Head-tail effect due to lattice nonlinearities in storage rings

Kazuhito Ohmi and Yukinori Kobayashi
KEK, High Energy Accelerator Research Organization Oho, Tsukuba, Ibaraki 305-0801, Japan
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The head-tail effect due to the transverse wake force has been studied in relation to the chromaticity by many people. We discuss another type of head-tail effect produced by lattice nonlinearities, especially by amplitude-dependent tune shifts. We show that damping of the transverse coherent motion due to the nonlinear smear (Landau damping) is strongly affected by this head-tail effect. We discuss a two-particle model and show results of multiparticle tracking including lattice nonlinearities and transverse wake force. An experiment that supports this effect has been already performed at KEK Photon Factory, and coincides with the results presented here. [S1063-651X(99)09401-5]

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I. INTRODUCTION

The wake force induced by vacuum chambers affects dynamics of a beam in storage rings. We present a transverse wake effect which couples to the lattice nonlinearities. The wake force is assumed to be linear in the dipole moment of the beam and is conventionally expressed by $W_1(z)$. The chromaticity, which is an energy-dependent tune shift, induces the ordinary head-tail effect. We discuss an another type of head-tail effect that is induced by the amplitude-dependent tune shift. Phenomenologically, we will observe this effect as a kind of interference between the Landau damping and head-tail damping; the damping of the dipole moment strongly depends on the sign of the amplitude-dependent tune shift. An asymmetry of the head-tail instability due to a sign of the amplitude-dependent tune shift has been studied [1–3] using a dispersion relation. We discuss this damping phenomenon using a two-particle model and multiparticle tracking. Our discussion is devoted to the study in electron storage rings, but a similar effect may occur in proton and other storage rings.

This work was motivated by experiments [4–7] in which the damping of the kicked beam has been observed. The most typical experiment was performed at the KEK Photon Factory (PF) using a fast kicker and a turn by turn monitor [4]. In the experiment, the amplitude of a bunch that had been kicked transversely was measured turn by turn for various strengths of the octupole magnets. The bunch coherent motion damps into the equilibrium orbit in a characteristic time. We know of three damping mechanisms: i.e., the Landau (nonlinear smear), the head-tail, and the radiation damping. The kicked bunch showed quite different behaviors as one changes the polarity of the octupole magnets in a parameter region where the Landau damping dominates. The results [5] are summarized as follows: (1) For a lower beam current (0.5 mA, $N_e = 1.95\times10^9$/bunch), only the Landau damping behavior was observed for both polarity of the octupole magnets. (2) For a higher beam current (a) Landau damping was observed for the negative polarity; (b) head-tail damping was observed for the positive polarity. (3) This phenomenon was more pronounced when the beam current was higher.

The damping rate seems to be far from the simple sum of the head-tail and Landau damping rates. The radiation damping does not seem to be important, since its characteristic time is much longer.

For simplicity, the motion of a beam that received only a horizontal (x) kick is studied here. The concrete study was performed for the PF ring whose parameters are shown in Table I. The current dependent tune shift is the measured value ($d\nu_{\beta,I}/dI \simeq 0.1A^{-1}$), and the wake force was estimated from the tune shift [8] as $W_1(z) = 1.0\times10^{17} z$ V/C/m. Here we assume that the transverse wake force contributes to the tune shift dominantly, and increases linearly in the longitudinal distance [$z(m)$].

The amplitude dependent tune shift is defined by

$$\omega_\beta(J) = \omega_{\beta0}(1 + aJ), \tag{1.1}$$

where J is a half of the Courant Snyder invariant: i.e., $J = (\gamma x^2 + 2\alpha x x' + \beta x'^2)/2$, and ω_β is the betatron frequency of the beam particles. The amplitude dependent tune shift causes a tune spread, because beam particles have a distribution in horizontal amplitude, which is characterized by the beam size. We would like to emphasize that not only the tune spread but also the tune shift [the sign of a in Eq. (1.1)] plays an important role in this head-tail effect.

