Pressure effects on the topology of magnetic fields in stellarators

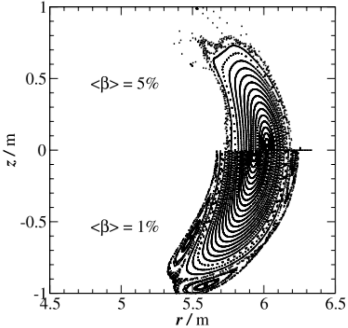
A. Baillod[1], J. Loizu[1], J. P. Graves[1], R. Jorge[2], A. Cerfon[3], D. Malhotra[3], J. Schilling[3], Z. Qu[4], M. Landreman[5]

[1] Ecole Polytechnique Fédérale de Lausanne (EPFL), Swiss Plasma Center (SPC), CH-1015 Lausanne, Switzerland
[2] Max-Planck-Institut für Plasmaphysik, Euratom Association, D-17491 Greifswald, Germany
[3] Courant Institute, New York University 251 Mercer St, New York, NY 10012, USA
[4] Mathematical Sciences Institute, the Australian National University, Canberra ACT 2601, Australia
[5] University of Maryland, College Park, MD, 20742, USA

October 11, 2021

Pressure effects need to be taken into account in stellarator design

- 3D toroidal field: **nested magnetic surfaces, magnetic islands, chaotic field lines**
- $\mathrm{j} \times \mathrm{B} = \nabla p \rightarrow$ pressure gradient generate currents (Pfirsch-Schlüter, diamagnetic)
- **Currents perturb vacuum field** and can destroy nested magnetic surfaces
- Magnetic surfaces are usually destroyed **before reaching MHD stability** limits [Helander, 2014]
- Studied in various configurations (LHD [Suzuki *et. al.*, 2020], W7-AS [Reiman, 2021.])



[Helander *et. al.*, 2012]

Definition: Equilibrium β-limit

$\beta = 2\mu_0 p / B^2$ above which volume occupied by nested magnetic surfaces decreases.

Question

Can we characterize these equilibrium β-limits?

3D magnetic equilibria are notoriously difficult to compute

- Starting from,

$$\mathrm{j} \times \mathrm{B} = \nabla p$$

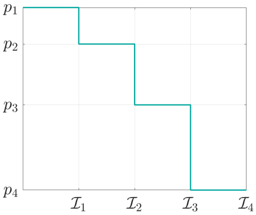
- Assuming nested flux surfaces, parallel current $Q = \mathrm{j} \cdot \mathrm{B}/B^2 = \sum Q_{mn} exp(im\theta - in\phi)$ with

$$Q_{mn} = \underbrace{-p' \frac{J_{mn}}{m\iota - n}}_{\text{Pfirsch-Schlüter "1/x" singularity}} + \underbrace{\hat{Q}_{mn}\delta(\psi - \psi_{mn})}_{\delta\text{-function current singularity}}$$

[A. Bhattacharjee *et. al.*, 1995]

with $\iota = 1/q$ the rotational transform

- Mathematically well defined solutions exist if
  1. $p(\psi)$ constant around $\iota = n/m \rightarrow p(\psi)$ **fractal** [Grad, 1967], or **stepped** [Bruno and Laurence, 1996]
  2. Non-resonant geometry [Weitzner, 2014]
  3. $\iota(\psi)$ irrational when $p' \neq 0$ [Loizu, 2015]
  4. Combination of 1. and 3. [Hudson, 2017]



Antoine Baillod

SPEC finds stepped-pressure equilibria

- Stepped pressure, sustained by N_{vol} **interfaces**

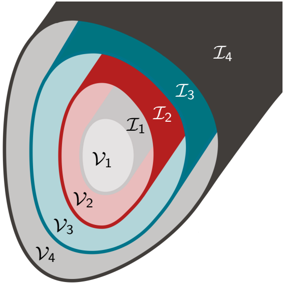
$$\left[\left[p + \frac{B^2}{2\mu_0}\right]\right]_l = 0 \qquad \text{across } \mathcal{I}_l$$

