Kinetic analysis of the collisional layer

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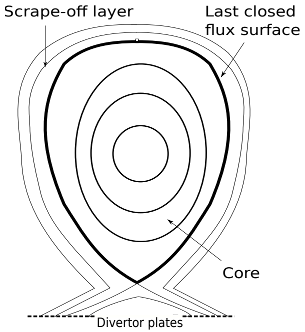
Motivation

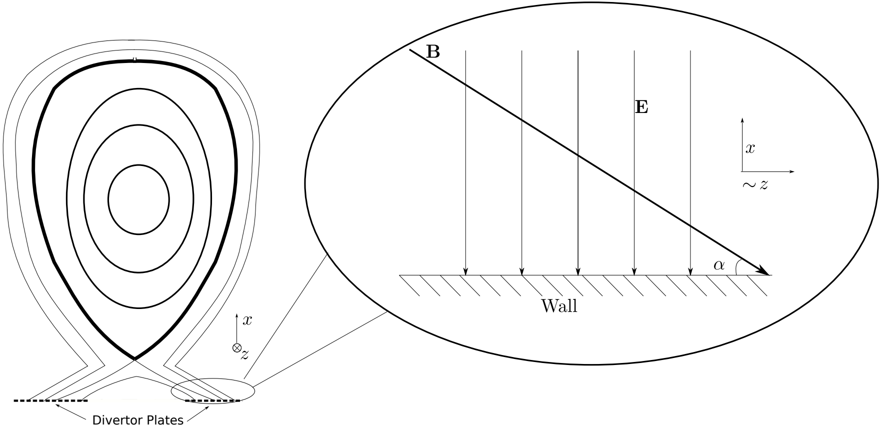
The goal of the project is to describe the distribution function of ions in the collisional plasma near the divertor region inside the tokamak.

- Plasma-wall interaction gives boundary conditions for the plasma core.
- Plasma-wall interaction determines the heat fluxes on plasma facing components of the device.



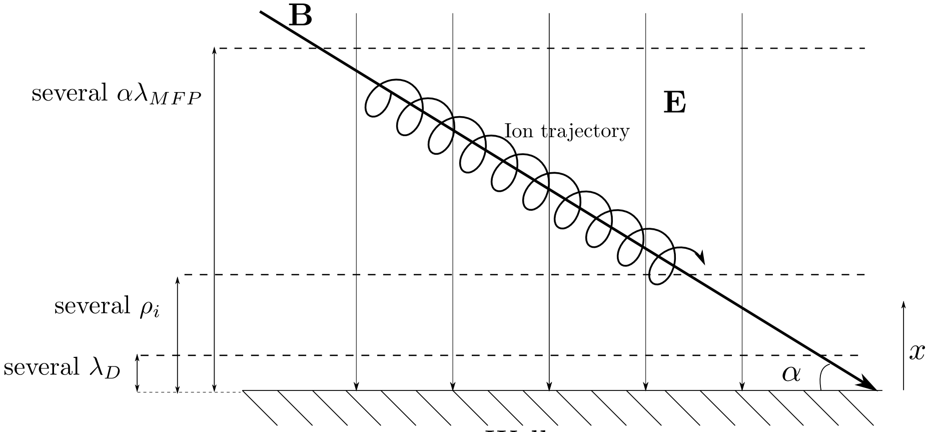
A sketch of magnetic flux surface contours in the poloidal plane of a fusion device.

Plasma-wall boundary near the divertor



Boundary layers

Near the divertor $\lambda_D \ll \rho_i \ll \alpha\lambda_{MFP} \ll L$, where L is device size, and so we can split the plasma-wall boundary into three separate layers: Debye sheath, magnetic presheath and the collisional presheath. In this project we focus on the collisional presheath.



Boltzmann Equation

We look at the case of a negatively charged wall. Here the electrons are repelled by the wall and most of them are not absorbed. Therefore we can describe electrons with a Boltzmann distribution

$$n_e = n_{e\infty} \exp\left(\frac{e(\phi - \phi_\infty)}{T_e}\right).$$

- n_e is the electron density.
- ϕ is the electric potential.
- ϕ_∞ is the electric potential far away from the wall.
- $n_{e\infty}$ is the electron density far away from the wall.
- T_e is the temperature far away from the wall.

