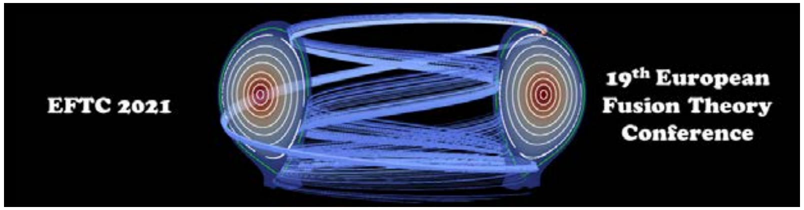

Scattering of radio frequency waves by turbulent cylindrical filaments in the plasma edge and radiation pressure on these filaments(*)

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Relevant study:

“Interaction of radio frequency waves with cylindrical density filaments - scattering and radiation pressure”

(S. I. Valvis, A. K. Ram, K. Hizanidis)

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Abstract (1/3)

Radio frequency (RF) waves:

- Are used in generating toroidal currents and they are useful for diagnostics
- Are effective in imparting toroidal momentum to electrons in the core of the confined plasma
- Are reflected, refracted and diffracted by turbulence in the plasma edge
- Are side scattered in the plasma edge, so that different wave modes from the launched RF are excited
- Have efficiency which is depending upon the wave characteristics (polarization, frequency, wavelength)

Abstract (2/3)

The overall effect of turbulence on the propagation of RF:

- Is changing with λ and with the dimensions of the turbulent structures
- Is important to be quantified for different cases

Our studies:

- Are both analytical and numerical
- Are valid for different density variations and filament sizes
- Are referring to ranges that vary, i.e lower-hybrid (LH) waves (4.6 GHz), helicon waves (476 MHz) and ion-cyclotron (IC) waves (120 MHz)
- Are referring to RF scattering due to the existence of turbulent structures as well as to the forces exerted to the turbulent structures by different RF waves in the plasma edge

Abstract (3/3)

Scattering of RF waves by cylindrical filaments is studied by:

- Using Maxwell's equations
- Considering the plasma cold and deriving the dispersion tensor
- Calculating permittivity inside and outside the cylinder
- Calculating electromagnetic fields and Poynting vector in all regions

Radiation pressure on these filaments:

- Can either pull the filament towards the RF source or push it away

Applications of these studies:

- ALCATOR C-mod plasma (LH waves, 4.6 GHz)
- Plasmas in medium sized tokamaks such as DIII-D (helicon waves, 476 MHz)
- High field concepts like SPARC (IC waves, 120 MHz)

Main assumptions and the geometry

Filaments are assumed to be:

- Cylindrical with infinite length
- Predominantly aligned along the direction of the externally imposed toroidal magnetic field

Plasma is assumed to be:

- Homogeneous
- Cold (thermal velocity of electrons and ions is considered zero)

Incident plane waves:

- Have their own orientation
- Are propagating in the ambient plasma region (with different electron density from the filament plasma region)
- Are hitting the cylindrical filament and are being scattered

Analytical theory (1/4)

Permittivity tensor and the Dispersion Relation

Cold plasma permittivity
in the magnetic field's
(or in the cylinder's axis)
frame of reference,
in Cartesian coordinates

$$\overset{\leftrightarrow}{\mathbf{K}}{}^{cart}_{mag} = \begin{pmatrix} K_\perp & -iK_\times & 0 \\ iK_\times & K_\perp & 0 \\ 0 & 0 & K_\parallel \end{pmatrix}$$

Maxwell –Faraday equation: $\varepsilon_0 \nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - \left(\frac{\omega}{c}\right)^2 \mathbf{D}(\mathbf{r}) = 0$

Dispersion Relation
in the cylinder's (filament)
frame of reference,
in cylindrical coordinates

$$\det\left(\overset{\leftrightarrow}{\mathbf{\Delta}}{}^{cyl}_{fila}\right) = 0$$

Analytical theory (2/4)

