Analytical instruments that can be useful to plasma theory: axial anomaly, conformal invariance, effective Larmor radius

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In tokamak the parallel divergence of the parallel current is non-zero due to the time variation of the vorticity. This has an interesting connection with the axial anomaly usually invoked in baryogenesis. In two-dimensions (good approximation for the plasma in strong magnetic field) the field of vorticity can be seen as a discrete set of positive and negative elementary vortices, of fixed magnitude and two orientations. Their density is the “matter”, their interaction is mediated by the field of velocity and the formalism is similar to a classical field theory. Consider the presence of an incipient, large scale vortex. The time variation of the topological “winding” of velocity field in the region around it is equivalent to generation of new elementary vortices. They are attracted by the initial vortex and this will grow. The current of vorticity along the main vortex has non-zero divergence which equals the time variation of the rotational of the velocity around (i.e. the time variation of the topology of the velocity field is equivalent to creation of new elementary vortices). One notes the possibility of a mapping to the formalism of the quantum axial anomaly: the main vortex is a scalar string, elementary vortices are fermions and the velocity field is the gauge field with nontrivial topology. In the quantum axial anomaly the topological winding of the gauge field is converted into fermions running along the scalar string. The model described above is parallel and purely classical. It may be useful to describe the processes of concentration of vorticity, in vortices or in the sheared rotation layers of barriers, including the H-mode layer.

The two-dimensional model (mentioned above) consisting of discrete elementary vortices has two formulations: (1) ideal fluid and (2) plasma/planetary atmosphere. In (1) the governing equation (Euler) does not exhibit any intrinsic length, it is conformally invariant. In (2) the equation is Charney Hasegawa Mima (CHM) and there is an intrinsic length (Larmor radius/ Rossby radius). The difference is essential. In the CHM case the Ertel’s theorem states that $(\omega+\Omega_c)/n$ is a Lagrangian invariant. In the ideal Euler fluid the connection between the vorticity ω and and density n is lost.

The distinction between ideal Euler (conformal) fluid and CHM plasma (fixed Larmor radius) becomes very important for the poloidally rotating plasma. The reason is a factor which occurs systematically in the perturbative treatment of the ion rotation , $(\rho_{eff})^{-2} = (\rho_s)^{-2}$ $(1-v_{dia}/u)$

. We call it “effective Larmor radius” because it replaces in some formula the basic Larmor radius, as if the system would base its dynamics on this modified parameter. It exhibits the fundamental change that occurs when the CHM dynamics reaches asymptotically the Euler dynamics. When v_{dia} is close to u the plasma behaves as Euler (rho-effective is very large) and the connection between the vorticity (shear of the rotation velocity) and the density is reduced. This places a particular limit on the pedestal evolution. In addition, since the ion “effective Larmor radius” is very large, a class of electron vortices (dependent on this condition) is excited, possibly as precursors of ELMs.