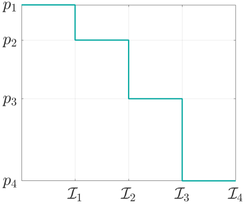
- **Force free-field** between interfaces

$$\nabla \times \mathrm{B} = \mu_l \mathrm{B} \qquad \text{in } \mathcal{V}_l$$

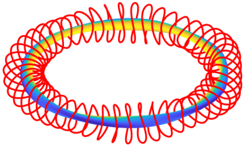
- Interfaces constrained to be nested surfaces
- In $\mathcal{V}_l$, **no constraints on topology** (island, chaos possible)
- SPEC input: Coils or Plasma boundary, $\{\psi_{t,l}, p_l, I^s_{\phi,l}, I^v_{\phi,l}\}$
  - $I^v_{\phi,l}$: externally driven currents, in $\mathcal{V}_l$
  - $I^s_{\phi,l}$: pressure driven currents, at $\mathcal{I}_l$
- SPEC output: B, geometry of $\mathcal{I}_l$

[Hudson *et. al.*, 2020]
[Baillod *et. al.*, 2021]





We consider a classical stellarator with bootstrap current



- Coils s.t. $B_c \cdot \hat{n} = 0$ in vacuum on a rotating ellipse

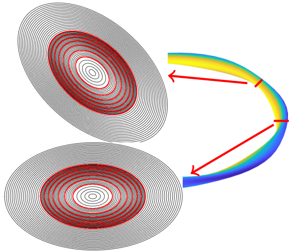
Antoine Baillod

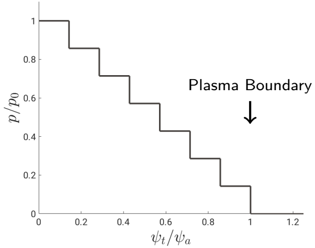
We consider a classical stellarator with bootstrap current

- Coils s.t. $\mathrm{B}_c \cdot \hat{\mathrm{n}} = 0$ in vacuum on a rotating ellipse
- Linear pressure profile

$$p(\psi_t) = p_0 \left(1 - \frac{\psi_t}{\psi_a}\right) \quad (1)$$

- $N_{vol} = 7$





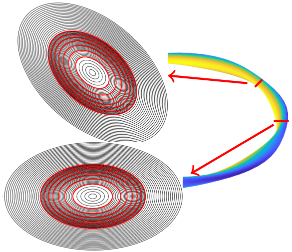
We consider a classical stellarator with bootstrap current

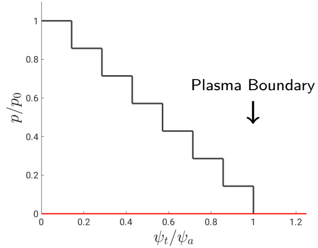
- Coils s.t. $B_c \cdot \hat{n} = 0$ in vacuum on a rotating ellipse
- Linear pressure profile

$$p(\psi_t) = p_0 \left(1 - \frac{\psi_t}{\psi_a}\right) \tag{1}$$

- $N_{vol} = 7$
- No externally driven currents

$$I^{v}_{\phi,l} = 0 \qquad \forall l \tag{2}$$





We consider a classical stellarator with bootstrap current

- Coils s.t. $\mathrm{B}_c \cdot \hat{\mathrm{n}} = 0$ in vacuum on a rotating ellipse
- Linear pressure profile

$$p(\psi_t) = p_0 \left(1 - \frac{\psi_t}{\psi_a}\right) \tag{1}$$

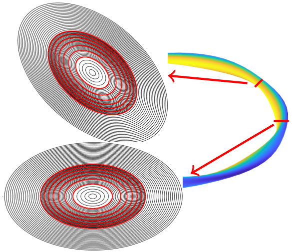
- $N_{vol} = 7$
- No externally driven currents

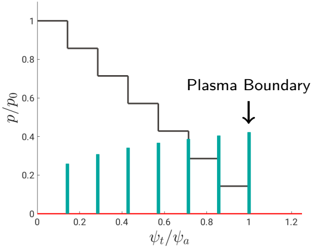
$$I^v_{\phi,l} = 0 \qquad \forall l \tag{2}$$

- High aspect ratio tokamak bootstrap current approximation

$$j_\phi = \sqrt{\epsilon} R_0 \frac{dp}{d\psi_p} \rightarrow I^s_{\phi,l} = C \left(\frac{\psi_t}{\psi_a}\right)^{1/4} [[p]]_l, \tag{3}$$