We discuss a two particle model in Sec. II. The model is helpful to understand this phenomenon qualitatively. Multiparticle tracking is shown in Sec. III, and its results are compared with experimental results.

TABLE I. Parameters of the PF ring used for the calculation.

Circumference (C)	187 m
Beam energy (E)	2.5 GeV
Current (I)	5 mA
Number of electrons (N_e)	1.9×10^{10}
Transverse tunes (ν_x, ν_y)	8.4, 3.305
Synchrotron tune (ν_s)	0.023
Natural bunch length (σ_z)	15 mm
Energy spread (σ_δ)	0.000 73
Emittances ($\varepsilon_x, \varepsilon_y$)	130, 1.5 nm

II. TWO-PARTICLE MODEL

We first consider two particles under a constant transverse wake. We take into account only the dipole mode of the beam, but not the longitudinal motion in this two-particle model. The head-tail interactions and synchrotron motion are essential, though they are not visible in the model. We discuss the interactions between the two particles with large and small amplitude. The horizontal positions of the two particles are represented by x_1 and x_2. The wake force which particles feel is a function of their barycenter. The equation of motion is expressed as

$$x''_{1,2}+\left(\frac{\omega_{\beta 0}(1+aJ_{1,2})}{c}\right)^2 x_{1,2}=\mathcal{W}\,\frac{x_1+x_2}{2}, \qquad (2.1)$$

where $\mathcal{W}=N_e e^2 W_1(2\sigma_z)/4EC$ (see Table I). We assume that it is possible to separate the solution into a slow changing factor and an exponential oscillation factor: i.e., $x_{1,2}(s)=X_{1,2}(s)\exp[-i\omega_{\beta,0}\int(1+aJ_{1,2})ds/c]$. We obtain a set of differential equations

$$X'_{1,2}=i\frac{\mathcal{W}}{4}\beta\left[X_{1,2}+\exp\left\{\pm ia\omega_{\beta_0}\int(J_1-J_2)ds\right\}X_{2,1}\right]. \qquad (2.2)$$

The solution for $a=0$ ($\omega_\beta(J)=\omega_{\beta,0}$) is well known [9]. Two eigenvectors, whose norms are approximately invariant, are given as follows:

$$X_+=\frac{X_1+X_2}{2}, \quad X_-=X_1-X_2,$$

$$X_+(C)=\exp(i\Upsilon\nu_s)X_+(0), \quad X_-(C)=X_-(0), \qquad (2.3)$$

where $\Upsilon=\mathcal{W}\beta C/2\nu_s$ and $\beta=c/\omega_\beta$ is the horizontal beta function where the wake source is installed. $\Upsilon\nu_s/2\pi$ corresponds to the tune shift due to the transverse wake force. The amplitude of each particle is expressed as

$$|X_{1,2}(C)|^2=\frac{|X_1(0)|^2+|X_2(0)|^2}{2} \pm\frac{|X_1(0)|^2-|X_2(0)|^2}{2}\cos(\Upsilon\nu_s) \mp\mathrm{Im}[X_1^*(0)X_2(0)]\sin(\Upsilon\nu_s). \qquad (2.4)$$

The amplitudes are modulated by the wake force with a period of $2\pi/\Upsilon\nu_s$ turns. We consider using the realistic parameters of Table I. If the source of wake is put at a position with $\beta=5$ m, then we have $\Upsilon=0.19$ and the amplitude modulates with a period of 1400 turns.

