Electromagnetic instabilities and plasma turbulence driven by electron-temperature gradient

T. G. Adkins[1], A. A. Schekochihin[1], C. M. Roach[2], and P. G. Ivanov[2]

[1]Rudolf Peierls Centre For Theoretical Physics,
University of Oxford, OXford, OX1 3PU, UK

[2]Culham Centre for Fusion Energy,
United Kingdom Atomic Energy Authority,
Abingdon, OX14 3EB, UK

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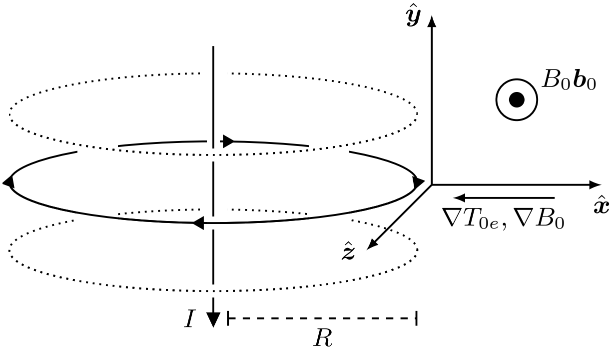
Local slab approximation



- Equilibrium gradients (constant throughout domain):

$$L_B^{-1} = -\frac{1}{B_0}\frac{\mathrm{d}B_0}{\mathrm{d}x}, \quad R^{-1} = -\left|\boldsymbol{b}_0 \cdot \nabla \boldsymbol{b}_0\right|, \quad L_T^{-1} = -\frac{1}{T_{0e}}\frac{\mathrm{d}T_{0e}}{\mathrm{d}x}.$$

Low-beta ordering

$$\omega \sim \underbrace{k_\parallel v_{\rm the}}_{\text{parallel streaming}} \sim \overbrace{\omega_{\rm KAW}}^{\text{kinetic Aflvén waves}} \sim \underbrace{\omega_{*e}}_{\rm ETG} \sim \overbrace{\omega_{de}}^{\text{Mag. drifts}} \sim \underbrace{k_\perp v_E}_{\boldsymbol{E}\times\boldsymbol{B}\text{ drifts}} \sim \overbrace{\nu_{ee} \sim \nu_{ei}}^{\text{collisions}},$$

- Lengthscales: flux-freezing scale is key

$$k_\perp d_e \sim 1, \quad k_\perp \rho_e \sim \sqrt{\beta_e}, \quad k_\parallel L \sim \sqrt{\beta_e}, \quad k_\parallel \lambda_e \sim 1, \quad \frac{k_\parallel}{k_\perp} \sim \epsilon\sqrt{\beta_e},$$

- Amplitudes: effects of magnetic compression are negligible

$$\frac{e\phi}{T_{0e}} \sim \frac{\delta n_e}{n_{0e}} \sim \frac{\delta n_i}{n_{0i}} \sim \frac{\delta T_e}{T_{0e}} \sim \frac{\delta T_i}{T_{0i}} \sim \epsilon, \quad \frac{\delta \boldsymbol{B}_\perp}{B_0} \sim \epsilon\sqrt{\beta_e}, \quad \frac{\delta B_\parallel}{B_0} \sim \epsilon\beta_e.$$

- What follows is all derived in an asymptotic limit of gyrokinetics, for $\epsilon \ll \sqrt{\beta_e} \ll 1$.

Low-beta ordering

$$\omega \sim \underbrace{\kappa k_\parallel^2}_{\text{thermal conduction}} \sim \overbrace{\omega_{\mathrm{KAW}}}^{\text{kinetic Aflvén waves}} \sim \underbrace{\omega_{*e}}_{\text{ETG}} \sim \overbrace{\omega_{de}}^{\text{Mag. drifts}} \sim \underbrace{k_\perp v_E}_{\boldsymbol{E}\times\boldsymbol{B}\text{ drifts}} \ll \overbrace{\nu_{ee} \sim \nu_{ei}}^{\text{collisions}},$$

- Lengthscales ($\chi^{-1} = \sqrt{\beta_e}\lambda_e/L$):

$$k_\perp d_e \sim \chi^{-1}, \quad k_\perp \rho_e \sim \chi^{-1}\sqrt{\beta_e}, \quad k_\parallel L \sim \sqrt{\beta_e}, \quad k_\parallel \lambda_e \sim \chi^{-1}, \quad \frac{k_\parallel}{k_\perp} \sim \chi\epsilon\sqrt{\beta_e},$$

- Amplitudes: effects of magnetic compression are negligible

$$\frac{e\phi}{T_{0e}} \sim \frac{\delta n_e}{n_{0e}} \sim \frac{\delta n_i}{n_{0i}} \sim \frac{\delta T_e}{T_{0e}} \sim \frac{\delta T_i}{T_{0i}} \sim \chi\epsilon, \quad \frac{\delta \boldsymbol{B}_\perp}{B_0} \sim \chi\epsilon\sqrt{\beta_e}, \quad \frac{\delta B_\parallel}{B_0} \sim \chi\epsilon\beta_e.$$

- Shall focus on the collisional limit, as the physics is more transparent. Important change is the replacement of parallel streaming with thermal conduction: $\kappa \sim v_{\mathrm{the}}^2/\nu_e$.

