Theory needs for future e^+e^- colliders

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- "Other" electroweak parameters ("input" parameters)
- Z pole & WW
- Higgs physics
- Calculational techniques

Need for theory input

- Comparison of EWPOs / HPOs with SM to **probe new physics**
  $\longrightarrow$ multi-loop corrections in full SM

- Extraction of EWPOs / HPOs (**pseudo-observables**) from **real observables**
  $\longrightarrow$ QED/QCD, MC tools $\longrightarrow$ talk by S. Jadach

- "Other" eletroweak parameters (**"input" parameters**)
  $\longrightarrow$ m_{t}, α_{s}, etc. extracted from other processes

Other electroweak precision parameters

Reviews: 1906.05379, 2012.11642

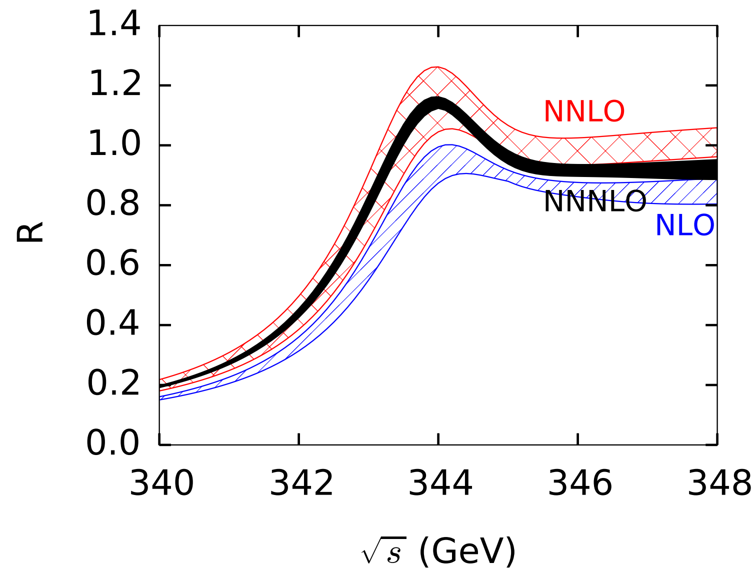
- M_Z, Γ_Z: From $\sigma(\sqrt{s})$ lineshape; δM_Z, $\delta\Gamma_Z \sim 0.1$ MeV at FCC-ee
  $\longrightarrow$ Main theory uncertainties: QED ISR $\longrightarrow$ talk by S. Jadach

- m_t: Current status $\delta m_t \sim 0.3$ GeV at LHC — PDG '20
  $\longrightarrow$ Additional theory uncertainties? — Butenschoen et al. '16; Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:

$$\delta m_t^{\overline{\text{MS}}} = [\quad]_{\text{exp}} \oplus [50\text{ MeV}]_{\text{QCD}} \oplus [10\text{ MeV}]_{\text{mass def.}} \oplus [70\text{ MeV}]_{\alpha_s} > 100\text{ MeV}$$



Beneke et al. '15

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From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:	**future:**	
$\delta m_t^{\overline{\rm MS}} = [\quad]_{\rm exp}$	$[20\text{ MeV}]_{\rm exp}$	
$\oplus\,[50\text{ MeV}]_{\rm QCD}$	$\oplus\,[30\text{ MeV}]_{\rm QCD}$	(h.o. resummation)
$\oplus\,[10\text{ MeV}]_{\text{mass def.}}$	$\oplus\,[10\text{ MeV}]_{\text{mass def.}}$	
$\oplus\,[70\text{ MeV}]_{\alpha_s}$	$\oplus\,[15\text{ MeV}]_{\alpha_s}$	$(\delta\alpha_s \lesssim 0.0002)$
> 100 MeV	$\lesssim 50$ MeV	

Other electroweak precision parameters

- $m_b,\ m_c$: From quarkonia spectra using Lattice QCD
  $\delta m_b^{\overline{\text{MS}}} \sim 30$ MeV, $\delta m_b^{\overline{\text{MS}}} \sim 25$ MeV LHC HXSWG '16

  $\longrightarrow$ estimated improvements $\delta m_b^{\overline{\text{MS}}} \sim 13$ MeV, $\delta m_b^{\overline{\text{MS}}} \sim 7$ MeV

  Lepage, Mackenzie, Peskin '14

- M_{H}: from kinematic constraint fits $HZ(\ell\ell)$, $H(b\bar{b})Z$
  $\longrightarrow$ $\delta M_{\text{H}} \sim$ 10...20 MeV
  $\longrightarrow$ theory errors subdominant

Strong coupling

- α_s: d'Enterria, Skands, et al. '15
  - Most precise determination using Lattice QCD:

    $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10

    $\alpha_s = 0.1185 \pm 0.0008$ ALPHA '17

    $\alpha_s = 0.1179 \pm 0.0015$ Takaura et al. '18

    $\alpha_s = 0.1172 \pm 0.0011$ Zafeiropoulos et al. '19

    $\longrightarrow$ Difficulty in evaluating systematics

  - e^+e^- event shapes and DIS: $\alpha_s \sim 0.114$

    Alekhin, Blümlein, Moch '12; Abbate et al. '11; Gehrmann et al. '13

    $\longrightarrow$ Subject to sizeable non-pertubative power corrections

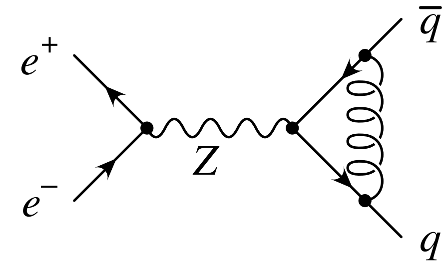
    $\longrightarrow$ Systematic uncertainties in power corrections?

