FAST PARTICLES RESONANCE WITH AXISYMMETRIC MODES IN SHAPED PLASMAS

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AXISYMMETRIC MODES IN ABSENCE OF FAST IONS

The general dispersion relation written in terms of quadratic forms is

$$-\gamma^2 \delta I = \delta W_{MHD} + \delta W_{fast} \tag{1}$$

Neglecting the fast ions term

- An elongated plasma is ideal-MHD unstable against axisymmetric perturbations. The whole plasma column undergoes a vertical shift that grows on a time scale proportional to the poloidal Alfvén frequency.
- The presence of an ideal wall nearby the plasma stabilizes the vertical instability thanks to eddy currents induced by the plasma movement (*Passive feedback*). In this case the axisymmetric modes are purely oscillatory with a frequency ω_0 proportional to the poloidal Alfvén frequency [1].
- The realistic case involves a wall with finite resistivity. In this situation the he relevant dispersion relation is cubic, involving three roots ([2]):

$$\gamma^3 + \gamma^2 \frac{1}{\tau_\eta} \frac{1 + a_w/b_w}{1 - \frac{e_0 b}{a+b} D} - \gamma \gamma_0^2 - \gamma_0^2 \frac{1}{\tau_\eta} \frac{1 + a_w/b_w}{1 - D} = 0 \tag{2}$$

  where the resistive wall time scale τ_η is proportional to the wall resistivity and $\omega_0^2 = -\gamma_0^2$ with $\omega_0 \approx e_0^{1/2} \tau_A^{-1} \sqrt{D-1}$. D is the so-called geometrical factor defined as $D = [\exp(4\mu_b) + 1] / [\exp(2\mu_w) + 1]$.

It is then possible to solve for γ in the limit of small wall resistivity $\gamma_0 \tau_\eta \gg 1$ in order to obtain the three relevant roots

$$\omega \approx \pm\omega_0 - i \frac{1}{2\tau_\eta} \frac{D}{(D-1)} = \pm\omega_0 - i\gamma_\eta \tag{3}$$

$$\gamma = \frac{1}{(D-1)\tau_\eta} \tag{4}$$

Under relevant tokamak conditions D must be larger than 1 to stabilize the two modes associated to the roots of Eq.(3), which would otherwise grow on Alfvén time scales. The third unstable root, is related to the so called resistive wall mode (RWM) with growth rate proportional to τ_η, which can be stabilized by means of *active feedback* stabilization.

In the following we will focus on the two stable modes which present a well determined oscillation frequency ω_0 and a damping rate γ_η related to the resistive wall time. Due to their oscillatory character, a resonant interaction between these modes and fast ions could take place leading to instability.

FAST IONS CONTRIBUTION

We proceed analytically using perturbative approach, assuming the energetic particles pressure (or *beta*) much smaller than the core plasma pressure $\beta_h/\beta_c \ll 1$. In this way $|\delta W_{MHD}| \gg |\delta W_{fast}|$ and, at zeroth order in such expansion parameter, the dispersion relation is the cubic of Eq.(2). It is then safe to neglect all real contribution of δW_{fast}, while keeping only its imaginary part.

$$\omega^2 = \omega_0^2 - 2i\omega_0\gamma_\eta + i\omega_0^2 \lambda_h + \mathcal{O}(\gamma^2/\omega_0^2) \tag{5}$$

$$\omega = \omega_0 + i\gamma_{tot}, \qquad \gamma_{tot} = \omega_0 \lambda_h / 2 - \gamma_\eta \tag{6}$$

With $\lambda_h = Im(\delta\hat{W}_h) \ll 1$, where $\delta\hat{W}_h$ the properly normalized mode-particle resonance term of δW_{fast}. It is clear that there is a competition between the resistive wall damping and the mode-particle resonance related term. In order to determine under which conditions the energetic particle term can overcome the damping, and thus to find stability thresholds, it is necessary to study δW_h.