$C \sim R_0/\iota B_0$. Example: $C_{ITER} \sim 18[\mathrm{A\ Pa}^{-1}]$





We consider a classical stellarator with bootstrap current

- Coils s.t. $\mathbf{B}_c \cdot \hat{\mathbf{n}} = 0$ in vacuum on a rotating ellipse
- Linear pressure profile

$$p(\psi_t) = p_0 \left(1 - \frac{\psi_t}{\psi_a}\right) \qquad (1)$$

- $N_{vol} = 7$
- No externally driven currents

$$I^v_{\phi,l} = 0 \qquad \forall l \qquad (2)$$

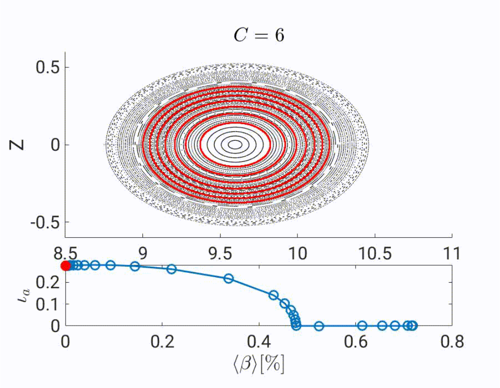
- High aspect ratio tokamak bootstrap current approximation

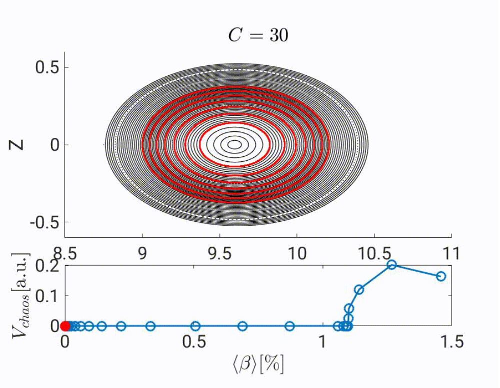
$$j_\phi = \sqrt{\epsilon} R_0 \frac{dp}{d\psi_p} \rightarrow I^s_{\phi,l} = C \left(\frac{\psi_t}{\psi_a}\right)^{1/4} [[p]]_l, \qquad (3)$$