  - Hadronic τ decays: $\alpha_s = 0.119 \pm 0.002$ PDG '18

    $\longrightarrow$ Non-perturbative uncertainties in OPE and from duality violation

    Pich '14; Boito et al. '15,18

- α_s: d'Enterria, Skands, et al. '15
  - Electroweak precision ($R_\ell = \Gamma_Z^{\text{had}}/\Gamma_Z^{\ell}$):
    $\alpha_s = 0.120 \pm 0.003$ PDG '18



$\longrightarrow$ No (negligible) non-perturbative QCD effects

FCC: $\delta R_\ell \sim 0.001$

$\Rightarrow \quad \delta\alpha_s < 0.0001$

Theory input: N^{3}LO EW corr. + leading N^{4}LO
to keep $\delta_{\text{th}} R_\ell \lesssim \delta_{\text{exp}} R_\ell$

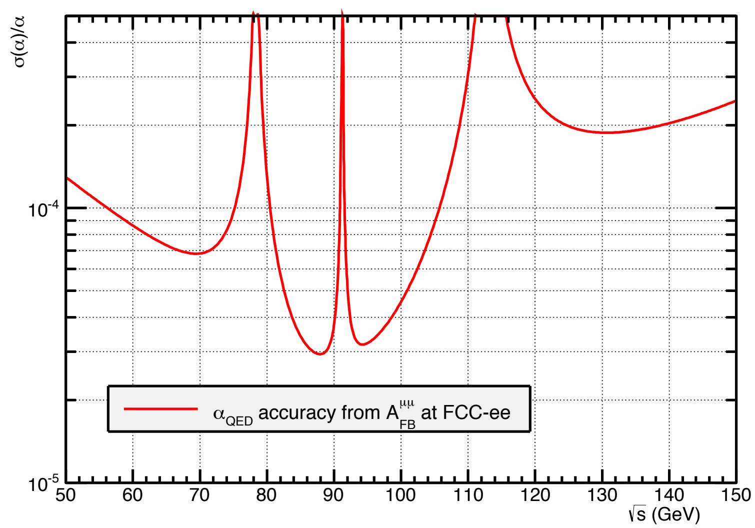
Caviat: R_ℓ could be affected by new physics

Shift of finestructure constant

- $\Delta\alpha_{\text{had}}$: Could be limiting factor
  a) From $e^+e^- \to$ had. using dispersion relation
  Current: $\delta(\Delta\alpha_{\text{had}}) \sim 10^{-4}$
  Improvement to $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$ likely
  b) Direct determination at FCC-ee from $e^+e^- \to \mu^+\mu^-$ off the Z peak
  (i.e. $A_{\text{FB}}^{\mu\mu}$ at $\sqrt{s} \sim 88$ GeV and $\sqrt{s} \sim 95$ GeV)
  $\to \delta_{\text{th}}(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$

Janot '15

Requires high-precision theory prediction for $e^+e^- \to \mu^+\mu^-$ including 2/3-loop corrections for γ-exchange and box contributions



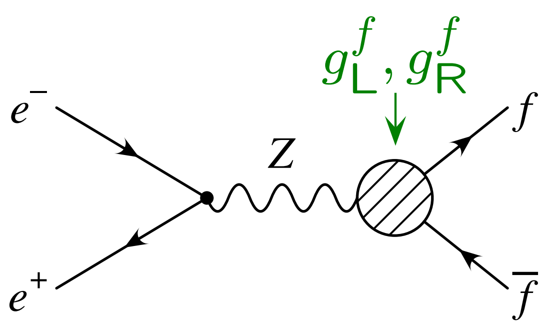
Z pole and WW

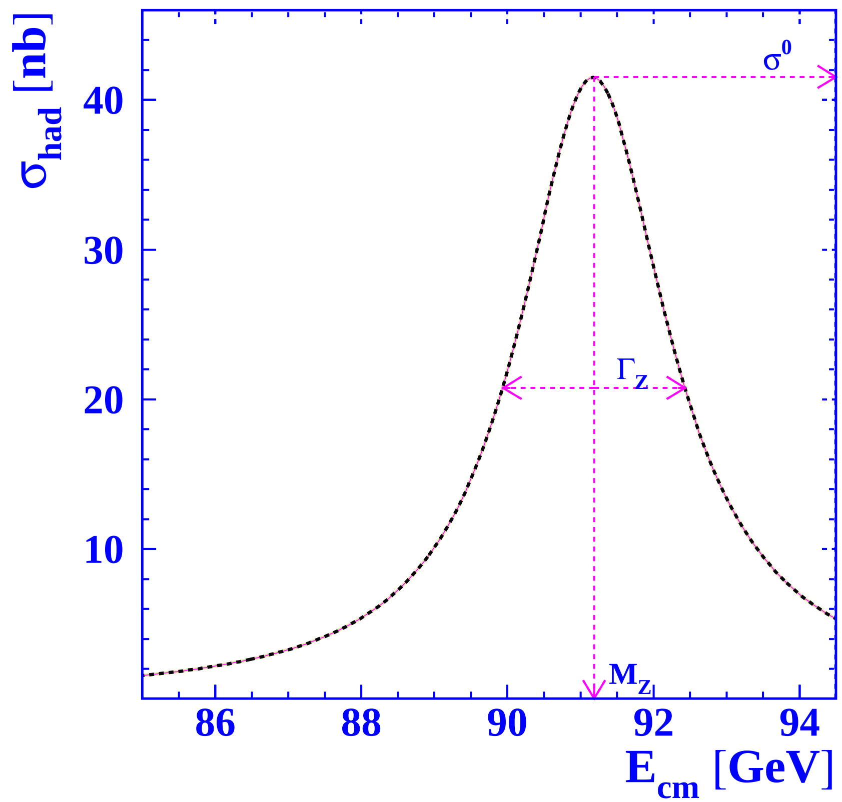
Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\text{CM}} \sim M_{\text{Z}}$:

- Mass M_{Z}
- Width $\Gamma_{\text{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\text{Z}}$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_{\text{Z}}^2)^2+M_{\text{Z}}^2\Gamma_{\text{Z}}^2} = \frac{12\pi}{M_{\text{Z}}^2} R_e R_f$

$$\Gamma_{ff} = C\left[(g_{\text{L}}^f)^2 + (g_{\text{R}}^f)^2\right]$$

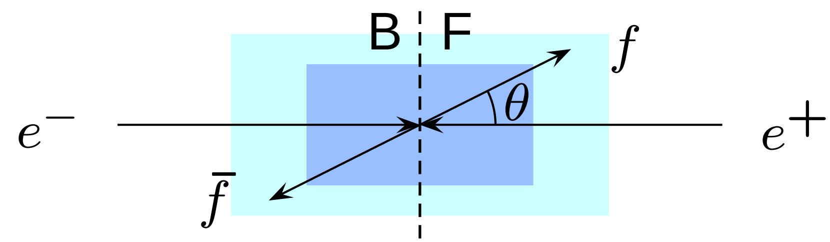




Z-pole asymmetries

Forward-backward asymmetry:

$$A_{\mathrm{FB}} \equiv \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f$$