Vector cylinder functions

Vector cylinder functions are used to express the fields:
(Z_n are Bessel functions: J_n for the incident and inside the filament
and Hankel of the first kind for the scattered fields)

$$\mathbf{m}_n(\eta_r\rho, \eta_z\zeta, \varphi) \equiv \left[in\frac{Z_n(\eta_r\rho)}{\rho}\hat{\mathbf{r}} - \frac{dZ_n(\eta_r\rho)}{d\rho}\hat{\boldsymbol{\varphi}} \right] \exp\left[i(\eta_z\zeta + n\varphi)\right]$$

$$\mathbf{l}_n(\eta_r\rho, \eta_z\zeta, \varphi) \equiv \left[\frac{dZ_n(\eta_r\rho)}{d\rho}\hat{\mathbf{r}} + in\frac{Z_n(\eta_r\rho)}{\rho}\hat{\boldsymbol{\varphi}} + i\eta_z Z_n(\eta_r\rho)\hat{\boldsymbol{\zeta}} \right] \exp\left[i(\eta_z\zeta + n\varphi)\right]$$

$$\mathbf{n}_n(\eta_r\rho, \eta_z\zeta, \varphi) \equiv \left\{ \frac{\eta_z}{\eta}\left[i\frac{dZ_n(\eta_r\rho)}{d\rho}\hat{\mathbf{r}} - n\frac{Z_n(\eta_r\rho)}{\rho}\hat{\boldsymbol{\varphi}} \right] + \frac{\eta_r^2}{\eta}Z_n(\eta_r\rho)\hat{\boldsymbol{\zeta}} \right\} \exp\left[i(\eta_z\zeta + n\varphi)\right]$$

By using the Vector cylinder functions,
we can take advantage of the existing cylindrical symmetry.

Analytical theory (3/4)

Electromagnetic fields and Poynting vector

The results are z independent because the cylinder has infinite length

Scattered (SC) and filament (FI) fields:

$$\tilde{e}(\rho,\varphi)_{(FI,SC)} = \sum_{M=O,X} \sum_{m=-\infty}^{m=\infty} \mathrm{i}^m \mathrm{e}^{\mathrm{i}m\varphi} \left[\mathcal{E}_{m\rho}^M(\rho)\hat{\rho} + \mathcal{E}_{m\varphi}^M(\rho)\hat{\varphi} + \mathcal{E}_{mz}^M(\rho)\hat{z}\right]_{(FI,SC)}$$

$$\tilde{h}(\rho,\varphi)_{(FI,SC)} = \frac{E_0}{H_0}\sqrt{\frac{\varepsilon_0}{\mu_0}} \sum_{M=O,X} \sum_{m=-\infty}^{m=\infty} \mathrm{i}^m \mathrm{e}^{\mathrm{i}m\varphi} \left[\mathcal{H}_{m\rho}^M(\rho)\hat{\rho} + \mathcal{H}_{m\varphi}^M(\rho)\hat{\varphi} + \mathcal{H}_{mz}^M(\rho)\hat{z}\right]_{(FI,SC)}$$

The parallel to the cylinder axis wave vector component stays the same for all regions

Incident fields are expressed in the same way, but are either O- or X-mode

$$\mathcal{E}_{mr}^M(\rho),\ \mathcal{E}_{m\varphi}^M(\rho),\ \mathcal{E}_{mz}^M(\rho), \mathcal{H}_{mr}^M(\rho), \mathcal{H}_{m\varphi}^M(\rho), \mathcal{H}_{mz}^M(\rho)$$

are functions only of ρ and can be calculated by an appropriate mathematical analysis.

Now, Poynting vector can be calculated in all regions:

$$\tilde{\boldsymbol{s}} = \frac{1}{2}\mathrm{Re}\{\tilde{\boldsymbol{e}} \times \tilde{\boldsymbol{h}}^*\}$$

Analytical theory (4/4)

Forces exerted on the cylinder

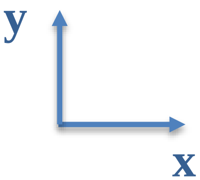
- RF waves exert forces on the turbulent cylindrical filament
- The forces exerted on the cylinder can be calculated from the Maxwell's stress tensor
- After an appropriate analysis, for the time averaged normalized radial stress one obtains:

$$\frac{cT_r}{|\tilde{s}|} = \frac{(K_\rho^{(f)} - K_\rho^{(a)})|p_\phi|^2 + (K_z^{(f)} - K_z^{(a)})|p_z|^2 + (K_\rho|p_r|^2)^{(a)} - (K_\rho|p_r|^2)^{(f)}}{2|\tilde{s}|}$$

