



Instabilities and turbulence in stellarators from the perspective of global codes

E. Sánchez¹, A. Bañón Navarro², F. Wilms², M. Borchardt³, R. Kleiber³

¹Laboratorio Nacional de Fusión / CIEMAT. Avda Complutense 40, 28040, Madrid, Spain.

²Max-Planck Institut für Plasmaphysik, Garching, Germany.

³Max-Planck Institut für Plasmaphysik, Greifswald, Germany.

19th European Fusion Theory Conference, 11 October 2021.



This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement number 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

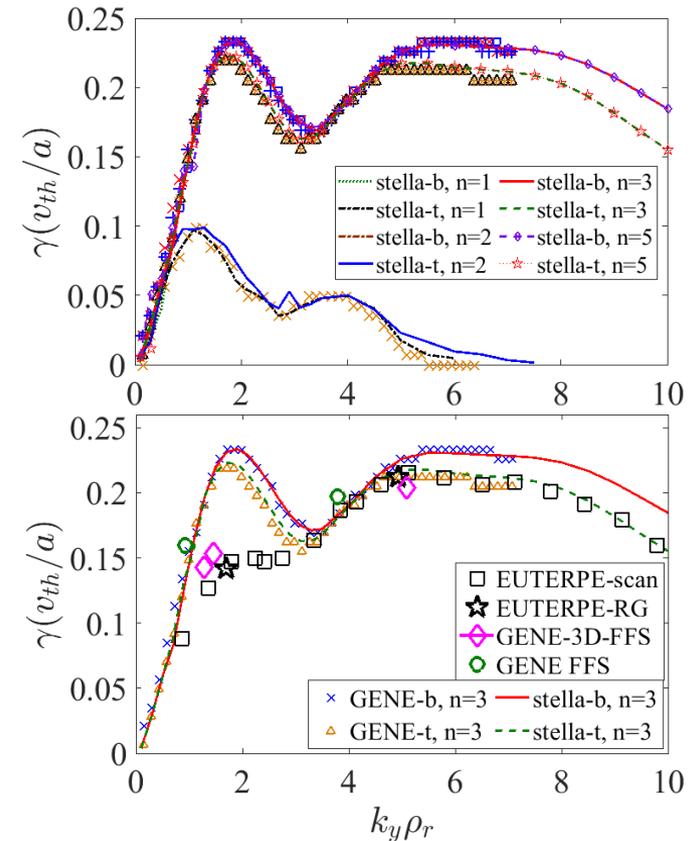
Background

- Recent works comparing GK simulations in different computational domains have shown limitations of the flux-tube model for stellarators (see Sanchez NF 2021, Smoniewski PoP 2021).
- Different flux tubes, or flux tubes with different lengths, provide different results for linear problems: ZF relaxation and ITG instabilities.
- At least full surface (or global) simulations are required in stellarators.

In this work:

- We compare the global codes EUTERPE and GENE-3D in stellarator configurations.
- We address two problems that can not be properly accounted for in flux tube simulations:
 - The localization of instabilities and turbulence over the flux surface.
 - The influence of an electric field.

Growth rate of ITG modes in global, full surface, and flux tube domains (Sanchez NF 2021).



- **Codes and magnetic configurations.**
- **Comparison of GENE-3D and EUTERPE in linear simulations.**
 - Adiabatic electrons.
 - Kinetic electrons.
- **Comparison in nonlinear simulations using model profiles.**
- **Localization of instabilities/turbulence over the flux surface.**
- **Influence of the electric field on**
 - Spatial localization of instabilities / turbulence.
 - Stabilization of linearly unstable modes.
 - Turbulent heat flux in nonlinear simulations.

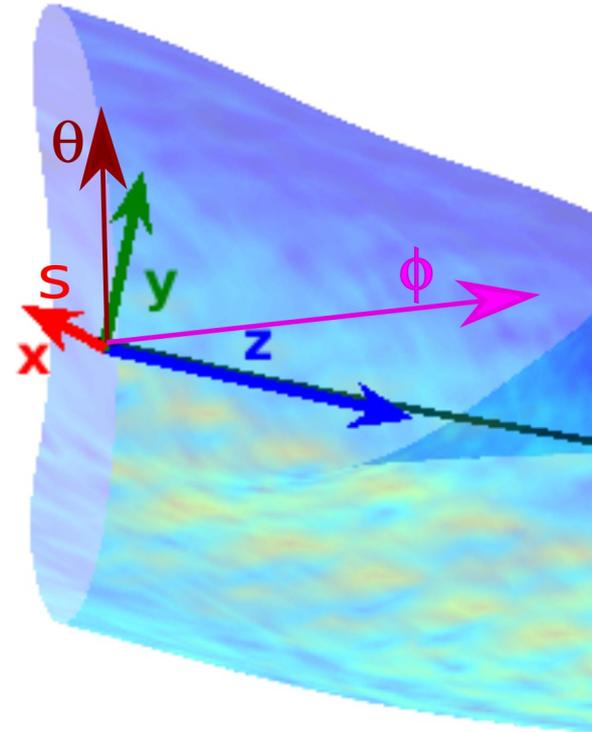
The gyrokinetic codes **EUTERPE** and **GENE-3D**

- **EUTERPE**

- Particle-in-cell, lagrangian, delta-f.
- Real space representation of the fields in PEST (s, θ, ϕ) coordinates. (cylindrical coordinates for markers).
- Full volume or radial annulus simulations.
- Electrostatic and electromagnetic.

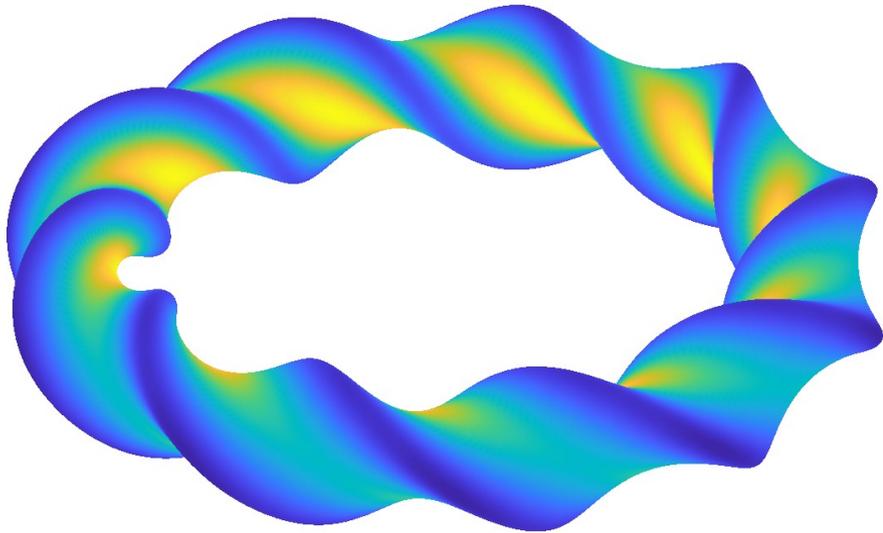
- **GENE-3D**

- Continuum, delta-f code.
- Field aligned coordinates x, y, z . ($x = \sqrt{s}$, $y = c_y(q\theta + \phi)$, $z = \theta$)
- Real space representation of the fields in x, y, z .
- Can be run in flux-tube, full surface or global domains.
- Electrostatic and electromagnetic.



Magnetic configurations of LHD and W7-X

LHD standard

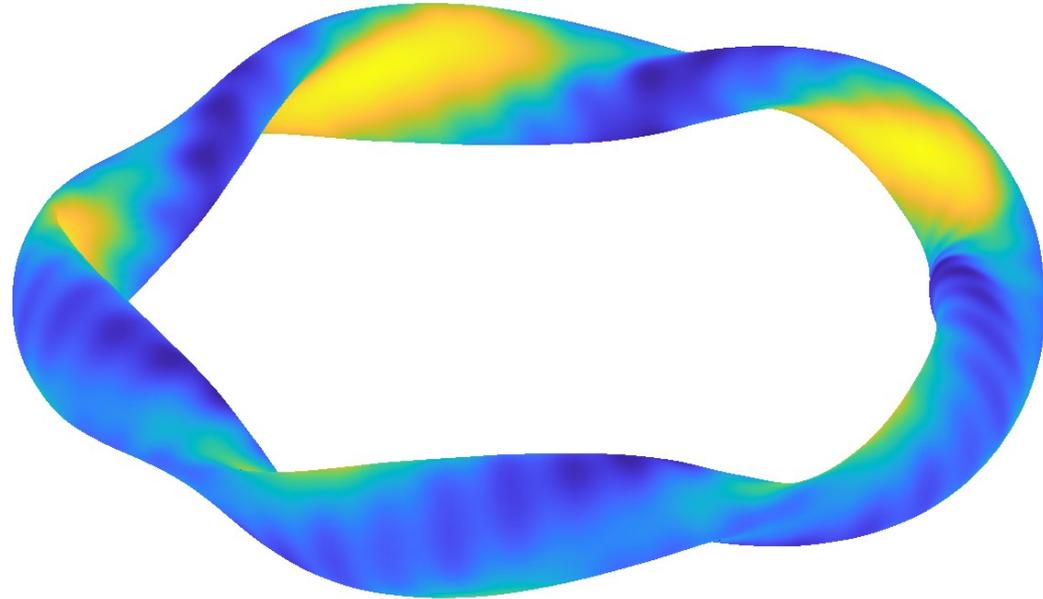


$R=3.7$ m, $a=0.6$ m ,10 periods

$\iota(0)-\iota(a) \sim 0.3-1.25$

magnetic shear $\sim 0.08-0.17$

W7-X Ref 168 (standard)



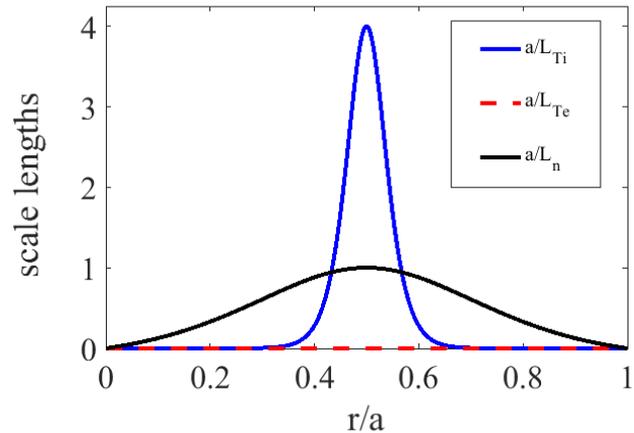
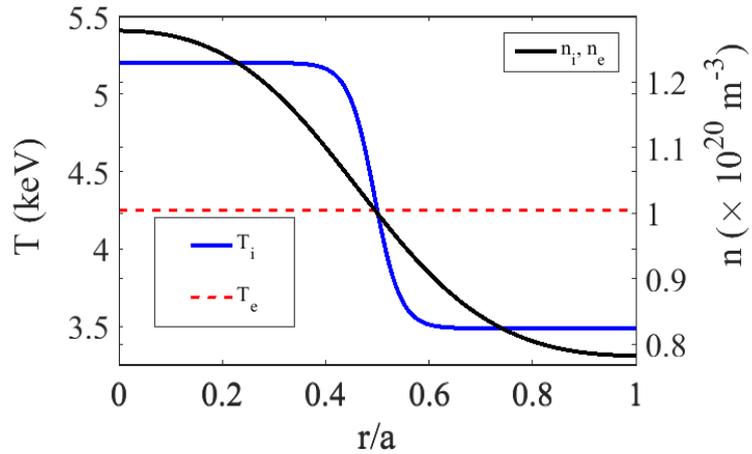
$R=5.5$ m, $a=0.52$ m, 5 periods

$\iota(0)-\iota(a) \sim 0.86-0.97$

magnetic shear < 0.0015

Linear stability of ITGs

Kinetic profiles for linear ITG simulations



- Model profiles for Ti and n.

$$X = X_* \exp \left[\frac{-\kappa_x}{1 - \text{sech}_x^2} \left(\tanh\left(\frac{r-r_0}{a\Delta_x}\right) - \text{sech}_X^2 \right) \right]$$

- Flat electron temperature profile ($a/L_{Te}=0$).
- Sharp ion temperature gradient at middle radius ($a/L_{Ti}=4$).
 $a/L_n=1, a/L_{Ti}=4 \Rightarrow \eta_i=4$ at r/a .

