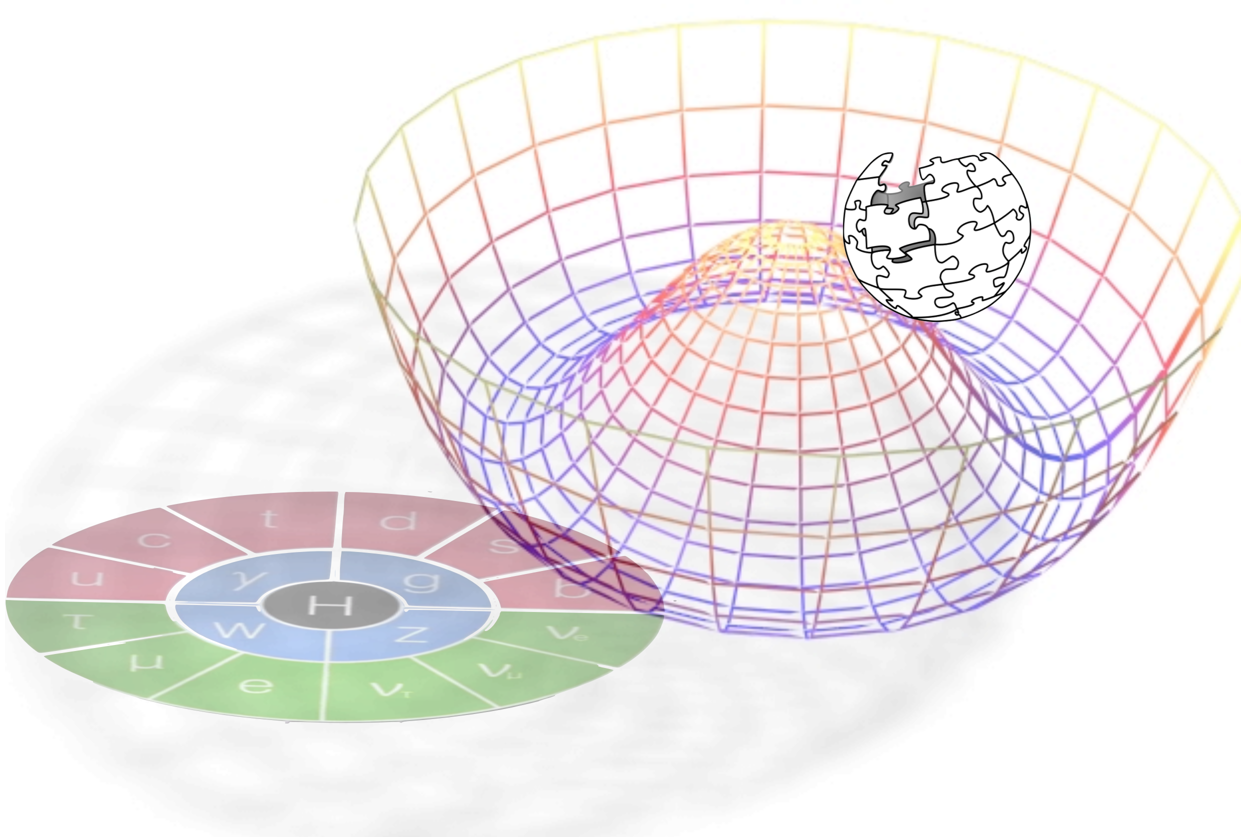
The Standard Model
of particle physics

CERN summer student lectures ~~2019~~ 2020

Q&A online session



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Lagrangians

Jonathan: "Why is a Lagrangian used to the describe particles and their interactions rather than a Hamiltonian?"

☛ The Hamiltonian formalism treats time differently than space coordinates. Therefore it is not so much appropriate to describe a Lorentz-invariant theory. It can be done, of course, but Lorentz invariance is less manifest than in the Lagrangian formalism.

Anna: "When the classical lagrangian was considered it was a difference between kinetic and potential energies, so [L]m=[Energy]m=1, but then we showed that [L]m=4. Why is it so?"