Collisional equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \underbrace{\nabla_\parallel u_{\parallel e}}_{\text{parallel compressions}} + \underbrace{\frac{\rho_e v_{\rm the}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}}}_{\text{magnetic drifts}} = 0,$$

$$\underbrace{\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\rm the}}{2}\frac{\partial\varphi}{\partial z}}_{\text{parallel electric field}} = \frac{v_{\rm the}}{2}\underbrace{\left(\nabla_\parallel\frac{\delta n_e}{n_{0e}} + \nabla_\parallel \log T_e\right)}_{\text{parallel pressure gradient}} + \underbrace{\nu_{ei}\frac{u_{\parallel e}}{v_{\rm the}}}_{e\text{-}i\text{ friction}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \underbrace{\kappa\nabla_\parallel^2 \log T_e}_{\text{thermal conduction}} + \underbrace{\frac{2}{3}\nabla_\parallel u_{\parallel e}}_{\text{compressional heating}} + \underbrace{\frac{\rho_e v_{\rm the}}{2L_T}\frac{\partial\varphi}{\partial y}}_{\boldsymbol{E}\times\boldsymbol{B}\text{ injection}} = 0, \quad \kappa = \frac{5v_{\rm the}^2}{18\nu_e},$$

- Quasineutrality (Boltzmann ions):

$$\frac{\delta n_e}{n_{0e}} = -\bar{\tau}^{-1}\varphi, \quad \varphi = \frac{e\phi}{T_{0e}}, \quad \bar{\tau} = \frac{\tau}{Z}, \quad \tau = \frac{T_{0i}}{T_{0e}}.$$

- Parallel Ampère's law:

$$\frac{u_{\parallel e}}{v_{\rm the}} = d_e^2\nabla_\perp^2\mathcal{A}, \quad \mathcal{A} = \frac{A_\parallel}{\rho_e B_0}.$$

- Parallel gradient of the total temperature $T_e = T_{0e} + \delta T_e$:

$$\nabla_\parallel \log T_e = \nabla_\parallel\frac{\delta T_e}{T_{0e}} + \frac{\delta B_x}{B_0}\frac{1}{T_{0e}}\frac{\mathrm{d}T_{0e}}{\mathrm{d}x} = \nabla_\parallel\frac{\delta T_e}{T_{0e}} - \frac{\rho_e}{L_T}\frac{\partial\mathcal{A}}{\partial y}.$$

Electrostatic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}} = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta T_e}{T_{0e}} + \nu_{ei}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \frac{\delta T_e}{T_{0e}} + \frac{2}{3}\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial \varphi}{\partial y} = 0.$$

- Below the resistive (flux-freezing) scale ($\chi^{-1} = \sqrt{\beta_e}\lambda_e/L$),

$$k_\perp d_e \chi \gg 1,$$

  $\Rightarrow$ magnetic field lines are no longer frozen into the electron velocity.
- Ignore inductive part of parallel electric field
- Parallel gradient of total temperature becomes:

$$\nabla_\parallel \log T_e = \nabla_\parallel \frac{\delta T_e}{T_{0e}}.$$

  $\Rightarrow$ no contributions due to finite magnetic field perturbations

Electrostatic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}} = 0,$$

$$\frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta T_e}{T_{0e}} + \nu_{ei}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \frac{\delta T_e}{T_{0e}} + \frac{2}{3}\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial \varphi}{\partial y} = 0.$$

- Collisional slab ETG:

$$\omega_{de} \ll \kappa k_\parallel^2 \ll \omega \ll \omega_{*e}, \quad \Rightarrow \quad \omega = \pm \frac{1 - i\,\mathrm{sgn}(k_y)}{\sqrt{2}} \left(\frac{k_\parallel^2 v_{\mathrm{th}e}^2 |\omega_{*e}| \bar{\tau}}{2\nu_{ei}} \right)^{1/2}.$$

- Equivalent of the usual (collisionless) slab ETG:

$$\omega_{de} \ll k_\parallel v_{\mathrm{th}e} \ll \omega \ll \omega_{*e}, \quad \Rightarrow \quad \omega = \mathrm{sgn}(k_y) \left(-1, \frac{1}{2} \pm i\frac{\sqrt{3}}{2} \right) \left(\frac{k_\parallel^2 v_{\mathrm{th}e}^2 |\omega_{*e}| \bar{\tau}}{2} \right)^{1/3}$$

- Similar feedback mechanism, though we shall not focus on that here.

Electrostatic regime

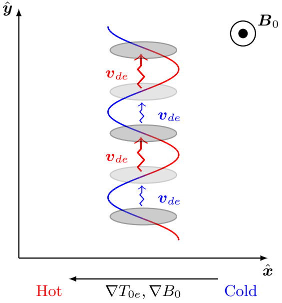
$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}} = 0,$$

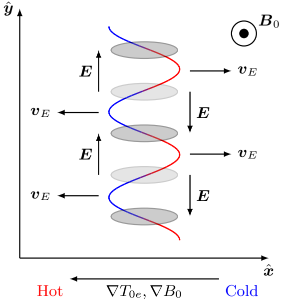
$$\frac{v_{\mathrm{the}}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{the}}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \frac{v_{\mathrm{the}}}{2}\nabla_\parallel \frac{\delta T_e}{T_{0e}} + \nu_{ei}\frac{u_{\parallel e}}{v_{\mathrm{the}}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa\nabla_\parallel^2 \frac{\delta T_e}{T_{0e}} + \frac{2}{3}\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{2L_T}\frac{\partial \varphi}{\partial y} = 0.$$

