Finite orbit width effects on neoclassical transport in large aspect ratio tokamaks

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Motivation

Transport barriers play an important role in tokamak performance and thus it is desirable to find a comprehensive transport model for these regions. Transport in the core is governed by turbulence, but in the pedestal and in transport barriers where large $E \times B$-shear quenches turbulent eddies, neoclassical transport becomes of importance. However, the applicability of standard neoclassical theory is limited because of sharp gradients of temperature, density and radial electric field [1].

In this work we include strong gradients in neoclassical theory by studying a large aspect ratio tokamak in which the orbit widths and gradient length scales are of the order of the poloidal gyroradius. This approach is different to previous studies that assumed small temperature gradients and neglected the poloidally varying part of the potential [2, 3].

Trapped and passing particles

$L_n \sim L_T \sim L_\phi \sim \rho_p = \frac{q\rho}{\epsilon}$ such that $u \equiv \frac{cI}{B}\frac{\partial\Phi}{\partial\psi} \sim v_t$

Particles are trapped for $w \equiv v_\parallel + u \sim \sqrt{\epsilon} v_t$.

'Orbit labels' ψ_f, $v_{\parallel f}$, which give the respective quantity at a fixed poloidal angle θ_f

$$f = \underbrace{f_{M_f} + h(\psi_f, \theta, v_{\parallel f}, \mu)}_{\text{orbits}} = \underbrace{f_M + g(\psi, \theta, v_\parallel, \mu)}_{\text{particles}} \tag{1}$$

The new variables follow from the conservation of energy, angular momentum and magnetic moment

$$\frac{1}{2}v_\parallel^2 + \mu B + \frac{Ze}{m}\Phi(\psi, \theta) = \frac{1}{2}v_{\parallel f}^2 + \mu B_f + \frac{Ze}{m}\Phi(\psi_f, \theta_f), \qquad \psi - \frac{Iv_\parallel}{\Omega} = \psi_f - \frac{Iv_{\parallel f}}{\Omega_f}. \tag{2}$$

⇒ **Deviations from normal variables are small**

Trapped particles: $v_{\parallel f} - v_\parallel \sim \sqrt{\epsilon} v_t$ and $\psi - \psi_f \sim \sqrt{\epsilon}\rho_p B_p R$

Passing particles: $v_{\parallel f} - v_\parallel \sim \epsilon v_t$ and $\psi - \psi_f \sim \epsilon \rho_p B_p R$

Solving the drift kinetic equation

The steady state drift kinetic equation:

$$\underset{\text{Poloidal velocity}}{(v_\parallel + u)}\hat{b} \cdot \nabla\theta \frac{\partial f}{\partial\theta}\bigg|_{v_{\parallel f}, \psi_f} = \underset{\text{Collision operator}}{C[f, f]} + \overset{\text{Source}}{\Sigma} \tag{3}$$

For low collisionality $qR\nu/v_t \ll 1$, the lowest order solution is a Maxwellian. In order to determine the next order solution $g(\psi, \theta, v_\parallel, \mu)$, a distinction must be made between the passing region $w \sim v_t$ and the trapped-barely passing region $w \sim \sqrt{\epsilon} v_t$.