In the right hand side of the equation, one can see the elements of the permittivity tensor for the filament (f) and the ambient plasma (a) and the polarizations of the wave

Results for Lower Hybrid (LH) plane waves (1/2)

Scattering of RF waves by a single turbulent filament



$Re(S_x)/max\{Re(S_{in})\}$ $Re(S_y)/max\{Re(S_{in})\}$ $Re(S_z)/max\{Re(S_{in})\}$

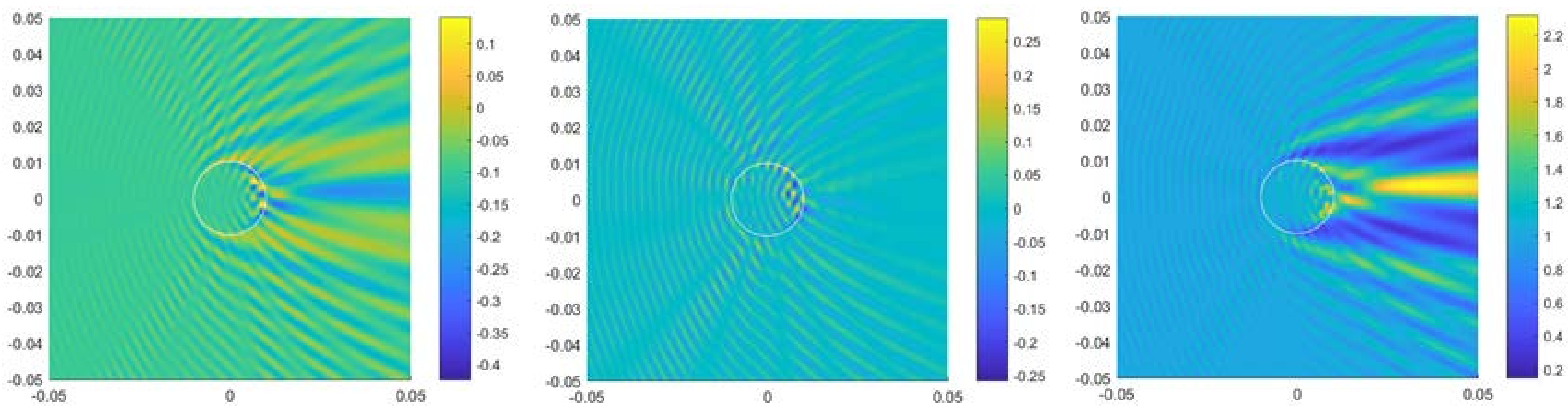


Figure 1a: Normalized Poynting vector components

f = 4.6 GHz, incident polarization: O-mode

$n_{e(ambi)} = 2x10^{19}\ m^{-3}$, $n_{e(fila)} = 2.25x10^{19}\ m^{-3}$, $B = 4.5$ T, $r = 0.01$ m

Results for Lower Hybrid (LH) plane waves (2/2)

