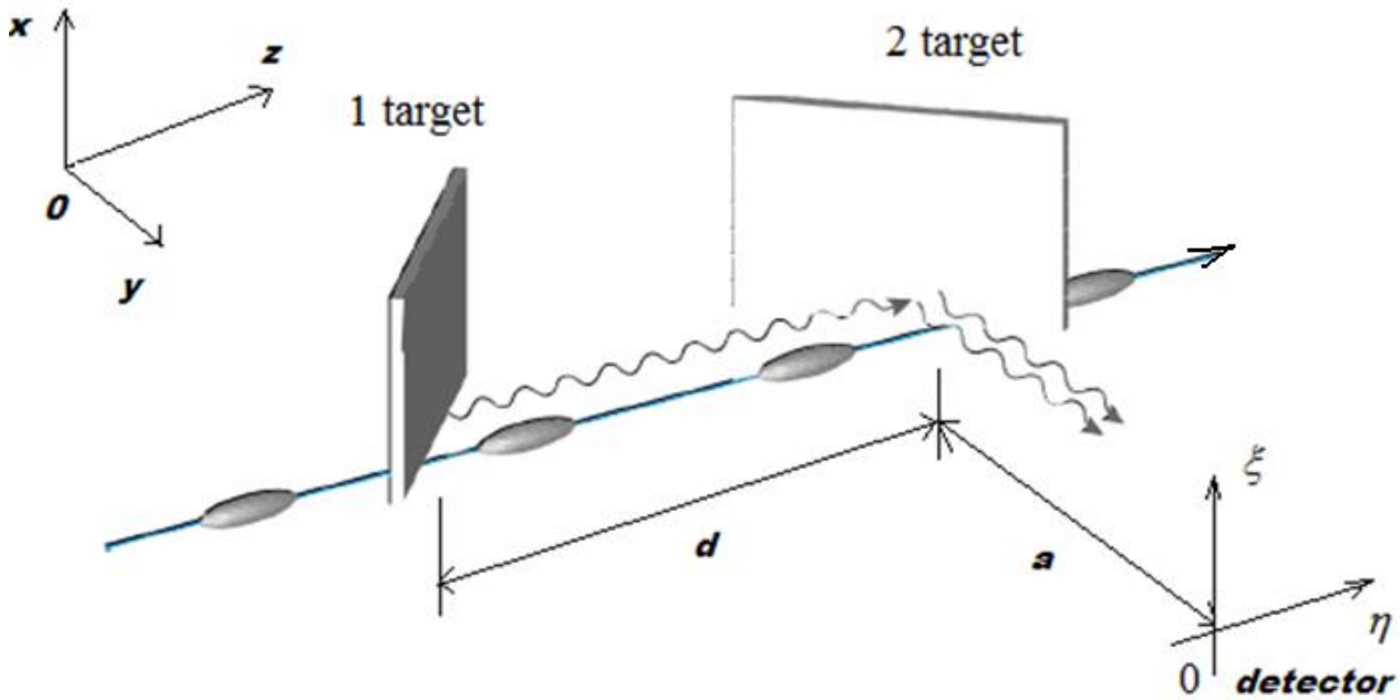
2. CDR experiment at CTF3 (CERN)



For the setup impact parameter is $h \approx 15mm \ll \gamma\lambda = 1175$ for $\gamma = 235$ and $\lambda = 5mm$

Recent upgrade has been performed, which included installation of the second target.

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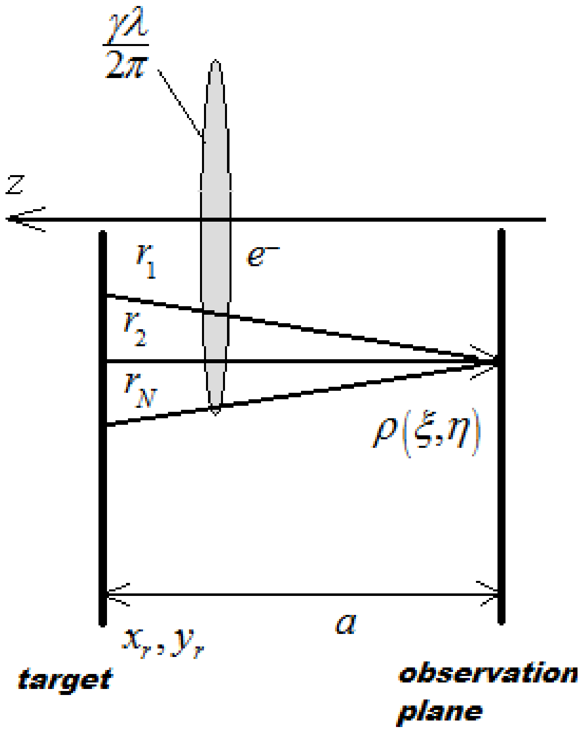
2.1 Methodology of calculations



$$E_{x,y}^{DR} = \frac{1}{4\pi^2}\iint E_{x,y}(x_r, y_r)\frac{e^{i\varphi}}{r}dy_r dx_r .* \quad (2.1.1)$$

$x_r = \rho_r \sin\psi_r ; y_r = \rho_r \cos\psi_r$ - the coordinates of a particle field.

$\rho_r ; \psi_r$ - radius and azimuthal angle of the particle pseudo photon field in the polar coordinates.

$E_{x,y}$ - amplitude of an arbitrary elementary source positioned on the target surface.

φ - phase advance of the photons emitted by each elementary source to the observation point.

r - distance from the elementary source on the target to the observation point.

