Three-dimensional Beltrami states for toroidal, shaped plasmas

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Abstract

Three-dimensional Beltrami states describing toroidal plasmas with D-shaped cross section, are constructed. The construction is carried out by perturbing two-dimensional axisymmetric single-Beltrami states with translationally symmetric Beltrami fields. The perturbation and the unperturbed magnetic field have a common Beltrami parameter λ, thus their superposition still satisfies the Beltrami equation. The boundary was imposed on the axisymmetric state upon using proper conditions for specific boundary points according to the shaping method of [1], [2]. The addition of the translationally symmetric component as a small perturbation has a noticeable impact on the equilibrium state, i.e. one can observe helical magnetic islands in Poincaré maps, which appear in certain rational magnetic surfaces. Furthermore, the conjecture of [3] is confirmed, according to which the surfaces of the resulting 3D configuration remain closed (toroidal) in the vicinity of the magnetic axis if the axisymmetric field has sufficiently high weight in the superposition.

Introduction

Plasmas in many astrophysical and laboratory systems tend to relax to minimum energy states, called Beltrami states, where the magnetic field is an eigenvector of the curl operator:

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \lambda \boldsymbol{B}, \tag{1}$$

where λ is a constant called Beltrami parameter. These states are also "Force-Free", in the sense that the Lorentz force vanishes. In that case, the axisymmetric Grad-Shafranov equation takes the following form:

$$\Delta^{\star}\Psi = -\lambda^2\Psi, \tag{2}$$

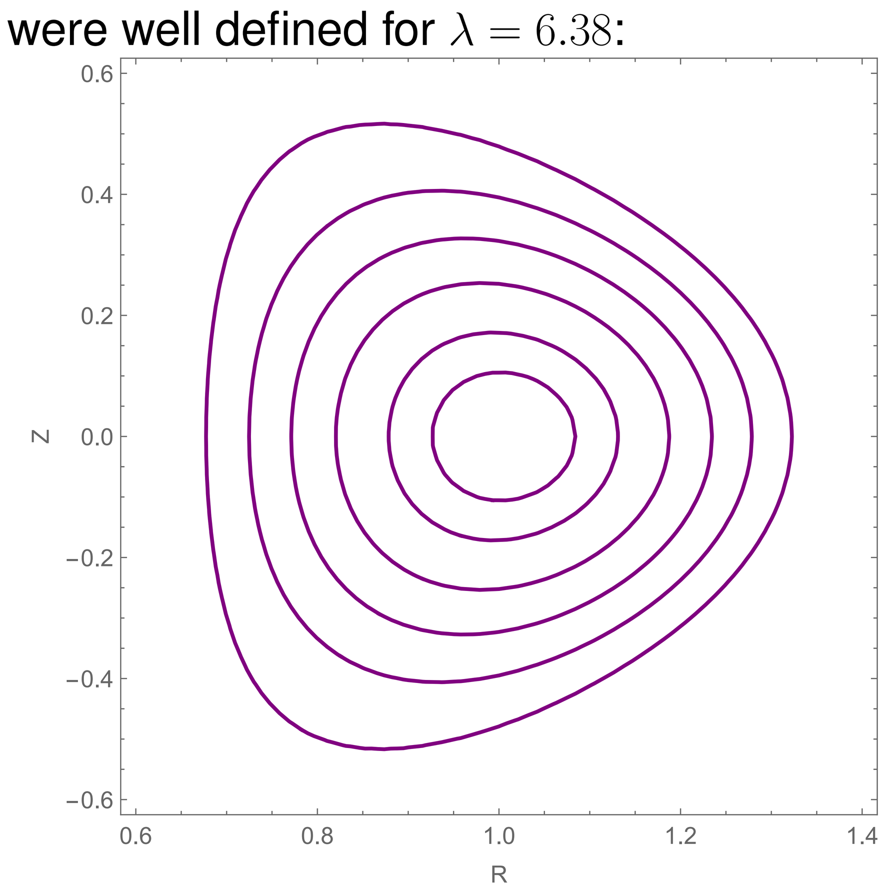
and the flux function Ψ can be written in the separable form: $\Psi = \rho(R)\zeta(Z)$, where:

$$\rho(R) = R\sum_k \left[C_k J_1(\sqrt{\lambda^2 - k^2}R) + D_k Y_1(\sqrt{\lambda^2 - k^2}R)\right] \quad \text{and} \tag{3}$$

$$\zeta(Z) = A\cos(kZ). \tag{4}$$

2D Axisymmetric Beltrami Equilibria Construction

We construct a Tokamak relevant equilibrium with D-Shaped boundary by using the above analytical solution and exploiting the shaping method of [1], [2]. Choosing ITER values for the geometrical parameters, we solved the system of algebraic equations numerically, and the coefficients C_k and D_k were specified. The magnetic surfaces were well defined for $\lambda = 6.38$:



3D Equilibria Construction

A Beltrami field that depends on all three cylindrical coordinates is constructed by adding a translationally symmetric perturbation: $\boldsymbol{B_{tr}} = \boldsymbol{B_{tr}}(r, \phi)$, that satisfies the Beltrami equation (1), i.e.:

$$\boldsymbol{B} = \boldsymbol{B_{ax}} + \gamma \boldsymbol{B_{tr}}, \tag{5}$$

where γ is the perturbation parameter. For that purpose, we will select:

$$\Psi_{tr} = J_m(\lambda R)\cos(m\phi) - 0.1J_0(\lambda R). \tag{6}$$

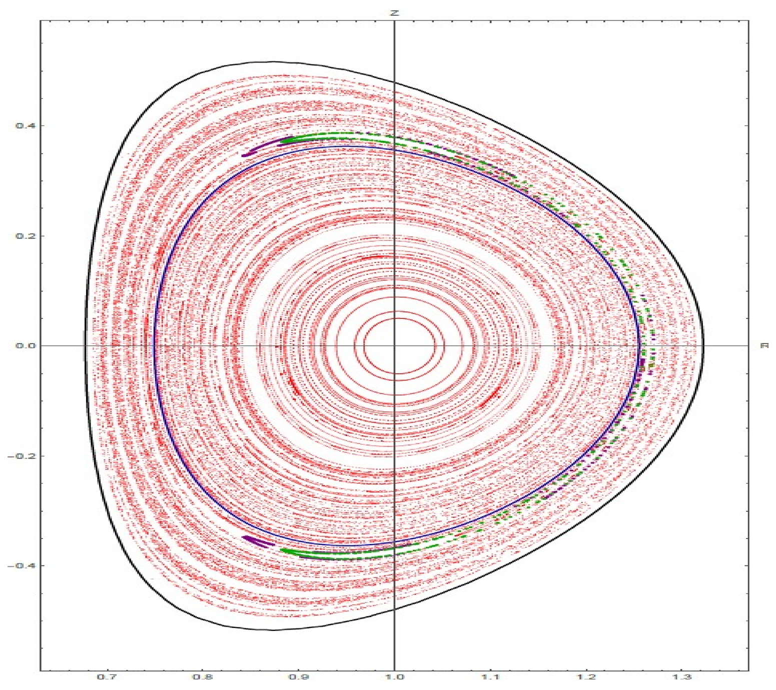
To vizualize the magnetic configuration, we trace the magnetic field line trajectories and create Poincaré maps, by numerically integrating the following system of Lagrange-Charpit ODEs:

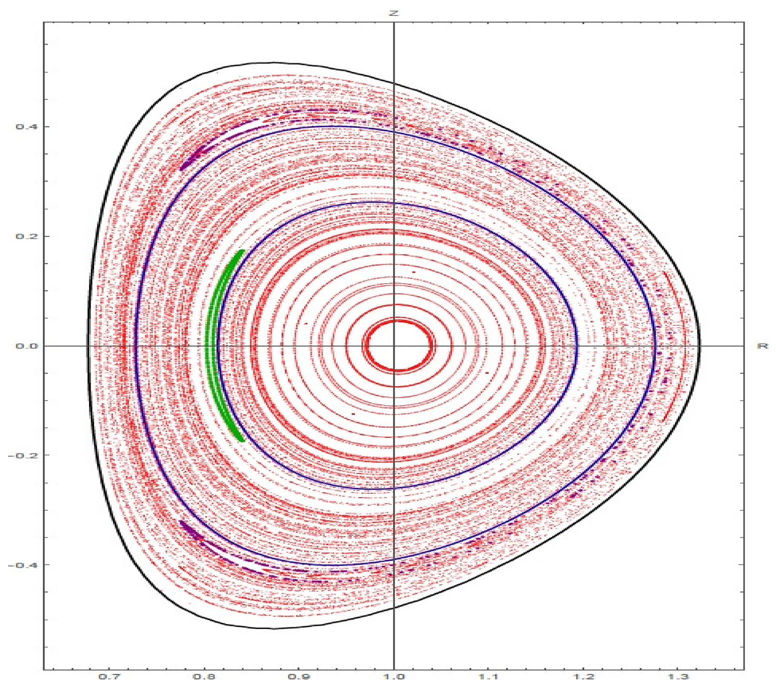
$$\frac{dR}{d\phi} = \frac{RB_R}{B_\phi} \quad \text{and} \quad \frac{dZ}{d\phi} = \frac{RB_Z}{B_\phi}. \tag{7}$$

The numerical integration is performed for 150 random initial points in the poloidal plane $\phi = 0$ and the corresponding field lines are traced for 500 toroidal twists. We choose: $\gamma = 0.0001$ and vary the value of separation constant m from 6 to 10.

Poincaré maps

We notice that the magnetic surfaces have been perturbed, due to the emerge of **magnetic islands:**

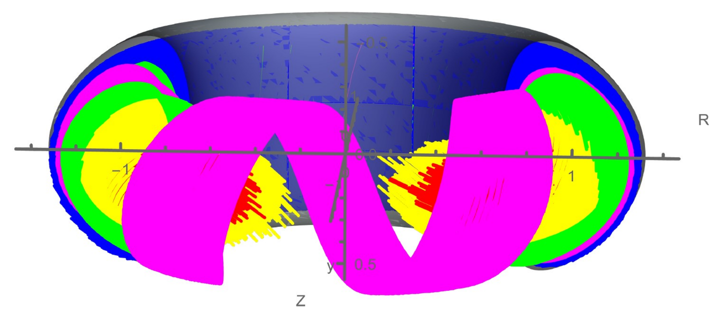


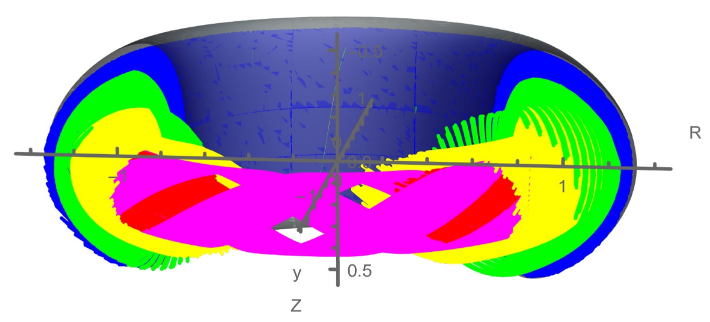


which appear in certain rational magnetic surfaces. That happens because a magnetic island has necessarily a rational safety factor, otherwise it would never be closed to itself. For illustration purposes we highlight a large island appearing at the $q = 1/6$ surface of the $m = 6$ equilibrium (left) and two islands that appear at $q = 1/8$ and $q = 2/8$ surfaces of the $m = 8$ equilibrium (right).

3D Plots

To visualize the helical twisting of the magnetic islands we also create three-dimensional plots of the magnetic field lines, for the $m = 6$ and $m = 9$ equilibria respectively:





Conclusions

To summarize, it is generally acknowledged that magnetic islands cause a deterioration in the magnetic confinement of plasma in fusion devices, as they are associated with enhanced transport and certain instabilities [4], [5]. Nevertheless, their emergence in a three-dimensional equilibria is inevitable, due to the presence of magnetic surfaces with rational values of the safety factor. With this work we provide a simple method for constructing such three-dimensional configurations which incorporate magnetic islands, by superimposing Beltrami magnetic fields with the same Beltrami parameter.

References

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This work has received funding from the National Programme for the Controlled Thermonuclear Fusion, Hellenic Republic.