We next include the amplitude dependent tune shift. The differential equations Eq. (2.2) have the integral $|x_1|^2+|x_2|^2=\mathrm{const}$. Introducing the new variable $\zeta\equiv|x_1|^2-|x_2|^2$, the following differential equation is obtained [10],

$$\zeta'''=\frac{\zeta'\zeta''}{\zeta}-\left(\frac{a}{2\beta^2}\right)^2\zeta^2\zeta'. \qquad (2.5)$$

The differential equation can be solved by elliptic functions. We obtained relations at $s=0$ as follows:

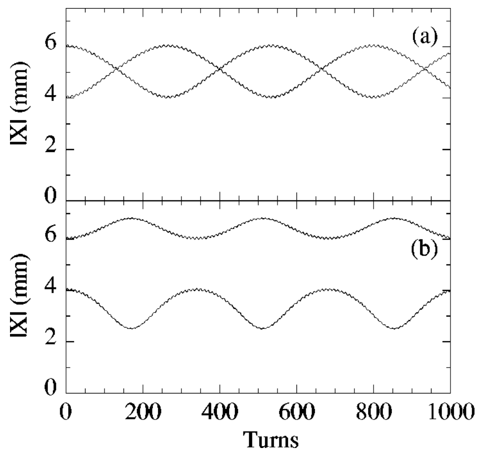

FIG. 1. Evolution of $|X_1|$ and $|X_2|$. (a) and (b) are evolution of $|X|$'s for a positive and negative amplitude-dependent tune shift.

$$\zeta''_0=-\frac{\mathcal{W}}{2}\beta\zeta_0\left\{\frac{a}{2\beta^2}\mathrm{Re}(X_1^*X_2)_0+\frac{\mathcal{W}}{2}\beta\right\}, \qquad (2.6)$$

$$\zeta'_0=-\frac{\mathcal{W}}{2}\beta\ \mathrm{Im}(X_1^*X_2)_0, \qquad (2.7)$$

where the 0 indicates initial values. When the two particles have the same initial betatron phase $[\mathrm{Im}(X_1^*X_2)_0=0]$, we find $\zeta'_0=0$. When $\zeta_0>0(|X_{1,0}|^2>|X_{2,0}|^2)$, ζ''_0 is expressed as

$$\zeta''_0\propto-\left(\frac{a\nu_\beta}{2\beta}\mathrm{Re}(X_1^*X_2)_0+\frac{\mathcal{W}\beta C}{4\pi}\right)\sim-a\nu_\beta J-\delta\nu_I. \qquad (2.8)$$

We found that the behavior of ζ near $s=0$ is determined by a competition between the amplitude dependent tune shift and the head-tail tune shift. If a is larger than a value determined by Eq. (2.8), the difference between $|X_1|^2$ and $|X_2|^2$ decreases near $s=0$; elsewhere it increases.

Here we will not go into the analytical approach any further, since we got the desired qualitative features. Numerical analysis is presented hereafter. We first solved the two-particle model of Eq. (2.1) numerically using the parameters in Table I. The amplitude dependent tune shift is examined for $a\nu_\beta=\pm 1000$. These two $a\nu_\beta$ correspond to negative and positive ζ''_0 of Eq. (2.8): i.e., $a\nu_\beta J=\pm 2.5\times 10^{-3}$ and $\delta\nu_I=0.85\times 10^{-3}$. We consider the case of a beam whose size is $\sigma_x\sim 1$ mm and which is kicked 5 mm horizontally. Initial amplitudes are $x_1=6$ mm and $x_2=4$ mm, and $x'_{1,2}=0$. Figure 1 shows evolutions of each $|X|$ obtained by solving the two-particle model. We obtained different behaviors of $|X|$ for each sign of a as is expected in Eq. (2.8). The amplitudes of two particles intersect each other for positive a, while they are separated for negative a. The modulation frequency for positive a ($\sim$550 turns) was faster than that of $a=0$ (1400 turns).

This modulation of $|X|$ is essential for the Landau damping due to the amplitude-dependent tune shift (spread). In the pure symplectic motion without the wake force, $|X|$ is conserved, and the betatron phases, whose phase advances are determined by the amplitudes of particles, are dispersed by the tune spread corresponding to the amplitude difference.