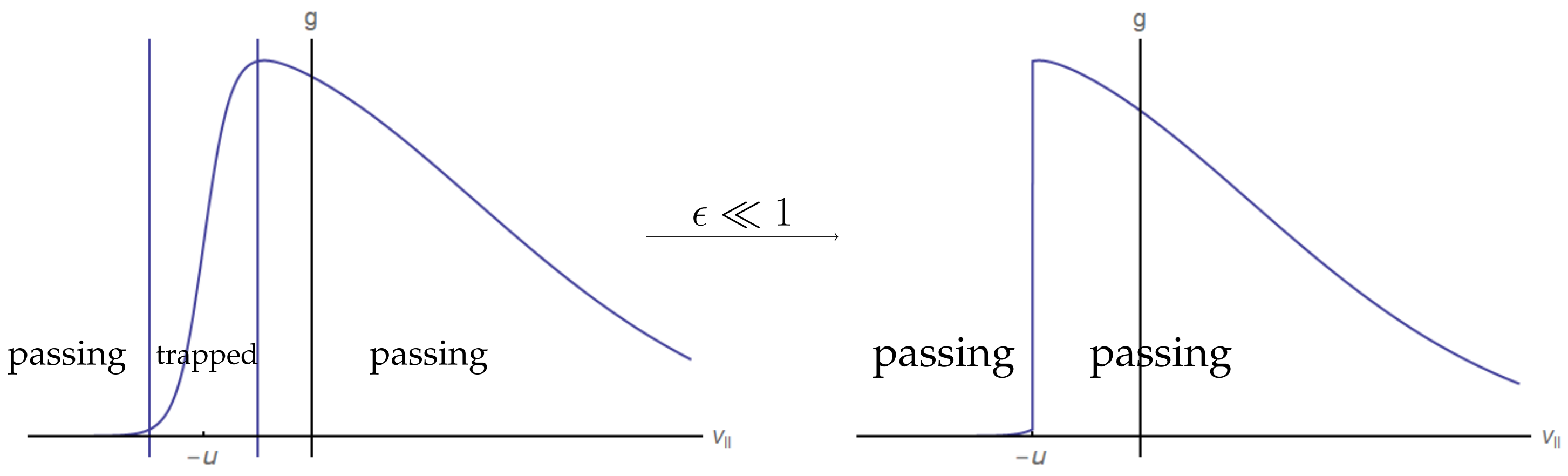

Figure 1: The full distribution function g can be reduced to g^p with a discontinuity in the trapped region.

The contribution from the trapped-barely-passing particles is very narrow for $\epsilon \ll 1$.

⇒ **Trapped particles reduce to discontinuity in the passing distribution function**

The jump and derivative discontinuity condition given by the trapped particle distribution function are

$$\Delta g^p = g_0^t(w \to \infty) - g_0^t(w \to -\infty), \qquad \Delta\left(\frac{\partial g^p}{\partial w}\right) = \frac{\partial g_1^t}{\partial w}\bigg|_{w\to\infty} - \frac{\partial g_1^t}{\partial w}\bigg|_{w\to-\infty}. \tag{4}$$

We solve (3) and (4) using

- Banana regime: $\frac{qR\nu}{v_t} \ll \epsilon^{3/2}$
- Large aspect ratio: $\epsilon \ll 1$
- Circular flux surfaces
- Expansion of g in $\sqrt{\epsilon}$

Poloidally varying electric potential

We keep the small poloidally varying part of the electric field $\Phi = \phi_r(\psi) + \phi(\psi, \theta)$, where $\phi/\phi_r \sim \epsilon$.

Quasineutrality:

$$Z \int \mathrm{d}^3 v (g - \langle g \rangle_\tau) = \frac{en_e}{T_e}\phi. \tag{5}$$

$\langle ... \rangle_\tau$ is the transit average and we assume $\phi = \phi_c(\psi)\cos(\theta)$.

References

[1] E. Viezzer et al., Nucl. Fusion **53** (2013), 053005
[2] P. Catto et al., Plasma Physics and Controlled Fusion **55** (2013), 045009
[3] P. Helander, Physics of Plasmas **5** (1998), 3999
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Transport Equations

The discontinuities give contributions when taking moments of the equation for g^p.

⇒ **Main contribution to transport from trapped particles**

Together with the quasineutrality equation for ϕ and assuming $\frac{\partial}{\partial\psi}\ln p = -\frac{m\Omega u}{IT}$, we find the particle flux, parallel momentum input and heat flux

$$\overset{\text{particle flux}}{\underbrace{\bar\Gamma_c}_{\to 0}} = -0.55\underset{\text{squeezing factor}}{\frac{\bar n_f^2}{S_f^{3/2}\bar T_f^{3/2}}}\left\{\left[-2\bar u_f - 2(\bar u_f + \underset{\text{mean flow}}{\bar V_f})\left(\underbrace{\frac{\partial \bar V_f}{\partial\bar\psi_f}}_{\to 0} - 1\right)\right]\underbrace{G_1(\bar u_f, \bar V_f, \bar\phi_f)}_{\to 1} - 1.17\frac{\partial\bar T_f}{\partial\bar\psi_f}\underbrace{G_2(\bar u_f, \bar V_f, \bar\phi_f)}_{\to 1}\right\} \tag{6}$$

$$\text{diffusion} \longrightarrow \bar u_f\frac{\partial\bar\Gamma_c}{\partial\bar\psi_f} + S_f\overset{\text{friction}}{\bar\Gamma_c} = -\bar\gamma_c \longleftarrow \text{parallel momentum input} \tag{7}$$