*M.L. Ter-Mikaelyan, High Energy Electromagnetic Processes in Condensed Media, Wiley-Interscience, New York , 1972

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2.1 DR spatial distribution

$$\frac{d^2W^{DR}}{d\omega d\Omega} = 4\pi^2 k^2 a^2 \left[\left|E_x^{DR}\right|^2 + \left|E_y^{DR}\right|^2 \right] \qquad (2.1.2)$$

E_x^{DR} - vertical polarization component of the DR from a target/targets

E_y^{DR} - horizontal polarization component of the DR

$k = \frac{2\pi}{\lambda}$ is a wave number where λ is a wavelength of DR

a - distance between the target and the observation point

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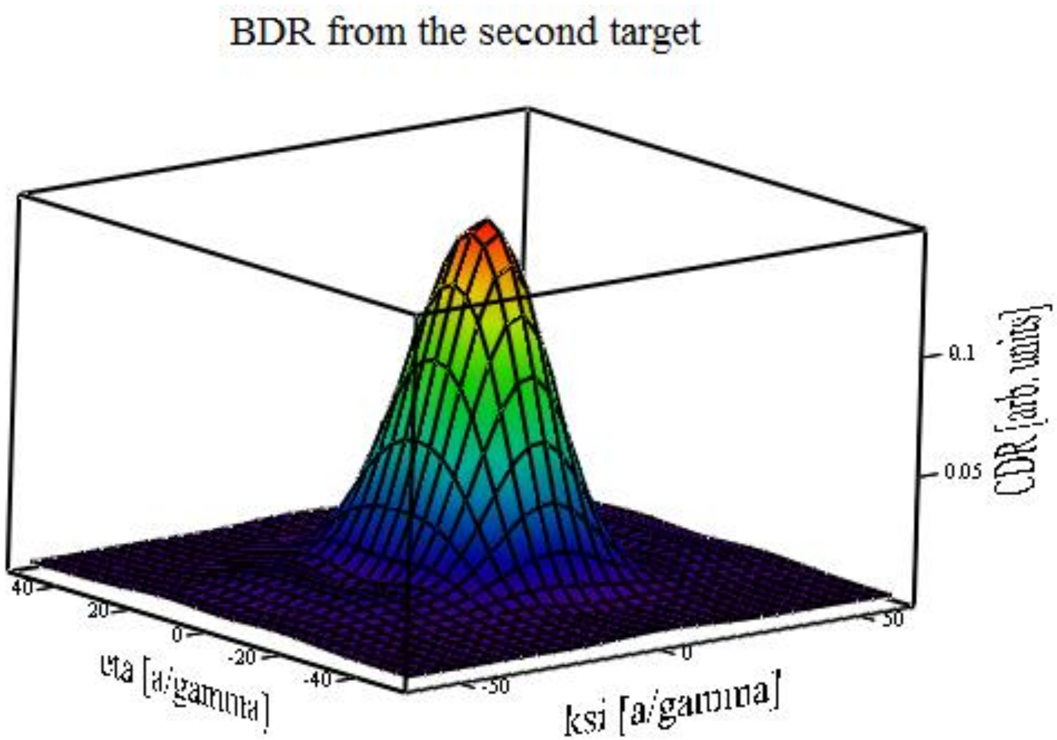
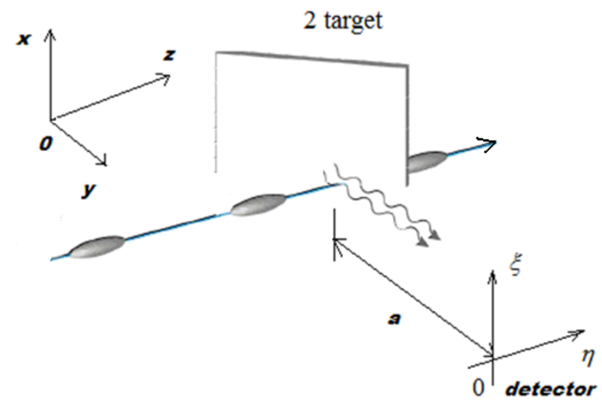
2.2 BDR from the second target only





Target dimensions: $40mm \times 60mm$

Impact parameter: $impact = h = 5mm$

Beam energy: $\gamma = 235$

Distance from the target to the detector: $a = 2m$

Wavelength of radiation: $\lambda = 5mm$

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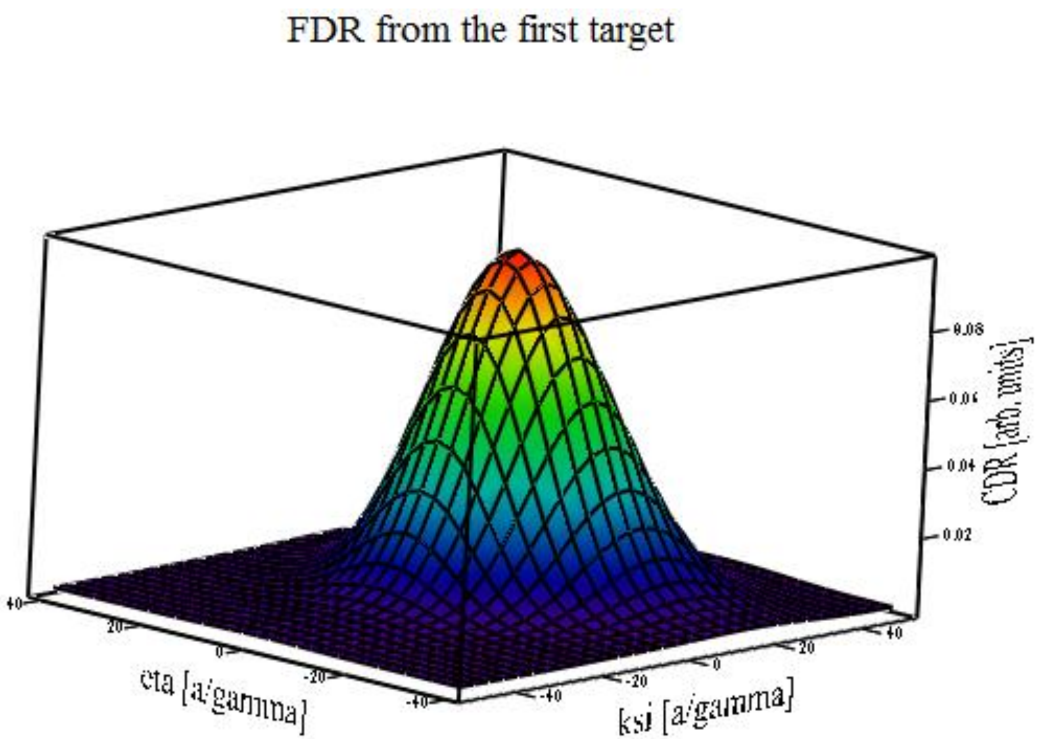
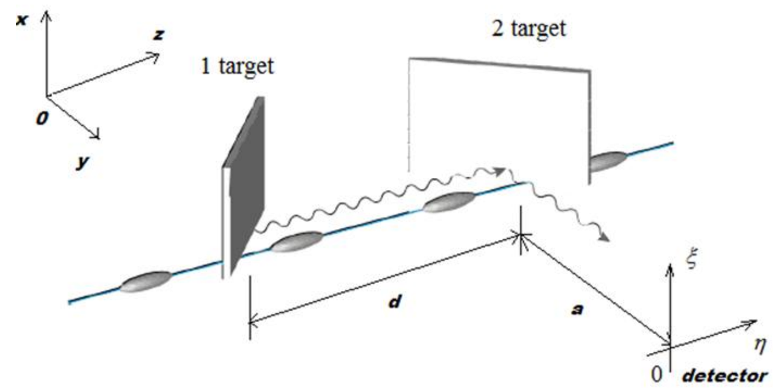
2.3 Distribution of FDR from the first target, diffracted at the second one