Forces exerted on a single filament due to the incident RF plane wave

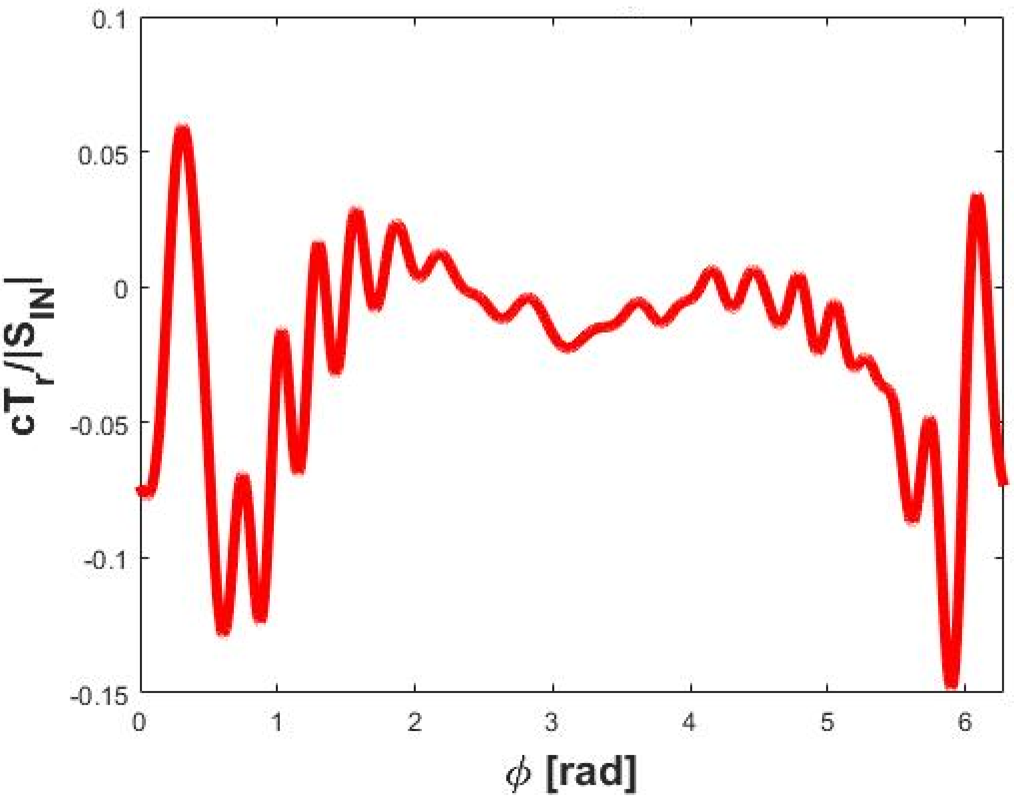


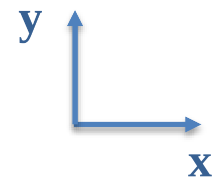
Figure 1b: Normalized radial stress for the same parameters with Figure 1a

After the calculation of the appropriate integrations over azimuthal phi, **the force per unit axial length in x (forward) direction, per unit power flux, is:**

$$-4.64\ \frac{\text{nN/m}}{\text{kW/m}^2}$$

Results for Helicon plane waves (1/4)

Scattering of RF waves by a single turbulent filament



$Re(S_x)/max\{Re(S_{in})\}$ $Re(S_y)/max\{Re(S_{in})\}$ $Re(S_z)/max\{Re(S_{in})\}$