- Curvature-mediated ETG (2D interchange mode)

$$\kappa k_\parallel^2 \ll \omega_{de} \ll \omega \ll \omega_{*e}, \quad \Rightarrow \quad \omega = \pm i\left(2\omega_{de}\omega_{*e}\bar{\tau}\right)^{1/2}.$$





Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}} = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \log T_e + \nu_{ei}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial \varphi}{\partial y} = 0$$

- Above the resistive (flux-freezing) scale ($\chi^{-1} = \sqrt{\beta_e}\lambda_e/L$):

$$k_\perp d_e \chi \ll 1,$$

  $\Rightarrow \delta \boldsymbol{B}_\perp$ is created as electrons move across field lines and drag them along
- Restore the inductive term and ignore resistive term
- Magnetic field contributions to parallel gradients:

$$\nabla_\parallel \log T_e = \nabla_\parallel \frac{\delta T_e}{T_{0e}} + \frac{\delta B_x}{B_0}\frac{1}{T_{0e}}\frac{\mathrm{d}T_{0e}}{\mathrm{d}x} = \nabla_\parallel \frac{\delta T_e}{T_{0e}} - \frac{\rho_e}{L_T}\frac{\partial \mathcal{A}}{\partial y}.$$

  $\Rightarrow$ introduces another mechanism by which the perturbations can go unstable due to L_T.

Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}} = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{the}}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \frac{v_{\mathrm{the}}}{2}\nabla_\parallel \log T_e + \nu_{ei}\frac{u_{\parallel e}}{v_{\mathrm{the}}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{the}}}{2L_T}\frac{\partial \varphi}{\partial y} = 0$$

- Dominant thermal conduction = isothermal along perturbed field line:

$$\nabla_\parallel \log T_e = \nabla_\parallel \frac{\delta T_e}{T_{0e}} - \frac{\rho_e}{L_T}\frac{\partial \mathcal{A}}{\partial y} = 0,$$

- Curvature-mediated *thermo-Alfvénic instability* (cTAI):

$$\omega = \pm i \left[2\omega_{de}\omega_{*e}(1+\bar{\tau})\right]^{1/2}.$$

  $\Rightarrow$ physically different, new instability.

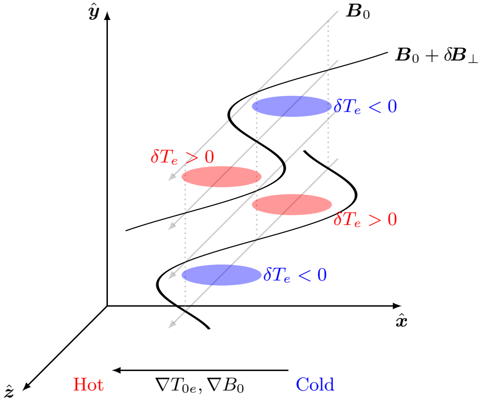
- Two key differences to cETG: (i) it relies on $k_\parallel \neq 0$, and (ii) it does not require the $\boldsymbol{E} \times \boldsymbol{B}$ feedback mechanism to be unstable.

Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{the}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}} = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{the}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{the}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \frac{v_{the}}{2}\nabla_\parallel \log T_e + \nu_{ei}\frac{u_{\parallel e}}{v_{the}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{the}}{2L_T}\frac{\partial \varphi}{\partial y} = 0$$



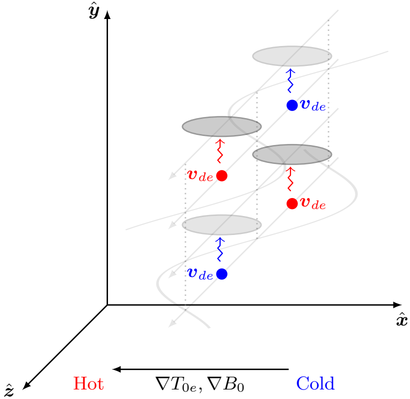
- A perturbation $\delta B_x = B_0 \rho_e \partial_y \mathcal{A}$ sets up a variation of total temp. along the perturbed field line as it makes excursions into hot and cold regions.
- Rapid thermal conduction along field lines creates a temperature perturbation that compensates for this.

Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}} = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \log T_e + \nu_{ei}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa \nabla_\parallel^2 \log T_e + \frac{2}{3}\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial \varphi}{\partial y} = 0$$



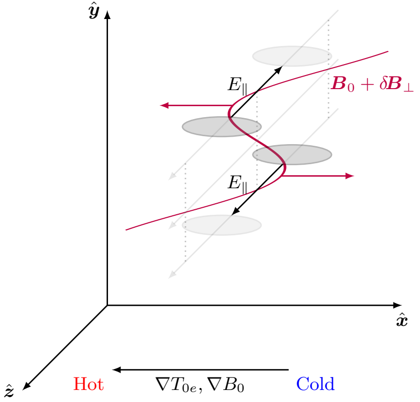
- Velocity dependence of magnetic drifts v_{de} creates an electron density perturbation (hot particles drift faster than cold ones).
- This electron density perturbation has both $k_y \neq 0$ and $k_\parallel \neq 0$.