Targets dimensions: $40mm \times 60mm$ Impact parameter: $h_1 = h_2 = 5mm$

Beam energy: $\gamma = 235$ Distance from the second target to the detector: $a = 2m$

Wavelength of the radiation: $\lambda = 5mm$ Distance between the targets: $b = 0.02m$

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2.3.1 Fresnel integrals approximation

In order to increase performance of calculations numeric approximation of Fresnel integrals was used.

For Fresnel integrals $C(z)=\int_0^z \cos\left(\frac{\pi}{2}t^2\right)dt$ and $S(z)=\int_0^z \sin\left(\frac{\pi}{2}t^2\right)dt$,

the following approximations were used:

$$C(z)=\begin{cases}\frac{1}{2}+f(z)\sin\left(\frac{\pi}{2}z^2\right)-g(z)\cos\left(\frac{\pi}{2}z^2\right), & if \quad z>0;\\ -\frac{1}{2}-f(|z|)\sin\left(\frac{\pi}{2}z^2\right)+g(|z|)\cos\left(\frac{\pi}{2}z^2\right), & otherwise.\end{cases} \qquad (2.3.2)$$

and

$$S(z)=\begin{cases}\frac{1}{2}-f(z)\cos\left(\frac{\pi}{2}z^2\right)-g(z)\sin\left(\frac{\pi}{2}z^2\right), & if \quad z>0;\\ -\frac{1}{2}+f(|z|)\cos\left(\frac{\pi}{2}z^2\right)+g(|z|)\sin\left(\frac{\pi}{2}z^2\right), & otherwise.\end{cases} \qquad (2.3.3)$$

where $f(z)\simeq\frac{1+0.926z}{2+1.792z+3.104z^2}$, $g(z)\simeq\frac{1}{2+4.142z+3.492z^2+6.67z^3}$, $0\le z\le\infty$

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2.4 Spatial distribution of CDR from two targets

$$\frac{d^2W^{CDR}}{d\omega d\Omega} = 4\pi^2 k^2 a^2 \left[\left(\operatorname{Re} E_1 - \operatorname{Re}\left[E_2 \exp\left(-\frac{ikd}{\beta} \right) \right] \right)^2 + \left(\operatorname{Im} E_1 - \operatorname{Im}\left[E_2 \exp\left(-\frac{ikd}{\beta} \right) \right] \right)^2 \right]$$

(2.4.1)

E_1 - calculated FDR from the first target

E_2 - calculated BDR from the second target

$\beta = v / c$ - electric charge speed in terms of speed of light

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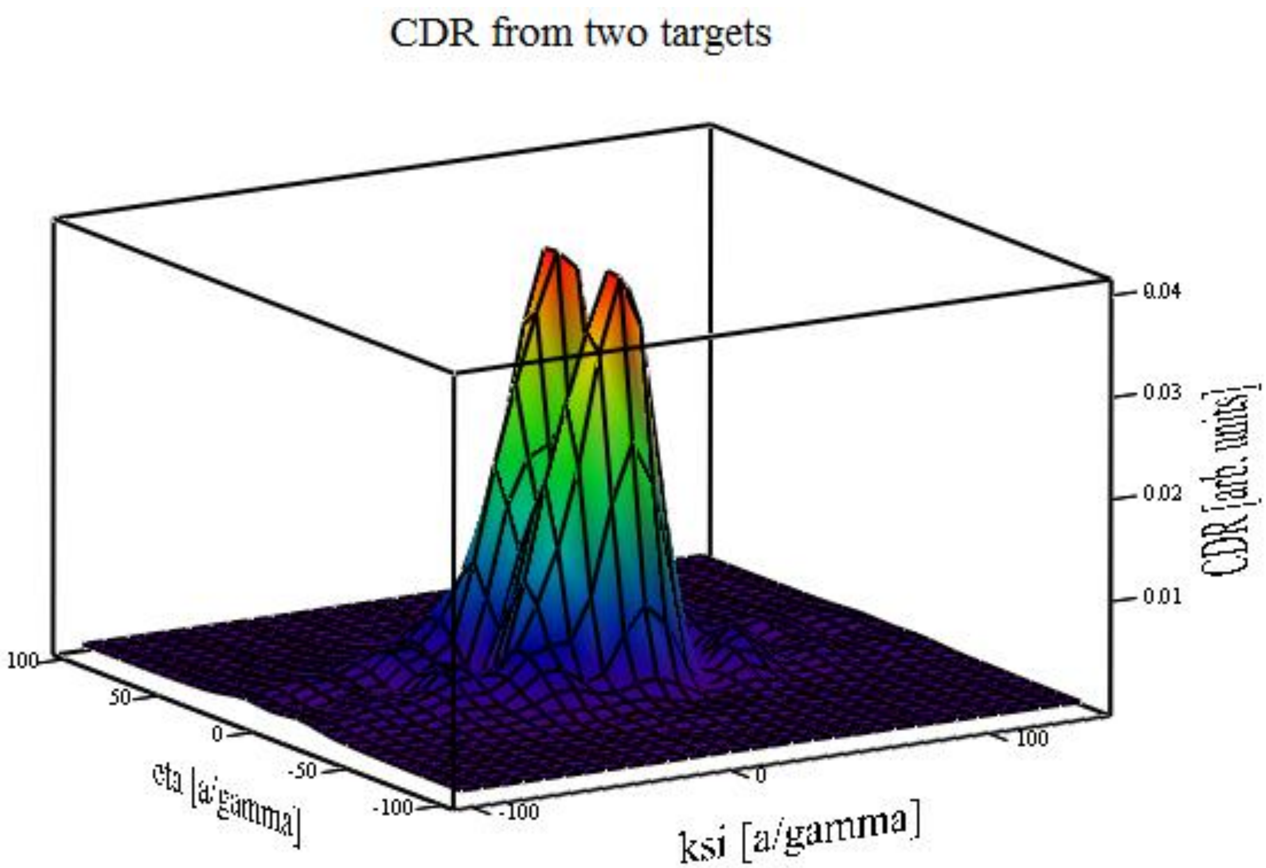
2.4 Spatial distribution of CDR from two targets