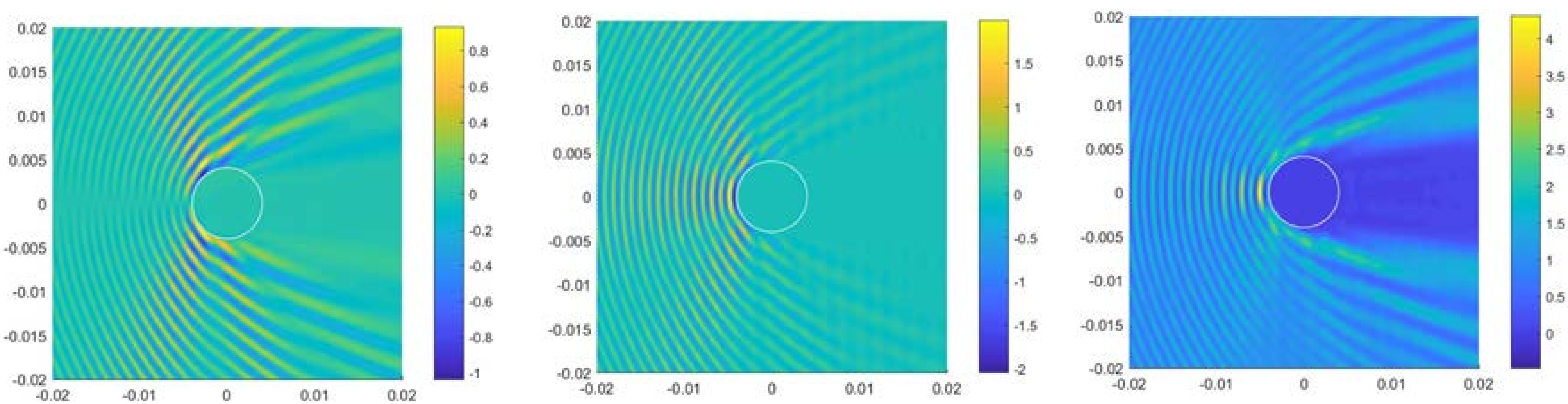


Figure 2a: Normalized Poynting vector components:

f = 476 MHz, incident polarization: O-mode

$n_{e(ambi)} = 10^{19}\ m^{-3}$, $n_{e(fila)} = 3x10^{19}\ m^{-3}$, B = 1.4 T, r = 0.004 m

Results for Helicon plane waves (2/4)

Forces exerted on a single filament due to the incident RF plane wave

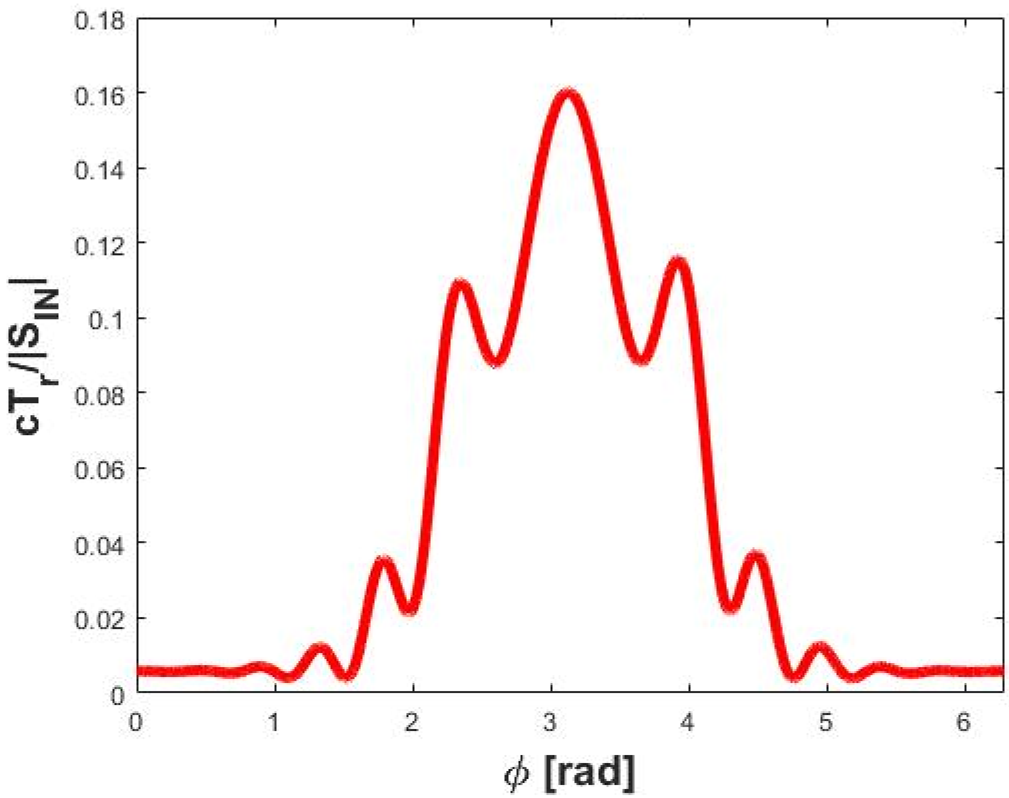


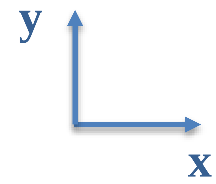
Figure 2b: Normalized radial stress for the same parameters with Figure 2a

After the calculation of the appropriate integrations over azimuthal phi, **the force per unit axial length in x (forward) direction, per unit power flux, is:**

$$3.69 \ \frac{\text{nN/m}}{\text{kW/m}^2}$$

Results for Helicon plane waves (3/4)