$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\mathrm{eff}})}{1 + (1 - 4\sin^2\theta^f_{\mathrm{eff}})^2} \qquad \sin^2\theta^f_{\mathrm{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$

Left-right asymmetry:

With polarized e^- beam: $$A_{\mathrm{LR}} \equiv \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \mathcal{A}_e$$

Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$: $$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\rm ini}(s,s') \otimes \sigma_{\rm hard}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\rm hard} = \sigma_{\rm Z} + \sigma_\gamma + \sigma_{\gamma{\rm Z}} + \sigma_{\rm box}$$

- Z-pole contribution:

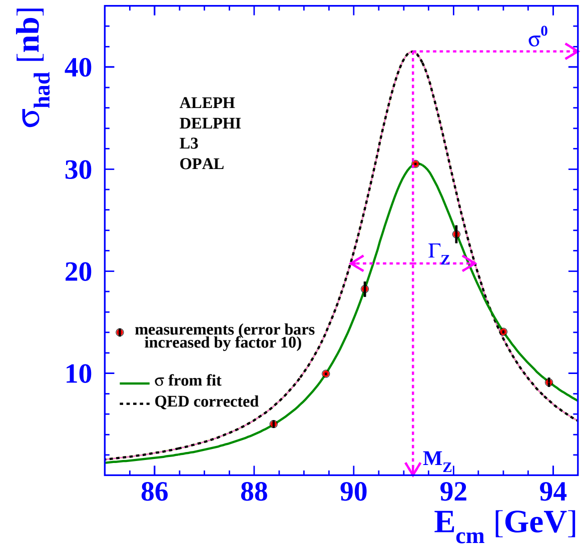
$$\sigma_{\rm Z} = \frac{R}{(s-\overline{M}_{\rm Z}^2)^2 + \overline{M}_{\rm Z}^2\Gamma_{\rm Z}^2} + \sigma_{\rm non-res}$$

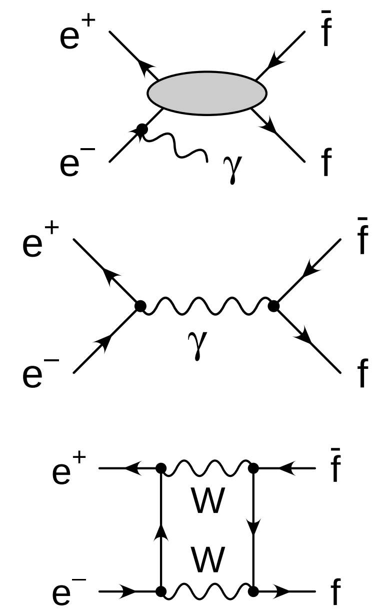
σ_γ, $\sigma_{\gamma{\rm Z}}$, $\sigma_{\rm box}$, $\sigma_{\rm non-res}$ known at NLO

$\longrightarrow$ need consistent pole expansion framework

$\longrightarrow$ leading NNLO may be needed for FCC-ee/CEPC

LEP EWWG '05



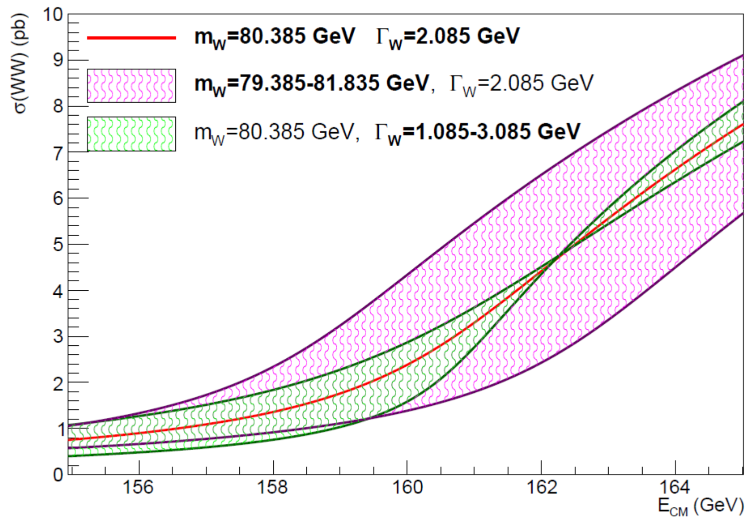


WW threshold

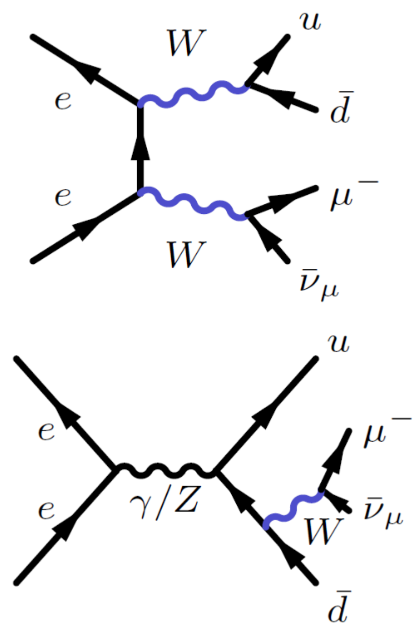
- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold
- a) Corrections near threshold enhanced by $1/\beta$ and $\ln\beta$

  $$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$

  b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$
  Denner, Dittmaier, Roth, Wieders '05
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$
  Beneke, Falgari, Schwinn, Signer, Zanderighi '07
  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{th}M_W \sim 3$ MeV
    Actis, Beneke, Falgari, Schwinn '08
  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{th}M_W \lesssim 0.6$ MeV



Comparison of EWPOs with theory

- To probe new physics, compare EWPOs with SM theory predictions

- Need to take theory error into account:

	Current exp.	Current th.	CEPC	FCC-ee
$M_{\mathrm{W}}{}^{*}$ [MeV]	15	4	1	1
Γ_Z [MeV]	2.3	0.4	0.5	0.1
$R_\ell = \Gamma_Z^{\mathrm{had}}/\Gamma_Z^{\ell}$ $[10^{-3}]$	25	5	2	1
$R_b = \Gamma_Z^{b}/\Gamma_Z^{\mathrm{had}}$ $[10^{-5}]$	66	10	4.3	6
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ $[10^{-5}]$	16	4.5	<1	0.5

* computed from G_μ

Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

- Full 2-loop results for M_{W}, Z-pole observables

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas '14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

- Approximate 3- and 4-loop results (enhance by Y_{t} and/or N_f)

