

Action-Angle formulation of the Guiding Center Kinetic Theory and Orbital Spectrum Analysis of Particle Energy and Momentum Transport

Y. Kominis, Y. Antonenas, T. Bournelis, G. Anastassiou, and P. Zestanakis

National Technical University of Athens, Athens, 157 80, Greece

The Guiding Center (GC) theory has been widely used for more than four decades as the basis for the study of single and collective particle dynamics in toroidal magnetic fields utilized in fusion devices. The Hamiltonian formulation of the theory has been originally given in terms of non-canonical variables and then extended to canonical ones. The canonical Hamiltonian description, apart from its elegant structure, is accompanied by an arsenal of powerful mathematical methods of practical importance, which are readily applicable when a final transformation to Action-Angle variables is performed. This transformation is possible for the case of GC motion in axisymmetric equilibria, where the corresponding equations are integrable. All symmetry-breaking perturbations related to non-axisymmetric and/or time-dependent modes and waves can be expressed in terms of the Action-Angle variables of the unperturbed axisymmetric system, facilitating the analytical and numerical study of the collective particle behaviour.

More specifically, the transformation provides the Orbital Spectrum of the GC orbits in terms of the bounce/transit frequencies as well as the bounce/transit-averaged toroidal precession and gyration frequencies. The respective Orbital Spectrum Analysis [1] gives an a priori knowledge of the exact location and the strength of the resonant particle interactions with multi-scale spatio-temporal perturbations in the GC phase space. This is of particular importance for understanding and predicting the modifications of the respective distribution functions of bulk and energetic particles and the synergetic effects between different types of perturbations. In addition, the unique advantage of the Action-Angle formulation, related to the clear separation of the different time scales of the particle motion in different degrees of freedom, enables the systematic dynamical reduction of the kinetic description to a hierarchy of evolution equations for the distribution functions [2] which extends standard bounce-averaged and quasilinear kinetic theories [3].

The practical use of the aforementioned advantages has been hindered mostly due to complications on the calculation of the transformation to Action-Angle variables. In fact, analytical results have been reported only for the case of a Large Aspect Ratio (LAR) equilibrium under the Zero Drift Width (ZDW) approximation and only for deeply trapped particles, which is the case described in many textbooks. Our work focuses on raising these restrictions in two ways: We obtain analytical results beyond the deeply trapped approximation [4], that can be further utilized in analytical perturbation theory. Moreover, we study Finite Drift Width (FDW) effects, which are important for energetic particles, and we consider the transformation to Action-Angle variables and the calculation of the Orbital Spectrum for realistic numerical and experimental equilibria.

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