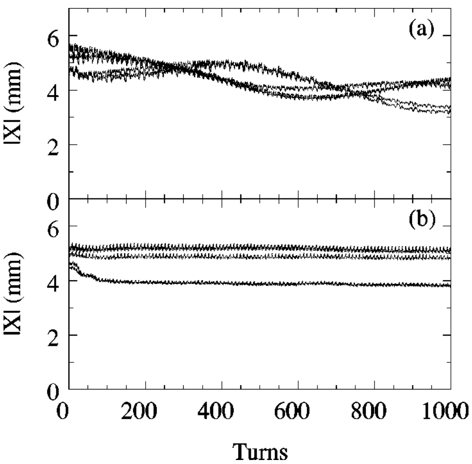

FIG. 2. Evolution of $|X| = \sqrt{2\beta J}$ for five macroparticles chosen arbitrarily. (a) and (b) are evolution of $|X|$'s for a positive and negative amplitude-dependent tune shift.

Now $|X|$ and J are not invariant due to the wake force. If $|X|$ exchange in the process, the phase which is dispersed is gathered again, with the result that the coherent amplitude does not decrease. Conversely, splitting $|X|$ amplifies the betatron phase dispersion and the damping of the coherent amplitude. To treat the phenomenon caused by this mechanism accurately, the two-particle model seems to be a bit inadequate. With the process of the Landau damping, the wake force is reduced by the damping of the dipole moment, and the modulation will be also reduced. In the two-particle model, the dipole moment could not be estimated with enough precision because of poor statistics. Multiparticle tracking is essential to understand the actual phenomenon due to this head-tail effect.

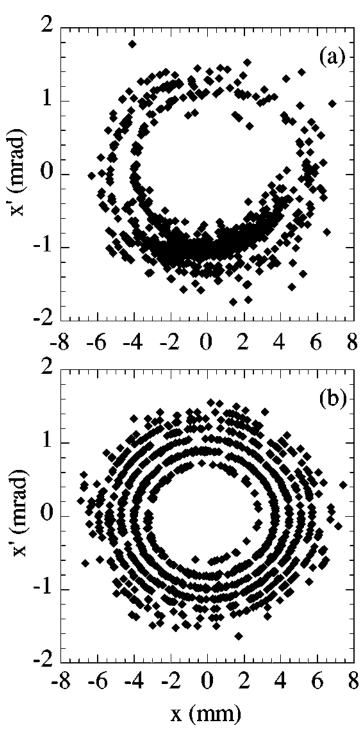

FIG. 3. Phase space distribution of macroparticles after 500 revolutions. (a) and (b) are distributions for a positive and negative amplitude-dependent tune shift.

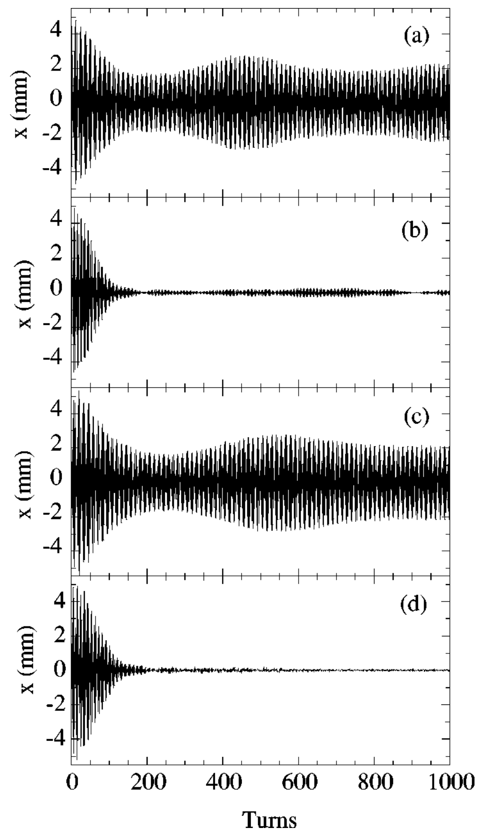

FIG. 4. Betatron oscillation affected by the wake force. (a) and (b) show betatron oscillation obtained by the simulation for positive and negative amplitude-dependent tune shift. (c) and (d) show betatron oscillation by the experiment at the PF ring with the same condition of (a) and (b).