Targets dimensions:

$40mm \times 60mm$

$\gamma = 235$

$a = 2m$

$b = 0.02m$

Impact parameters:

$h_1 = 5mm,\ h_2 = 5mm$

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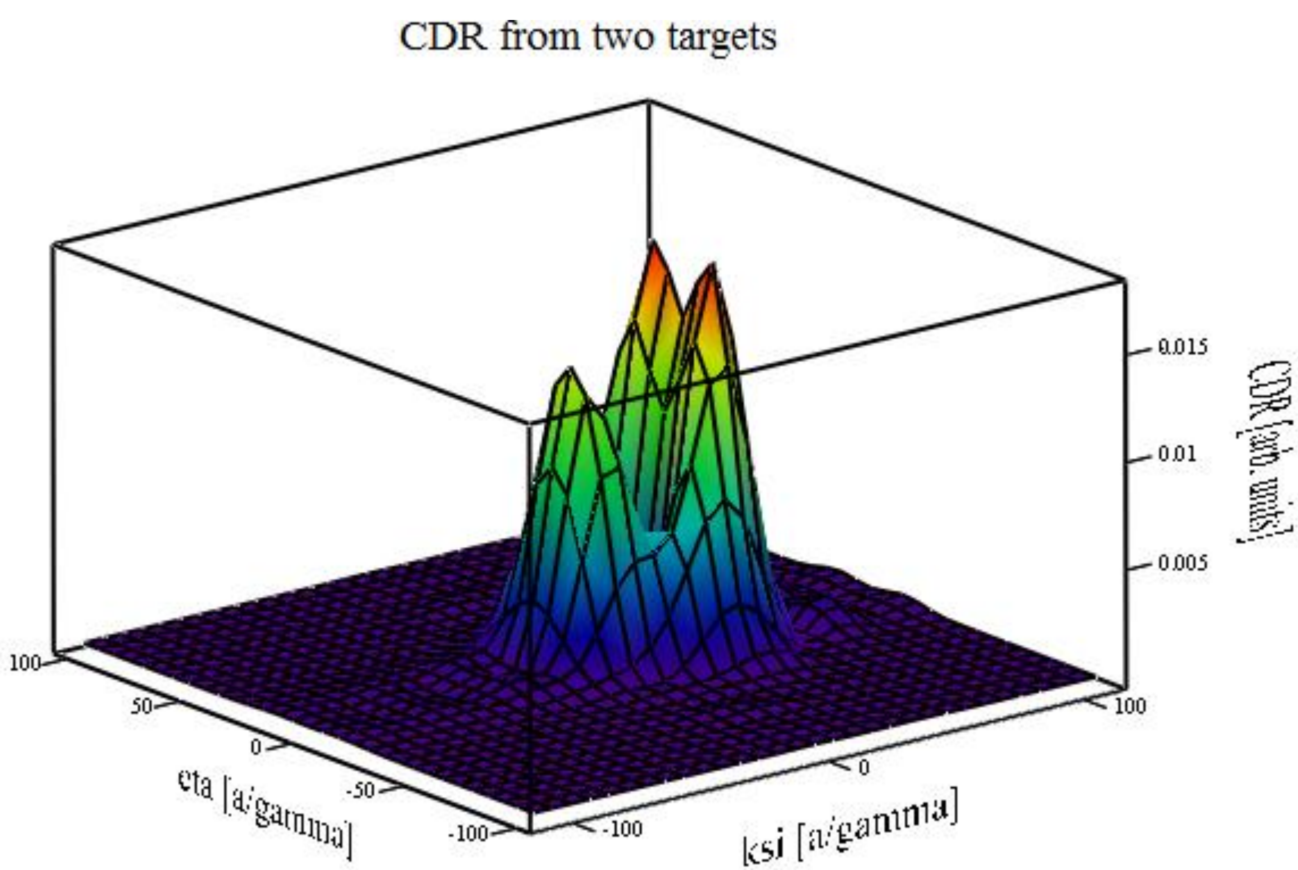
2.4 Spatial distribution of CDR from two targets



Targets dimensions:

$$40mm \times 60mm$$

$$\gamma = 235$$

$$a = 2m$$

$$b = 0.02m$$

Impact parameters:

$$h_1 = 5mm,\ h_2 = 15mm$$

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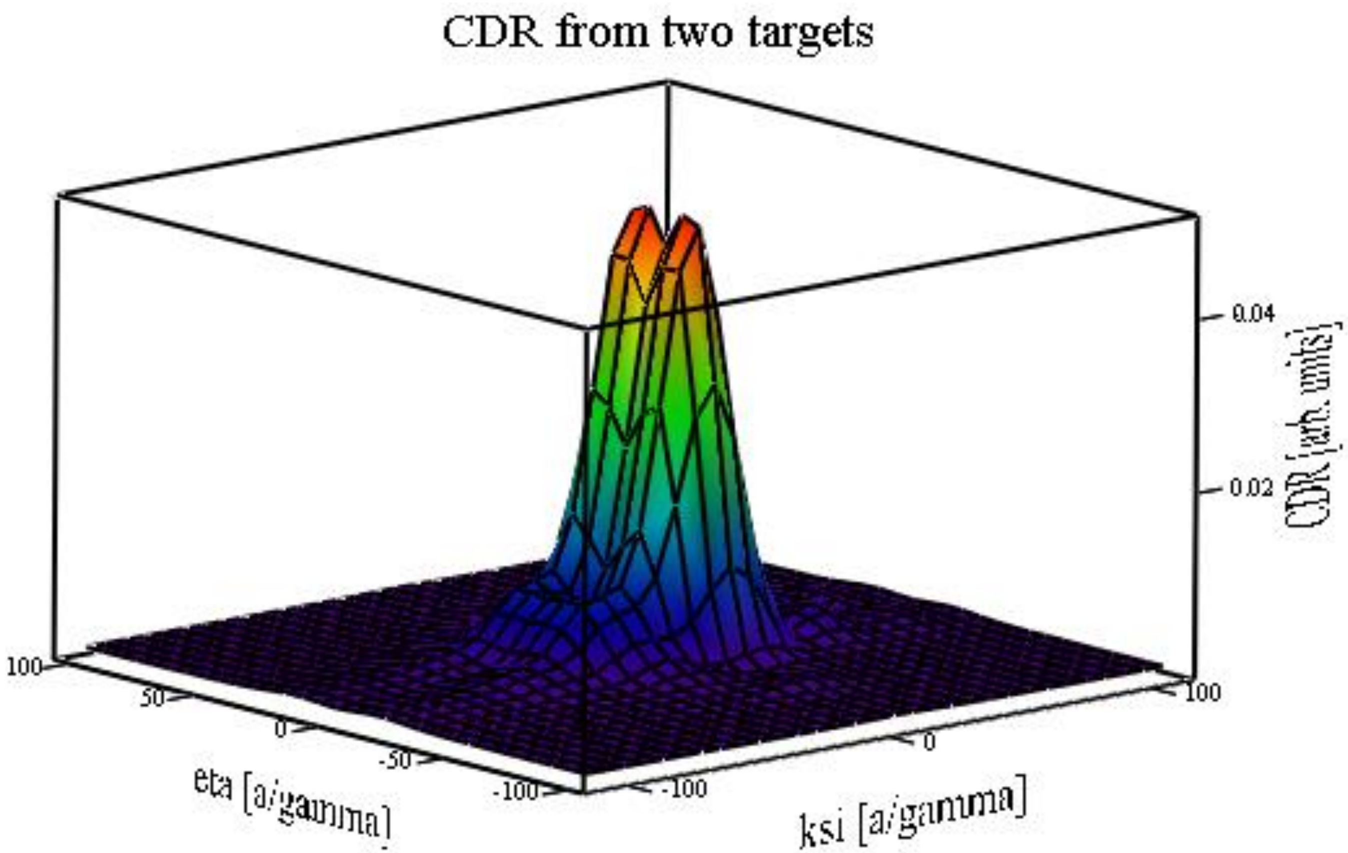
2.4 Spatial distribution of CDR from two targets



Targets dimensions:

$$40mm \times 60mm$$

$$\gamma = 235$$

$$a = 2m$$

$$b = 0.02m$$

Impact parameters:

$$h_1 = 15mm,\ h_2 = 5mm$$

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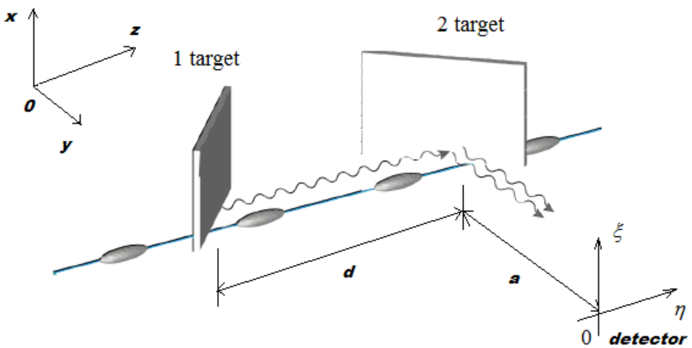
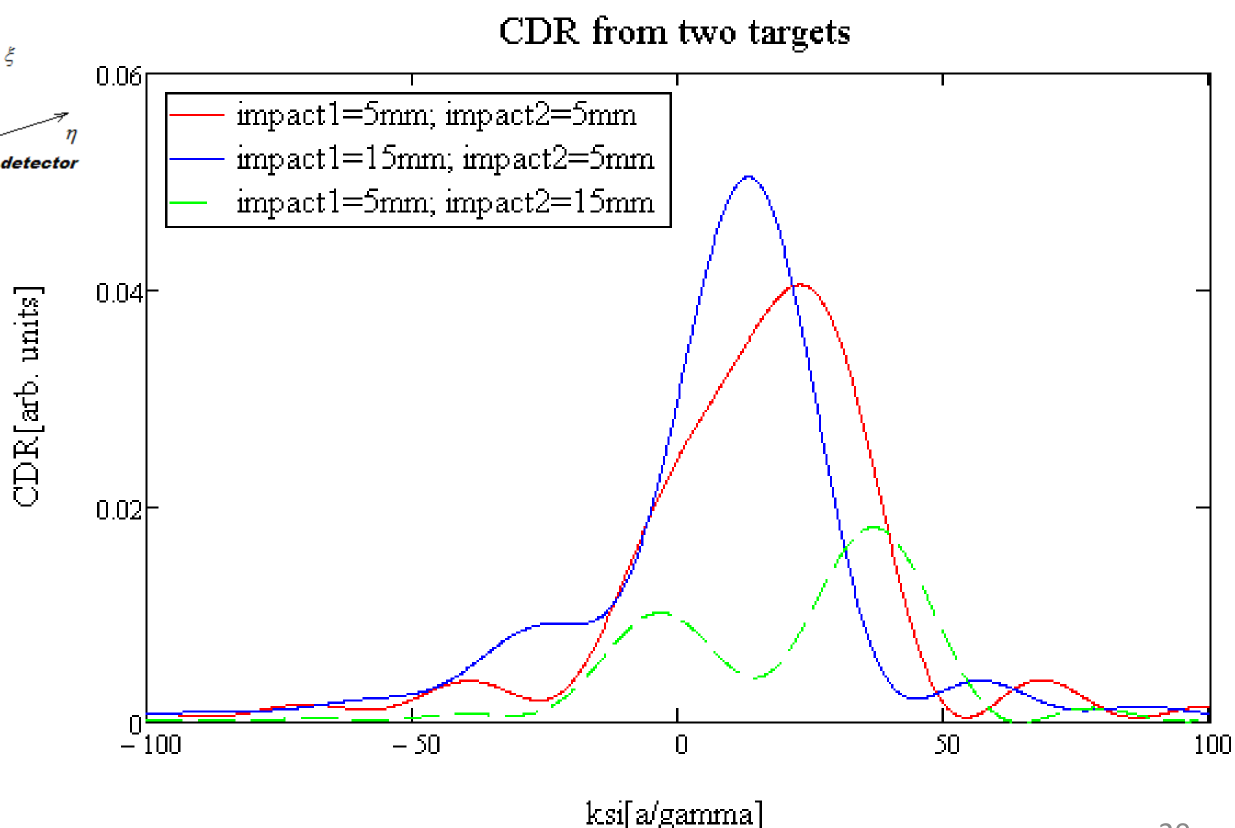
2.4 Spatial distribution of CDR from two targets