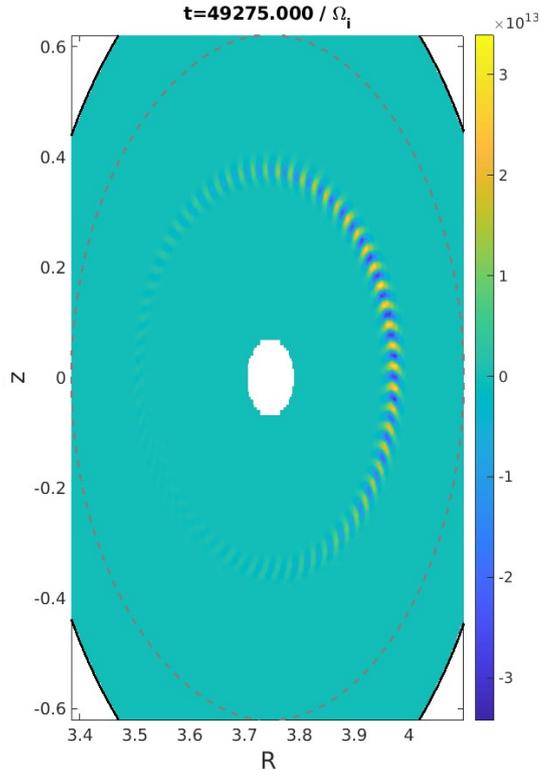
- With these settings we run linear simulations (with adiabatic electrons) and find the most unstable mode

We compare the mode structure and characterize its growth rate and frequency.

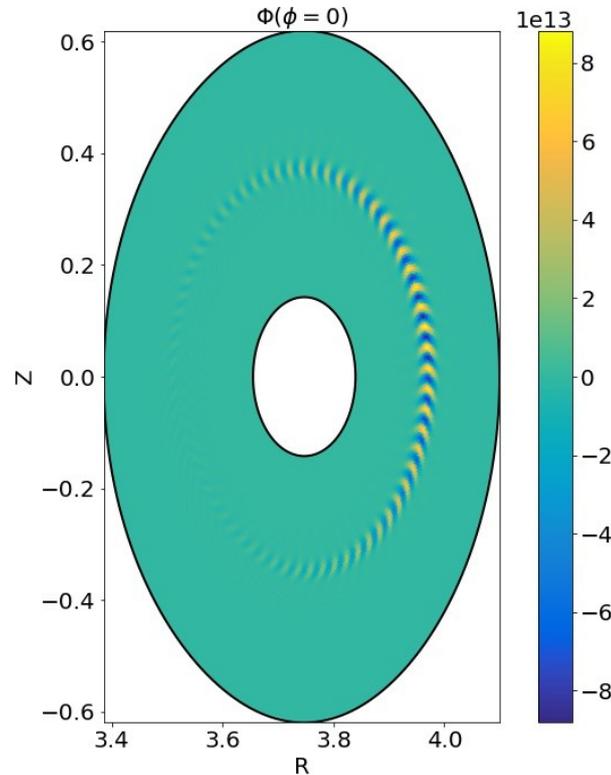
We will study the influence of an electric field on the linearly unstable modes.

Linear simulations of ITGs in LHD

EUTERPE



GENE-3D



- Simulations restricted in radius

GENE-3D: ($r/a=0.2-0.8$)

EUTERPE: ($r/a=0-1$)

← Electrostatic potential at $\phi=0$.

- **Very good agreement on the most unstable mode:**

Mode number.

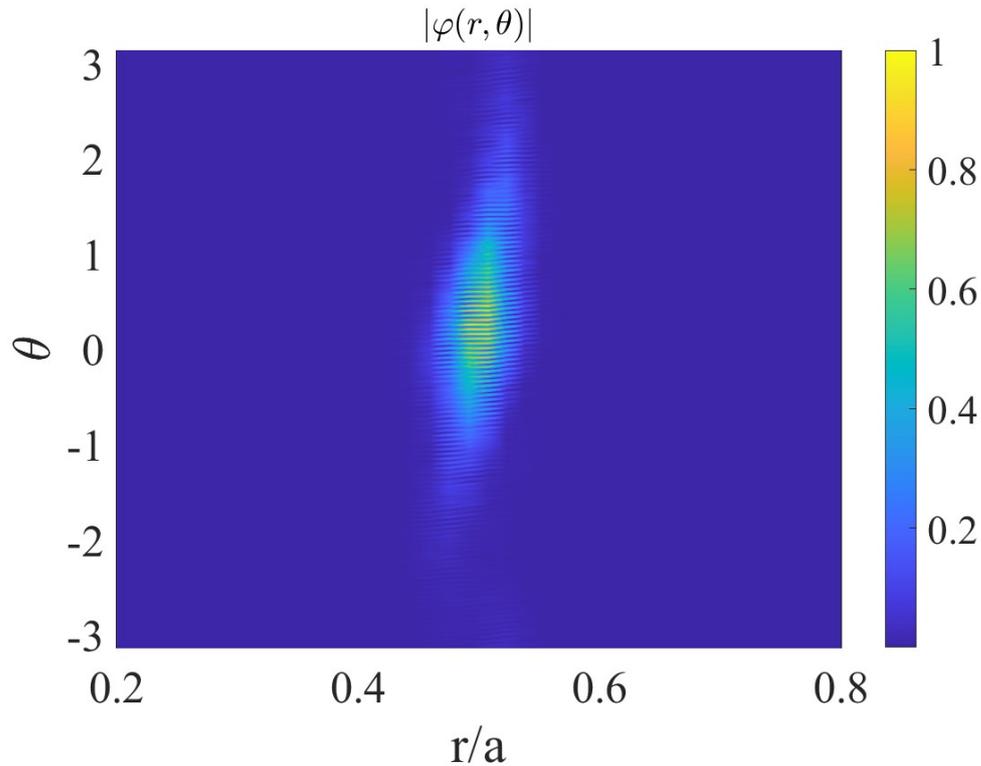
Location.

Mode structure.

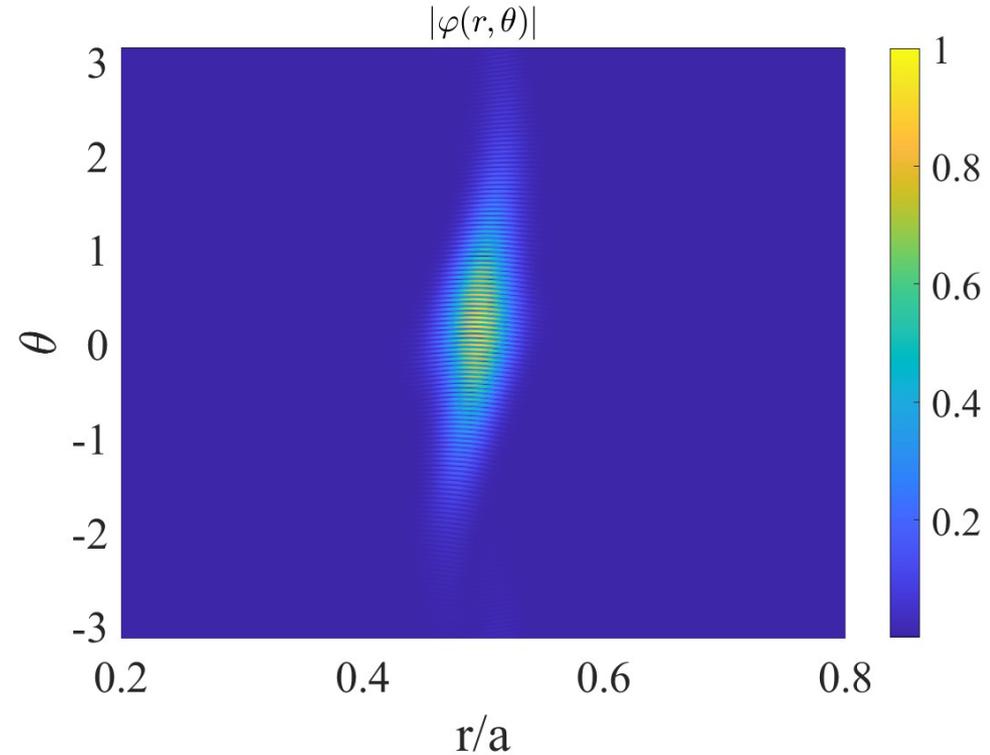
Growth rate and frequency.