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06
  Chen, Freitas '20

Theory and parametric uncertainties

	CEPC	perturb. error with 3-loop[†]	Param. error CEPC*	main source
M_W [MeV]	1	1	2.1	m_t, $\Delta\alpha$
Γ_Z [MeV]	0.5	0.15	0.15	m_t, α_s
R_b [10^{-5}]	4.3	5	< 1	
$\sin^2\theta^\ell_\text{eff}$ [10^{-5}]	<1	1.5	2	m_t, $\Delta\alpha$

[†] **Theory scenario:** $\mathcal{O}(\alpha\alpha_\text{s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_\text{s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_\text{s})$, leading 4-loop

(N_f^n = at least n closed fermion loops)

Parametric inputs:

***CEPC:** $\delta m_t = 600$ MeV, $\delta\alpha_\text{s} = 0.0002$, $\delta M_\text{Z} = 0.5$ MeV,
$\delta(\Delta\alpha) = 5 \times 10^{-5}$

Theory and parametric uncertainties

	CEPC	perturb. error with 3-loop[†]	Param. error CEPC*	main source
M_{W} [MeV]	1	1	0.6	$\Delta\alpha$
Γ_Z [MeV]	0.5	0.15	0.1	α_{s}
R_b $[10^{-5}]$	4.3	5	< 1	
$\sin^2\theta^{\ell}_{\text{eff}}$ $[10^{-5}]$	<1	1.5	1	$\Delta\alpha$

[†] **Theory scenario:** $\mathcal{O}(\alpha\alpha_{\text{s}}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_{\text{s}})$, $\mathcal{O}(N_f^2\alpha^2\alpha_{\text{s}})$, leading 4-loop

(N_f^n = at least n closed fermion loops)

Parametric inputs:

***FCC-ee:** $\delta m_t = 50$ MeV, $\delta\alpha_{\text{s}} = 0.0002$, $\delta M_{\text{Z}} = 0.5$ MeV,

$\delta(\Delta\alpha) = 3 \times 10^{-5}$

Higgs physics: SM predictions for Higgs decays

Reviews: 1404.0319, 1906.05379

hbb: [CEPC: 2.0%, FCC-ee: 0.8%]

- $\mathcal{O}(\alpha_s^4)$ QCD corrections — Baikov, Chetyrkin, Kühn '05
- $\mathcal{O}(\alpha)$ QED+EW — Dabelstein, Hollik '92; Kniehl '92
- leading $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$ for large m_t — Kwiatkowski, Steinhauser '94; Butenschoen, Fugel, Kniehl '07
  $\longrightarrow$ Use for error estimate

Current theory error: $\Delta_{th} < 0.4\%$

With full 2-loop: $\Delta_{th} \sim 0.2\%$

Parametric error:

$$\left.\begin{array}{l} \delta m_b = 0.030 \text{ GeV} \\ \delta\alpha_s = 0.001 \end{array}\right\} \to \Delta_{par} \approx 1.4\%$$

$$\left.\begin{array}{l} \delta m_b = 0.013 \text{ GeV} \\ \delta\alpha_s = 0.0002 \end{array}\right\} \to \Delta_{par} \approx 0.6\%$$

SM predictions for Higgs decays

h$\tau\tau$: [CEPC: 2.4%, FCC-ee: 1.1%]

With full 2-loop (no QCD): $\Delta_{\mathrm{th}} < 0.1\%$

Parametric error negligible

hWW*/hZZ*: [CEPC: 2.2%, FCC-ee: 0.4%]

- complete $\mathcal{O}(\alpha) + \mathcal{O}(\alpha_s)$ for $h \to 4f$ — Bredenstein, Denner, Dittmaier, Weber '06
- leading $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha\alpha_s^2)$ for large m_t — Djouadi, Gambino, Kniehl '97; Kniehl, Spira '95; Kniehl, Steinhauser '95; Kniehl, Veretin '12

  $\longrightarrow$ Small (0.2%) effect

Theory error: $\Delta_{\mathrm{th,EW}} < 0.3\%$, $\Delta_{\mathrm{th,QCD}} < 0.5\%$

With NNLO final-state QCD corrections: $\Delta_{\mathrm{th,QCD}} < 0.1\%$

Parametric error:

$\delta M_H \sim 10$ MeV $\quad \longrightarrow \Delta_{\mathrm{par}} \approx 0.1\%$

Note: Distributions affected by corrections $\longrightarrow$ implementation into MC tools

SM predictions for Higgs decays

hgg: [CEPC: 2.4%, FCC-ee: 1.6%]

- $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ (in large m_t-limit) QCD corrections — Baikov, Chetyrkin '06; Schreck, Steinhauser '07
- $\mathcal{O}(\alpha)$ EW — Aglietti, Bonciani, Degrassi, Vicini '04; Degrassi, Maltoni '04

Theory error (dominated by QCD): $\Delta_{th} \approx 3\%$
With $\mathcal{O}(\alpha_s^4)$ in large m_t-limit (4-loop massless QCD diags.): $\Delta_{th} \approx 1\%$

Parametric error: $\delta\alpha_s = 0.001 \rightarrow \Delta_{par} \approx 3\%$
$\delta\alpha_s = 0.0001 \rightarrow \Delta_{par} \approx 0.3\%$

h$\gamma\gamma$: [CEPC: 3.2%, FCC-ee: 3.0%]