Cross – section of 3D distributions at $\eta = 13[a/gamma]$. $a = 2m; \gamma = 235; \lambda = 5mm$





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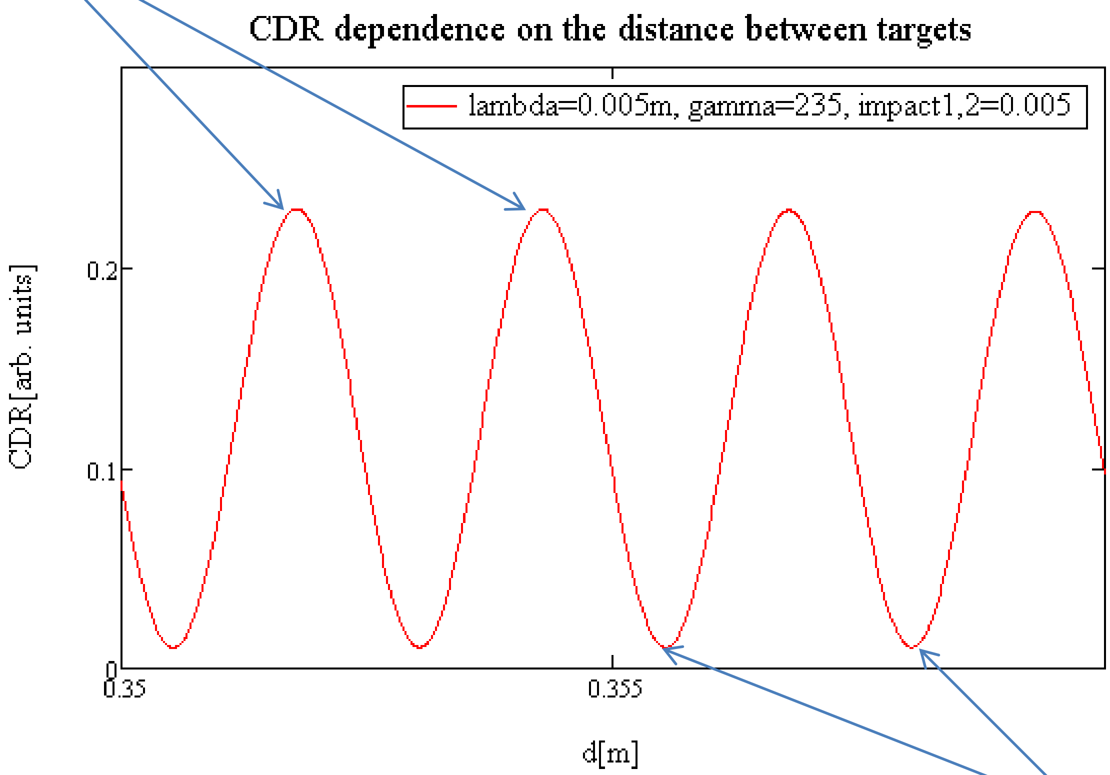
2.4.1 Constructive and destructive interference

Variation of CDR intensity at $\xi = 13[a/gamma]; \eta = 10[a/gamma]; \quad a = 2m; \gamma = 235$

Constructive interference, the signals sum up and come in phase.



Destructive interference, the signals cancel out and come out of phase.

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3.1 Kramers-Kronig analysis, bunch form - factor

Kramers- Kronig relation is used to derive longitudinal particle distribution in a bunch from a form factor:

$$\rho^2(k) = \frac{S_{coh}(k)}{N_e^2 S_e(k)}, \qquad (3.1.1)$$

$S_{coh}(k)$ - experimentally measured CDR spectrum;

N_e - number of particles in a bunch;

k - wave number;

$\rho^2(k)$ - bunch form factor;

$S_e(k)$ -single electron spectrum.

where $\rho(k)$ is a Fourier transform of a longitudinal charge distribution in a bunch.

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3.1 Normalized bunch distribution, phase factor

Normalized bunch distribution function can be determined as:*

$$S(z)=\frac{1}{\pi}\int_0^{\infty}\rho(k)\cos(\psi(k)-zk)\,dk. \qquad (3.1.2)$$

z - longitudinal coordinate; $\rho(k)$ - form-factor amplitude; $\psi(k)$ - phase factor

The phase factor and the form factor are related by the Kramers-Kronig relation. If the form factor is measured at all wave numbers the phase factor can be obtained as follows:*

$$\psi(k)=-\frac{2c}{\pi}\int_0^{\infty}\frac{\ln(\rho(x)/\rho(k))}{x^2-k^2}. \qquad (3.1.3)$$

x - integration variable in a wave number domain

*R. Lai, A.J. Sievers, Determination of a charge-particle-bunch shape from the coherent far infrared spectrum, Phys. Rev. E 50, R3342 (1994)