Linear simulation of ITGs in LHD: electrostatic potential at $\phi=0$

EUTERPE



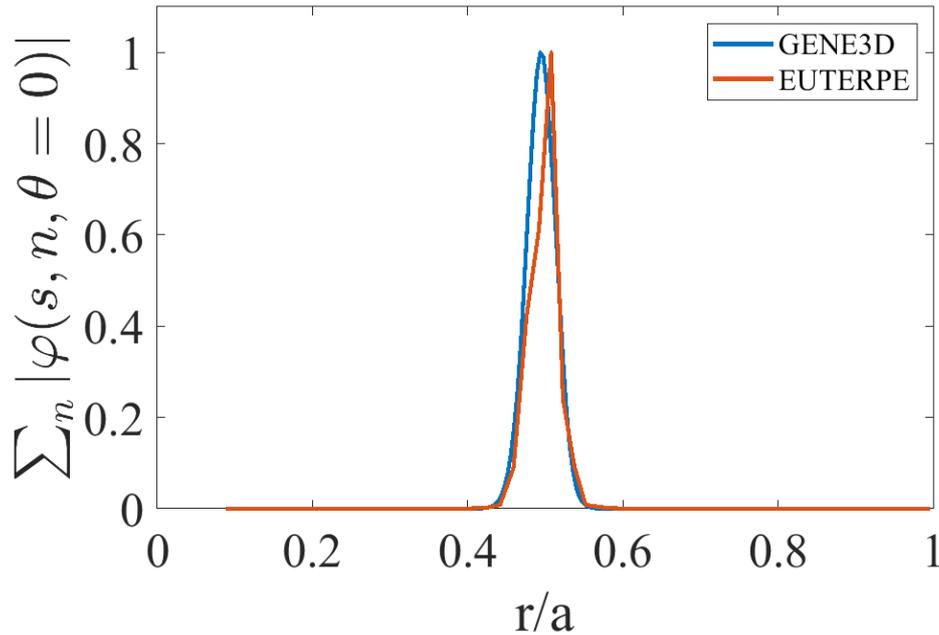
GENE-3D



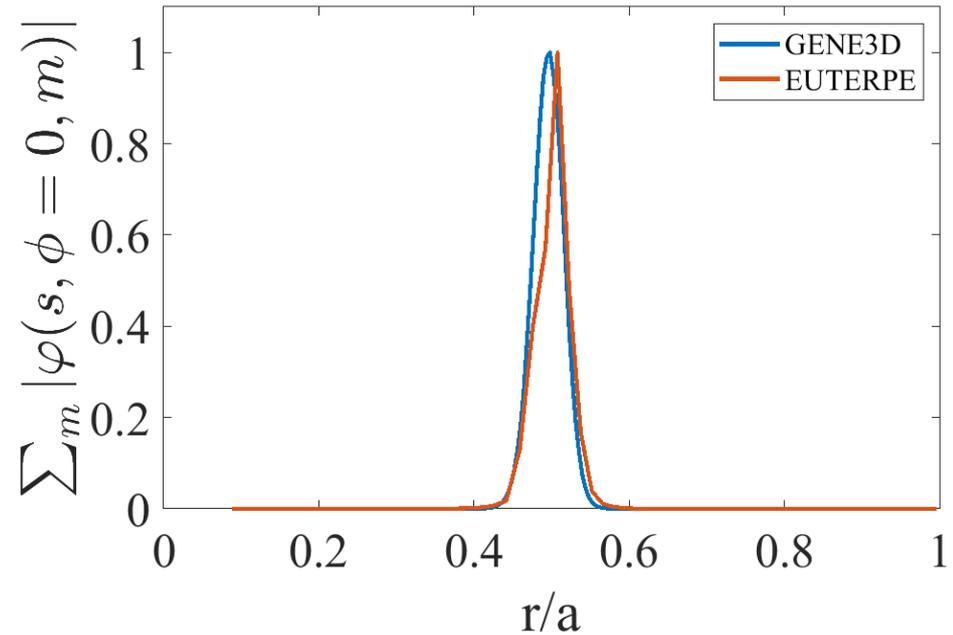
- **Very good agreement of the location of the mode.**

Linear simulations of ITGs in LHD: mode location

Mode amplitude at $\theta = 0$



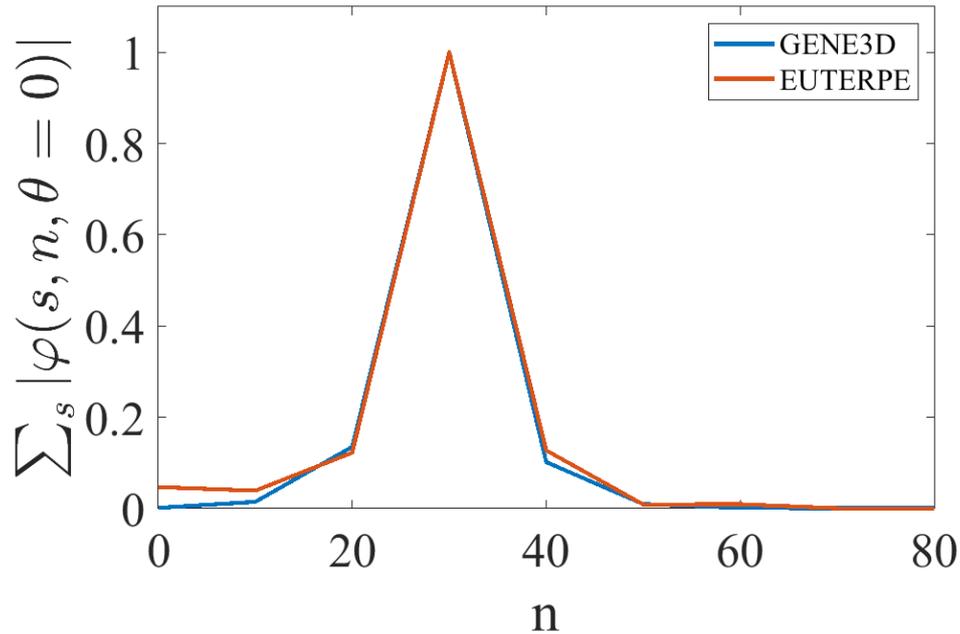
$\phi = 0$



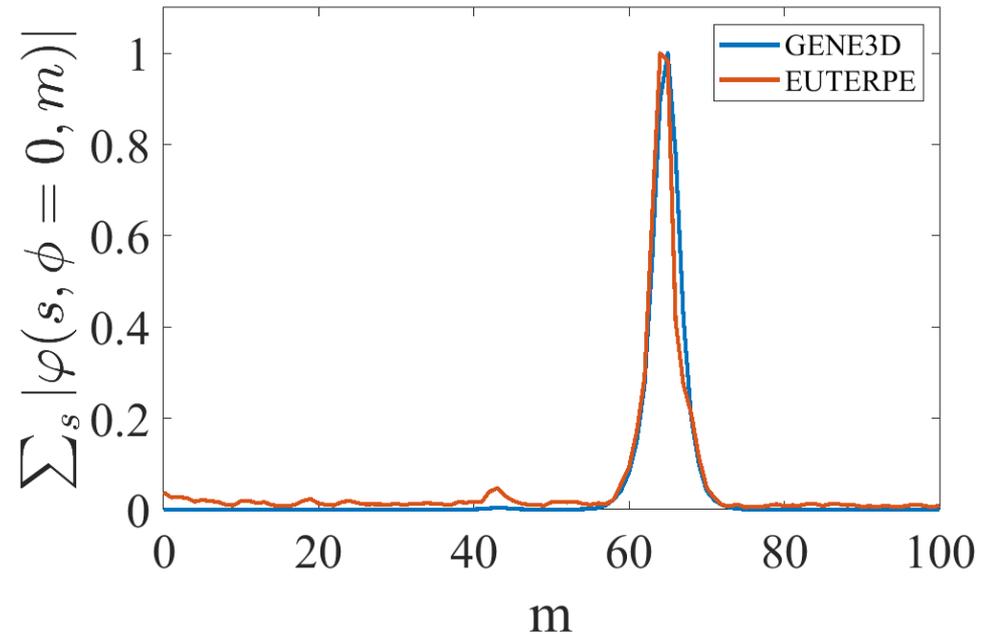
- **Very good agreement between both codes on the radial localization of modes.**

Linear simulations of ITGs in LHD: Fourier analysis

Fourier spectrum at $\theta = 0$



$\phi = 0$



- A mode around $n=30$, $m=64$ is found in both codes.
- **Very good agreement on the mode wavenumbers.**

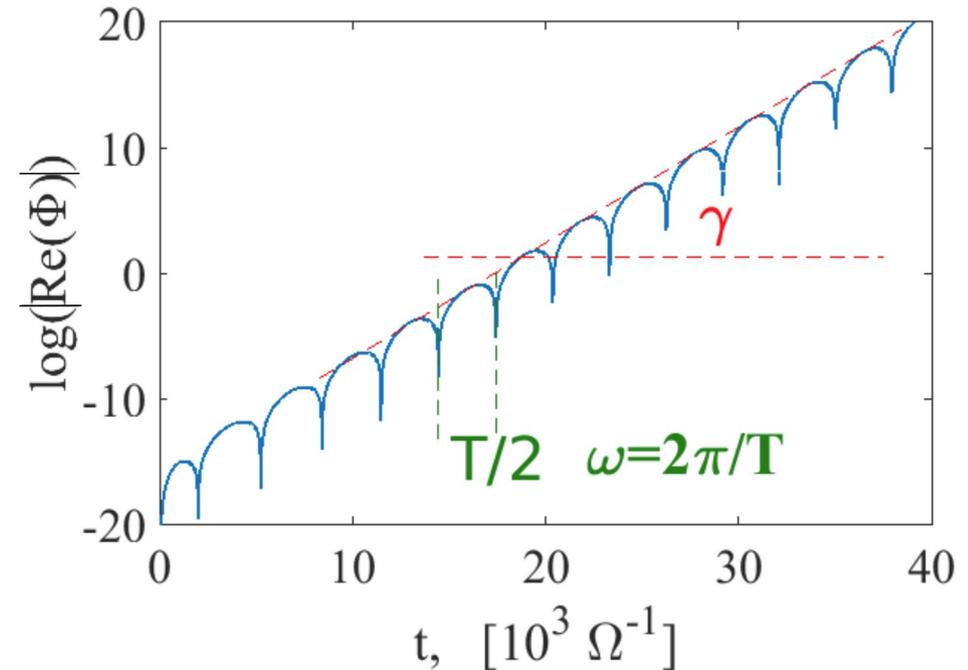
Linear simulations of ITGs in LHD: quantitative comparison

- **Growth rate** and **frequency** of the unstable modes are compared.
 - extracted from a fit of the time signal (an approximation can be made from the total electrostatic energy time evolution)

$$\phi_k(t) = A_k e^{(\gamma t + i\omega)t}$$

- The most unstable mode is chosen as the one with largest amplitude at the end of the simulation/analysis.

	EUTERPE	GENE-3D
n	30	30
m	64	65
γ (v_{th}/a)	0.21	0.196
ω (v_{th}/a)	0.39	0.405



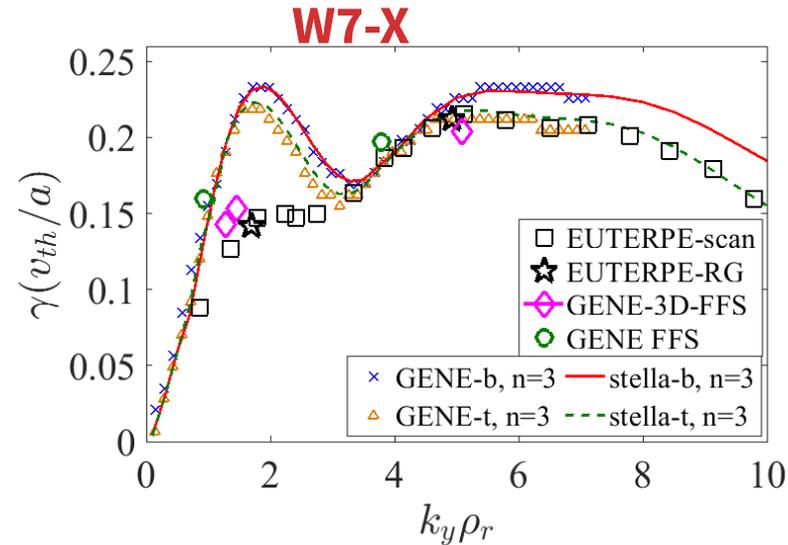
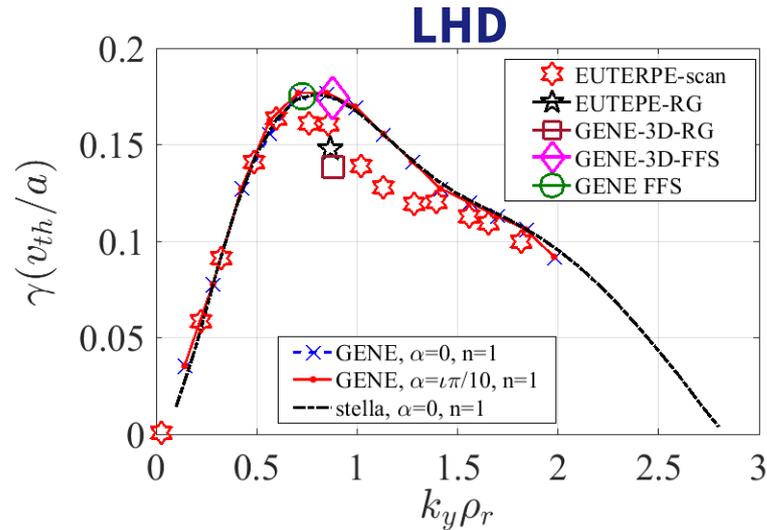
Very good agreement on growth rate and frequency of the modes.