Electromagnetic regime

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}} = 0,$$

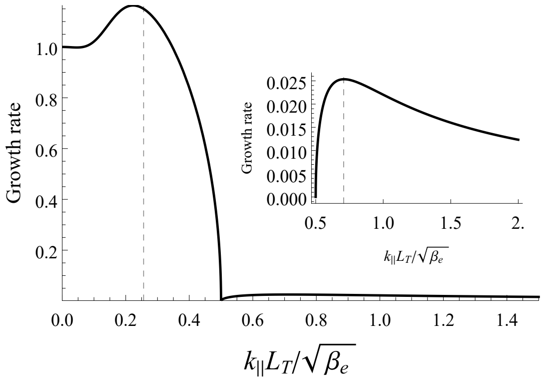
$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial \varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \frac{\delta n_e}{n_{0e}} + \frac{v_{\mathrm{th}e}}{2}\nabla_\parallel \log T_e + \nu_{ei}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa\nabla_\parallel^2 \log T_e + \frac{2}{3}\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial\varphi}{\partial y} = 0$$



- The parallel density gradient must be balanced by the parallel electric field.
- Inductive part leads to an increase in δB_x, deforming the field line further into the hot and cold regions $\Rightarrow$ feedback.
- NB: δT_e and δB_x are frozen into different flows in this limit.

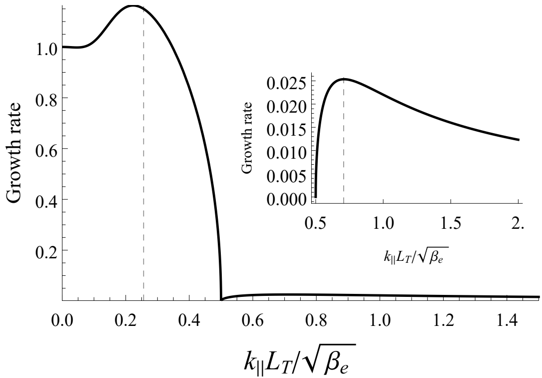
Electromagnetic regime



- Clear enhancement of cETG at finite $k_\parallel$:

$$\frac{k_{\parallel\max}L_T}{\sqrt{\beta_e}} = \left[\frac{81}{50}\frac{3+4\bar{\tau}}{(1+\bar{\tau})^2}\frac{L_T}{L_B}\right]^{1/6}\left(\frac{k_y^2 d_e}{k_\perp}\chi\right)^{1/3}.$$

Electromagnetic regime

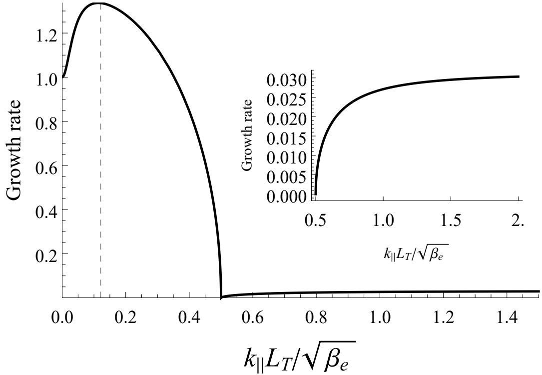


- For $\omega_{*e} \sim \kappa k_{\|}^2$, general TAI dispersion relation:

$$\omega^2 = -\left(2\omega_{de}\omega_{*e} - \omega_{\mathrm{KAW}}^2\right)\left(\bar{\tau} + \frac{1}{1+i\xi_*}\right), \quad \xi_* = \frac{\omega_{*e}}{\kappa k_{\|}^2}.$$

- Zielinski et. al. (2017) found this dispersion relation for $\xi_* = 0$, but the maximum was for $k_{\|} = 0$.

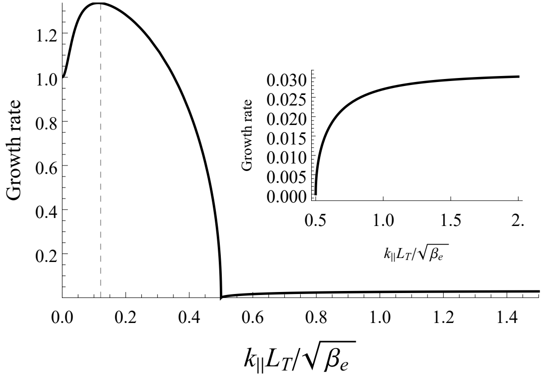
Electromagnetic regime



- For $\omega_{*e} \sim k_{\|} v_{\text{the}}$, general TAI dispersion relation:

$$\omega^2 = -\left(2\omega_{de}\omega_{*e} - \omega_{\text{KAW}}^2\right)\left(\bar{\tau} + \frac{1}{1+i\xi_*}\right), \quad \xi_* = \frac{\sqrt{\pi}}{2}\frac{\omega_{*e}}{k_{\|} v_{\text{the}}}.$$