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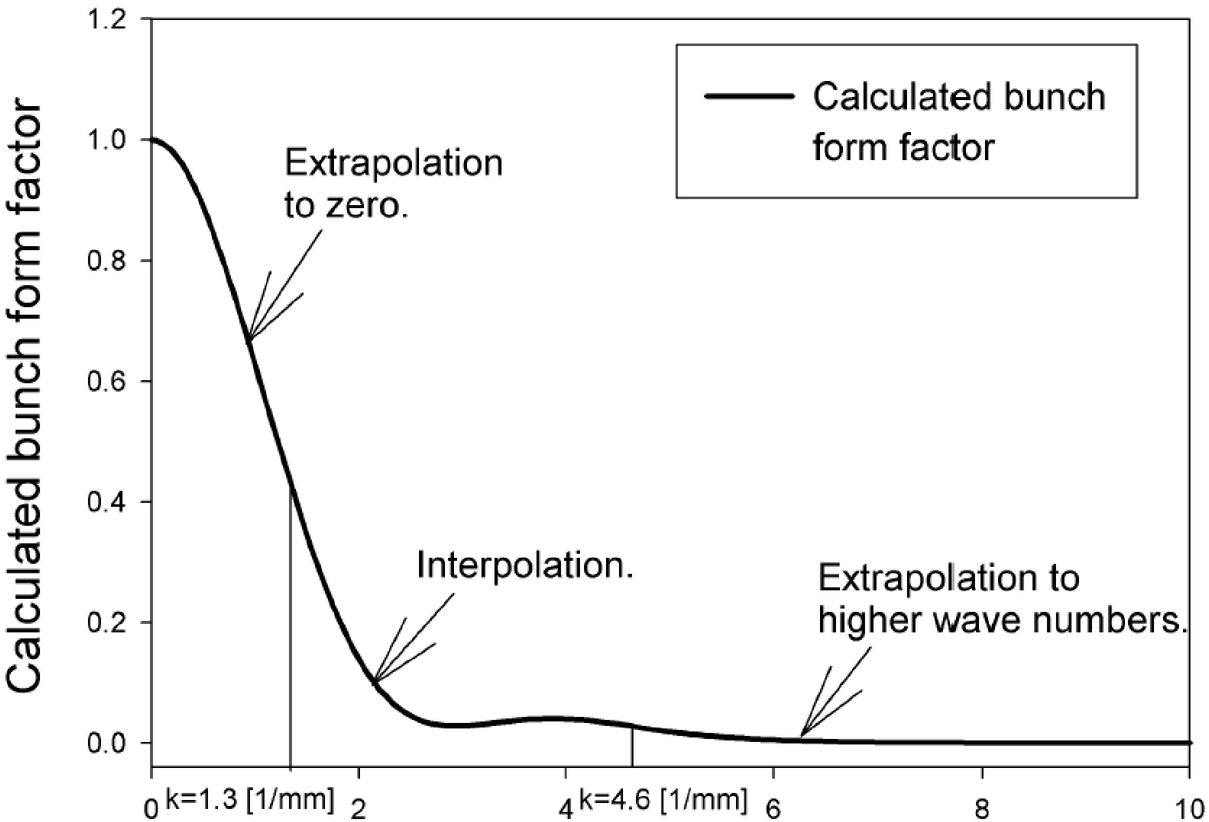
3.2 Calculated bunch form - factor



- Interpolation and extrapolation procedures were applied for the reconstruction.
- The data area confined within two vertical lines was assumed to be a given data set.

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3.3 Interpolation method

For the interpolation between the form factor data points the following function was applied:

$$\rho_{\text{int}}^2(k) = \frac{\sum_{n=0}^{N} \rho^2(k_n) \exp\left(-\frac{(k_n - k)^2}{2\sigma^2} \right)}{\sum_{n=0}^{N} \exp\left(-\frac{(k_n - k)^2}{2\sigma^2} \right)}. \qquad (3.3.1)$$

$\rho^2(k_n)$ - form factor data; σ - smoothing parameter

For presented form factor reconstruction σ was chosen to be $\frac{k_n - k_{n-1}}{3}$, to avoid significant smoothing of the data.

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3.4.1 Extrapolation method, low wave numbers

Low wave number extrapolation:*

$$\rho_{low}^{2}(k) = \rho_{\text{int}}^{2}(k_0)\exp\left(-ak^2 + bk + c\right), \qquad (3.4.1)$$

$$a = \left(\ln\rho_{\text{int}}^{2}(k_0) - k_0\frac{s}{\rho_{\text{int}}^{2}(k_0)}\right)\frac{1}{k_0^2},\ b = \frac{s}{\rho_{\text{int}}^{2}(k_0)} + 2ak_0,\ c = -\ln\rho_{\text{int}}^{2}(k_0)$$

$\rho_{\text{int}}^{2}(k_0)$ - interpolation function value corresponding to the lowest wave number

k_0 - lowest wave number

s - slope derived from the interpolation function.

$$s = \frac{\rho_{\text{int}}^{2}(k_4) - \rho_{\text{int}}^{2}(k_0)}{k_4 - k_0}. \qquad (3.4.2)$$

*V. Blackmore, Determination of the Time Profile of Picosecond-Long Electron Bunches through the use of Coherent Smith-Purcell Radiation, PhD Thesis, 2008

3.4.2 Extrapolation method, large wave numbers

The following function was used to extrapolate towards the large wave numbers:

$$\rho_{large}^{2}(k) = \exp\left(-\beta k^{2} + \gamma k + \delta\right), \qquad (3.4.3)$$

where β, γ, δ are chosen to smoothly join the large wave numbers.

- $\rho_{large}^{2}(k)$ must match the data at the largest wave number.

- The first and the second derivatives of $\rho_{large}^{2}(k)$ must match the first and the second derivatives of $\rho_{\text{int}}^{2}(k)$ at the largest wave number.