Agreement on γ within 7% and ω within 4%.

Linear simulation of ITGs in W7-X

- From previous works: **The spectrum of ITG modes is much wider in W7-X than in LHD**

(see Regaña JPP 2020, Sanchez NF 2021)

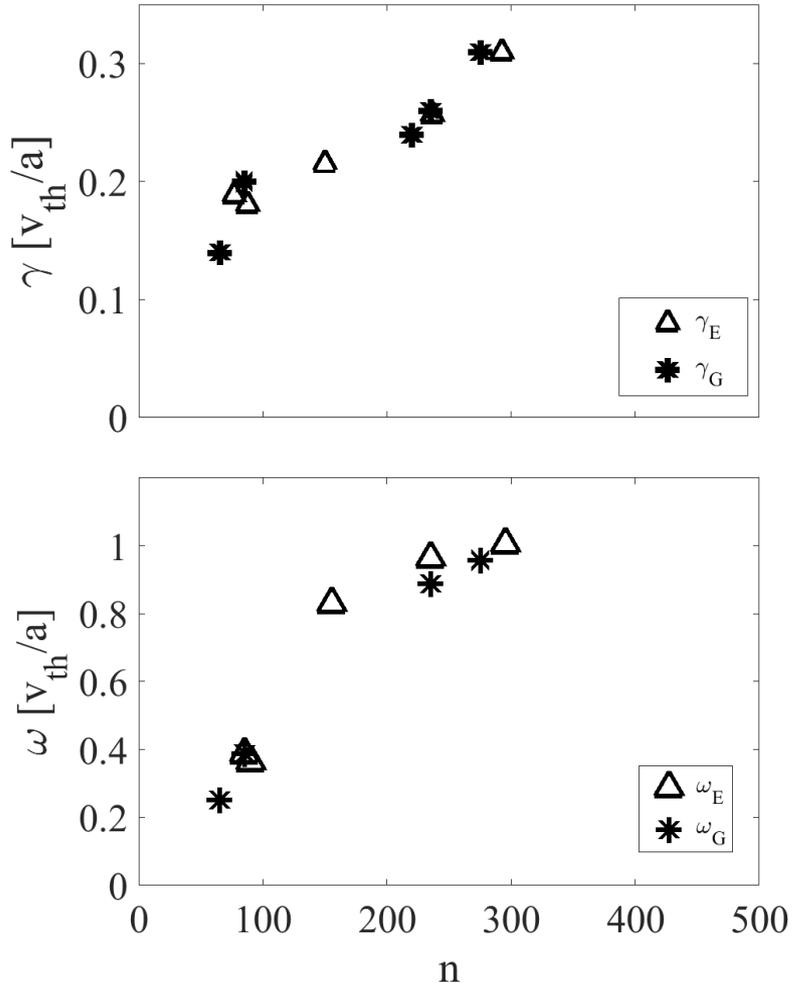


Comparison of simulations in different domains:

- GENE-3D and EUTERPE: radially global (**RG**).
- GENE-3D and GENE: full flux surface (**FFS**).
- GENE and stella: flux tube (**b, t, $\alpha=0, \alpha=\ell\pi/10$**).

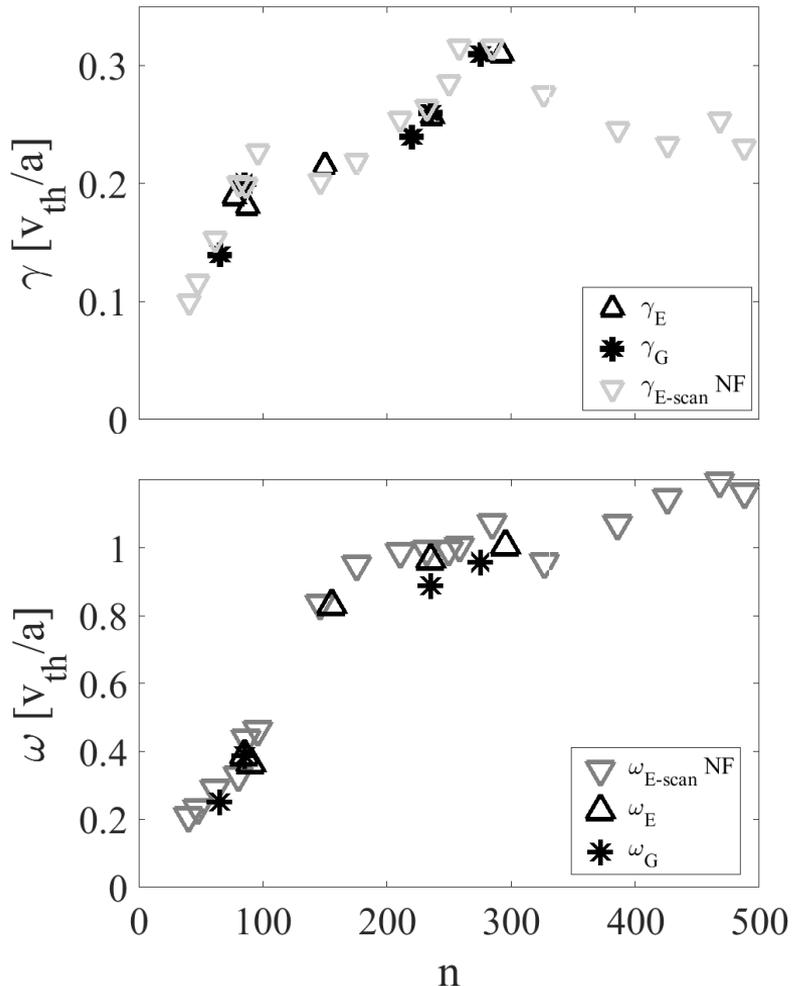
- Huge resolution required to resolve the most unstable modes in W7-X with a global code.
- Different modes can be obtained by different codes due to numerical differences.
- A single-point comparison can be suited for LHD but not for W7-X.

Linear simulations of ITGs in **W7-X**



- Set of simulations with both codes with increasing resolutions over the flux surface.
 - GENE-3D: $n_y=64, 128, 192, 256, 384$.
 - EUTERPE: $n_\theta=128, 256, 384, 512, 768$.
 - Same kinetic profiles as those used in LHD.
 - Different modes are obtained by both codes in single simulations
 - but
- Good agreement is found in the γ -n and ω -n curves and the most unstable modes are identified.

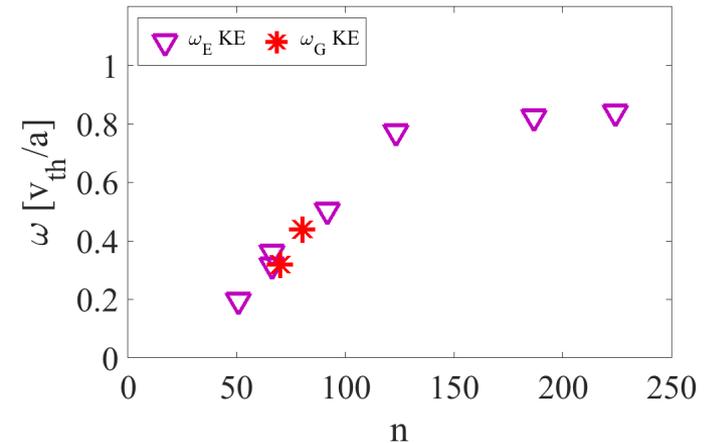
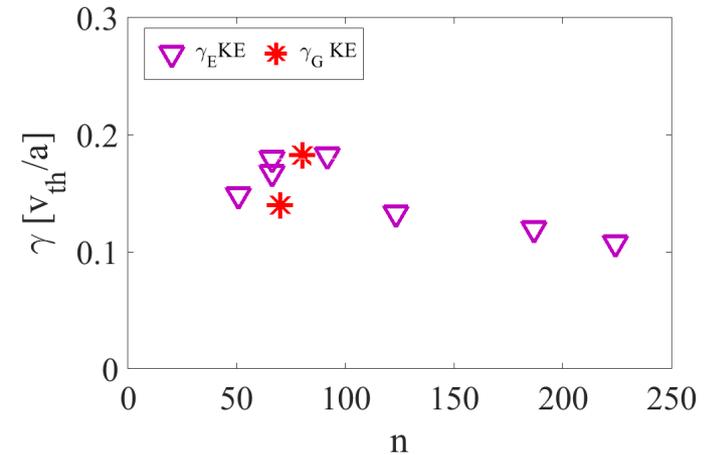
Linear simulations of ITGs in **W7-X**



- **Set of simulations with both codes with increasing resolutions over the flux surface.**
 - GENE-3D: $n_y=64, 128, 192, 256, 384$.
 - EUTERPE: $n_\theta=128, 256, 384, 512, 768$.
- **Same kinetic profiles as those used in LHD.**
- **Different modes are obtained by both codes in single simulations**
but
- **Good agreement is found in the γ -n and ω -n curves and the most unstable modes are equally identified.**
- **Exploration using a narrow filter with EUTERPE and using a phase factor (∇)**
 - agrees with results with wider filters (GENE/EUTERPE).
 - confirms most unstable modes for $n < 300$.

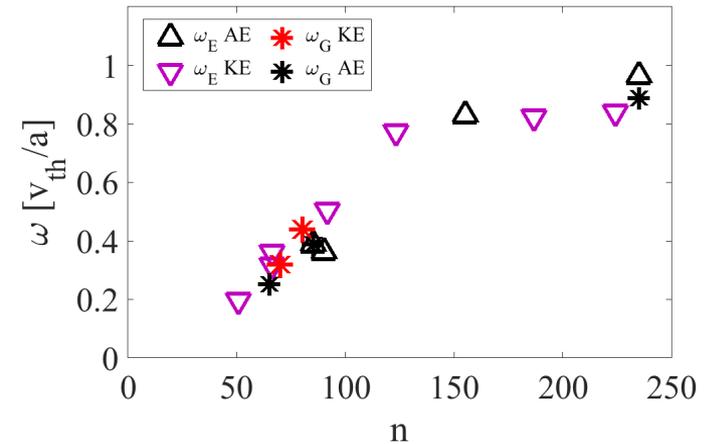
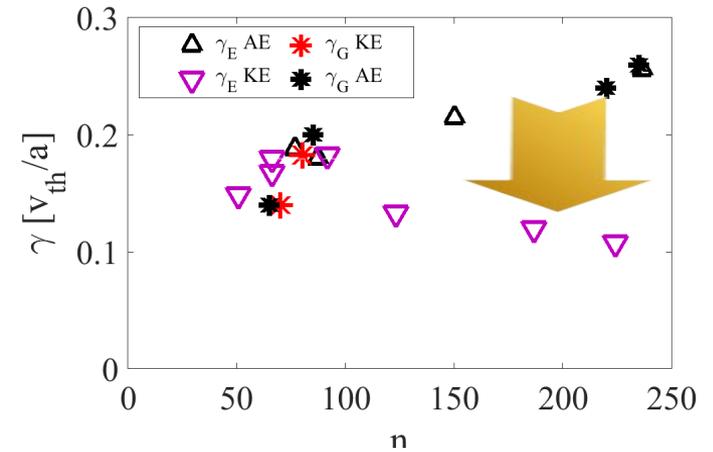
Linear simulations in **W7-X** with kinetic electrons