☛ In classical physical, the action is the time integral of the Lagrangian: $S = \int_{t_1}^{t_2} dt \mathcal{L}(x, \dot{x})$

Since in natural unit, the action is dimensionless, the Lagrangian has a mass dimension 1

In QFT, the action is the space-time integral of the Lagrangian density: $S = \int d^4x \, \mathcal{L}$

The Lagrangian density has now a mass dimension 4

Goldstone Theorem & Higgs Mechanism

Artem: "In the fifth lecture you explained that after ElectroWeak Symmetry Breaking, the SU(2)xU(1) gauge group is broken i.e. Lagrangian stays the same, but the vacuum changes under the group. Now only U(1)Q group is unbroken - saves the vacuum. So, we have only one massless particle - photon. This is interesting because Goldstone theorem predicts the number of massless particle to be the number of broken generators 3+1-1 = 3. But, instead of three massless, we have only one. It seems like Goldstone theorem just can not be applied here for gauge group. Am I wrong? What is the correct explanation?"

☛ The Goldstone th. states that to any broken generator of the *global* continuous symmetry corresponds a massless scalar field (a Goldstone boson). When the symmetry is gauged, like $SU(2)_L xU(1)_Y$, the Goldstone bosons are "eaten" to become the longitudinal component of the corresponding massive gauge boson.

Remember your QM classes:
"a particle of spin s has 2s+1 polarisation states"
but you learnt in optics that the photon has only 2 (transverse) polarisation!
Is optics incompatible with QM? No!!
As you might have understood in the QFT lectures, a massless spin-1 field needs to couple to a conserved current, i.e., it is associated to a gauge symmetry. And the Ward identities (=the quantum version of classical symmetry) makes one of the polarisation unphysical. It is only when the symmetry is spontaneously broken that this polarisation remains physical

massless spin-1 field: 2 degrees of freedom massive spin-1 field: 3 degrees of freedom

Higgs Boson

Before EW symmetry breaking

- 4 massless gauge bosons for SU(2)x(1): 4 x 2 = 8 dofs
- Complex scalar doublet: 4 dofs

After EW symmetry breaking

- 1 massless gauge boson, photon: 2 dofs
- 3 massive gauge bosons, $W^\pm$ and Z: 3 x 3 = 9 dofs
- 1 real scalar: 1 dof

$$H = \left(\begin{array}{c} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{array} \right)$$

h(x) describes the Higgs boson
(the fluctuation above the VEV).
The other components of the Higgs doublet H become
the longitudinal polarisations of the $W^\pm$ and Z

EW Symmetry Breaking

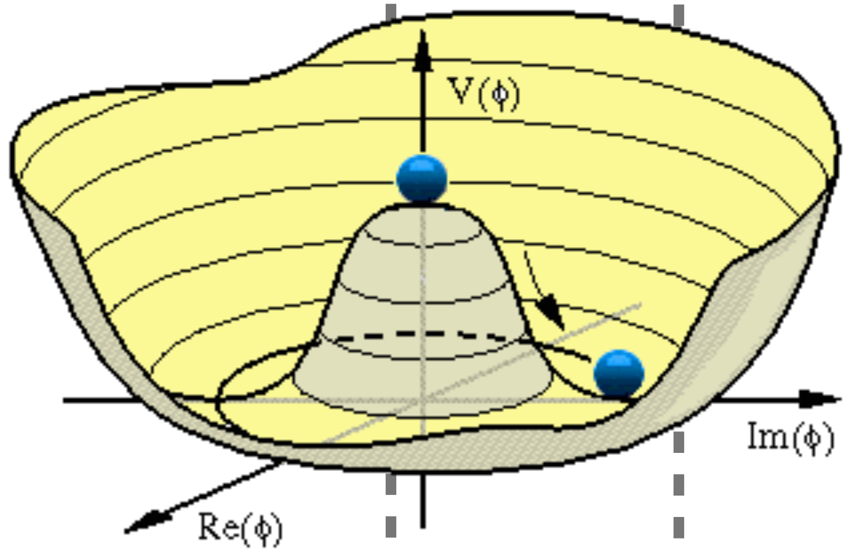
Carolina: "In the SM Lagrangian, what we have are the SU(2)L X U(1)Y terms? If so, how can we talk about the photon, Z and W bosons when calculating the amplitudes of the processes since they are the results of symmetry breaking and the bosons appearing in the terms would be a "combination" of them?"