Scattering of RF waves by a single turbulent filament



$Re(S_x)/max\{Re(S_{in})\}$ $Re(S_y)/max\{Re(S_{in})\}$ $Re(S_z)/max\{Re(S_{in})\}$

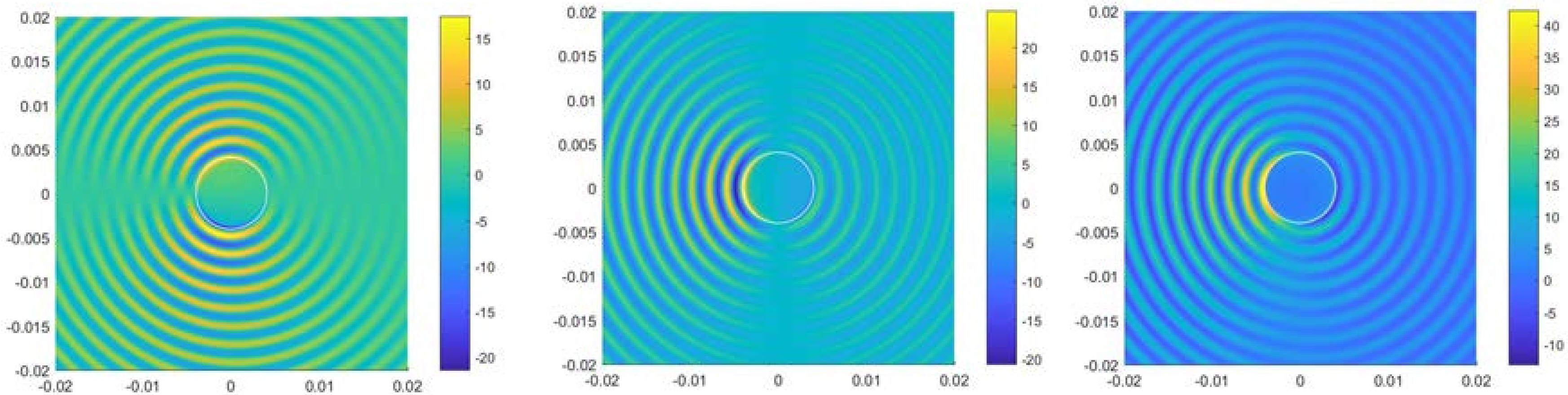


Figure 3a: Normalized Poynting vector components
f = 476 MHz, incident polarization: X-mode
$n_{e(ambi)} = 10^{19}$ m^{-3}, $n_{e(fila)} = 3x10^{19}$ m^{-3}, $B = 1.4$ T, $r = 0.004$ m

Results for Helicon plane waves (4/4)