- Simulations using the same n and T profiles as for adiabatic electrons
 - **GENE-3D** simulations are run with a wide filter and resolutions $n_y = 64, 128$ and 256 .
($n_y = 128$ and 256 provide exactly the same results).
 - **EUTERPE** simulations with $n_\theta = 64, n_\phi = 32$ and moving the Fourier filter center for scanning the Fourier spectrum (using phase factor).
- **Agreement between codes in growth rate and frequency of the modes.**



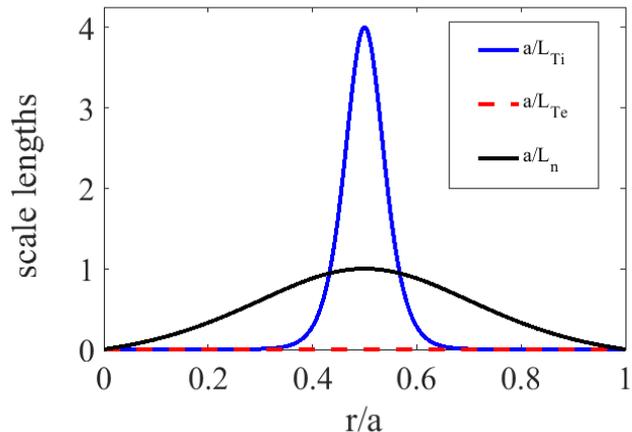
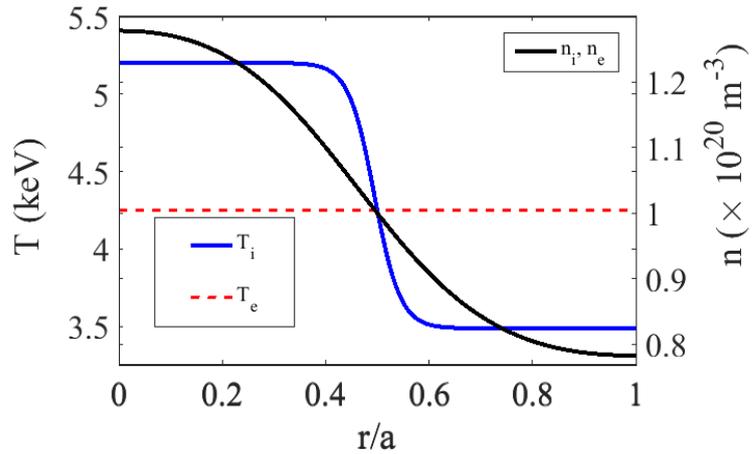
Linear simulations in **W7-X**: kinetic vs adiabatic electrons

- Simulations using the same n and T profiles as for adiabatic electrons
 - **GENE-3D** simulations are run with a wide filter and resolutions $n_y = 64, 128$ and 256 .
($n_y = 128$ and 256 provide exactly the same results).
 - **EUTERPE** simulations with $n_\theta = 64, n_\phi = 32$ and moving the Fourier filter center for scanning the Fourier spectrum (using phase factor).
- **Rough agreement between codes in growth rate and frequency of the modes.**
- **Stabilization of smaller scales by kinetic electrons.**
 - The frequency is not much affected.
 - The growth rate rate is significantly reduced for modes with $n > 100$ with respect to the adiabatic-electron case.



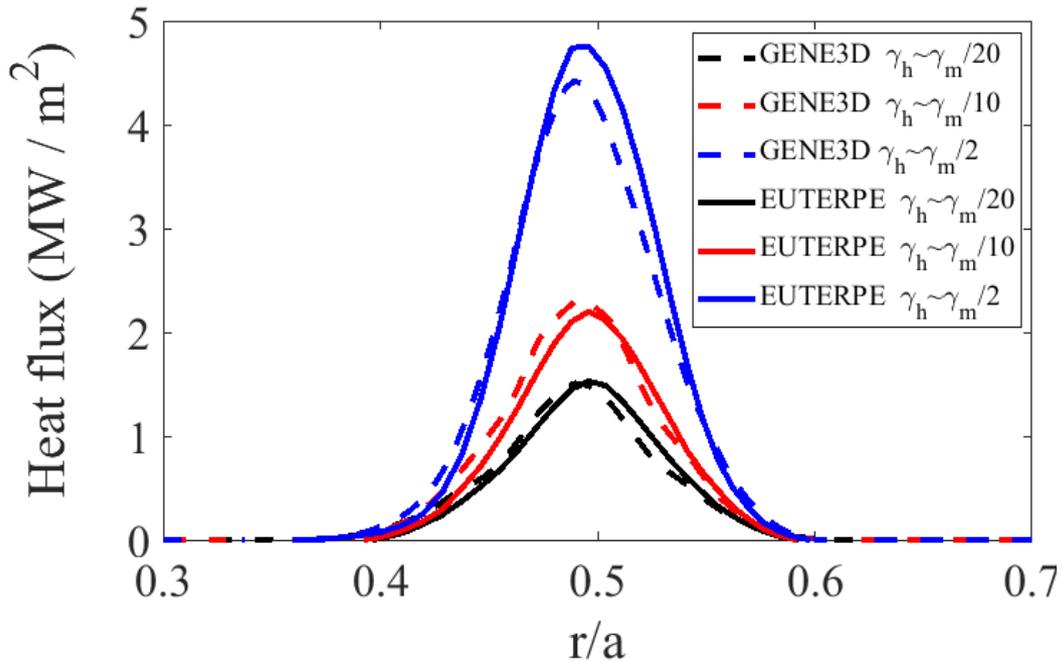
Non linear simulations of ITGs

Nonlinear simulations in LHD



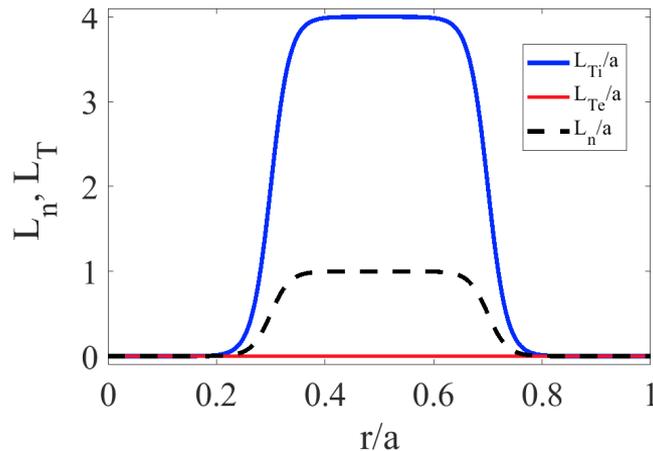
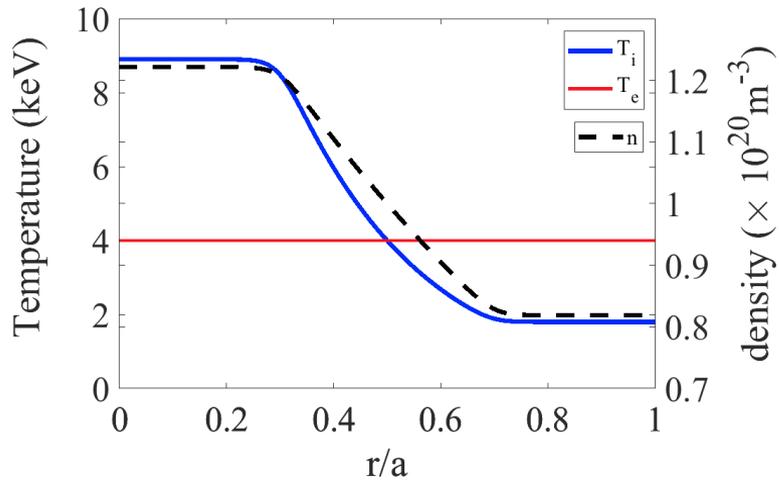
- **Same kinetic profiles as in linear simulations**
 - Strong T_i gradient at middle radius.
 - Localized source of turbulence.
- **Nonlinear simulations are restricted in radius to reduce computational cost ($0.1 < r/a < 0.9$).**
 - Adiabatic electrons.
 - A source term is used to sustain the profiles (see [McMillan PoP 2009](#)).
 - Weight smoothing is used in EUTERPE for improving the S/N ratio (see [Sánchez JPP 2020](#)).

Nonlinear simulations in LHD: turbulent heat flux



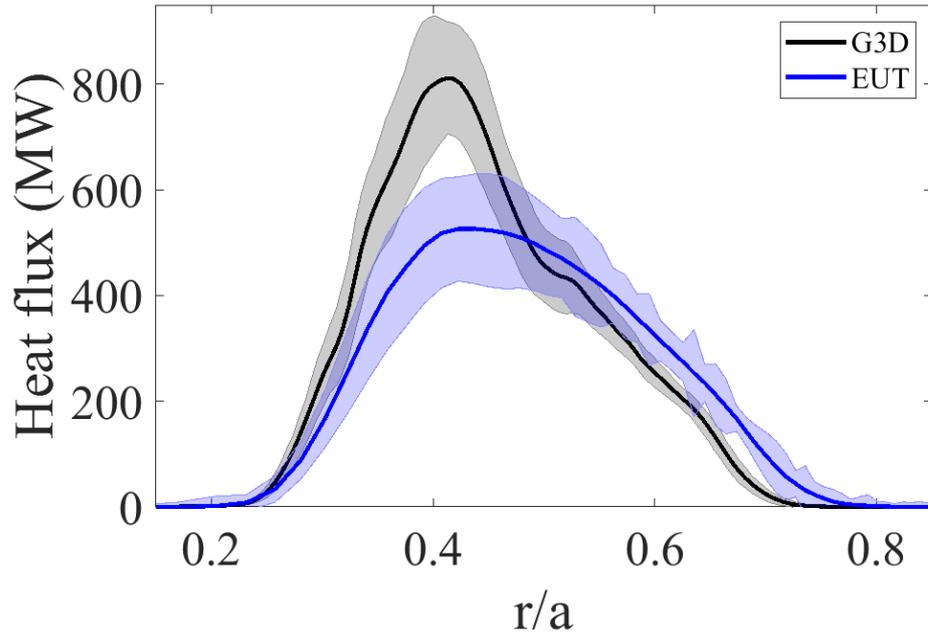
- **The density and temperature profiles relax as the turbulence saturates.**
 - Heating source is used to sustain the profiles (see [McMillan PoP 2009](#)).
- **Series of simulations with different heating sources.**
 - Strong dependence on the heating source used to sustain the profiles.
- **Good agreement between codes for all values of the heating source.**

Nonlinear simulations in W7-X



- **Kinetic profiles similar to those in McMillan PoP 2009:**
 - Facilitate keeping the density and temperature profiles after nonlinear saturation.
- **Nonlinear simulations are restricted in radius to reduce computational cost ($0.1 < r/a < 0.9$).**
- **Adiabatic electrons.**
- **A source term is used to sustain the profiles (see [McMillan PoP 2009](#)).**
 - Source term frequency set to 1/10 of the maximum growth rate.
- **Weight smoothing is used in EUTERPE for improving the S/N ratio (see [Sánchez JPP 2020](#)).**

Nonlinear simulations in W7-X



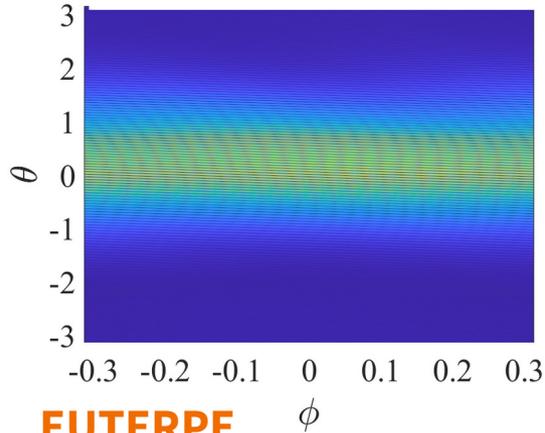
- Non linear simulations using the same heating source to sustain the profiles in both codes.
- The turbulent heat flux is averaged in a steady state time window.
Error shade corresponds to standard deviation.
- The peak heat flux is larger in GENE-3D than in EUTERPE by ~40%.