Drift kinetic approximation

In the collisional layer, for ions, we can use the drift kinetic approximation

$$\frac{\partial f}{\partial t} - v_\parallel \sin\alpha \frac{\partial f}{\partial x} + \frac{Ze}{m_i} \sin\alpha \frac{\partial \phi}{\partial x} \frac{\partial f}{\partial v_\parallel} = C[f, f], \quad (1)$$

where f is the gyroaveraged ion distribution function and $C[f, f]$ is the Fokker-Planck collision operator for ion-ion collisions. The electric field is $\vec{E} = -\frac{\partial \phi}{\partial x} \hat{x}$.

Together with quasineutrality $Zn_i = n_e$ and the adiabatic electron approximation this gives us a system of equations for the unknowns $f(t, x, v_\parallel, v_\perp)$ and $\phi(t, x)$.

Boundary conditions

Far away from the wall, at $x \to \infty$, the plasma is highly collisional and the distribution function, to lowest order, is a Maxwellian.

$$f(x \to \infty, v_{\parallel} > 0, v_{\perp}, t) = f_M = n_{i\infty} \left(\frac{m_i}{2\pi T_{i\infty}} \right)^{3/2} \exp\left\{ -\frac{m_i (v_{\parallel} - u_{\parallel\infty})^2 + m_i v_{\perp}^2}{2T_{i\infty}} \right\}. \quad (2)$$

Once ions enter the wall at $x = 0$ they can no longer return as they are absorbed by the negatively charged wall.

$$f(x = 0, v_{\parallel} \leq 0, v_{\perp}, t) = 0. \quad (3)$$

Distribution far away from the wall

In steady state the system of equations we want to solve is

$$-v_{\parallel} \sin\alpha \frac{\partial f}{\partial x} + \frac{Ze}{m_i} \sin\alpha \frac{\partial \phi}{\partial x} \frac{\partial f}{\partial v_{\parallel}} = C[f, f], \quad (4)$$

$$n_{e\infty} \exp(e\phi/T_e) = Zn_i, \quad (5)$$

where we chose the potential at infinity to be zero. At large x we expect f to be a Maxwellian plus a small perturbation. We can choose the distribution function and the potential to have a perturbation of the form

$$f = f_M + F \exp(-x/\lambda \sin\alpha), \quad (6)$$

$$\phi = \Phi \exp(-x/\lambda \sin\alpha), \quad (7)$$

where $\lambda \sin\alpha$ is the decay length of the perturbation and $\lambda \sin\alpha / x \ll 1$.

Distribution far away from the wall

Keeping terms to lowest order in $\exp(-x/\lambda \sin\alpha)$, we then perform a subsidiary expansion in two new expansion parameters

- $\epsilon = \lambda_{MFP}/\lambda \ll 1$, this means that we are ordering our lengthscales as $\lambda_{MFP} \ll \lambda \ll x$.
- $\Delta_s = (u_{\parallel\infty} - c_s)/c_s \ll 1$,

where $c_s = \sqrt{5T_{i\infty}/3m_i + ZT_e/m_i}$ is the adiabatic speed of sound in plasma at $x \to \infty$. They satisfy $\epsilon \sim \Delta_s \ll 1$.

Distribution far away from the wall

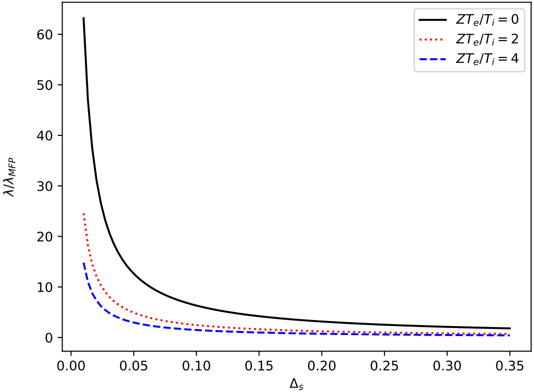
Expanding our equation in Δ_s and ϵ and solving the equations order by order gives us

$$\frac{\Delta_s}{\epsilon} = \frac{1.76}{5 + 3ZT_e/T_{i\infty}} + \frac{0.28}{(1 + 3ZT_e/5T_{i\infty})^{3/2}}. \quad (8)$$

Since ϵ must be positive for a solution that does not diverge exponentially far away from the wall, Δ_s must also be positive, therefore $u_{\|\infty} > c_s$. **The flow along the field lines of the Maxwellian at the boundary far away from the wall has to be at least sonic.**

Perturbation lengthscale

Perturbation lengthscale $\epsilon^{-1} = \lambda/\lambda_{MFP}$ as a function of $\Delta_s = (u_{\|\infty} - c_s)/c_s$, for a range of $ZT_e/T_{i\infty}$. It tells us that for larger Δ_s the perturbation to the Maxwellian solution decays quicker.