$$\overset{\text{heat flux}}{\bar q_c} - \left(\tfrac{5}{2}\bar T_f + \bar u_f^2\right)\underbrace{\bar\Gamma_c}_{\to 0} = 0.65\frac{\bar n_f^2}{S_f^{3/2}\bar T_f^{1/2}}\left\{\left[-2\bar u_f - 2(\bar u_f + \bar V_f)\left(\underbrace{\frac{\partial \bar V_f}{\partial\bar\psi_f}}_{\to 0} - 1\right)\right]\underbrace{H_1(\bar u_f, \bar V_f, \bar\phi_f)}_{\to 1} - 2.22\frac{\partial\bar T_f}{\partial\bar\psi_f}\underbrace{H_2(\bar u_f, \bar V_f, \bar\phi_f)}_{\to 1}\right\} \tag{8}$$

usual neoclassical limit

The ion particle flux

Due to equation (7), particle flux $\bar\Gamma_c \neq 0$ only if momentum input $\bar\gamma_c \neq 0$.

Proof: For $\bar\gamma_c = 0$, $\bar\Gamma_c \sim \exp\left(-\int \mathrm{d}\bar\psi_f\ S_f/\bar u_f\right)$ decreases and is even smaller inside a transport barrier than outside

Two options:

- Turbulent transport of particles never decreases and hence neoclassical transport of particles remains very small.
- The turbulence supplies parallel momentum to enable neoclassical particle transport.

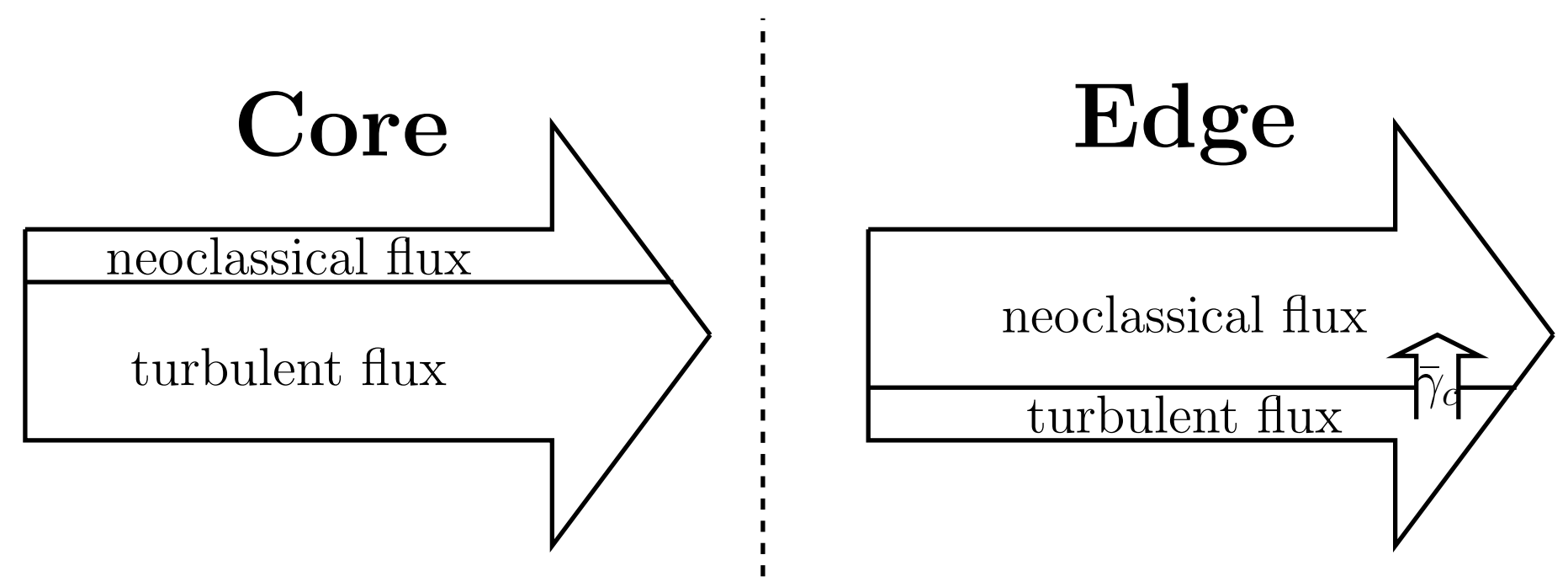

Figure 2: In the core, turbulent transport dominates but as turbulence get quenched when entering a transport barrier and the total flux must be kept, the neoclassical fluxes increase..