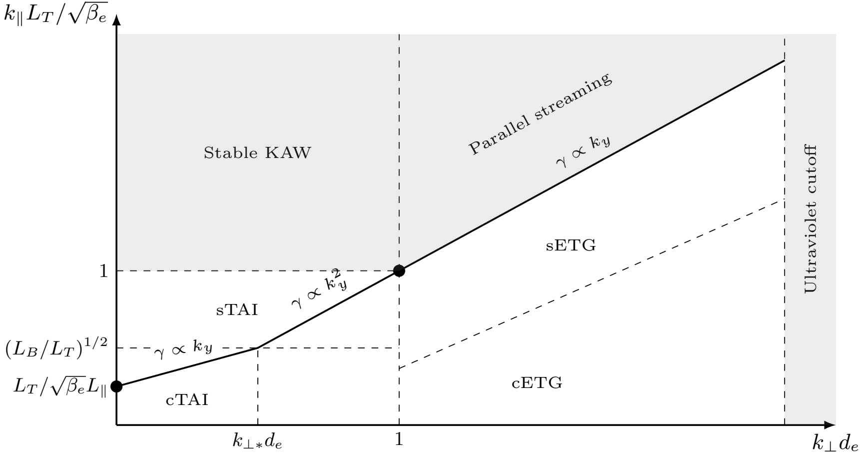
Electromagnetic regime



- Clear enhancement of cETG at finite $k_{\|}$:

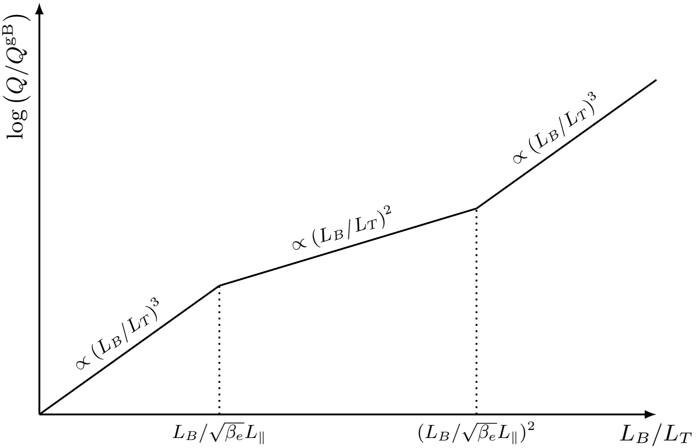
$$\frac{k_{\|\max}L_T}{\sqrt{\beta_e}} = \left[\frac{\pi}{64}\frac{3+4\bar{\tau}}{(1+\bar{\tau})^2}\frac{L_T}{L_B}\right]^{1/4}\left(\frac{k_y^2 d_e}{k_\perp}\right)^{1/2}.$$

Turbulent transport



- At low-k_y, the dominant injection is at finite $k_\parallel$.
- Finite $k_\parallel$ at the cTAI maximum allows us to fix the injection scale.
- Can construct a theory of turbulence by constant flux arguments [Barnes et. al., (2011)]

Turbulent heat fluxes



- cTAI stiffens electron transport for $L_B/L_T \geqslant L_B^2/(\beta_e L_\parallel^2)$ (large scale injection).
- Occurs on sub ρ_i scales, assuming $\beta_e \gtrsim m_e/m_i$.

Summary and future work

- We considered the turbulent state of a low-beta, magnetised plasma allowing electromagnetic perturbations and a variation of the equilibrium magnetic field
- In the electromagnetic regime, it was shown that the system supports the *thermo-Alfvénic instability (TAI)* that extracts free energy from the equilibrium temperature gradient through finite perturbations to the magnetic field direction.
- This TAI enhances the (two-dimensional) cETG mode, reaching a maximum at a finite parallel wavelength.
- Constant flux arguments show that the resultant turbulent heat flux dominates over that due to the conventional slab ETG for $L_B/L_T \geqslant L_B^2/(\beta_e L_\parallel^2)$.
- All of this is subject to generalisations to include the effects of kinetic ions, magnetic shear, toroidal geometry, elongation, triangularity, Shafranov shift, plasma shaping, trapped particles, impurities, fast particles, runaway electrons, etc.
- Pre-print available on request: toby.adkins@physics.ox.ac.uk

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Backup slides

- Ordering of frequencies:

$$\frac{\omega}{\Omega_e} \sim \epsilon\beta_e, \quad \frac{\omega}{\Omega_i} \sim \epsilon.$$

- Ordering of lengthscales ($L \sim L_{n_s} \sim L_{T_s} \sim L_B \sim R$):

$$k_\perp \rho_i \sim k_\perp d_e \sim 1, \quad k_\perp \rho_e \sim \sqrt{\beta_e}, \quad k_\parallel L \sim \sqrt{\beta_e},$$
$$k_\parallel \lambda_e \sim 1, \quad \frac{k_\parallel}{k_\perp} \sim \epsilon\sqrt{\beta_e}.$$

- Ordering of fluctuation amplitudes:

$$\frac{e\phi}{T_{0e}} \sim \frac{\delta n_e}{n_{0e}} \sim \frac{\delta n_i}{n_{0i}} \sim \frac{\delta T_e}{T_{0e}} \sim \frac{\delta T_i}{T_{0i}} \sim \epsilon, \quad \frac{\delta \boldsymbol{B}_\perp}{B_0} \sim \epsilon\sqrt{\beta_e}, \quad \frac{\delta B_\parallel}{B_0} \sim \epsilon\beta_e.$$

- Non-adiabatic response of the ions:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + \boldsymbol{v}_{di} \cdot \nabla_\perp\right) g_i + \frac{c}{B_0} \left\{ \langle \phi \rangle_{\boldsymbol{R}_i} - \phi, g_i \right\} + \langle \boldsymbol{v}_E \rangle_{\boldsymbol{R}_i} \cdot \nabla_\perp f_{0i}$$
$$= C \left[g_i + \frac{q_i \langle \phi \rangle_{R_i}}{T_i} f_{0i} \right],$$

where

$$g_i = h_i - \frac{q_i \langle \phi \rangle_{R_i}}{T_i} f_{0i}.$$

- Quasineutrality and parallel Ampere's law:

$$\frac{\delta n_e}{n_e} = -\bar{\tau}^{-1} \varphi + \frac{1}{n_i} \int \mathrm{d}^3 \boldsymbol{v} \ \langle g_i \rangle_{\boldsymbol{r}} \,, \quad \frac{u_{\parallel e}}{v_{\mathrm{th}e}} = d_e^2 \nabla_\perp^2 \mathcal{A},$$

where we have defined the normalised variables $\varphi = e\phi/T_e$ and $\mathcal{A} = A_\parallel / \rho_e B_0$.