Symmetry of the Lagrangian

$$SU(2)_L \times U(1)_Y$$

Higgs Doublet

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$



Symmetry of the Vacuum

$$U(1)_{e.m.}$$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV}$$

$$D_\mu H = \partial_\mu H - \frac{i}{2} \begin{pmatrix} g W_\mu^3 + g' B_\mu & \sqrt{2} g W_\mu^+ \\ \sqrt{2} g W_\mu^- & -g W_\mu^3 + g' B_\mu \end{pmatrix} H \quad \text{with} \quad \mathrm{W}_\mu^\pm = \frac{1}{\sqrt{2}} \left(\mathrm{W}_\mu^1 \mp \mathrm{W}_\mu^2 \right)$$

- Gauge boson spectrum
  - electrically charged bosons $\quad M_W^2 = \frac{1}{4} g^2 v^2$
  - electrically neutral bosons

$$Z_\mu = c W_\mu^3 - s B_\mu$$
$$\gamma_\mu = s W_\mu^3 + c B_\mu$$

Weak mixing angle

$$c = \frac{g}{\sqrt{g^2 + g'^2}} \qquad s = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2$$

$$M_\gamma = 0$$

SU(2)xU(1) vs SU(2)

Artem: "When deciding between SU(2) or SU(2)xU(1) for electroweak theory. You explained the first exprerimentally proven wrong. Because SU(2) has traceless generators, sum of the charges should be zero. So not only electron, and neutrino, but also another charged particle X exists. The question is: How they understood that at current collider energies it should be possible to create X (if it existed)? How could one at least roughly predict the mass of X? Or should it manifest itself as a virtual particle, then in what process?"

1. No additional "force": (Georgi, Glashow '72) $\Rrightarrow$ extra matter

$SU(2)$

$$[T^a, T^b] = i\epsilon^{abc}T^c$$

$$[T^+, T^-] = Q \qquad [Q, T^\pm] = +\pm T^\pm$$

$$T^\pm = \frac{1}{\sqrt{2}}(T^1 \pm iT^2)$$

$\mathrm{Tr}_{\mathrm{irrep}} T^3 = 0$ $\Rrightarrow$ extra matter $\begin{pmatrix} X_L \\ \nu_L \\ e_L \end{pmatrix}$ $\begin{pmatrix} X_R \\ \nu_R \\ e_R \end{pmatrix}$

$SU(1,1)$

$$[T^+, T^-] = -Q$$

$$[Q, T^\pm] = +\pm T^\pm$$

non-compact

unitary rep. has dim ∞

E_2

2D Euclidean group

$$[T^+, T^-] = 0$$

$$[Q, T^\pm] = +\pm T^\pm$$

only one unitary rep.

of finite dim. = trivial rep.

2. No additional "matter" (SM: Glashow '61, Weinberg '67, Salam '68): SU(2)xU(1)

$\Rrightarrow$ extra force

$Q = T^3?$

as Georgi-Glashow

$\Rrightarrow$ extra matter

$Q = Y?$

$Q(e_L) = Q(\nu_L)$

$Q = T^3 + Y!$

Gell-Mann '56, Nishijima-Nakano '53

Neutrino Masses

Erdenebulgan: "Should we write Dirac mass term for neutrino? For the Majorana mass term, there is isospin violation. Is it right? If so, how could we solve this problem?"

Dauke: "How does lepton number conservation apply in neutrino oscillations? If neutrinos turn out to be Majorana particles, how would one assign them a lepton number?"