Forces exerted on a single filament due to the incident RF plane wave

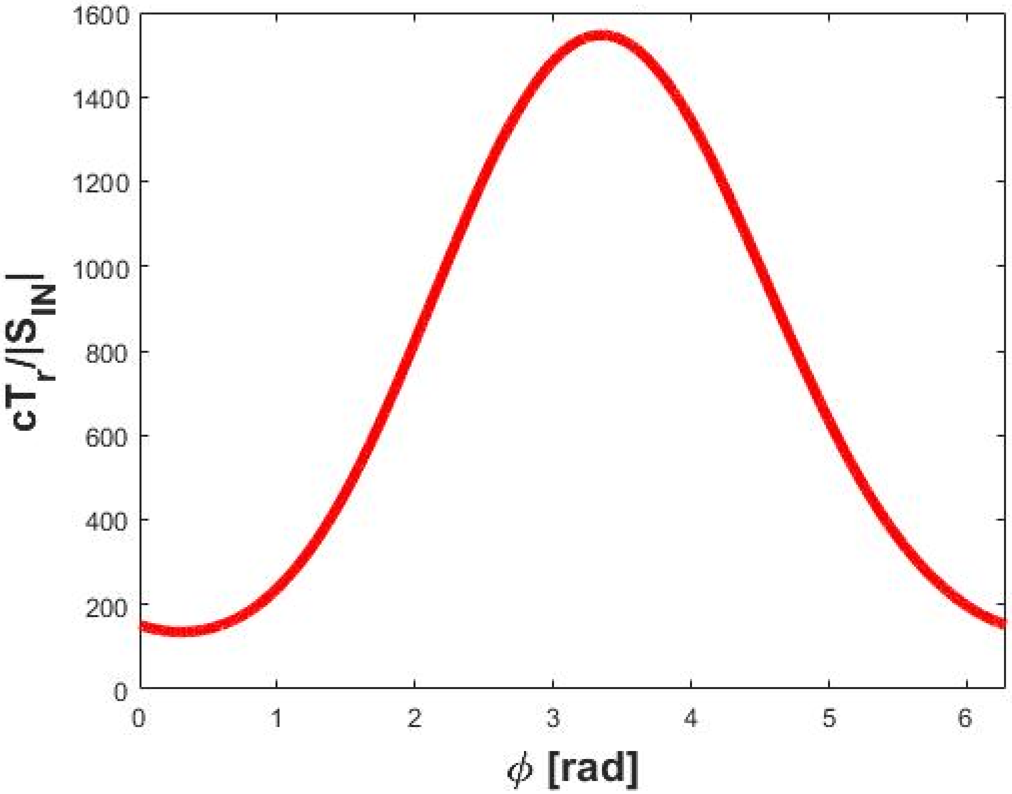


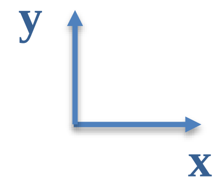
Figure 3b: Normalized radial stress
for the same parameters with Figure 3a

After the calculation of the appropriate integrations over azimuthal phi, **the force per unit axial length in x (forward) direction, per unit power flux, is:**

$$63410.50 \ \frac{\mathrm{nN/m}}{\mathrm{kW/m^2}}$$

Results for Ion Cyclotron (IC) plane waves (1/2)

Scattering of RF waves by a single turbulent filament



$Re(S_x)/max\{Re(S_{in})\}$ $Re(S_y)/max\{Re(S_{in})\}$ $Re(S_z)/max\{Re(S_{in})\}$

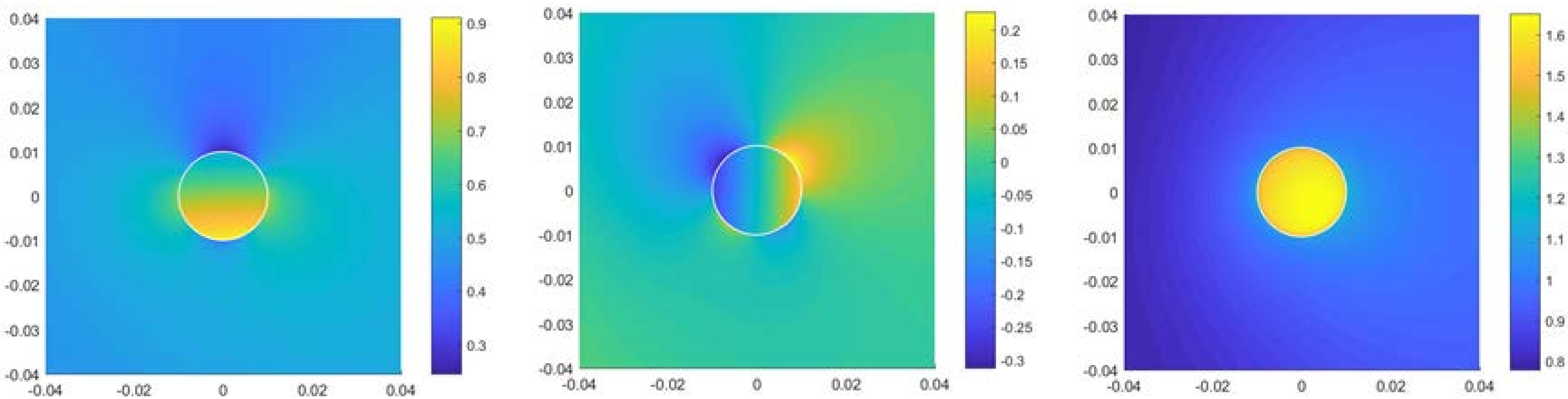


Figure 4a: Normalized Poynting vector components

f = 120 MHz, incident polarization: X-mode

$n_{e(ambi)} = 4x10^{19}\ m^{-3}$, $n_{e(fila)} = 7x10^{19}\ m^{-3}$, $B = 9.3$ T, $r = 0.01$ m