Time evolution

For general x we cannot solve the drift kinetic equation analytically, therefore we try and find a numerical solution to equation

$$\frac{\partial f}{\partial t} - v_{\|} \sin\alpha \frac{\partial f}{\partial x} + \frac{Ze}{m_i} \sin\alpha \frac{\partial \phi}{\partial x} \frac{\partial f}{\partial v_{\|}} = C[f, f]. \tag{9}$$

The left hand side of the drift kinetic equation is approximated with the first order Euler method, and the collision operator is treated implicitly.

$$\begin{aligned} f(t_\mu, x(t_\mu), v_{\|}(t_\mu)) - f(t_{\mu-1}, x(t_{\mu-1}), v_{\|}(t_{\mu-1})) \\ = \Delta t C[f(t_\mu), f(t_\mu)]. \end{aligned} \tag{10}$$

Velocity space

In **velocity space** we approximate the distribution function $f(v_\perp, v_\parallel)$ with the finite element basis $\{\psi_i(v_\perp, v_\parallel)\}_{i=1,..,N}$ and weights a_i

$$f(x > 0, v_\perp, v_\parallel, t) = \exp\left(\sum_{i=1}^{N} a_i(x,t)\psi_i(v_\perp, v_\parallel)\right). \tag{11}$$

This expansion ensures positivity of f.
For the element at the wall we have a boundary condition $f(x = 0, v_\parallel < 0) = 0$. To satisfy this, at the wall

$$f(x = 0, v_\perp, v_\parallel > 0, t) = v_\parallel \exp\left(\sum_{i=1}^{N} a_i(x = 0, t)\psi_i(v_\perp, v_\parallel)\right). \tag{12}$$

We use Galerkin method to solve the equations numerically.

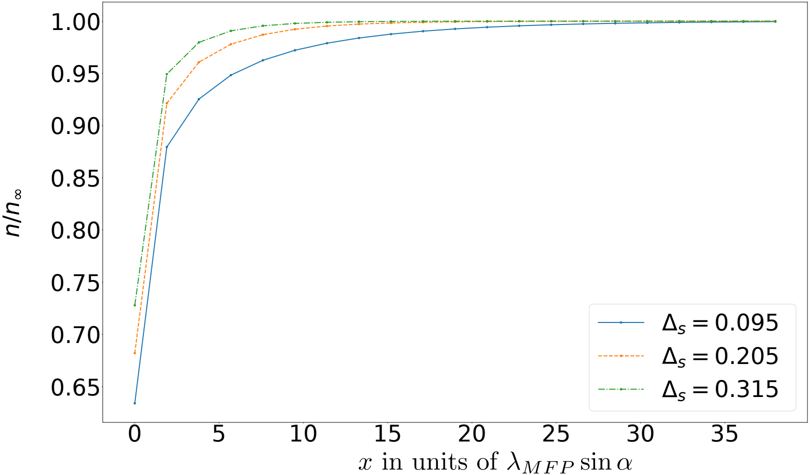
Evolution of f

So far we have only found converged solutions for the equation

$$\frac{\partial f}{\partial t} - v_{\parallel} \sin\alpha \frac{\partial f}{\partial x} = C[f, f]. \tag{13}$$

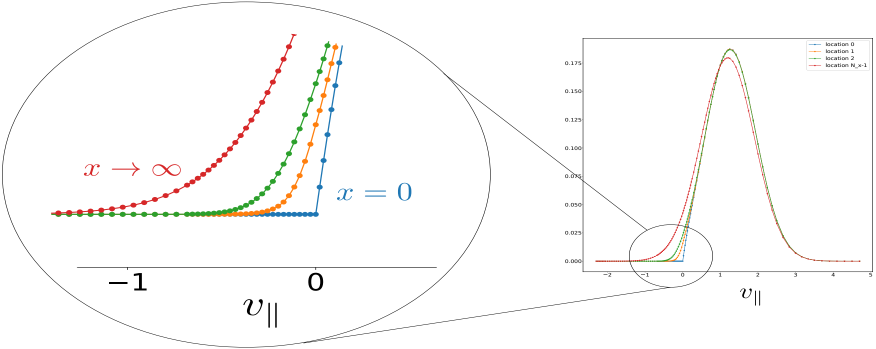
This is the lowest order equation in the limit of cold electrons $T_e/T_{i\infty} \sim e\phi/T_{i\infty} \ll 1$.