$C \sim R_0/\iota B_0$. Example: $C_{ITER} \sim 18[\text{A Pa}^{-1}]$

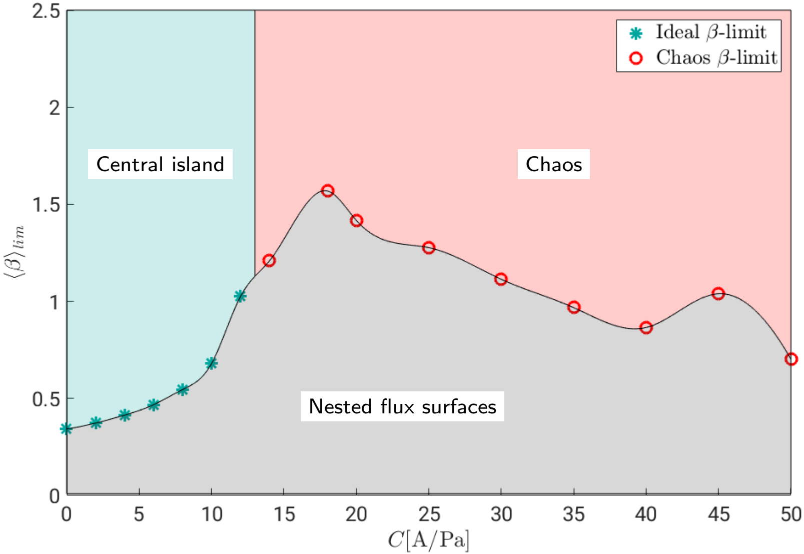
Parameters of interest:
$\{C, p_0\}$

Changes in the topology of the magnetic equilibrium can be detected





The equilibrium β-limit is extracted as a function of C

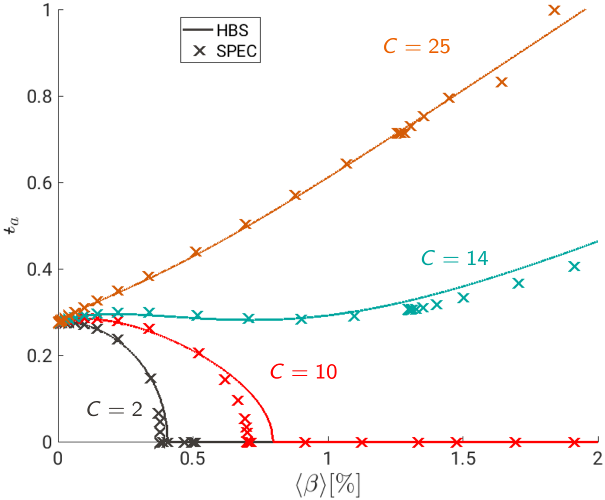


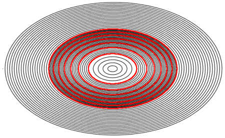
The HBS expansion predicts the rotational transform at the plasma edge

$$\iota_a = [\iota_I(\langle\beta\rangle) + \iota_v]\sqrt{1 - \frac{\langle\beta\rangle^2}{\epsilon_a^2[\iota_I(\langle\beta\rangle) + \iota_v]^4}}$$

$$\iota_I = \frac{R_0}{2\pi a^2 B_0}\mu_0 I_\phi(\langle\beta\rangle)$$

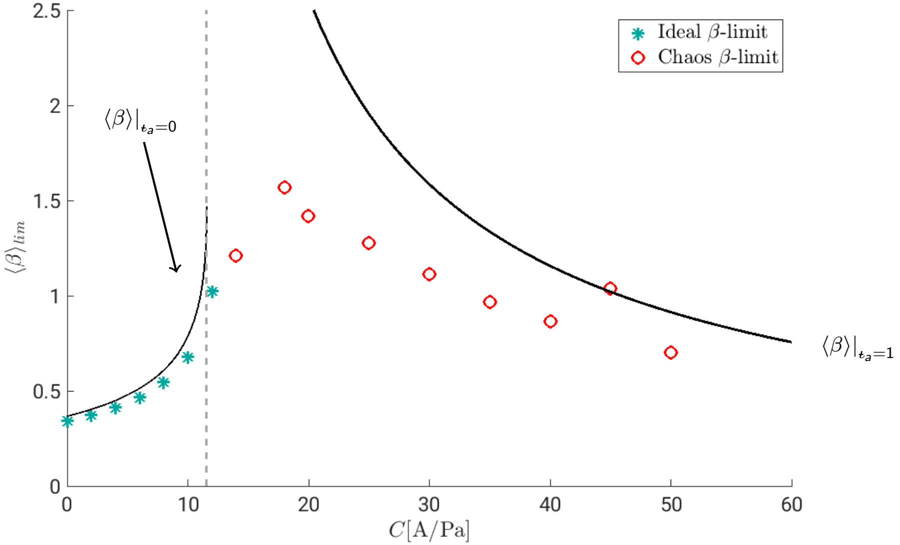
[Freidberg, *ideal MHD*, 2014]