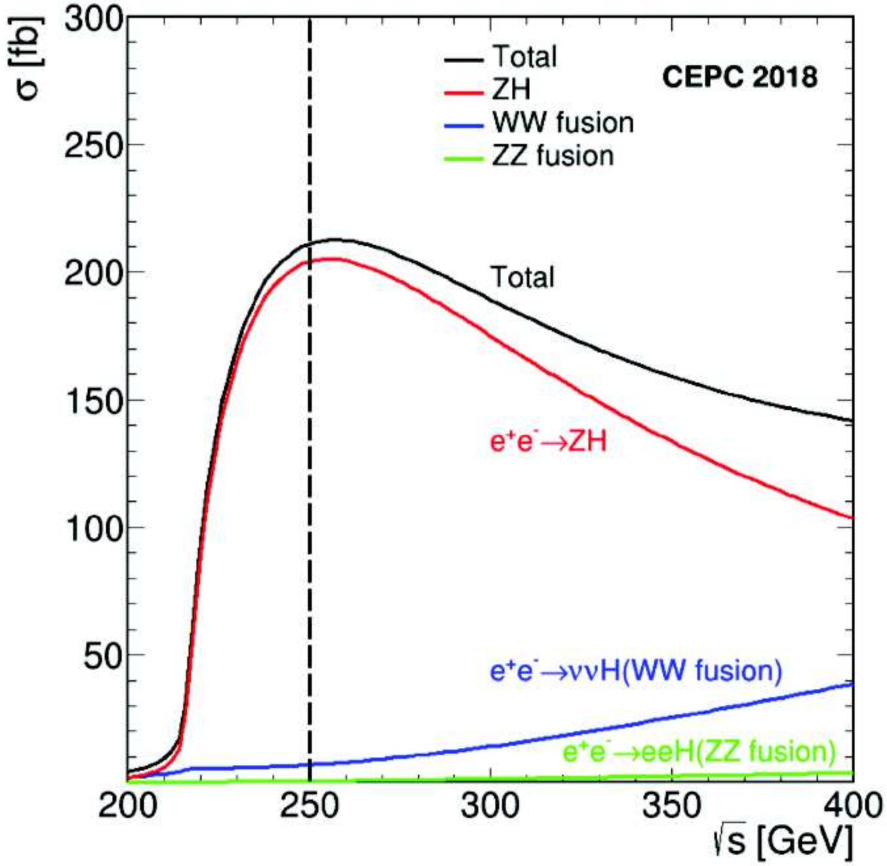
- $\mathcal{O}(\alpha_s^2)$ QCD corrections — Zheng, Wu '90; Djouadi, Spira, v.d.Bij, Zerwas '91; Dawson, Kauffman '93; Maierhöfer, Marquard '12
- $\mathcal{O}(\alpha)$ EW — Aglietti, Bonciani, Degrassi, Vicini '04; Degrassi, Maltoni '04; Actis, Passarino, Sturm, Uccirati '08

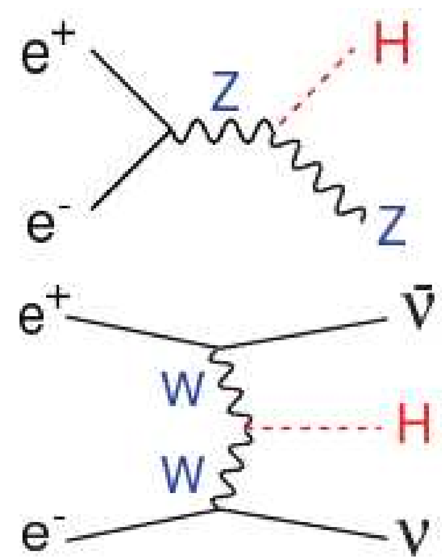
Theory error: $\Delta_{th} < 1\%$

Parametric error negligible

SM predictions for Higgs production

- hZ production: dominant at $\sqrt{s} \sim 240$ GeV
- WW fusion: sub-dominant but useful for constraining h width Han, Liu, Sayre '13

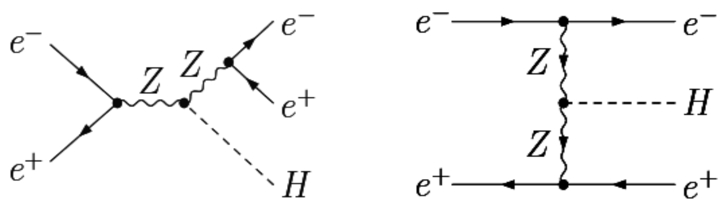




SM predictions for Higgs production

hZ production: [CEPC: 0.5%, FCC-ee: 0.3%]

- $\mathcal{O}(\alpha)$ corr. to hZ production and Z decay — Kniehl '92; Denner, Küblbeck, Mertig, Böhm '92; Consoli, Lo Presti, Maiani '83; Jegerlehner '86; Akhundov, Bardin, Riemann '86
- Technology for $\mathcal{O}(\alpha)$ with off-shell Z-boson available — Boudjema et al. '04; Denner, Dittmaier, Roth, Weber '03



- Can be combined with h.o. ISR QED radiation — Greco et al. '17
- $\mathcal{O}(\alpha\alpha_s)$ corrections — Gong et al. '16; Chen, Feng, Jia, Sang '18

Theory error: $\Delta_{th} \sim O(1\%)$

With full 2-loop corrections for $ee \to HZ$: $\Delta_{th} \lesssim O(0.3\%)$

Parametric error: negligible if $\delta M_H < 100$ MeV

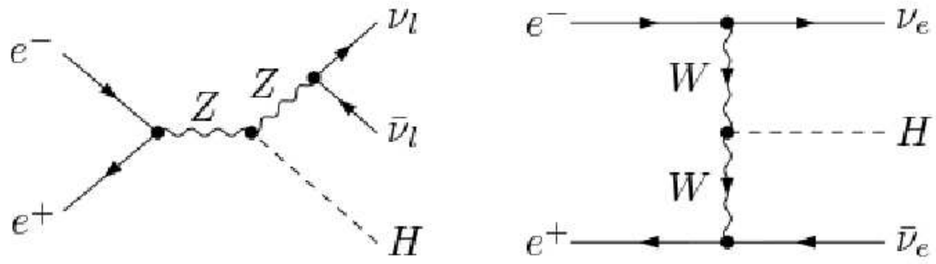
SM predictions for Higgs production

WW fusion:

- $\mathcal{O}(\alpha)$ corrections with h.o. ISR — Belanger et al. '02; Denner, Dittmaier, Roth, Weber '03

Theory error: $\Delta_{\text{th}} \sim O(1\%)$?

Parametric error: negligible

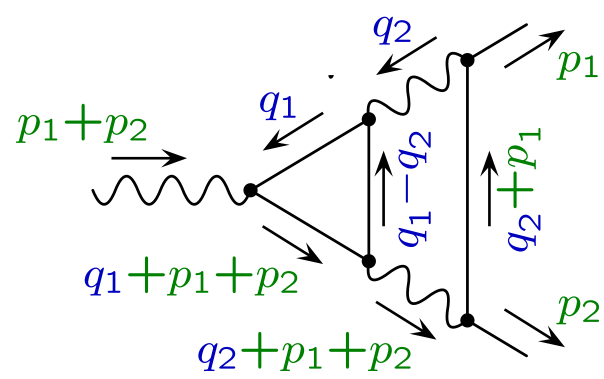


Full $\mathcal{O}(\alpha^2)$ calculation for 2$\rightarrow$3 process is very challenging

$\longrightarrow$ Contributions with closed fermion loops maybe feasible

Calculational techniques

Experimental precision requires inclusion of **radiative corrections** in theory (1-loop, 2-loop, and partial 3-loop)



Integrals over loop momenta:

$$\int d^4q_1 d^4q_2 \; f(q_1,\, q_2,\, p_1,\, p_2,\, \ldots,\, m_1,\, m_2,\, \ldots)$$

Computer algebra tools:

- Generation of diagrams, $\mathcal{O}(100) - \mathcal{O}(10000)$
- Lorentz and Dirac algebra
- Integral simplification (and expansion)

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods are more general, but computing intensive
- Special numerical techniques can balance precision and evaluation time

Analytic calculations

- Mostly used for diagrams with few mass scales
- Reduce to **master integrals** with integration-by-parts and other identities

  Chetyrkin, Tkachov '81; Gehrmann, Remiddi '00; Laporta '00; ...