III. MULTIPARTICLE TRACKING

We performed a realistic simulation for the PF ring. All magnets and cavities are expressed by six-dimensional symplectic maps. An element inducing the wake force was installed at one position in the ring. Needless to say, the synchrotron motion and the head-tail interactions are taken into account by the maps. A simplified model using a linear map, an octupole magnet and the wake force element showed the same results qualitatively, therefore the exact lattice information was not needed. It is better to present the result using the exact map in order to compare with the experiment more quantitatively. Macroparticles were generated randomly using a Gaussian distribution in the six-dimensional phase space with the beam size in Table I. Macroparticles are shifted 5 mm to the x direction at the launching point. The octupole magnets were excited by the same field strength for both polarity. The amplitude dependent tune shift is $a\nu_\beta = 2344$ and -2848 [8], and the difference of their absolute values is offset due to the sextupole magnets. We tracked 1000 macroparticles in normal simulation, and tracked also 2000 particles to check the statistics. There was no difference between the two cases.

Figure 2 shows evolutions of $|X| = \sqrt{2\beta J}$ for five particles chosen to be arbitrary. We found intersections of $|X|$ around the 200th and 500th turns for $a > 0$, as seen in the two-particle model. For $a < 0$, $|X|$'s of some particles decrease and do not increase again. The difference from the two particle model is due to the statistics. Though there are a few differences between the two-particle model and the exact simulation, the important point of view is kept: that is, positive a causes the mixing of the amplitudes, while negative a causes a split.

Figure 3 shows the phase-space distributions of the macroparticles after 500 turns. For $a<0$, the betatron phases of macroparticles are ordered like a vortex, and the nonlinear smear evolves smoothly as is expected. On the other hand, for $a>0$, many particles are localized around a betatron phase, and the nonlinear smear is much weaker than for $a<0$.

The evolution of x obtained by the simulation [(a) and (b)] and the experiment [(c) and (d)] are shown in Fig. 4. We first focus at the result of the simulation. Those for $a>0$ and $a<0$ are evidently different. We found very long damping time for $a>0$, and something like an echo at approximately 450 turns. This structure will occur due to the exchange of $|X|$ shown in Figs. 1 and 2. On the other hand, very rapid damping was found for $a<0$. We next look at the result of experiment. The behavior of the betatron oscillation for each sign of a completely coincided with the simulation. The echo was observed at approximately 550 turns at the experiment. It is certain that the phenomenon discussed above actually comes into existence in the experiment.

IV. CONCLUSION

We investigated a head-tail effect due to the transverse wake force. The head-tail effect is caused by the amplitude-dependent tune shift, and is closely related to the Landau damping. The wake force works by mixing the amplitudes of particles. The positive amplitude-dependent tune shift amplifies the mixing, while negative shift suppresses the mixing or leads to a split of the amplitudes. The amplitude mixing works to suppress the Landau damping. The betatron phases of the particles, which are normally dispersed by the amplitude-dependent tune shift, are not dispersed, though they are modulated by the mixing. On the other hand, the splitting of the amplitude amplifies the dispersion of the betatron phase. The Landau damping is suppressed for the positive amplitude tune shift, while it is amplified for the negative tune shift. The multiparticle tracking simulation reproduced this mechanism, and coincided with the experiment at the PF ring. Typically the phenomenon occurs in a region where the amplitude dependent tune shift and head-tail tune shift are comparable. This phenomenon will be instructive in the sense that the corrective effect interferes with the Landau damping.

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