- The electrons are drift kinetic, since $k_\perp \rho_e \sim \sqrt{\beta_e} \ll 1$, vis.:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + v_\parallel \nabla_\parallel + \boldsymbol{v}_{de} \cdot \nabla_\perp\right) \delta f_e = -\boldsymbol{v}_\chi \cdot \nabla_\perp f_{0e} - \frac{v_\parallel e E_\parallel}{T_e} + C[\delta f_e].$$

- We choose to expand δf_e in Hermite-Laguerre moments $g_{\ell,m}$:

$$g_{\ell,m}(\boldsymbol{r}, t) = \frac{1}{n_{0e}} \int \mathrm{d}^3\boldsymbol{v}\, (-1)^\ell \frac{H_m(v_\parallel/v_{\mathrm{the}}) L_\ell(v_\perp^2/v_{\mathrm{the}}^2)}{\sqrt{2^m m!}}\, \delta f_e,$$

$$\delta f_e(\boldsymbol{r}, v_\parallel, v_\perp^2, t) = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\infty} (-1)^\ell \frac{H_m(v_\parallel/v_{\mathrm{the}}) L_\ell(v_\perp^2/v_{\mathrm{the}}^2) f_{0e}}{\sqrt{2^m m!}}\, g_{\ell,m},$$

where

$$H_m = \text{Hermite polynomials of order } m,$$
$$L_\ell = \text{Laguerre polynomials of order } \ell.$$

- The Hermite-Laguerre transform allows us to express the electron gyrokinetic equation in terms of a series of 'fluid moments':

$$\frac{\mathrm{d}g_{\ell,m}}{\mathrm{d}t} + \frac{v_{\mathrm{the}}}{\sqrt{2}}\nabla_{\parallel}(\sqrt{m+1}\, g_{\ell,m+1} + \sqrt{m}\, g_{\ell,m-1}) - C[g_{\ell,m}] + \omega_{de}[g_{\ell,m}] = I_{\ell,m}.$$

- We have defined:

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{v}_E \cdot \nabla_{\perp} = \frac{\partial}{\partial t} + \frac{\rho_e v_{\mathrm{the}}}{2}\{\varphi, ...\},$$

$$\nabla_{\parallel} = \boldsymbol{b}\cdot\nabla = \frac{\partial}{\partial z} + \frac{\delta\boldsymbol{B}_{\perp}}{B_0}\cdot\nabla_{\perp} = \frac{\partial}{\partial z} - \rho_e\{\mathcal{A}, ...\},$$

which are the time derivative in the frame moving with the $\boldsymbol{E}\times\boldsymbol{B}$ flow, and the derivative along the exact, perturbed magnetic field line respectively.

- Electron-electron and electron-ion collisions ($\nu_e = \nu_{ee} + \nu_{ei}$)

$$\begin{aligned} C[g_{\ell,m}] = &- \nu_e(m+2\ell)g_{\ell,m} + \nu_{ee} g_{0,1}\delta_{0,1} \\ &+ \frac{1}{3}\nu_e \left(\sqrt{2}g_{0,2} + 2g_{1,0}\right)\left(\sqrt{2}\delta_{0,2} + 2\delta_{1,0}\right). \end{aligned}$$

- Magnetic drifts:

$$\begin{aligned} \omega_{de}[g_{\ell,m}] = \frac{\rho_e v_{\mathrm{the}}}{2L_B}\frac{\partial}{\partial y}\Big[&\sqrt{(m+1)(m+2)}g_{\ell,m+2} + 2(m+\ell+1)g_{\ell,m} \\ &+ \sqrt{m(m-1)}g_{\ell,m-2} + (\ell+1)g_{\ell+1,m} + \ell g_{\ell-1,m}\Big]. \end{aligned}$$

- Energy/momentum injection:

$$\begin{aligned} I_{\ell,m} = &- \frac{\rho_e v_{\mathrm{the}}}{2L_{n_e}}\frac{\partial \varphi}{\partial y}\left[\delta_{0,0} + \eta_e\left(\delta_{1,0} + \frac{1}{\sqrt{2}}\delta_{0,2}\right)\right] \\ &+ \frac{\sqrt{2}\rho_e v_{\mathrm{the}}}{2L_{n_e}}\frac{\partial \mathcal{A}}{\partial y}\left[\delta_{0,1} + \eta_e\left(\delta_{0,1} + \delta_{1,1} + \sqrt{\frac{3}{2}}\delta_{0,3}\right)\right] \\ &+ \frac{v_{\mathrm{the}}}{\sqrt{2}}\left(\frac{2}{v_{\mathrm{the}}}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{\partial \varphi}{\partial z}\right)\delta_{0,1} + \frac{\rho_e v_{\mathrm{the}}}{2L_B}\frac{\partial \varphi}{\partial y}\left[\sqrt{2}\delta_{0,2} + \delta_{1,0} + 2\delta_{0,0}\right]. \end{aligned}$$