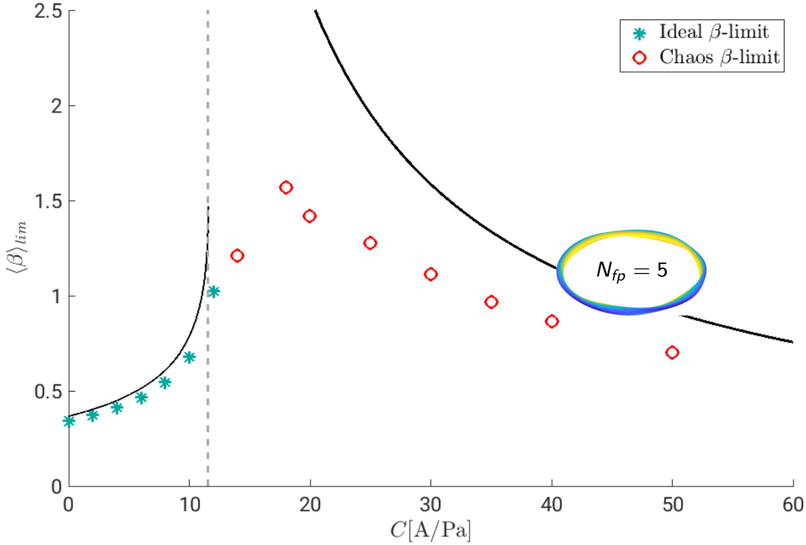


Ideal limit $\iota_a = 0$
Chaos limit $\iota_a \sim 1$

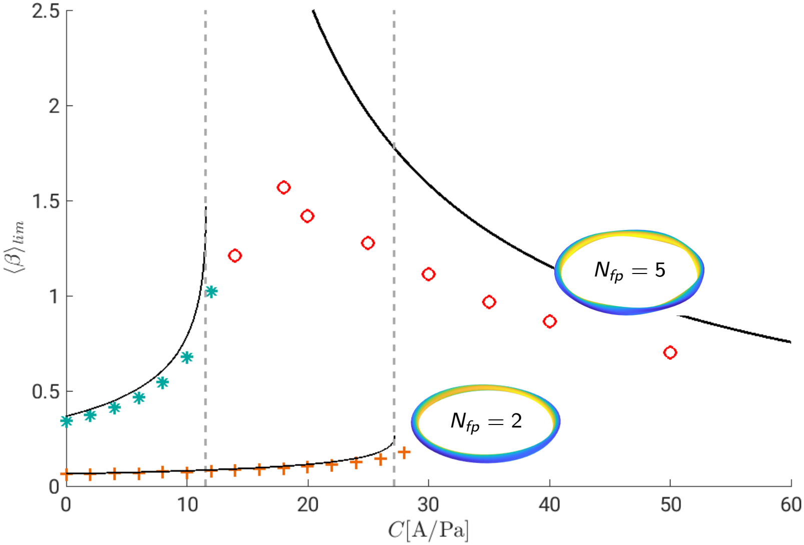
Both equilibrium β-limit are analytically predicted with HBS



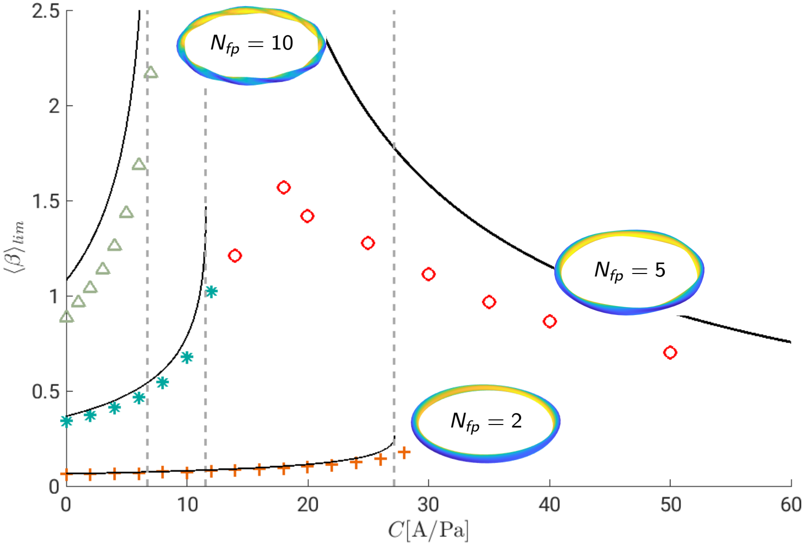
Effect of important design parameters can be measured



Effect of important design parameters can be measured

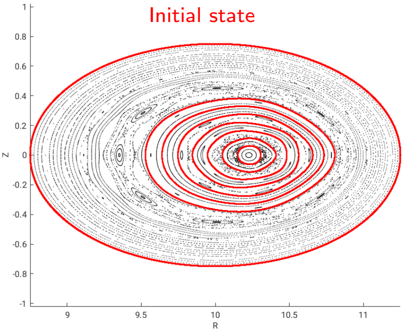


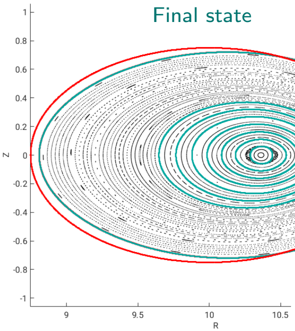
Effect of important design parameters can be measured