Localization of instabilities/turbulence

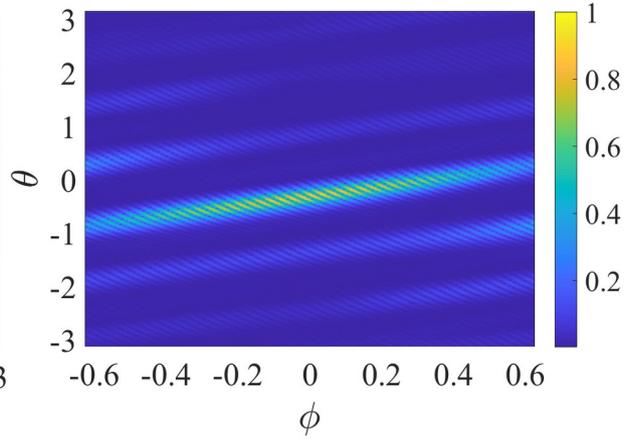
Localization of ITG modes over the flux surface

GENE-3D

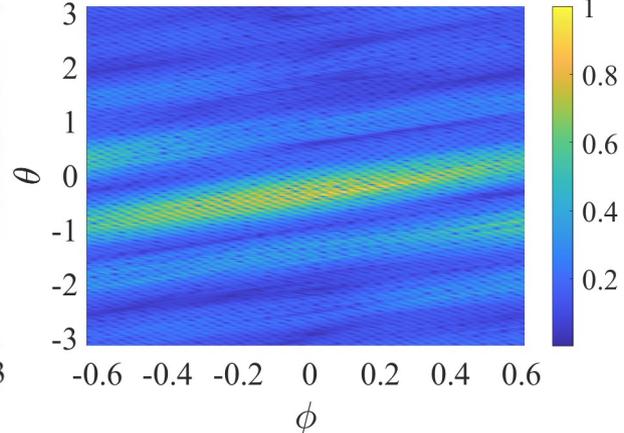
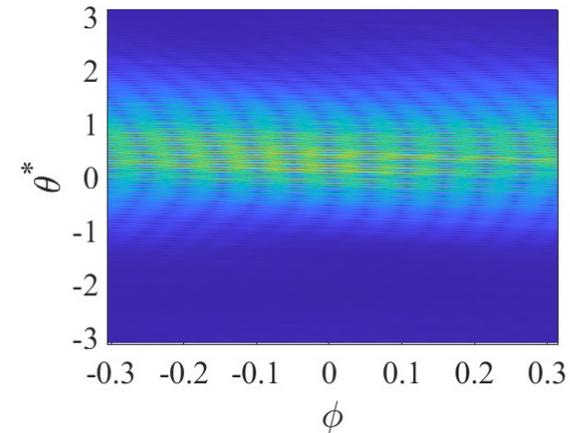
LHD



W7-X



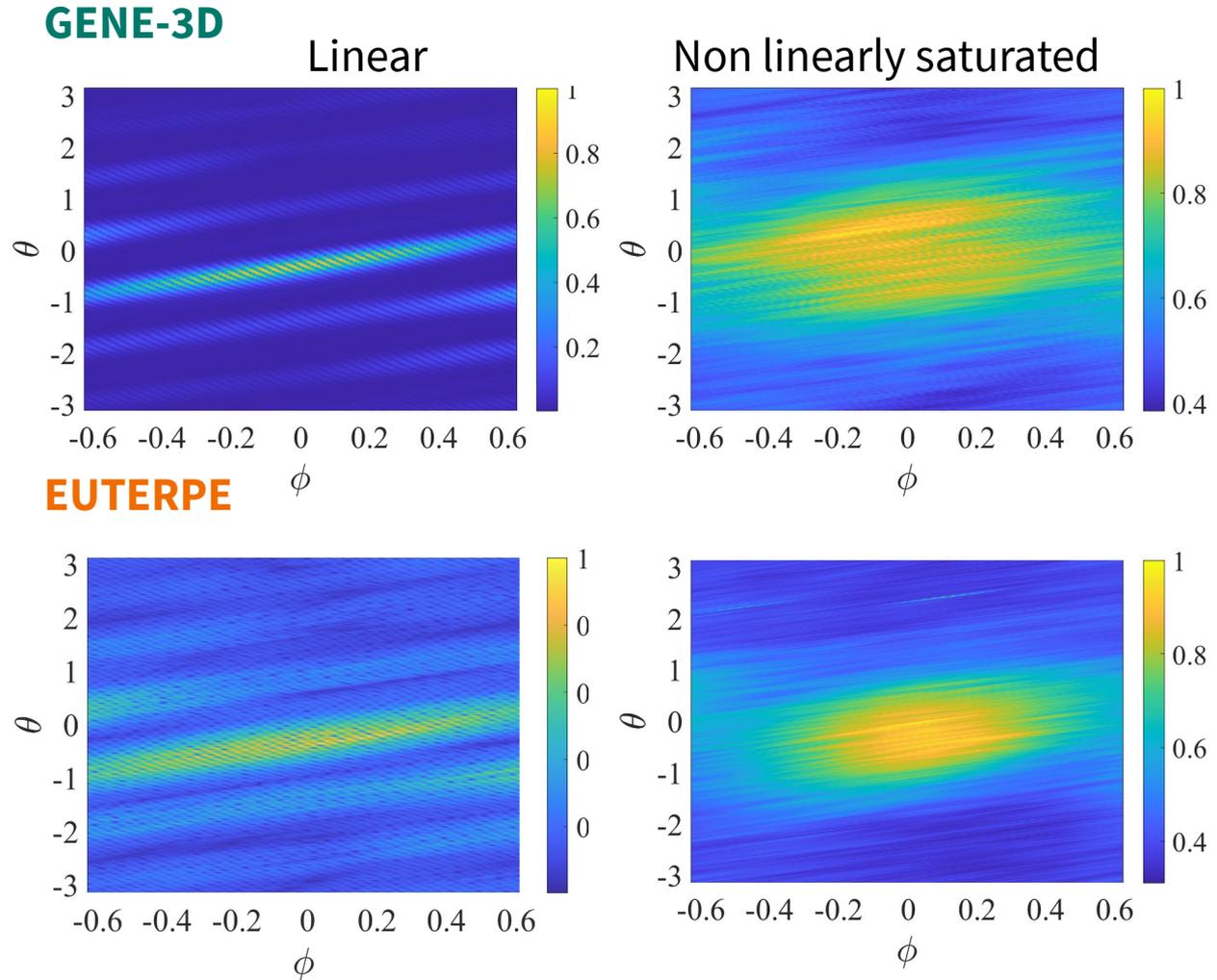
EUTERPE



- **Electrostatic potential at $r/a=0.5$** (normalized to its maximum value).
- **The ITG modes appear strongly localized**
 - More localized in W7-X than in LHD.
 - The maximum amplitude is aligned with the field lines.
 - **Agreement with previous works** ([Nadeem PoP 2001](#), [Kornilov PoP 2004](#))
- **Qualitative agreement between codes in the localization of modes.**

Localization of ITG turbulence over the flux surface (W7-X)

- **Amplitude (left) and RMS (right) of density at $r/a=0.5$ in linear/nonlinear simulations** (normalized to its maximum value).
- **The maximum of fluctuations is more spread as compared to linear modes.**
- **Qualitative agreement between codes on the localization.**



Influence of the electric field

Influence of the electric field on instabilities localization (W7-X)