- Reduced collisionless equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_B}\frac{\partial}{\partial y}\left(\frac{\delta T_{\parallel e}}{T_{0e}} + \frac{\delta T_{\perp e}}{T_{0e}}\right) = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathcal{A} - \frac{u_{\parallel e}}{v_{\mathrm{th}e}}\right) = -\frac{v_{\mathrm{th}e}}{2}\left[\frac{\partial\varphi}{\partial z} - \nabla_\parallel\left(\frac{\delta n_e}{n_{0e}} + \frac{\delta T_{\parallel e}}{T_{0e}}\right) + \frac{\rho_e}{L_T}\frac{\partial\mathcal{A}}{\partial y}\right],$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_{\parallel e}}{T_{0e}} + \nabla_\parallel\left(\frac{\delta q_{\parallel e}}{n_{0e}T_{0e}} + 2u_{\parallel e}\right) + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial\varphi}{\partial y} = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_{\perp e}}{T_{0e}} + \nabla_\parallel\frac{\delta q_{\perp e}}{n_{0e}T_{0e}} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial\varphi}{\partial y} = 0.$$

- Reduced collisional equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta n_e}{n_{0e}} + \nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{L_B}\frac{\partial}{\partial y}\frac{\delta T_e}{T_{0e}} = 0,$$

$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} + \frac{v_{\mathrm{th}e}}{2}\frac{\partial\varphi}{\partial z} = \frac{v_{\mathrm{th}e}}{2}\left(\nabla_\parallel\frac{\delta n_e}{n_{0e}} + \nabla_\parallel \log T_e\right) + \nu_{ei}\frac{u_{\parallel e}}{v_{\mathrm{th}e}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\delta T_e}{T_{0e}} - \kappa\nabla_\parallel^2\log T_e + \frac{2}{3}\nabla_\parallel u_{\parallel e} + \frac{\rho_e v_{\mathrm{th}e}}{2L_T}\frac{\partial\varphi}{\partial y} = 0, \quad \kappa = \frac{5v_{\mathrm{th}e}^2}{18\nu_e}.$$

- The growth rate $\gamma = \mathrm{Im}(\omega)$ and the (real) frequency $\omega_r = \mathrm{Re}(\omega)$ of the growing TAI modes can be written as

$$\gamma^2 = \left|2\omega_{de}\omega_{*e} - \omega_{\mathrm{KAW}}^2\right| \bar{\tau}\, f_+(\xi_*), \quad \omega_r^2 = \left|2\omega_{de}\omega_{*e} - \omega_{\mathrm{KAW}}^2\right| \bar{\tau}\, f_-(\xi_*),$$

where

$$f_\pm(\xi_*) = \frac{1}{2\bar{\tau}} \left[\sqrt{\left(\bar{\tau} + \frac{1}{1+\xi_*^2}\right)^2 + \frac{\xi_*^2}{(1+\xi_*^2)^2}} \pm \mathrm{sgn}\left(2\omega_{de}\omega_{*e} - \omega_{\mathrm{KAW}}^2\right) \left(\bar{\tau} + \frac{1}{1+\xi_*^2}\right) \right],$$

and

$$\xi_* = \frac{\sqrt{\pi}}{2} \frac{\omega_{*e}}{k_\parallel v_{\mathrm{the}}}, \quad \xi_* = \frac{\omega_{*e}}{\kappa k_\parallel^2}.$$

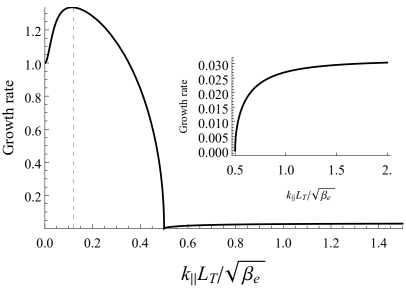
- Location of cTAI maximum:

$$\frac{k_{\|\max} L_T}{\sqrt{\beta_e}} = \begin{cases} \left[\frac{\pi}{64}\frac{3+4\bar{\tau}}{(1+\bar{\tau})^2}\frac{L_T}{L_B}\right]^{1/4}\left(\frac{k_y^2 d_e}{k_\perp}\right)^{1/2}, & \text{collisionless,} \\ \left[\frac{81}{50}\frac{3+4\bar{\tau}}{(1+\bar{\tau})^2}\frac{L_T}{L_B}\right]^{1/6}\left(\frac{k_y^2 d_e}{k_\perp}\chi\right)^{1/3}, & \text{collisional,} \end{cases}$$

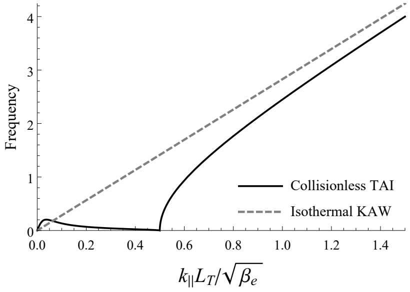
  in the collisionless and collisional limits, respectively.
- Transition wavenumber between isothermal and isobaric regimes:

$$k_{\perp *} d_e = \begin{cases} \frac{4}{\sqrt{\pi}}\left(\frac{L_T}{L_B}\right)^{1/2}, & \text{collisionless,} \\ \frac{5}{9}\frac{L_T}{L_B}\chi^{-1}, & \text{collisional,} \end{cases}$$