☛ A mass term corresponds in the Lagrangian to an operator that is quadratic in the field. in QFT, this operator needs to be Lorentz invariant and invariant under the local/gauge symmetries. For spin-1/2 field, there are two types of mass terms:

- a Dirac mass:

$$m\bar{\psi}\psi = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

- a Majorana mass:

$$m\bar{\psi}_C\,\psi = m\left(\bar{\psi}_{L_C}\psi_L + \bar{\psi}_{R_C}\psi_R\right) \quad \text{where} \quad \psi_C = i\gamma^2\psi^* \quad \text{is the charge conjugated spinor}$$

Fermion Masses

SM is a chiral theory ($\neq$ QED that is vector-like)

$$\underset{\text{Y=1/2}}{m_e\bar{e}_L}\,\underset{\text{Y=-1}}{e_R} + h.c. \qquad \text{is not gauge invariant}$$

The SM Lagrangian cannot contain fermion mass term.
Fermion masses are emergent quantities
that originate from interactions with Higgs VEV

$$\mathcal{L} = y_e \underset{\text{Y=1/2}}{\begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix}} \cdot \underset{\text{Y=1/2}}{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}} \underset{\text{Y=-1}}{e_R} \overset{H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}}{=} \frac{y_e\, v}{\sqrt{2}} \left(\bar{e}_L e_R + \frac{1}{v}\, \bar{e}_L e_R \underset{\text{Higgs Boson}}{\boxed{h}} \right)$$

Higgs couplings proportional to the mass of particles

Neutrino Masses

The same construction doesn't work for neutrinos
since in the SM there are only Left Handed neutrinos

For a uncharged particle, it is possible to write a Majorana mass
another Lorentz-invariant quadratic term in the Lagrangian
(it involves the charge-conjugate spinor, see lecture #2)

$$\mathcal{L}_{\text{Majorana}} = m\bar{\psi}_C\,\psi = m\left(\bar{\psi}_{L_C}\psi_L + \bar{\psi}_{R_C}\psi_R\right)$$

can build such a term with LH field only

$$\mathcal{L} = \frac{y_\nu}{\Lambda}\underbrace{\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}_C}_{\text{mass}^{3/2}}\cdot\underbrace{\begin{pmatrix}H^+\\ H^0\end{pmatrix}}_{\text{mass}}\underbrace{\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}}_{\text{mass}^{3/2}}\cdot\underbrace{\begin{pmatrix}H^+\\ H^0\end{pmatrix}}_{\text{mass}} = \frac{y_\nu\, v^2}{\Lambda}\nu_{LC}\nu_L$$

Seesaw: $m_\nu = \dfrac{y_\nu v^2}{\Lambda}$ Order eV for y_ν~1 and Λ~10^{14}GeV

A Majorana mass violates lepton number ($\Delta L=2$) but it is fully gauge (and isospin) invariant

Dimensional analysis

Anna: “This question is about the formula Γ=Gf^2*m^5. Why is it a mass? And not for example a (lambda)^-5 or any combination of different quantities?” & “How were coupling constants calculated?”

$[G_N]=\text{mass}^{-1}\,L^3\,T^{-2}$, $[\hbar]=\text{mass}\,L^2\,T^{-1}$, $[c]=L\,T^{-1}$

In High Energy Physics, it is current to use a system of units for which $\hbar=1$ and $c=1$

$\boxed{\text{Mass} \sim \text{distance}^{-1} \sim \text{time}^{-1}}$

E	**T**	**L**
1eV	10^{-16}s	10^{-7}m
10^{-16}eV	1s	10^{9}m
10^{-7}eV	10^{-9}s	1m

Particle lifetime of a (decaying) particle: $[\tau]_m = -1$ Width: $[\Gamma = 1/\tau]_m = 1$

Cross-section (“area” of the target): $[\sigma]_m = -2$

$$\underbrace{\mathcal{L}}_{[\text{mass}]^4} = \underbrace{G_F}_{[\text{mass}]^{-2}} \underbrace{\psi^4}_{[\text{mass}]^{3/2\times 4}}$$

$$\Rightarrow \quad \Gamma \propto G_F^2 \underbrace{m^5}_{[\text{mass}]}$$

A priori, the coupling constants are input parameters that need to be taken from experimental measurements

Dimensionality of π

Dauke: "Would you mind elaborating on the dimensionality of pi?"

Anna: "How to explain the 1/16pi^2 coefficient in the cross-sections of the decays with loops?"