$$\delta W_h = -\frac{2\pi^2 c}{Zem^2} \sum_\sigma \int dP_\phi d\mathcal{E} d\mu_\perp \tau_\Omega \omega \frac{\partial F}{\partial \mathcal{E}} \sum_{p=-\infty}^{+\infty} \frac{|\Upsilon_p|^2}{\omega + p\omega_\Omega} \tag{7}$$

The behaviour of Eq.7 depends mainly on:

- The resonant denominator $\omega + p\omega_\Omega$, introducing the imaginary part of δW_h
- The derivative of the equilibrium distribution function of fast ions $\partial F/\partial\mathcal{E}$, which decides the sign of δW_h
- The Fourier coefficients Υ_p, describing the contribution to the resonance of the different harmonics in the particles orbit periodicity

RESONANT DENOMINATOR AND $\partial F/\partial\mathcal{E}$

Resonant denominator

- Particles orbit frequencies can be rewritten as $\omega_\Omega = v \cdot h_\Omega(r,\Lambda)/(R_0 q)$, with dimensionless function h_Ω involving elliptic integrals and is different for trapped and circulating particles.
- The resonant denominator introduces a pole in the velocity (energy) for all harmonics with the exception of p=0.
- For each harmonic the resonant velocity is $v_p^* = \omega_0 R_0 q / [p \cdot h_\Omega(r,\Lambda)]$.

Equilibrium distribution function

- The sign of λ_h depends only on $\partial F/\partial\mathcal{E}$. An instability would require a positive λ_h, large enough to overcome the resistive wall damping, and consequently $\partial F/\partial\mathcal{E} > 0$.
- This can be obtained including losses of fast particles. Considering a simplified situation with monocromatic fast ions source at velocity v_{fast} ($\delta(v_{fast} - v)$) and a velocity-independent loss frequency ν_{loss} the *Equilibrium distribution function reads*:

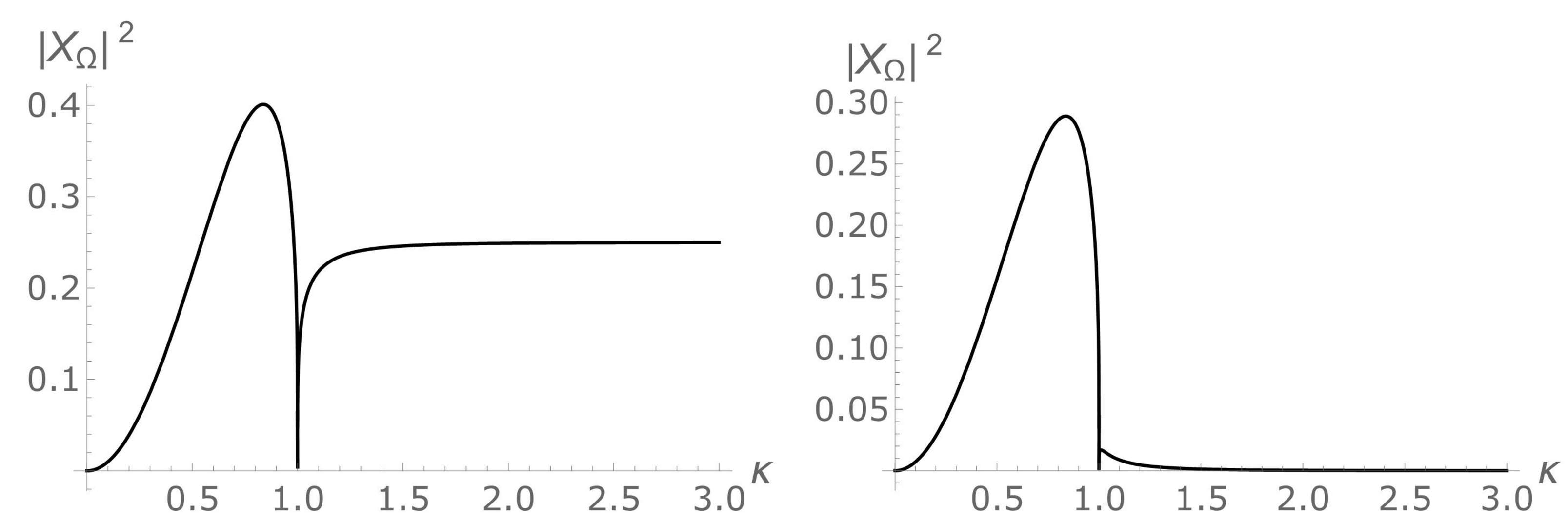
$$f_h(v) = C \cdot \frac{H(v_{fast} - v)}{(v^3 + v_c^3)^{1-\alpha}} \tag{8}$$