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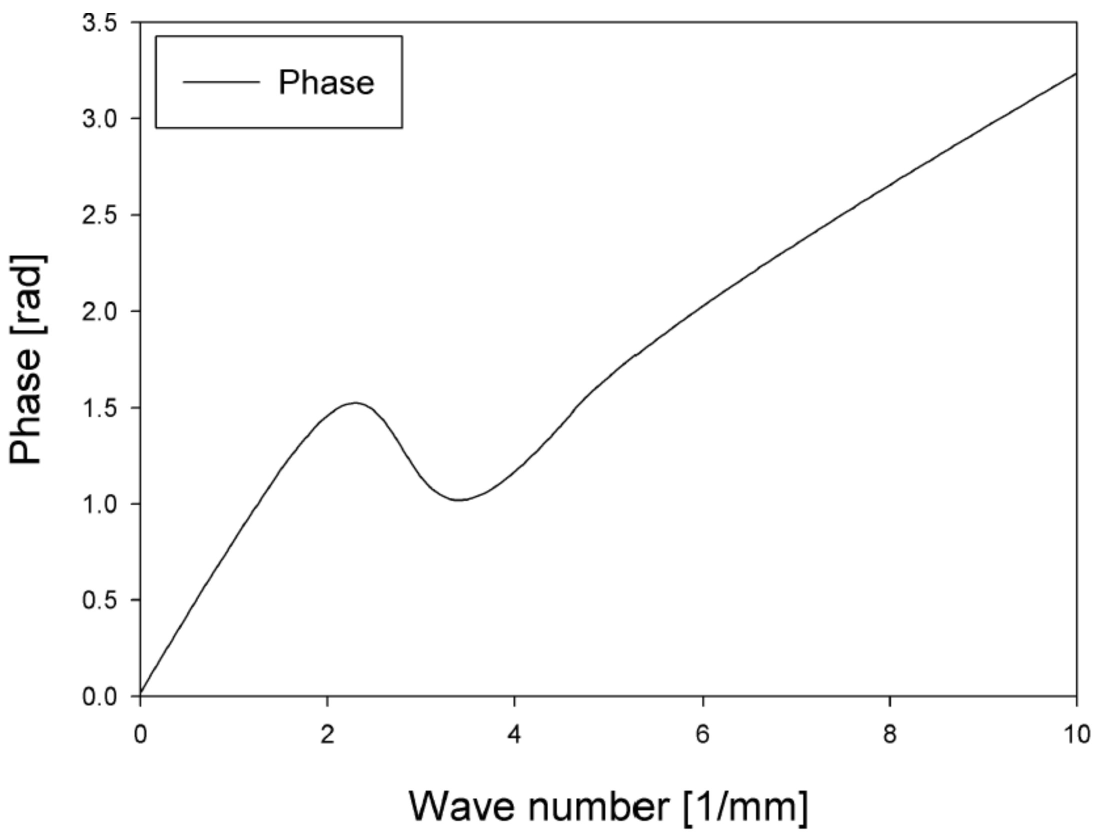
3.5 Phase reconstruction



Sufficiently large spectral detector coverage is very important. If the spectral range is too short, especially towards the large wave numbers the method doesn't reconstruct the initial phase accurately enough.

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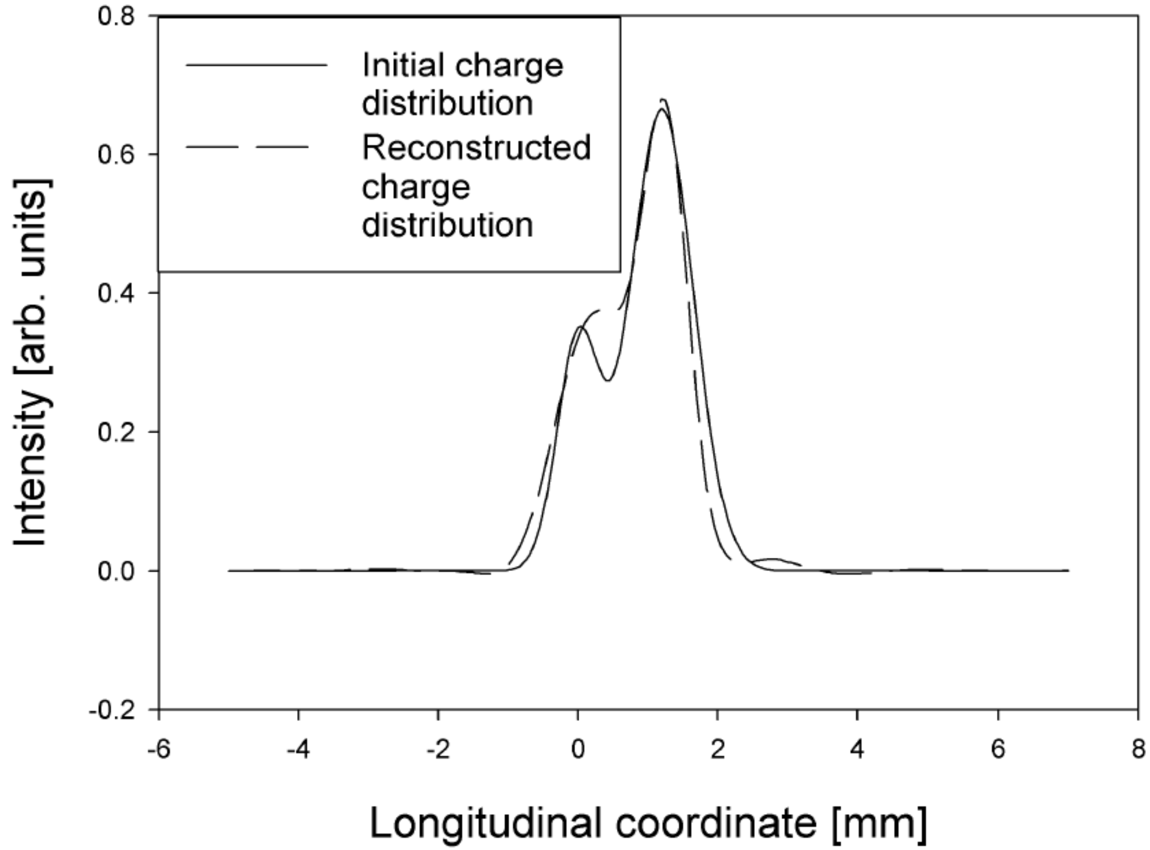
3.6 Longitudinal charge distribution, reconstruction



$$S_{initial}(z) = \frac{\exp\left(-\frac{z^2}{2\sigma_1^2}\right)}{4\sqrt{2\pi}\sigma_1} + \frac{3\exp\left(-\frac{(z-z_0)^2}{2\sigma_1^2}\right)}{4\sqrt{2\pi}\sigma_1},$$ where $z_0 = 1,2mm; \sigma_1 = 0,3mm; \sigma_2 = 0,45mm$

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Summary

- Simulations on CDR from two targets have been performed.

- Next step will be the spectrum reconstruction from the radiation spatial distribution.

- Studies on Kramers- Kronig analysis as a tool for bunch profile reconstruction from the measured spectrum have been shown.

Tools based on Coherent Diffraction Radiation are very useful for longitudinal beam diagnostics in modern and future accelerator machines, as they are

- non-invasive;

- have instantaneous emission and large emission angles;

- give information about the longitudinal dimensions and structure.

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