  Public programs:

`Reduze`	von Manteuffel, Studerus '12
`FIRE`	Smirnov '13,14
`LiteRed`	Lee '13
`KIRA`	Maierhoefer, Usovitsch, Uwer '17

  $\longrightarrow$ Large need for computing time and memory

- Evaluate master integrals with differential equations or Mellin-Barnes rep.

  Kotikov '91; Remiddi '97; Smirnov '00,01; Henn '13; ...

  $\longrightarrow$ Result in terms of Goncharov polylogs / multiple polylogs

  $\longrightarrow$ Some problems need iterated elliptic integrals / elliptic multiple polylogs

  Broedel, Duhr, Dulat, Trancredi '17,18

  Ablinger er al. '17

  $\longrightarrow$ Even more classes of functions needed in future?

Asymptotic expansions

- Exploit large mass ratios,
  e. g. $M_Z^2/m_t^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

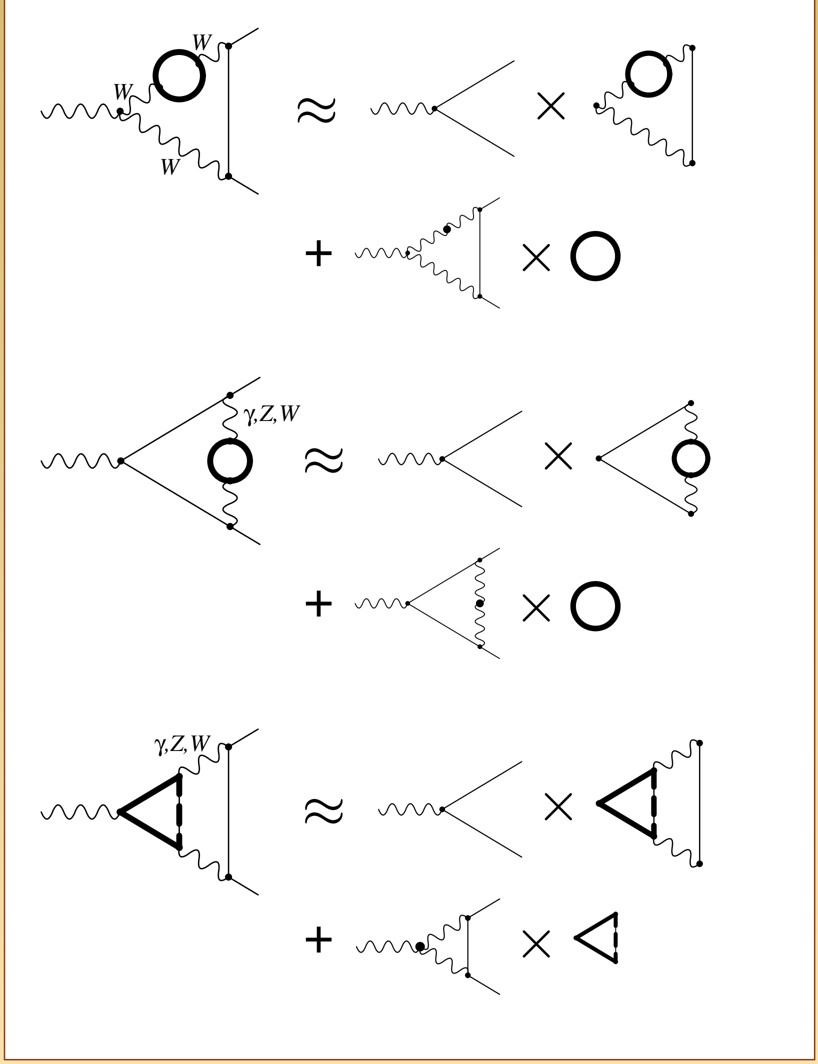
$\longrightarrow$ Used in some 2/3-scale problems

$\longrightarrow$ Public programs:

exp Harlander, Seidensticker, Steinhauser '97
asy Pak, Smirnov '10

$\longrightarrow$ Possible limitations:

- Difficult coefficient integrals
- bad convergence



Numerical integration

Two general approches:

⟶ Automated treatment of UV/IR divergencies

⟶ No restriction on number of loops or legs

- **Sector decomposition:**

  Public programs:

`SecDec`	Carter, Heinrich '10; Borowka et al. '12,15,17
`FIESTA`	Smirnov, Tentyukov '08; Smirnov '13,15

- **Mellin-Barnes representations:**

  Public programs:

`MB/MBresolve`	Czakon '06; Smirnov, Smirnov '09
`AMBRE/MBnumerics`	Gluza, Kajda, Riemann '07 Dubovyk, Gluza, Riemann '15 Usovitsch, Dubovyk, Riemann '18