- When $\alpha = \nu_{loss}\tau_s/3$ is larger than 1 the usual slowing down distribution of energetic particles cannot form and the distribution function of fast ions presents $\partial F/\partial\mathcal{E}$.
- There are experimental evidences that a distribution function with $\partial F/\partial\mathcal{E} > 0$ can be obtained by a modulation of the fast particles source: In Ref.[3] this is done modulating the neutral beam injection period, while Ref.[4] the modulation of the fast particle source is induced by sawthoot oscillations.

FOURIER COEFFICIENTS Υ_p

The Fourier coefficients are defined as $\Upsilon_p(\mathcal{E},\mu,P_\varphi) = \oint d\tau \tilde{\mathcal{L}} exp(ip\omega_\Omega\tau)/\tau_{b/t}$ where the perturbed Lagrangian is $\tilde{\mathcal{L}} \approx \epsilon^2 \mathcal{E}(2-\Lambda)\xi \sin(\theta)/r$. With the exception of θ, all quantities in $\tilde{\mathcal{L}}$ are constant along the orbit.
Thus we are interested in the quantity $|X_\Omega|^2 = |\langle \sin(\theta) exp(ip\omega_\Omega\tau)\rangle|^2$.

- For **passing particles** (pitch angle $0 < \Lambda < 1-\epsilon$) $X_\Omega = i(1/4\mathcal{K}(1/\kappa^2)) \int_0^{2\pi} d\theta \, [\sin(\theta)\sin(p\theta)] / \sqrt{1 - \sin^2(\theta/2)/\kappa^2}$, and reduces to $X_\Omega = i(1/2\pi) \int_0^{2\pi} d\theta \sin(\theta)\sin(p\theta)$ for $\Lambda = 0$. All harmonics higher then $p=1$ are negligible.
- For **trapped particles** (pitch angle $1-\epsilon < \Lambda < 1-\epsilon$) $X_\Omega = \kappa/\mathcal{K}(\kappa^2) \int_{-\pi/2}^{\pi/2} d\zeta \sin(\zeta)\sin(p\zeta)$, where $\sin(\theta/2) = \kappa\sin(\zeta)$. All odd harmonics, with the exception of $p=1$, are identically zero, while all even harmonics with $p \geq 4$ are negligible with respect to lower harmonics or passing particles contributions.



(a) First harmonic p=1 (b) Second harmonic p=2

Figure 1: Plots of $|X_\Omega|^2 = |\langle \sin(\theta) exp(ip\omega_\Omega\tau)\rangle|^2$ as a function of κ (defined as $\kappa^2 = 1/2 + (1-\Lambda)/(2\epsilon)$) for $p=1$ and $p=2$ harmonics. (Trapped particles for $0 < \kappa < 1$ and passing particles for $\kappa > 1$)

HARMONICS CONTRIBUTIONS TO λ_h

It is then possible to obtain stability thresholds for the mode in terms of the density ration n_h/n_c:

$$\lambda_h = \lambda' \frac{n_h}{n_c} \rightarrow \left(\frac{n_h}{n_c}\right)_{crit} = \frac{2\gamma_\eta}{\lambda'\omega_0} \tag{9}$$

where λ' is a numerical coefficient. It is possible to distinguish the contributions coming trapped and passing particles and from different harmonics to the numerical factor: $\lambda' = \sum_{p=1,2} \sum_{\Omega=b,t} \lambda'_{p,\Omega}$.
If the fast particle energy threshold $E_{fast} = m_h v_{fast}^2/2$ is not large enough, the bounce frequency of the particles could be too small compared to the Alfvénic mode frequency, and only passing particles will resonate with the mode (See Fig.2).