Profiles and Fluxes

Sources of particles, momentum and heat represent interaction with turbulence in addition to external injection

⇒ **Found no solutions for certain choices of sources and boundary conditions**

Proof: $\bar V_f'$ near $\bar V_f + \bar u_f = 0$ is

$$\bar V_f' = \frac{\bar V_f + c}{\bar V_f + \bar u_f} \tag{9}$$

For constant c and $\bar u_f$, $\bar V_f$ behaves in three possible ways

$$\frac{\partial\bar V_f}{\partial\bar\psi_f} \longrightarrow 1, \qquad \frac{\partial\bar V_f}{\partial\bar\psi_f} \longrightarrow \pm\infty, \qquad \text{or} \qquad \frac{\partial\bar V_f}{\partial\bar\psi_f} \longrightarrow 0. \tag{10}$$

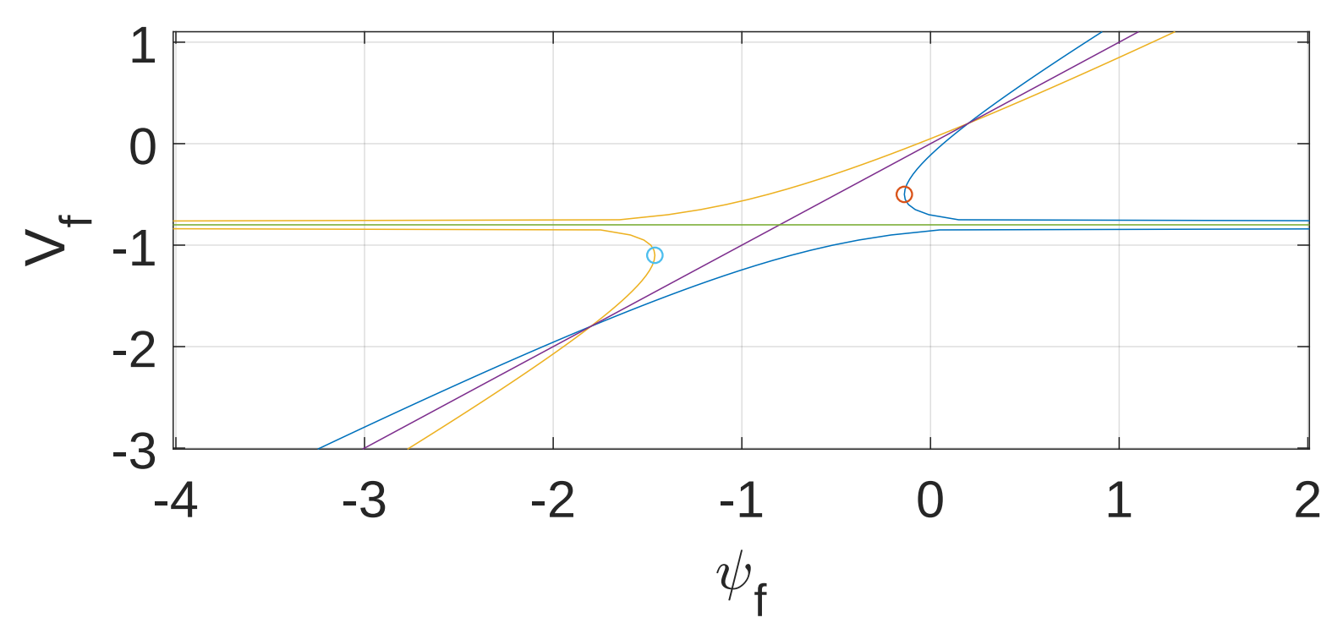

Figure 3: The solution for $\bar V_f$ when setting c and $\bar u_f$ constant. The blue (yellow) lines represent solutions for $\bar u_f + c < 0$ ($\bar u_f + c > 0$).

To determine typical $\bar q_c$, $\bar u_f$ and $\bar\phi_f$ profiles, take measured profiles of $\bar n_f$ and $\bar T_f$ ([4]), and assume $\bar\Gamma_c = 0$ throughout the barrier and $\bar u_f + \bar V_f = 0$ at the top of the barrier.

