Braginskii Equations for Hot Plasmas: Weakly Relativistic Approach

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Relativistic effects in astrophysical objects and fusion plasmas do not necessarily require extremely high temperatures and energies. They appear to be non-negligible even for electron temperatures T_e of the order of tens of keV, i.e. when $T_e \ll m_e c^2$. Relativistic effects in transport physics appear due to macroscopic features of the relativistic thermodynamic equilibrium given by the Maxwell–Jüttner distribution function. In fusion devices such as ITER, DEMO and especially in future aneutronic fusion schemes, relativistic effects are surely not important for ions, but electrons require more careful attention. In particular, it was shown recently [1,2] that relativistic effects in electron transport for temperatures characteristic of fusion reactor leads to noticeably different results for both tokamaks and stellarators compared with the corresponding non-relativistic calculations. Nevertheless, practically all transport codes applied for simulation of the fusion reactor scenarios are based on the non-relativistic approach.

In the literature on relativistic plasmas one can find two different approaches. The most rigorous one is based on the covariant formulations of the kinetics and hydrodynamics of plasmas (see, for example [3] and the references therein). This formalism is required for the correct treatment of astrophysical objects. Another approach, aimed at the description of the laboratory fusion plasma, does not satisfy Lorentz invariance but takes features of relativistic thermodynamic equilibrium into account [1,2,4]. For different reasons, both these approaches are not convenient enough for implementation into reactor transport codes.

The present work is focused on transport processes in hot plasmas with relativistic electrons with characteristic velocities of fluxes $V \ll v_{te}$ and $v_{te} < c$. The main goal is to derive Braginskii equations in a weakly relativistic approach for the fluxes, i.e. neglecting terms of the higher order than V^2/c^2 and $v_{te}V/c^2$, while the bulk electrons are described as fully relativistic. The final equations are mathematically very similar to the non-relativistic ones and have a transparent physical interpretation.

For closure of the transport equations, it is proposed to use generalized Laguerre polynomials of order $3/2 + \mathcal{R}(T_e)$, where $\mathcal{R}(T_e) = (15/8)(T_e/m_e c^2) + \mathcal{O}(T_e^2/m_e^2 c^4)$. Just like Sonine polynomials in non-relativistic approach, these polynomials give a finite number of terms for the right-hand-side of the relativistic linearized kinetic equation. In contrast to earlier attempts to use Sonine polynomials for relativistic transport (see also [5]), this makes the proposed presentation more convenient and practical.

References:

[1] I. Marushchenko, N. A. Azarenkov, and N. B. Marushchenko, *Phys. Plasmas* 19, 112109 (2012); *Plasma Phys. Control. Fusion* **55**, 085005 (2013)
[2] G. Kapper, S. V. Kasilov, W. Kernbichler, and M. Aradi, *Phys. Plasmas* **25**, 122509 (2018)
[3] J. M. TenBarge, R. D. Hazeltine, and S. M. Mahajan, *Phys. Plasmas* **15**, 062112 (2008)
[4] T. Mettens, and R. Balescu, *Phys. Fluids* **2**, 2076 (1990)
[5] D. I. Dzhavakhishvili and N. L. Tsintsadze, Sov. Phys. JETP **37**, 666 (1973)