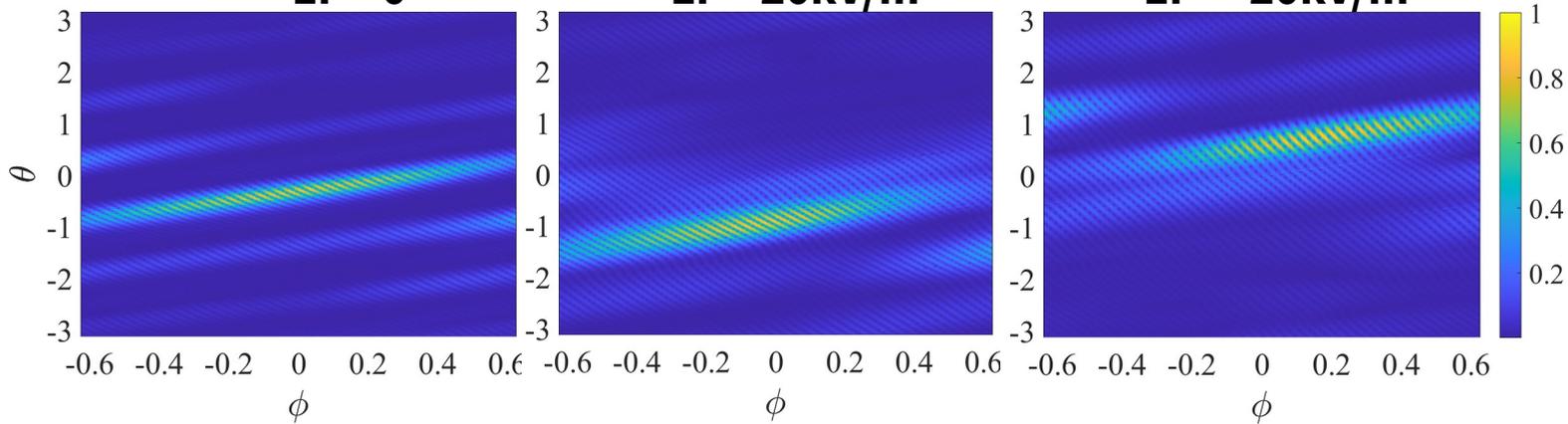
GENE-3D

Instant value of density

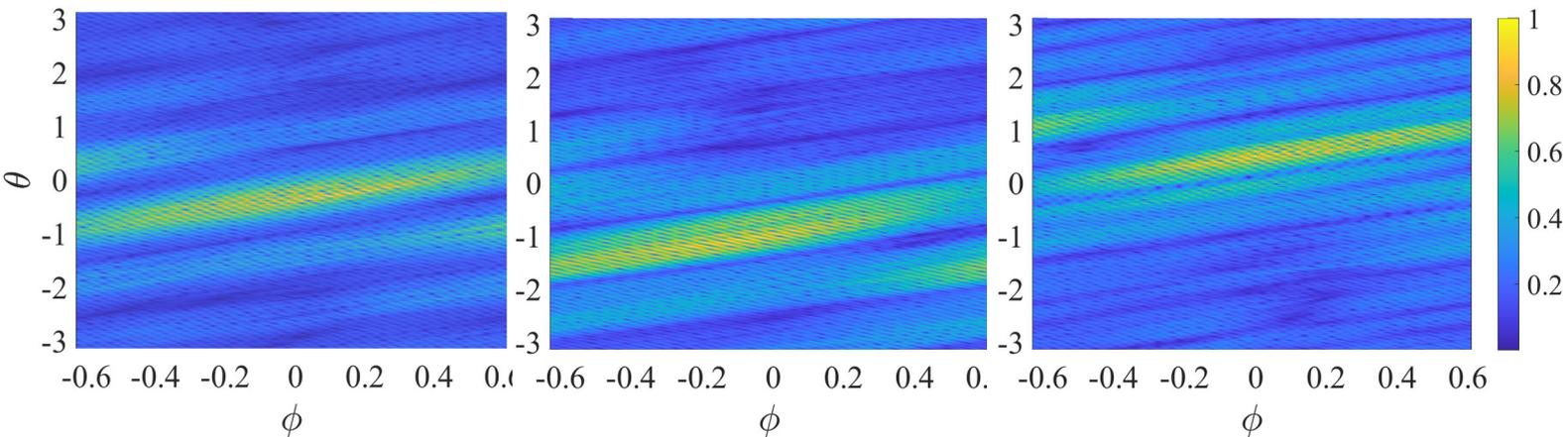
$E_r = 0$

$E_r = 20\text{kV/m}$

$E_r = -20\text{kV/m}$



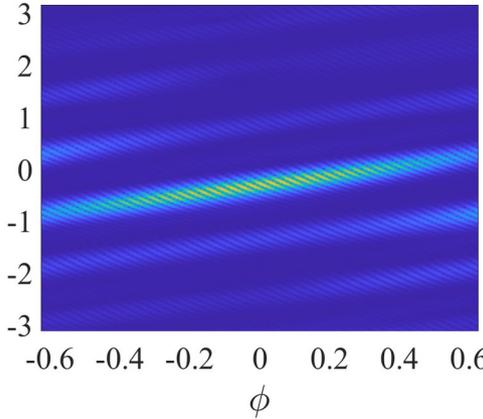
EUTERPE



Influence of the electric field on instabilities localization (W7-X)

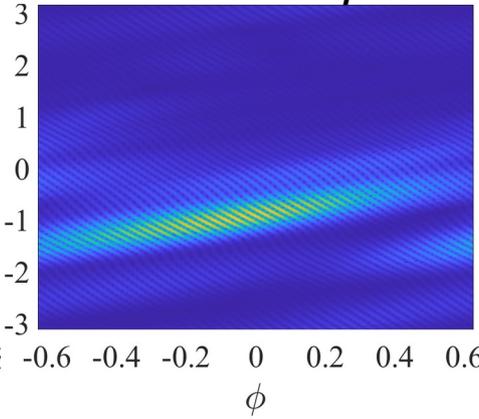
GENE-3D

$E_r = 0$

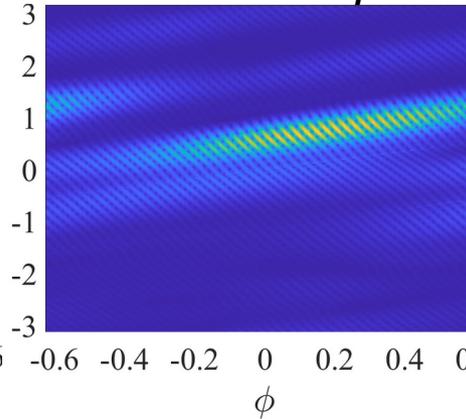


Instant value of density

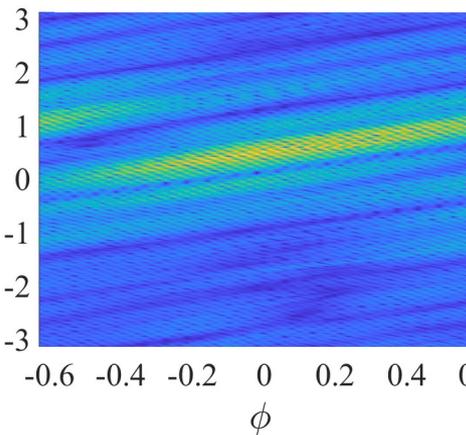
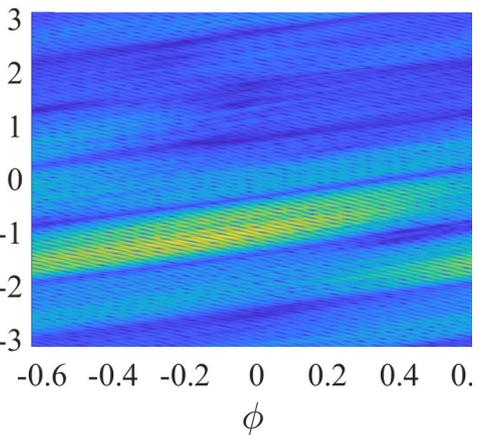
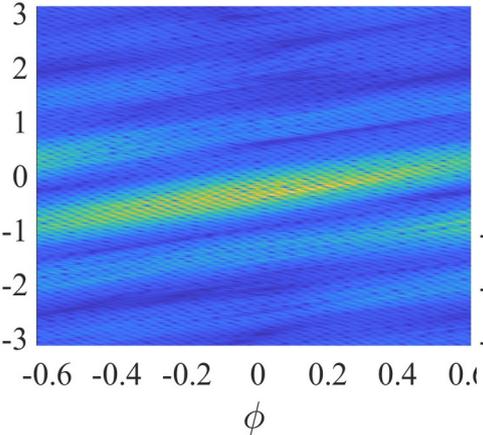
$E_r = 20\text{kV/m}$



$E_r = -20\text{kV/m}$



EUTERPE

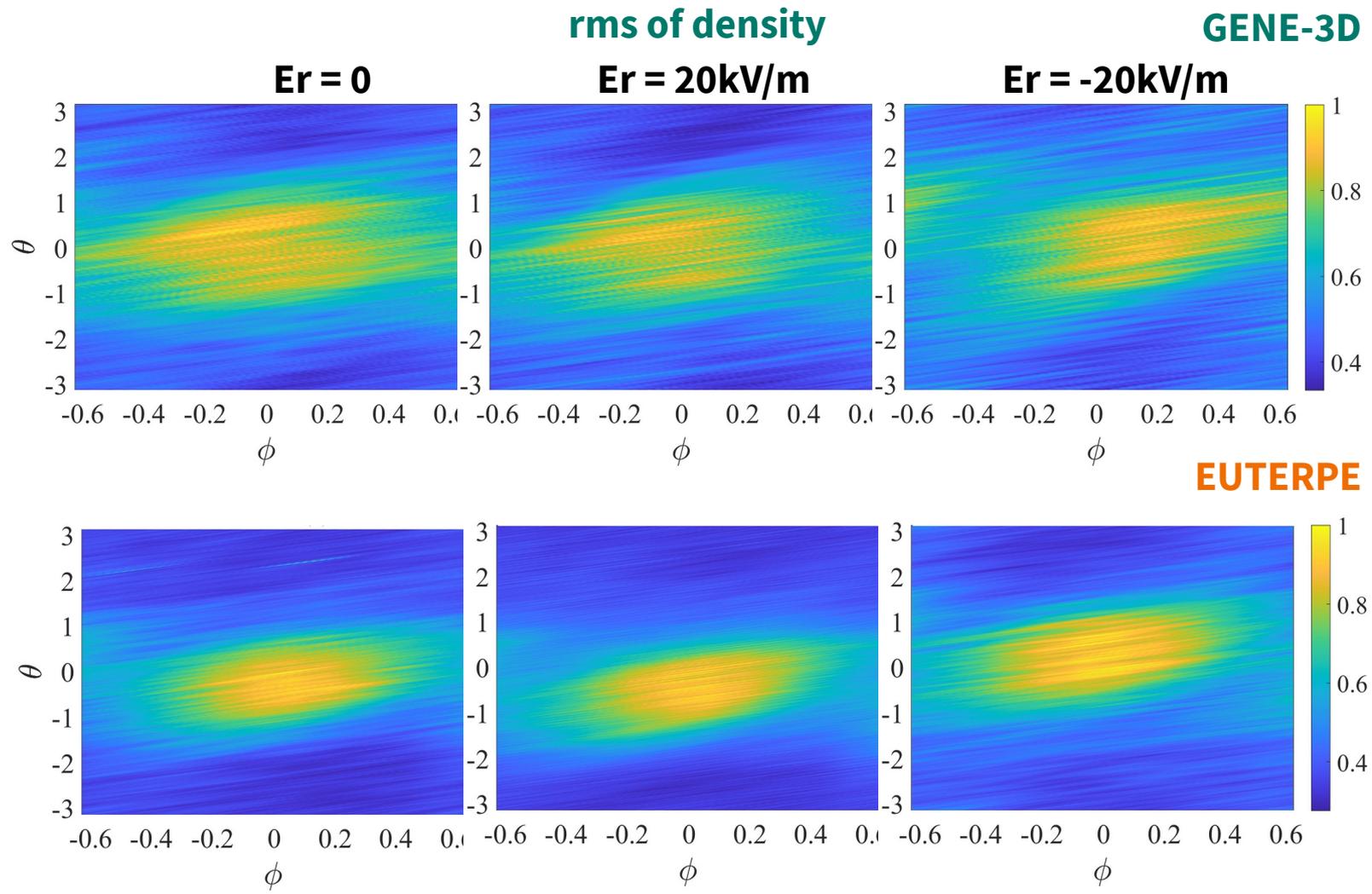


- The electric field displaces the location of maximum instability in the poloidal direction.

agreement with previous works (Riemann PPCF 2015).

- The sign of the displacement changes with the sign of E_r .
- There is a qualitative agreement between the codes.