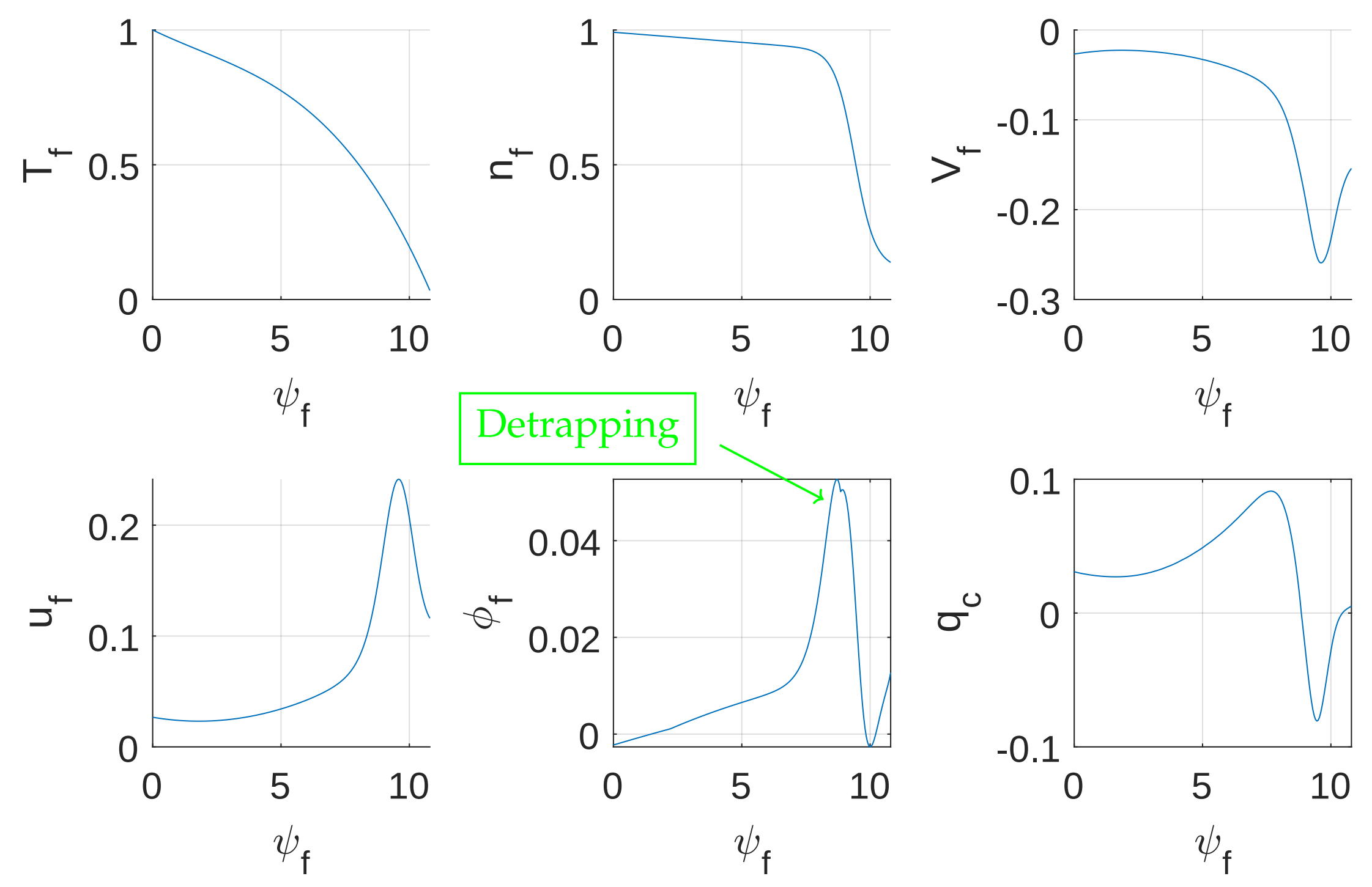

Figure 4: Poloidal velocities, $\bar\phi_c$ and $\bar q_c$ peak in the strong gradient region. The potential can detrap deeply trapped particles.

Summary

We set gradients to be of the same size as the poloidal gyroradius and expanded in ϵ. Transport is dominated by trapped particles and the poloidally varying component of the electric potential enters the equations for particle and heat flux. A source of parallel momentum is required for non-zero particle flux. The equations give divergent solutions of $\bar V_f$ for certain fluxes and boundary conditions. We present example profiles of poloidal velocities, potential and heat flux for measured temperature and density profiles.