Optimization can improve β-limit

- SPEC coupled with SIMSOPT [Landreman *et. al.*, 2021]
- Proof of principle for equilibrium β-limit optimization

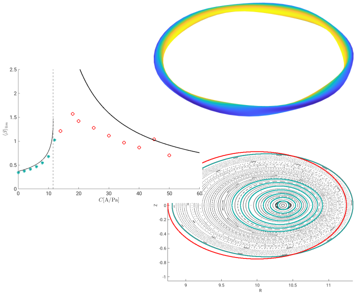




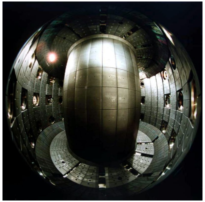
SIMONS FOUNDATION

Conclusions

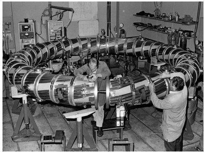
- Shown: equilibrium β-limit can be **evaluated with SPEC**
- Obtained **analytical understanding** of the equilibrium β-limit in classical stellarators
- Improved equilibrium β-limits via **optimization**
- Outlooks
  - **More shaped geometries** (W7-AS, W7-X). Benchmark against PIES [A. Reimann, 2021]
  - Extract equilibrium β-limits dependencies in standard QA, QH and QI configurations.



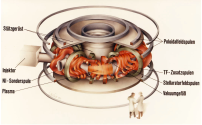
Tokamaks



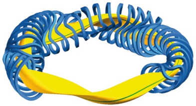
W7-A



W7-AS

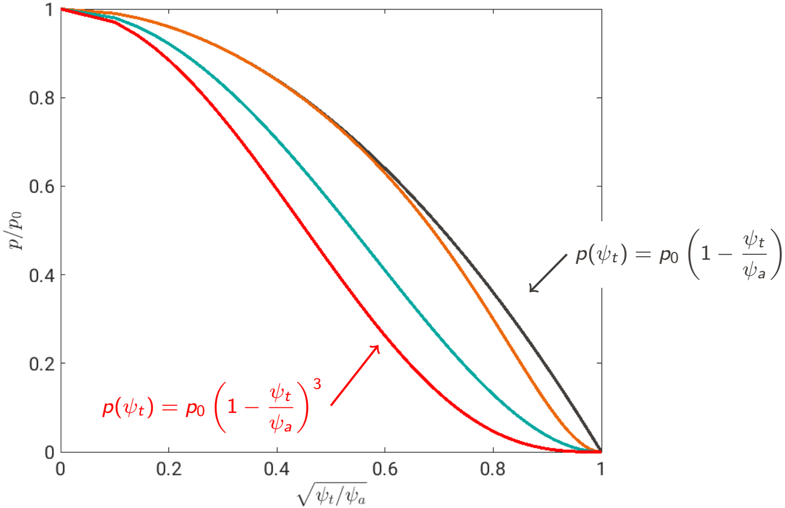


W7-X



Backup slides

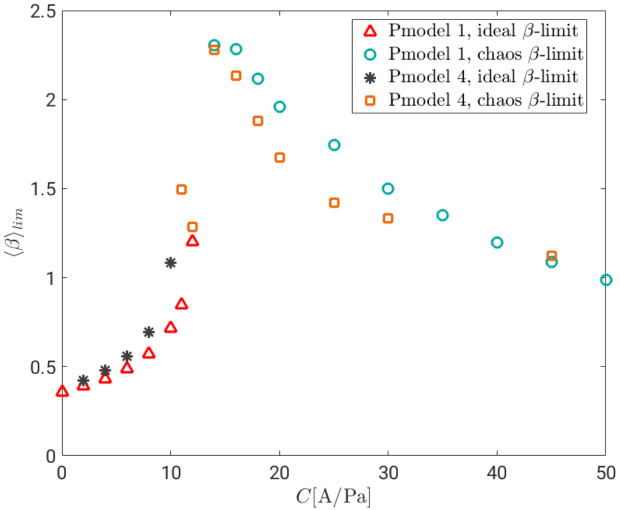
Different pressure models were considered



Antoine Baillod

EFTC 2021

Choice of pressure profile does not affect strongly the β-limit



Chaos β-limit dependence on α