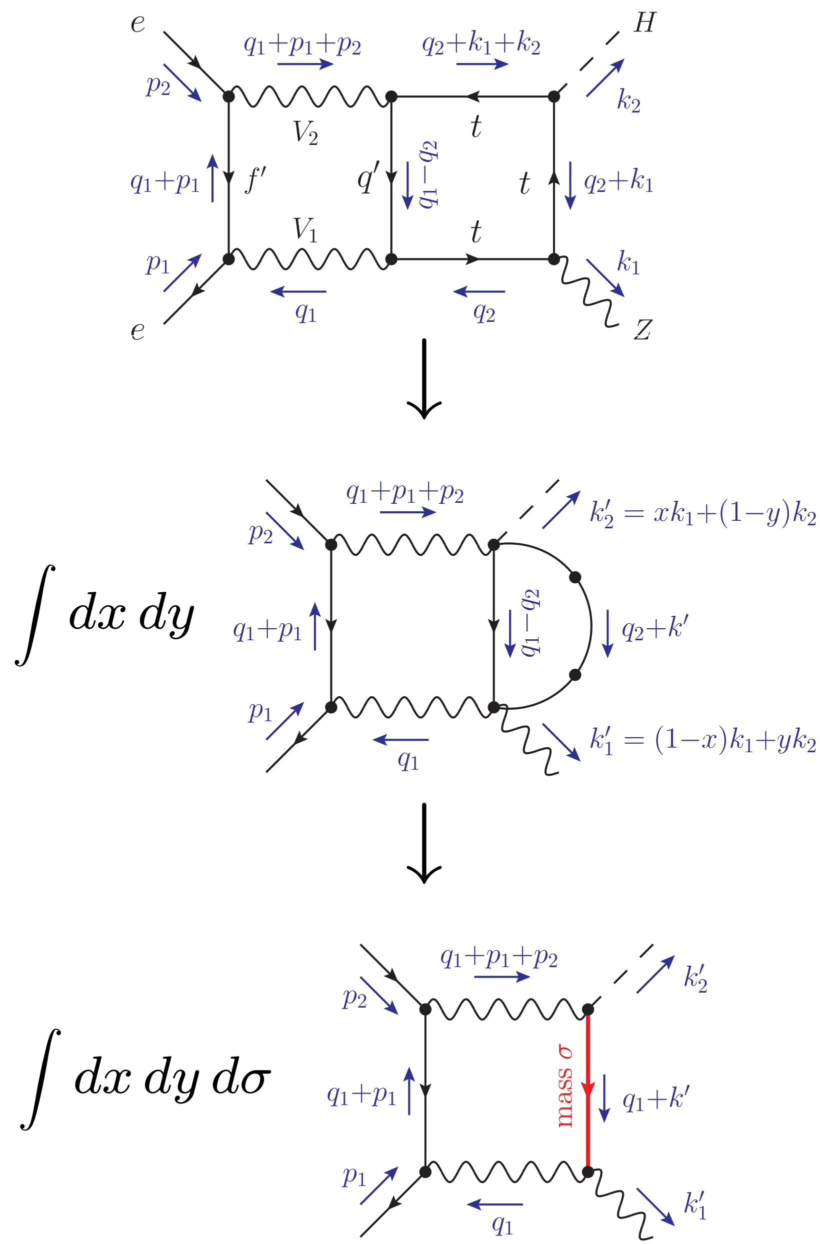
- Diagrams with internal thresholds can cause numerical instabilities
- Specialized techniques (for some type of diagrams) often improve computing time, robustness, precision (but not automated)

Specialized numerical techniques

Example: HZ double boxes

Song, Freitas '21

- Introduce Feynman parameters and disp. rel.
- Expressions for second loop from, e.g., `LoopTools`
  Hahn, Perez-Victoria '98
- 3-dim. numerical integral with adaptive Gaussian integration
- $\mathcal{O}(0.1\%)$ precision in $\mathcal{O}$(min.) on laptop



Summary

For Higgs and WW physics:

- Full $\mathcal{O}(\alpha^2)$ for $2 \to 2$ processes
- $\mathcal{O}(\alpha_s^4)$ QCD corrections
- Matching to Monte-Carlo tools
- Also need $O(\alpha)$ (or better?) corrections for backgrounds: $e^+e^-b\bar{b}$, $\nu\bar{\nu}b\bar{b}$, etc.

  $\longrightarrow$ Technology exists, but work needed   Denner, Dittmaier, Roth, Wieders '05

For Z pole:

- 3-loop EW and mixed EW-QCD corrections for Zff vertices
- Leading 4-loop effects
- Initial-final QED effects / mergering multi-loop and Monte-Carlo

Input parameters:

- Direct determination of α_s, m_t, $\alpha(M_Z)$ at e^+e^- colliders is important
- Perturbative and non-perturbative theory uncertainties need improvement
- Lower-energy experiments can provide additional input (BELLE II, BES, ...)

Backup slides

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\mathrm{ini}}(s,s') \otimes \sigma_{\mathrm{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\mathrm{hard}} = \sigma_{\mathrm{Z}} + \sigma_\gamma + \sigma_{\gamma\mathrm{Z}} + \sigma_{\mathrm{box}}$$

- Z-pole contribution:

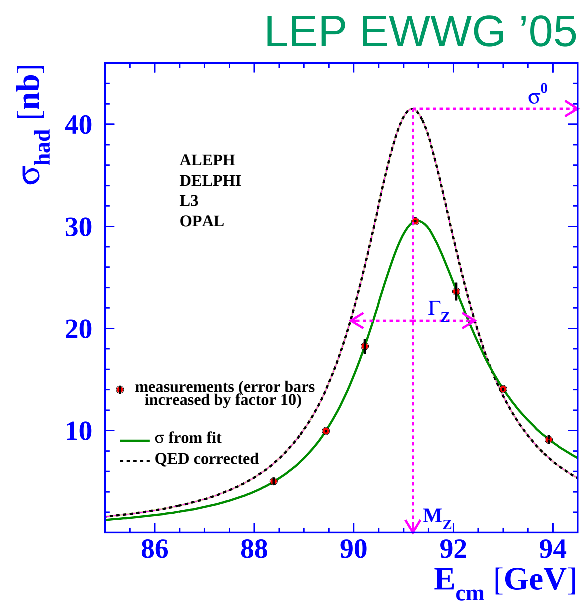
$$\sigma_{\mathrm{Z}} = \frac{R}{(s-\overline{M}_{\mathrm{Z}}^2)^2 + \overline{M}_{\mathrm{Z}}^2\overline{\Gamma}_{\mathrm{Z}}^2} + \sigma_{\mathrm{non-res}}$$