Particle number density profiles for three different values of $u_{\parallel\infty}$



$f(v_\parallel)$ at different x locations

Large gradients develop near the wall. This could be a result of large differences in the distribution function near the wall at small negative velocities.



Distribution function at the wall ($x = 0$), points next to the wall and far away from the wall ($x = N_x - 1$).

Distribution near the wall

The region of interest is $x/\lambda_{MFP} \ll 1$ and $-v_{\|}/v_{ti} \ll 1$. Here collision operator can be simplified to be $C[f,f] \approx D\frac{\partial^2 f}{\partial v_{\|}^2}$. This simplifies our steady state equation to

$$-v_{\|}\frac{\partial f}{\partial x} = D\frac{\partial^2 f}{\partial v_{\|}^2} \tag{14}$$

This gives

- $f \propto x^{1/3}$
- $n \propto x^{2/3}$
- $\frac{\mathrm{d}n}{\mathrm{d}x} \propto x^{-1/3}$, which diverges as $x \to 0$, explaining the large gradients in our density plot

Future work

- Get a converged solution for the distribution functions at $T_e \sim T_i$.
- Find the boundary condition at the presheath exit ($x = 0$) (Expect to recover Chodura condition).
- Use our solution for the distribution function at the entrance of the magnetic presheath together with the theory and code developed by A Geraldini to accurately predict the effect of ion collisions with the wall.
- Add neutrals to the model.
- Get an analytic result for $x \to \infty$ for perturbations that are not $\sim \exp\{-x/\lambda\}$

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Backup slides

Boundary layers

At the plasma-wall boundary, in the direction perpendicular to the wall, the length scales of interest are:

- Debye length $\lambda_D \sim \sqrt{\frac{T_e}{n_e}} \sim 10^{-5}m$
- Ion gyroradius $\rho_i \sim \sqrt{\frac{T_i m_i}{B}} \sim 10^{-3}m$
- Collisional mean free path projection in the direction normal to the wall $\lambda_{MFP} \sin\alpha \simeq \alpha\lambda_{MFP} \sim \alpha n_i T_i^2 \sim 10^{-1}m$
- Device size $L \sim 1m$

Galerkin method

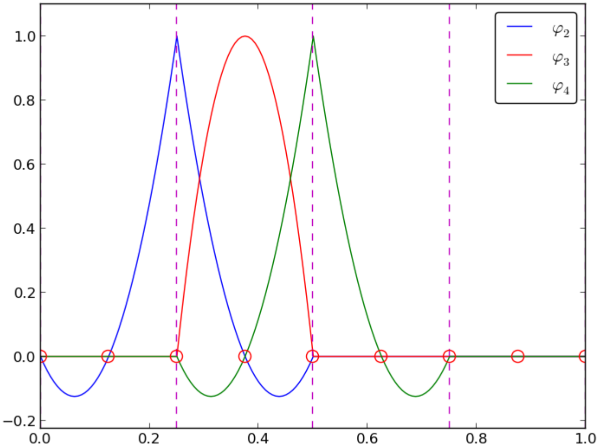
Galerkin method solves the equation by requiring that the equation is orthogonal with respect to the chosen basis functions, that is

$$\int d^3v \frac{\mathrm{d}f}{\mathrm{d}t}\psi_i = \int d^3v C[f,f]\psi_i, \text{ for } i \ = \ 1, ..., N. \tag{15}$$

This gives a system of equation that can be solved numerically.

Finite elements

Quadratic finite element basis functions for a uniform 1D grid. Dashed lines indicate the boundaries between adjacent elements.



Conservation properties

Even though the collision operator satisfies the required properties, the method used to estimate the left hand side of the drift kinetic equation

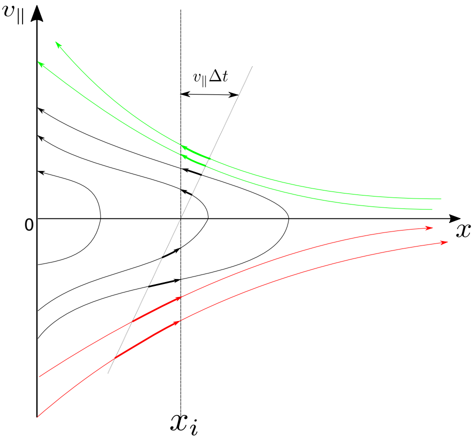
$$\frac{\mathrm{d}f}{\mathrm{d}t} = f(t_\mu, x(t_\mu), v_\parallel(t_\mu), v_\perp(t_\mu)) - f(t_{\mu-1}, x(t_{\mu-1}), v_\parallel(_{\mu-1}), v_\perp(_{\mu-1}) \tag{16}$$

does not satisfy the steady state continuity equation

$$\frac{n(x_{i+1})u_\parallel(x_{i+1}) - n(x_i)u_\parallel(x_i)}{\Delta x_i} = 0. \tag{17}$$