Nonlinear simulations of ITGs in **W7-X**: mode localization



Nonlinear simulations of ITGs in **W7-X**: mode localization

- **Agreement between the codes EUTERPE and GENE-3D.**

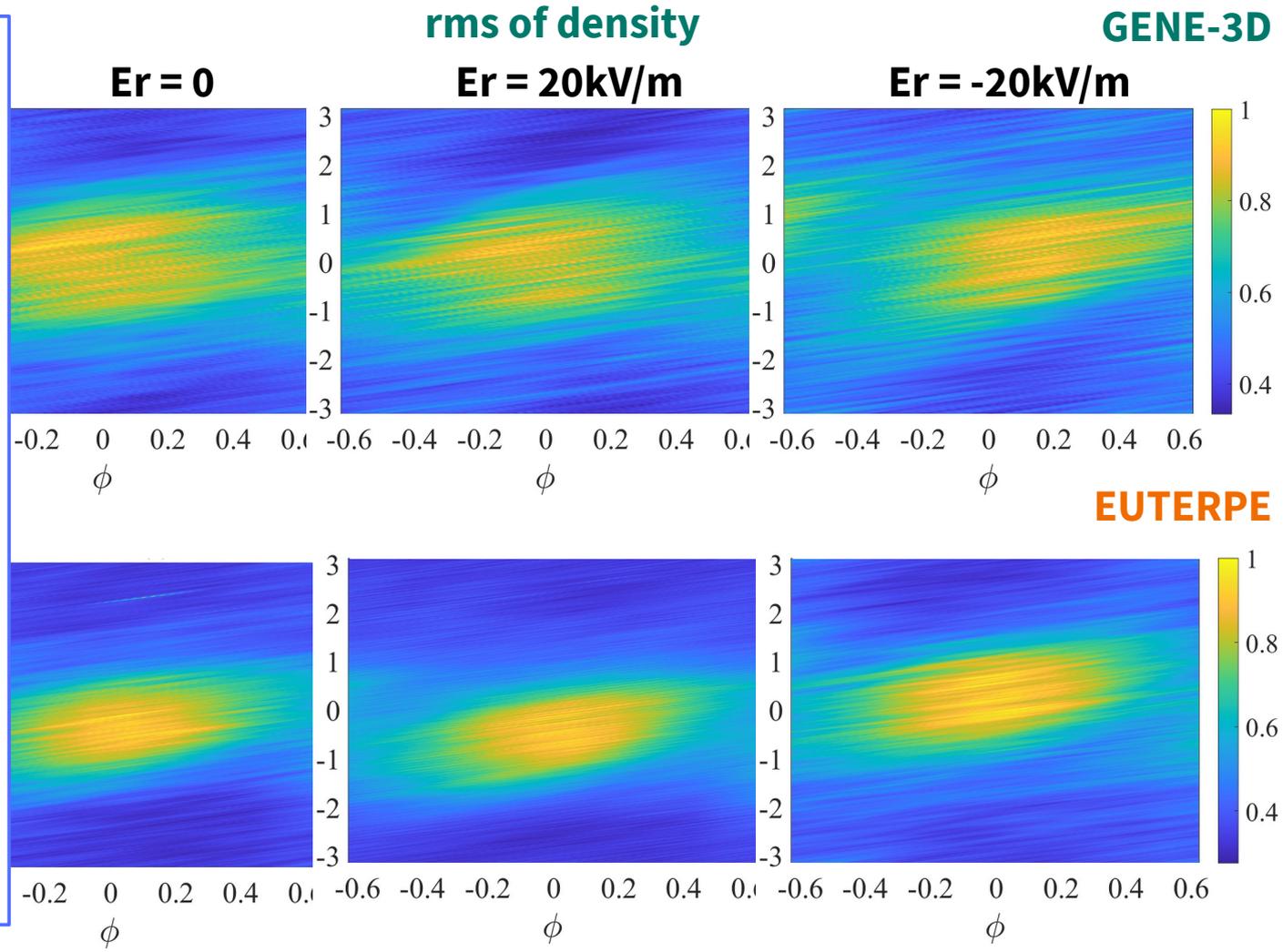
- qualitative agreement on the localization of maximum fluctuations.
- quantitative agreement on the ratio max/min rms.

- **The localization of turbulence is much weaker than that of instabilities.**

- weaker than previously reported in flux surface simulations.

- **Displacement due to E_r is less clear in the nonlinear regime.**

- weaker than previously reported in full surface simulations.



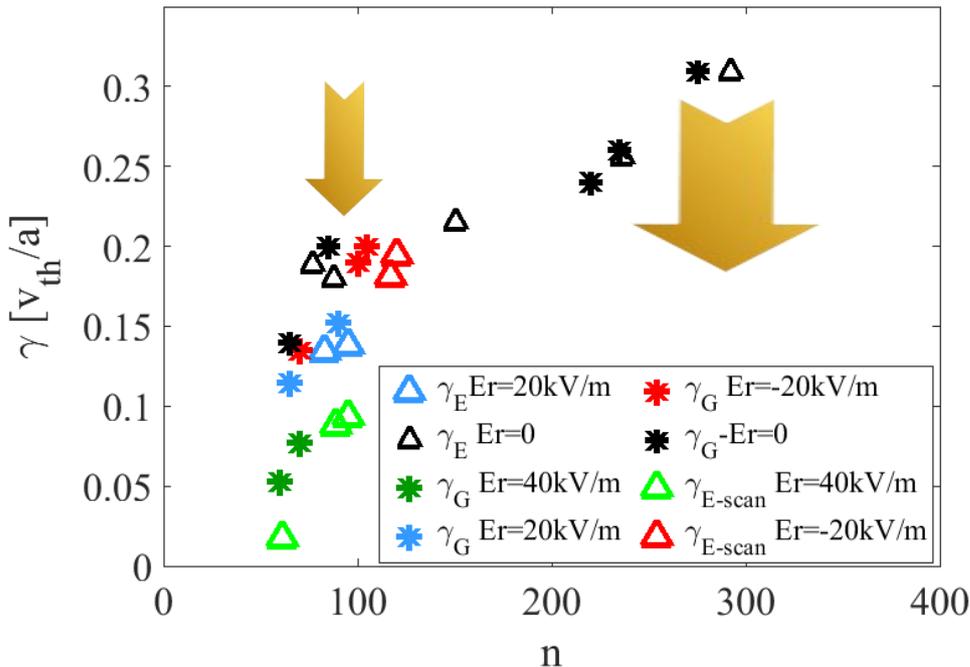
Influence of E_r on growth rate and frequency in LHD

- Set of linear simulations with different strength of the electric field:
 - $E_r=0, 6\text{kV/m}, -6\text{kV/m}, 12\text{kV/m}, -12\text{kV/m}$
 - The growth rate (γ) and frequency (ω) of the most unstable mode are compared.
- The mode number is not much affected.
- Positive E_r reduces γ (stabilization) and increases ω .
- Negative E_r only slightly increase γ but reduces ω .
- **Very good agreement between codes.**

LHD		EUTERPE	GENE-3D
$E_r=0$	n, m	n=30, m=64	n=30, m=65
	γ (v_{th}/a)	0.21	0.196
	ω (v_{th}/a)	0.39	0.405
$E_r=6$ kV/m	ν, μ	n=30, m=65	n=30, m=65
	γ (v_{th}/a)	0.189	0.184
	ω (v_{th}/a)	0.921	0.93
$E_r=-6$ kV/m	ν, μ	n=30, m=64	n=30, m=65
	γ (v_{th}/a)	0.2154	0.2
	ω (v_{th}/a)	0.1457	0.143
$E_r=12$ kV/m	ν, μ	n=30, m=66	n=30, m=65
	γ (v_{th}/a)	0.1799	0.17
	ω (v_{th}/a)	1.494	1.49
$E_r=-12$ kV/m	ν, μ	n=30, m=64	n=30, m=64
	γ (v_{th}/a)	0.2158	0.202
	ω (v_{th}/a)	0.6721	0.67

Influence of E_r on the growth rate in W7-X

- Set of simulations with increasing resolution in angles over the flux surface.
- E_r : 0, 20kV/m, -20kV/m, 40kV/m



- Including the electric field reduces the growth rate (linear stabilization).

The reduction increases with E_r strength

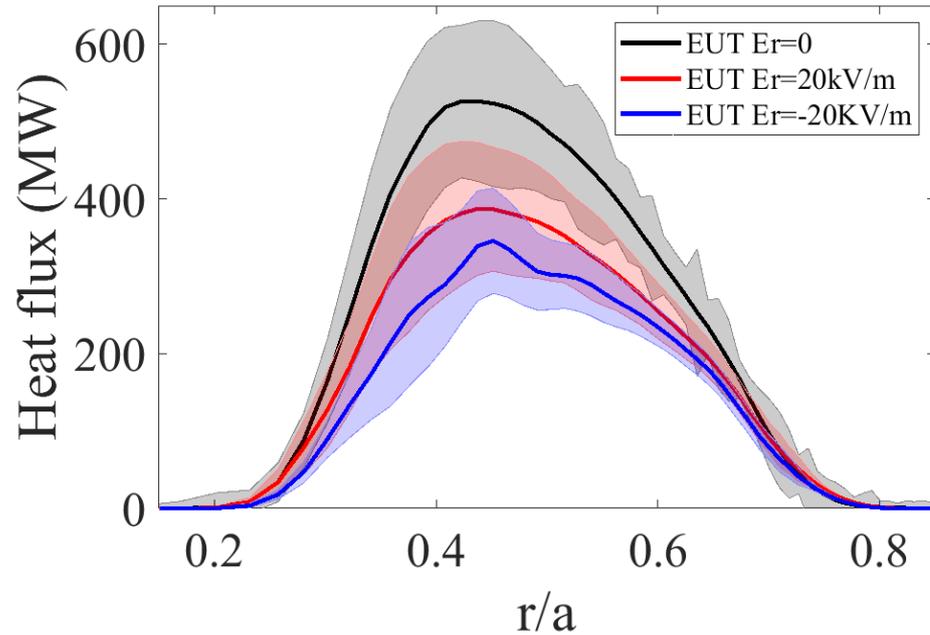
$$\gamma_{40\text{kV/m}} < \gamma_{20\text{kV/m}} < \gamma_0$$

- The stabilization is stronger in large wave numbers (small scales).
- The sign of E_r matters (20kV/m vs -20kV/m). $\gamma_{20\text{kV/m}} < \gamma_{-20\text{kV/m}} \sim \gamma_0$

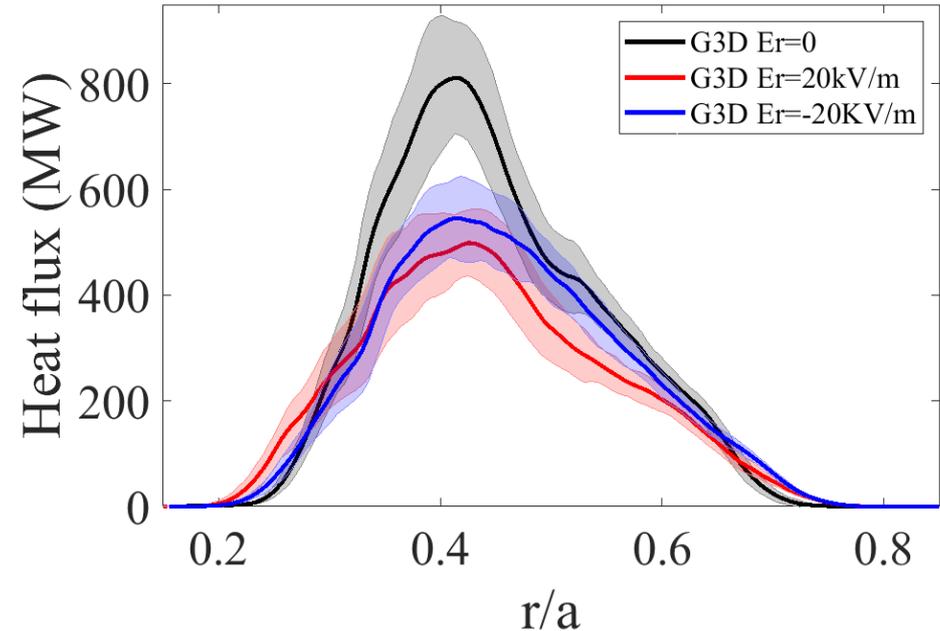
Reasonable agreement between codes.

Influence of E_r on turbulent transport (**W7-X**)

EUTERPE



GENE-3D



- Significant reduction of turbulent heat flux ($\sim 35\%$) including an electric field.
- The reduction of heat flux is almost independent of the sign of E_r .

Summary and conclusions

•Linear simulations

- Very good agreement on γ/ω of the most unstable mode in LHD.
- The mode structure is perfectly reproduced by both codes in LHD.
- The dependence of γ/ω with n is equally captured by both codes in W7-X.

•Nonlinear simulations

- Very good agreement between codes in LHD with respect to the turbulent heat flux.
- In W7-X the peak turbulent heat flux obtained with GENE-3D is larger than with EUTERPE by ~40%.

•Localization of instabilities/turbulence

- The linearly unstable modes are highly localized in LHD, and even more in W7-X.
- The localization of saturated turbulence is much smaller than that of linear instabilities.

•Influence of E_r

- The effect of E_r on and the localization of instabilities is captured by both codes in LHD and W7-X.
- A linear stabilization of ITG modes by E_r is found in both codes and both devices.
- A reduction of turbulent transport around 35% by E_r (W7-X) is observed in both codes.