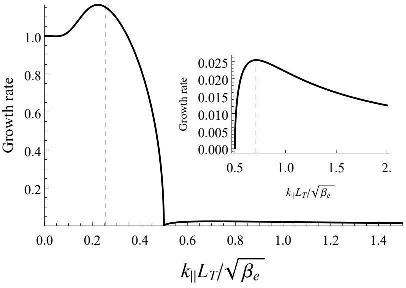
  with $\chi^{-1} = \sqrt{\beta_e}\lambda_e/L_T$.



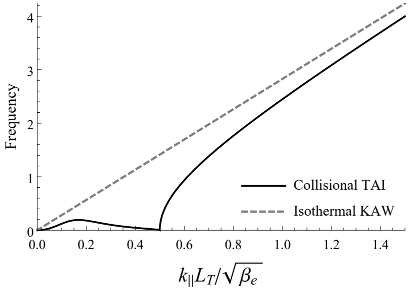
(a) $L_B/L_T = 4$, $k_y d_e = 0.1$



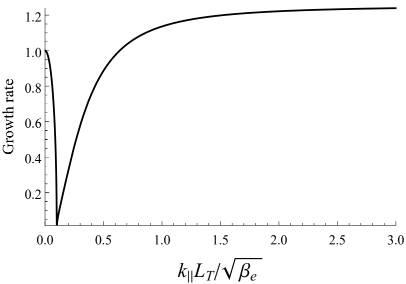
(b) $L_B/L_T = 4$, $k_y d_e = 0.1$



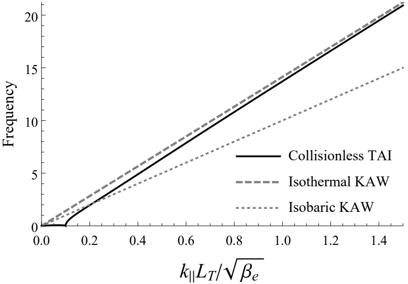
(c) $L_B/L_T = 4$, $k_y d_e \chi = 0.01$



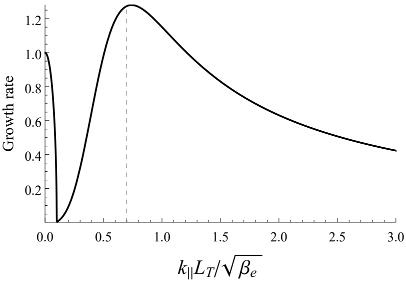
(d) $L_B/L_T = 4$, $k_y d_e \chi = 0.01$



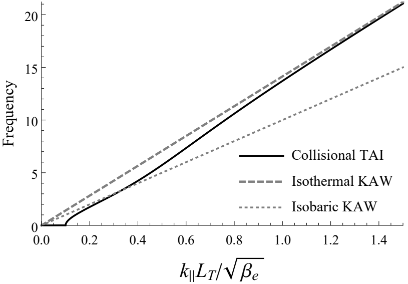
(a) $L_B/L_T = 100$, $k_y d_e = 0.8$



(b) $L_B/L_T = 100$, $k_y d_e = 0.8$

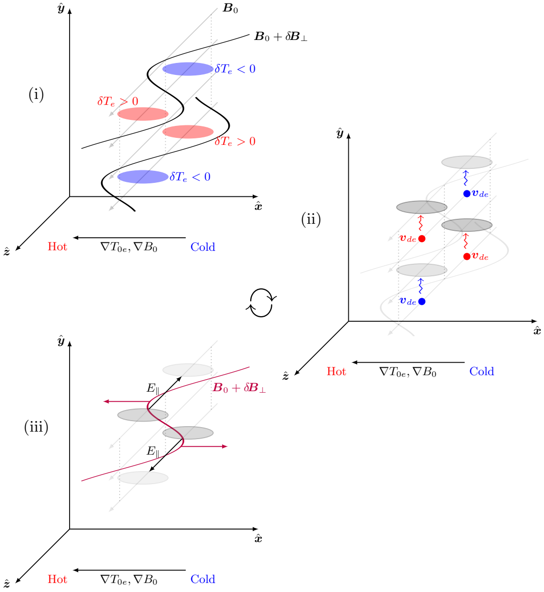


(c) $L_B/L_T = 100$, $k_y d_e \chi = 0.1$



(d) $L_B/L_T = 100$, $k_y d_e \chi = 0.1$

Curvature mediated thermo-Alfvénic instability (cTAI)



- $\gamma = \pm\left[2\omega_{de}\omega_{*e}(1+\bar{\tau})\right]^{1/2}$
- Intrinsically electromagnetic instability
- $\delta B_x = B_0\rho_e\partial_y\mathcal{A}$ sets up a variation of total temp. along the perturbed field line as it makes excursions into hot and cold regions.
- Rapid thermal conduction along field lines creates a compensating temp. pert.
- Velocity dependence of mag. drifts $\boldsymbol{v}_{de}$ creates an electron density pert.
- Parallel density gradient must be balanced by $E_\parallel$, the inductive part of which leads to an increase in δB_x $\Rightarrow$ feedback.