☛ In HEP natural units, we set c=ħ=1, such that [length]=[time]=[mass]$^{-1}$=[energy]$^{-1}$
But these fundamental constants are dimensionful. And it might be useful to keep track of the ħ-dimensions in addition to the mass dimension of any physical quantity

		M^n	$\hbar^n$
scalar field	ϕ	1	$1/2$
fermion field	ψ	$3/2$	$1/2$
vector field	A_μ	1	$1/2$
mass	m	1	0
gauge coupling	g	0	$-1/2$
quartic coupling	λ	0	-1
Yukawa coupling	y_f	0	$-1/2$

$$\mathcal{S} = \int d^4x \left(\mathcal{L}_0 + \hbar\mathcal{L}_1 + \hbar^2\mathcal{L}_2 + \ldots\right)$$

$[\mathcal{L}_0]_\hbar = 1$ $\quad$ $[\mathcal{L}_1]_\hbar = 0$ $\quad$ $[\mathcal{L}_2]_\hbar = -1$

$[\mathcal{L}_0]_M = 4$ $\quad$ $[\mathcal{L}_1]_M = 4$ $\quad$ $[\mathcal{L}_2]_M = 4$

example:
tree-level generated operator

$[\cdot]_\hbar = -1$ $\quad$ $[\cdot]_\hbar = 2$

$$\frac{1}{M^2} g_*^2 \left(\partial^\mu |H|^2\right)^2$$

example:
one-loop generated operator

$$\frac{1}{M^2}\frac{g^2}{16\pi^2} g'^2 |H|^2 B_{\mu\nu}B^{\mu\nu}$$

The factors of π are very often associated to loop factors which are counting the ħ-dimension
Remember the normalisation of the states in QFT: $d^4k/(2\pi)^4$

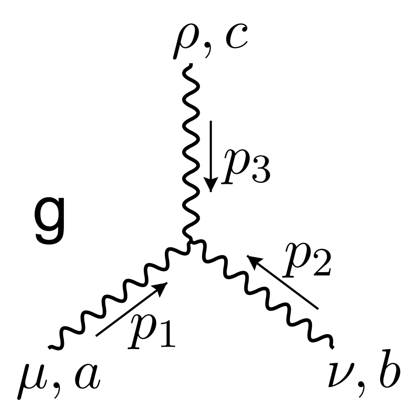
Symmetries and Diagrams

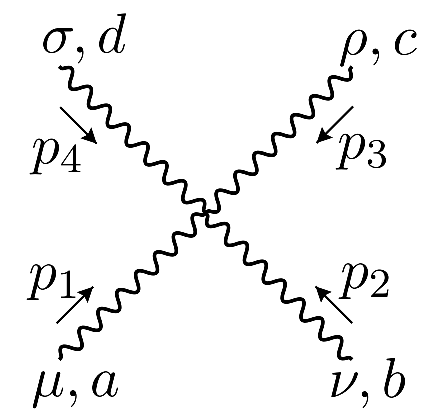
Anna: "How symmetries can influence the diagrams of the processes? I mean there can be different diagrams with the same cross-sections if an interaction does not distinguish between any particles."

☛ Gauge invariance is a dynamical principle, i.e., it dictates/predicts some interactions.

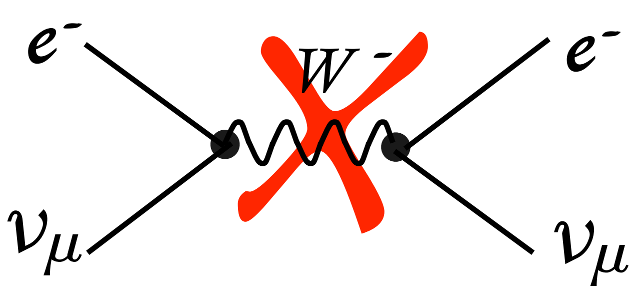
$$\mathcal{L} \propto F_{\mu\nu}F^{\mu\nu} \supset g\partial AAA + g^2 AAAA$$