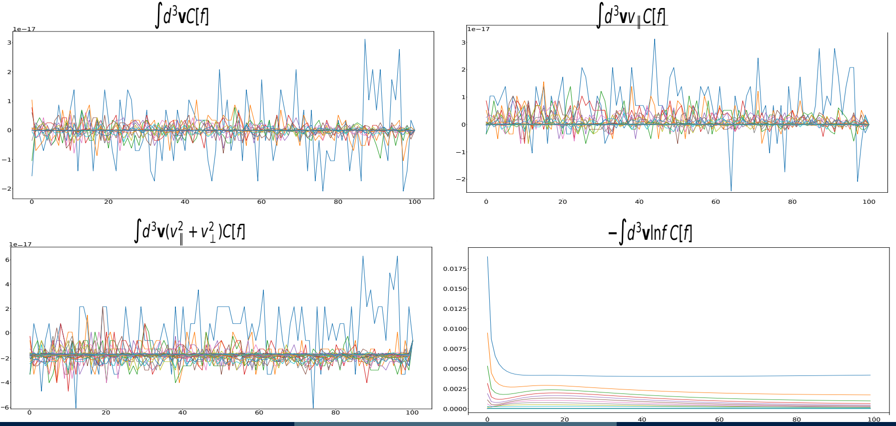
Time evolution

Along the characteristic $df/dt = [f(t_{\mu+1}, x(t_{\mu+1})) - f(t_\mu, x(t_\mu))]/\Delta t$. For $x(t_{\mu+1}) = x_i$ we need information t_μ and $x(t_\mu) = x_i + v_\parallel \Delta t$.

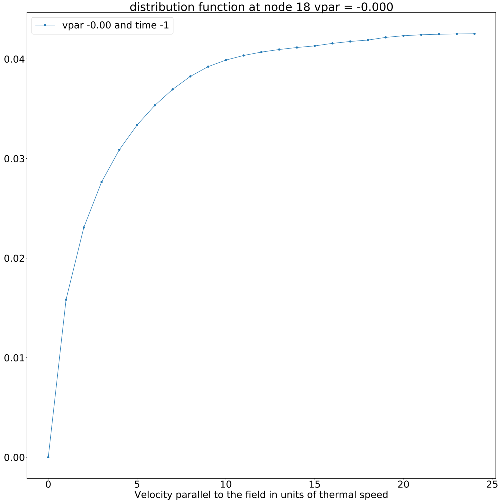


Conservation properties

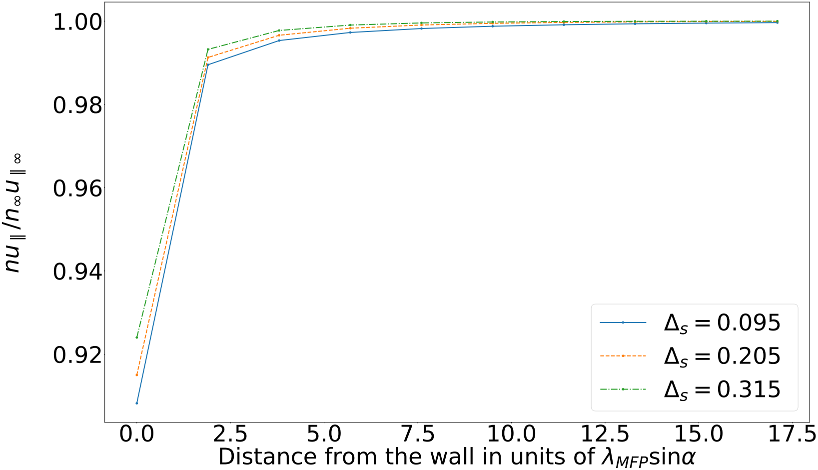
The Galerkin method allows the conservation properties of collision operator to be satisfied exactly. Below are moments of collision operator showing conservation of particles, momentum and energy, and entropy production as a function of time, with different curves representing different points in x space.



f as a function of x at $v_\parallel = 0$



Particle flux density profiles for three different values of $u_{\|\infty}$



Particle flux density profiles for two different resolutions in x