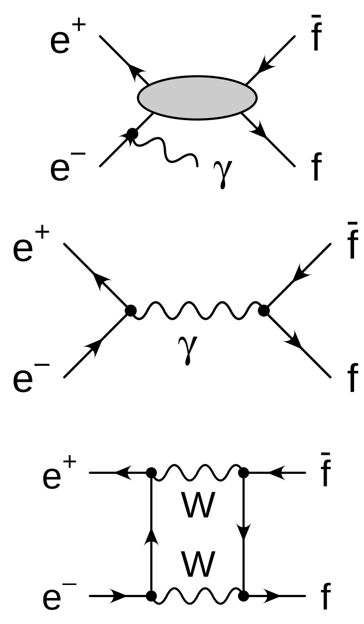
- In experimental analyses:

$$\sigma \sim \frac{1}{(s-M_{\mathrm{Z}}^2)^2 + s^2\Gamma_{\mathrm{Z}}^2/M_{\mathrm{Z}}^2}$$

$$\overline{M}_{\mathrm{Z}} = M_{\mathrm{Z}} \Big/ \sqrt{1+\Gamma_{\mathrm{Z}}^2/M_{\mathrm{Z}}^2} \approx M_{\mathrm{Z}} - 34\ \mathrm{MeV}$$

$$\overline{\Gamma}_{\mathrm{Z}} = \Gamma_{\mathrm{Z}} \Big/ \sqrt{1+\Gamma_{\mathrm{Z}}^2/M_{\mathrm{Z}}^2} \approx \Gamma_{\mathrm{Z}} - 0.9\ \mathrm{MeV}$$





"Hard" matrix element

Consistent (gauge-invariant) theory setup:

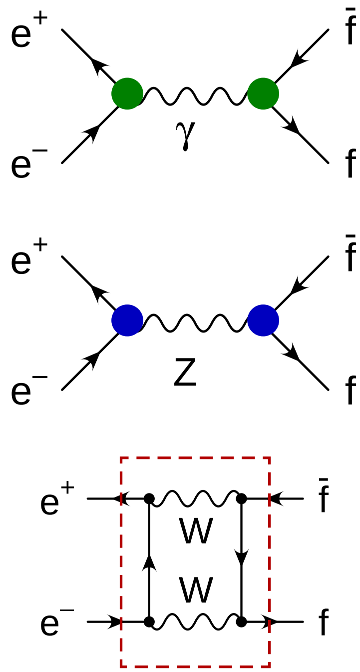
Expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_Z^2 - iM_Z\Gamma_Z$:

$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)T + \dots$$

$$R = g_Z^e(s_0)g_Z^f(s_0)$$

$$S = \left[\frac{1}{M_Z^2} g_\gamma^e g_\gamma^f + g_Z^e g_Z^{f\,\prime} + g_Z^{e\,\prime} g_Z^f + S_{\text{box}}\right]_{s=s_0}$$

$g_V^f(s)$: effective $Vf\bar{f}$ couplings

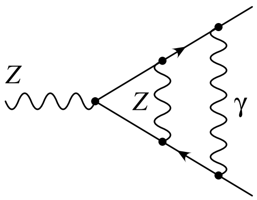


At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

Z decay

Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi} \left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2 \right) \frac{1}{1 + \text{Re}\,\Sigma'_Z} \right]_{s=\overline{M}_Z^2}$$



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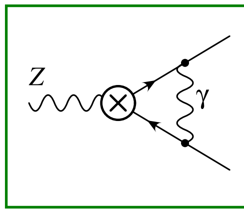
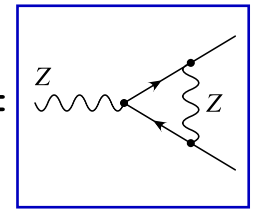
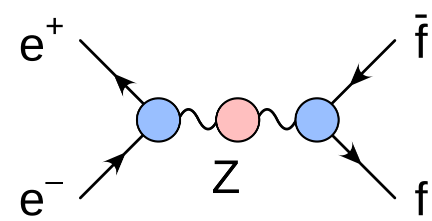


+ finite, with $\otimes =$


$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;
known to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$

Kataev '92
Chetyrkin, Kühn, Kwiatkowski '96
Baikov, Chetyrkin, Kühn, Rittinger '12

g_V^f, g_A^f, Σ'_Z: Electroweak corrections



Z decay

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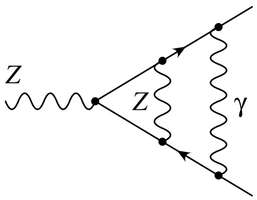

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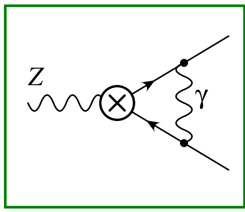
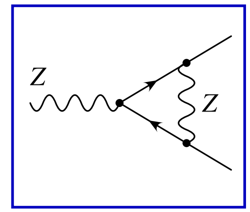
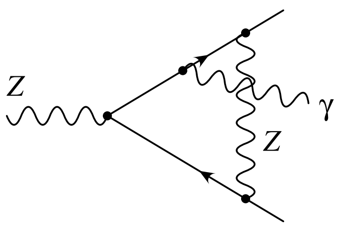


$+$ finite, with $\otimes =$


Additional non-factorizable contributions, e.g.



$\longrightarrow$ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn ’96
Harlander, Seidensticker, Steinhauser ’98

$\longrightarrow$ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty on Γ_Z, σ_Z, maybe larger for A_b

$\longrightarrow$ How to account for in MC simulations?

Z-pole asymmetries

Blondel scheme: (if e^- and e^+ polarization available) Blondel '88

Four independent measurements for $P_{e^+}/P_{e^-} = ++, +-, -+, --$

$$A_{\mathrm{LR}} = \sqrt{\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}$$

Note: No need to know $|P_{e^\pm}|$!

Main systematic uncertainties:

- Difference of $|P|$ for $P > 0$ and $P < 0$
- Difference of $\mathcal{L}$ for $P > 0$ and $P < 0$

$\delta A_{\mathrm{LR}} \approx 10^{-4}$ $\Rightarrow$ $\delta \sin^2 \theta^{\ell}_{\mathrm{eff}} \approx 1.3 \times 10^{-5}$ Mönig, Hawkings '99

Theory calculations: Uncertainties

	Experiment	Theory error	Main source
M_W	80.379 ± 0.012 MeV	4 MeV	α^3, $\alpha^2\alpha_\mathrm{s}$
Γ_Z	2495.2 ± 2.3 MeV	0.4 MeV	α^3, $\alpha^2\alpha_\mathrm{s}$, $\alpha\alpha_\mathrm{s}^2$
R_ℓ	20.767 ± 0.025	0.005	α^3, $\alpha^2\alpha_\mathrm{s}$
R_b	0.21629 ± 0.00066	0.0001	α^3, $\alpha^2\alpha_\mathrm{s}$
$\sin^2\theta^\ell_\mathrm{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_\mathrm{s}$

- Theory error estimate is not well defined, ideally $\Delta_\mathrm{th} \ll \Delta_\mathrm{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors:

$$\mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z \alpha^2 \sim 0.1 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi} \alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.5$ MeV