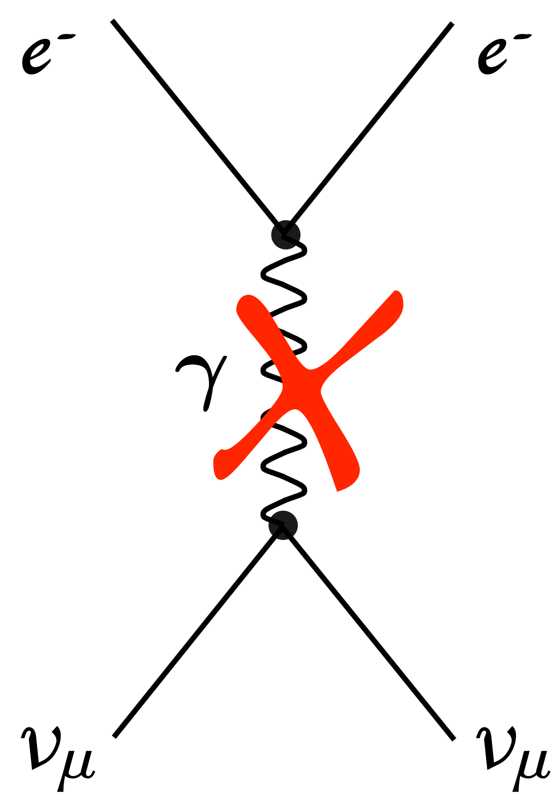
g

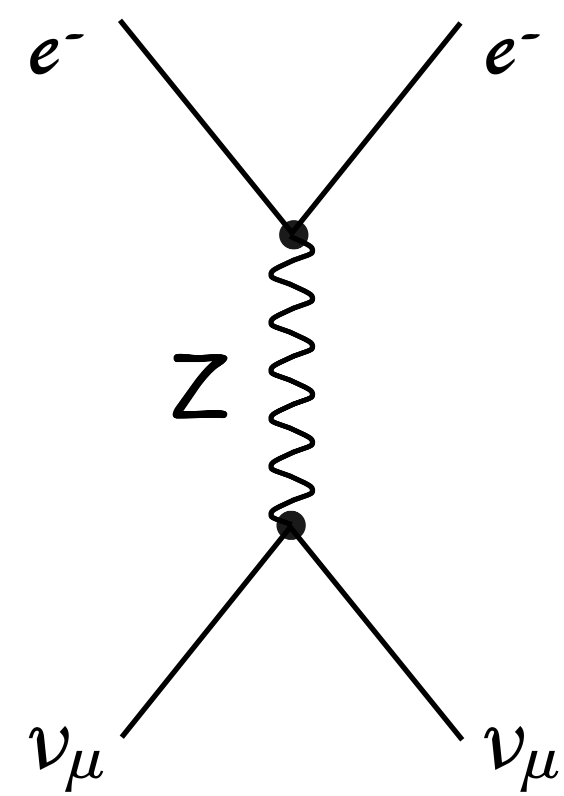




g^2







Symmetries and Diagrams

Joshua: "From gauge theory to Fermi theory" is this a similar process, but in Lagrangian and not Hamiltonian formalism, to taking the phonon-electron scatter and obtaining the electron-electron effective interaction? (i.e. is it equivalent to using a canonical transform on your Hamiltonian to obtain a fermion-fermion interaction from a boson-fermion one?"

$$\mathcal{L} = G_F \, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \dots$$

For the **muon**, the relevant mass scale is the muon mass m_μ=105MeV:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 10^{-19}\,\text{GeV} \quad \text{i.e.} \quad \tau_\mu \sim 10^{-6}\,\text{s}$$

For the **neutron**, the relevant mass scale is (m_n-m_p)≈1.29MeV:

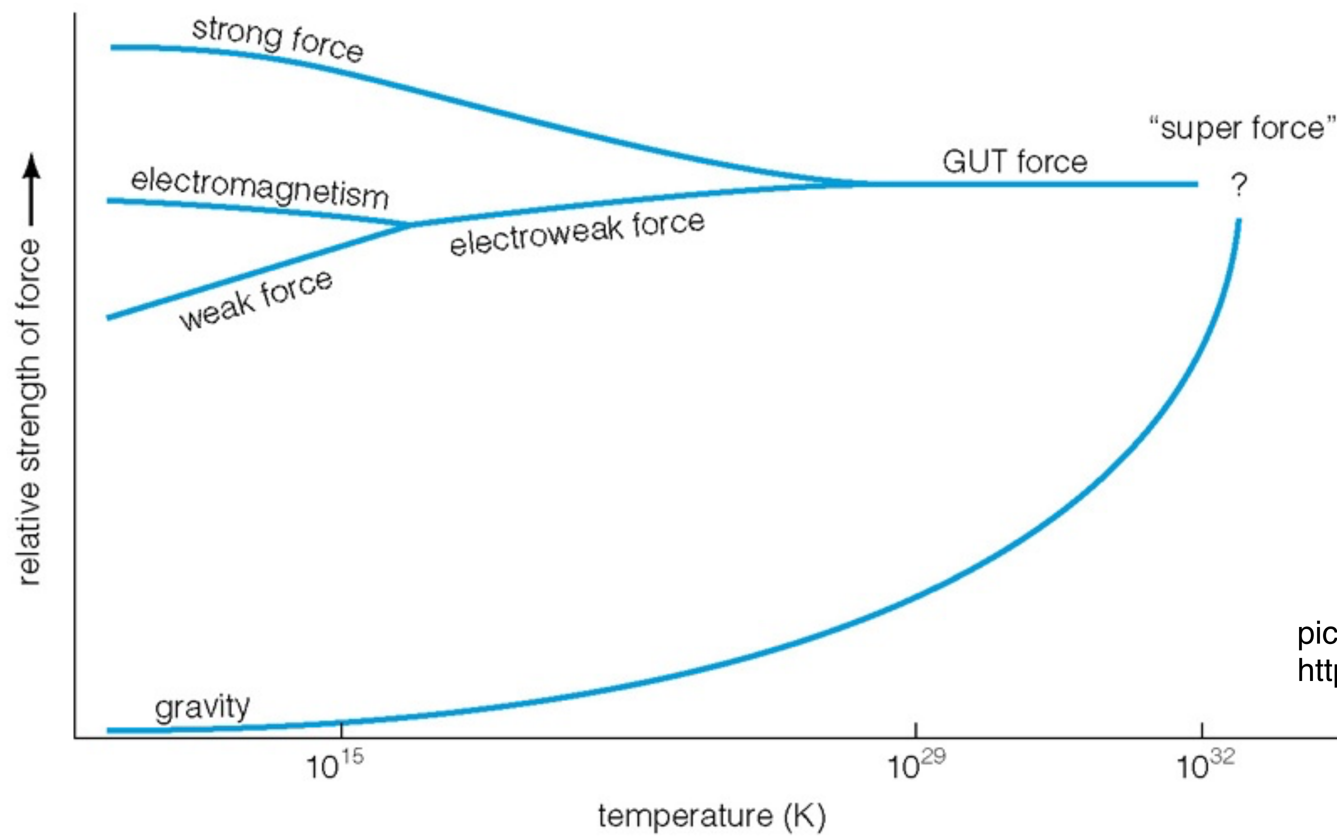
$$\Gamma_n = \mathcal{O}(1)\frac{G_F^2 \Delta m^5}{\pi^3} \sim 10^{-28}\,\text{GeV} \quad \text{i.e.} \quad \tau_\text{n} \sim 10^3\,\text{s}$$

The same interaction describes different physical processes

SM & Gravity

Richik: "Why can gravity not be added to the standard model even though gravity has been reduced to graviton particles?"

☛ It is actually possible to couple the SM to gravity and to quantise the graviton. The issue is that gravity is not renormalisable and to get ride of infinities in loop computation, one needs to add more and more counter-terms that are not present originally in the classical GR Lagrangian. At most gravity can be treated as an effective field theory and there are arguments that show that its UV completion is unlikely to be a quantum field theory but rather a theory of more complicated objects like matrices or strings. There is an important difference between gauge (spin-1) interactions and gravity: the gauge couplings of the former exhibit a logarithmic evolution with the energy of the process, while the strength of gravity grows like E^2. An important question is to figure out the scale of quantum gravity: is it M_{Planck}~10^{19}GeV? it could be lower down to few TeVs if there are (large or highly curved) extra dimensions. In that case, totally new phenomena could be observed at colliders… see the BSM lectures



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