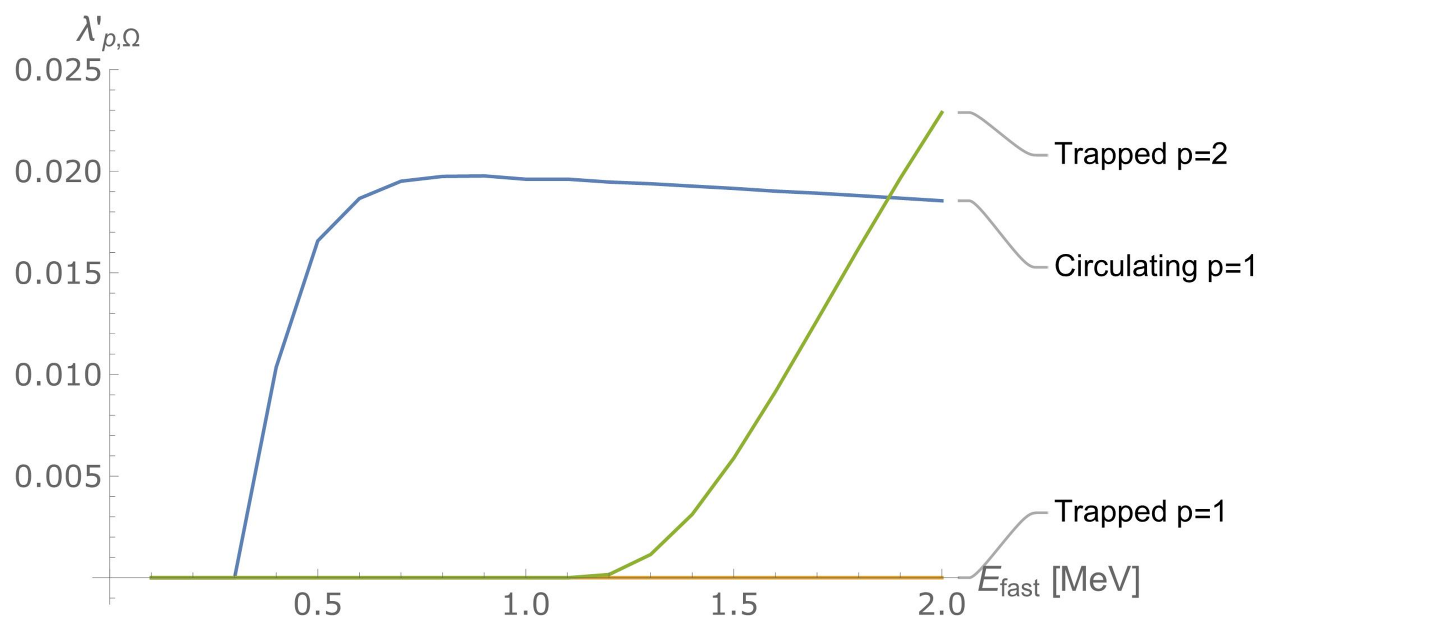


Figure 2: Plot of $\lambda'_{p,\Omega}$ as a function of E_{fast} for JET-like deuterium plasma with parameters : R_0= 3,a=0.9,q= 1, $r_h = a/2$ = 0.45,= 2, $f_0 =_0 /2\pi = 300kHz$.

CONCLUSIONS

Considering the same plasma parameters of Fig.(2), $E_{fast} = 1.5MeV$ and $\gamma_\eta \sim 1\times10^2 - 1\times10^3$, the critical value for the stability threshold is of the order of $n_h/n_c|_{crit} \sim 4\times10^{-3} - 4\times10^{-2}$.
Although conclusive evidence is not yet available, we suggest that the this fast-ion-driven axisymmetric mode (in brief, FIDAM) may provide a possible explanation for the observation of $n=0$ modes observed in recent JET experiments in presence of fast ions with energies around $1MeV$ [5].

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