Results for Ion Cyclotron (IC) plane waves (2/2)

Forces exerted on a single filament due to the incident RF plane wave

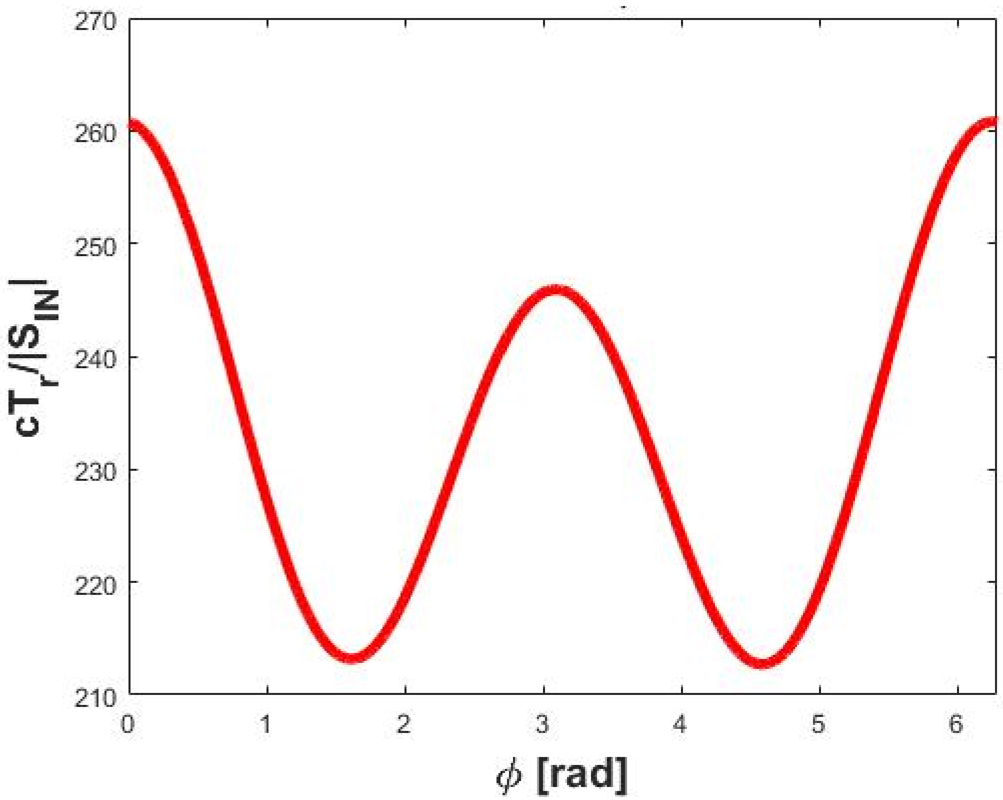


Figure 4b: Normalized radial stress
for the same parameters with Figure 4a

After the calculation of the appropriate integrations over azimuthal phi, **the force per unit axial length in x (forward) direction, per unit power flux, is:**

$$48901.30 \ \frac{\text{nN/m}}{\text{kW/m}^2}$$

Summary

- The scattering of three different types of RF waves (Lower Hybrid, Helicon and Ion Cyclotron) by turbulent cylindrical filament structures in the plasma edge was discussed

- The results showed that the scattering process is depending upon the wave characteristics (polarization, frequency, wavelength) and the plasma characteristics (density, filament's radius, density contrast), too

- The forces exerted on the cylinder due to the incidence of RF waves, were studied and the results showed that the norm of the force exerted on the filament by the incident RF wave as well as the sign (the direction) of this force vary

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Thank you for your attention