



# Signals of a Quantum Universe

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arXiv:2001.09149 [Phys. Rev. Lett. 124, 25]



Structure in the Universe

(Cosmic) Bell Test

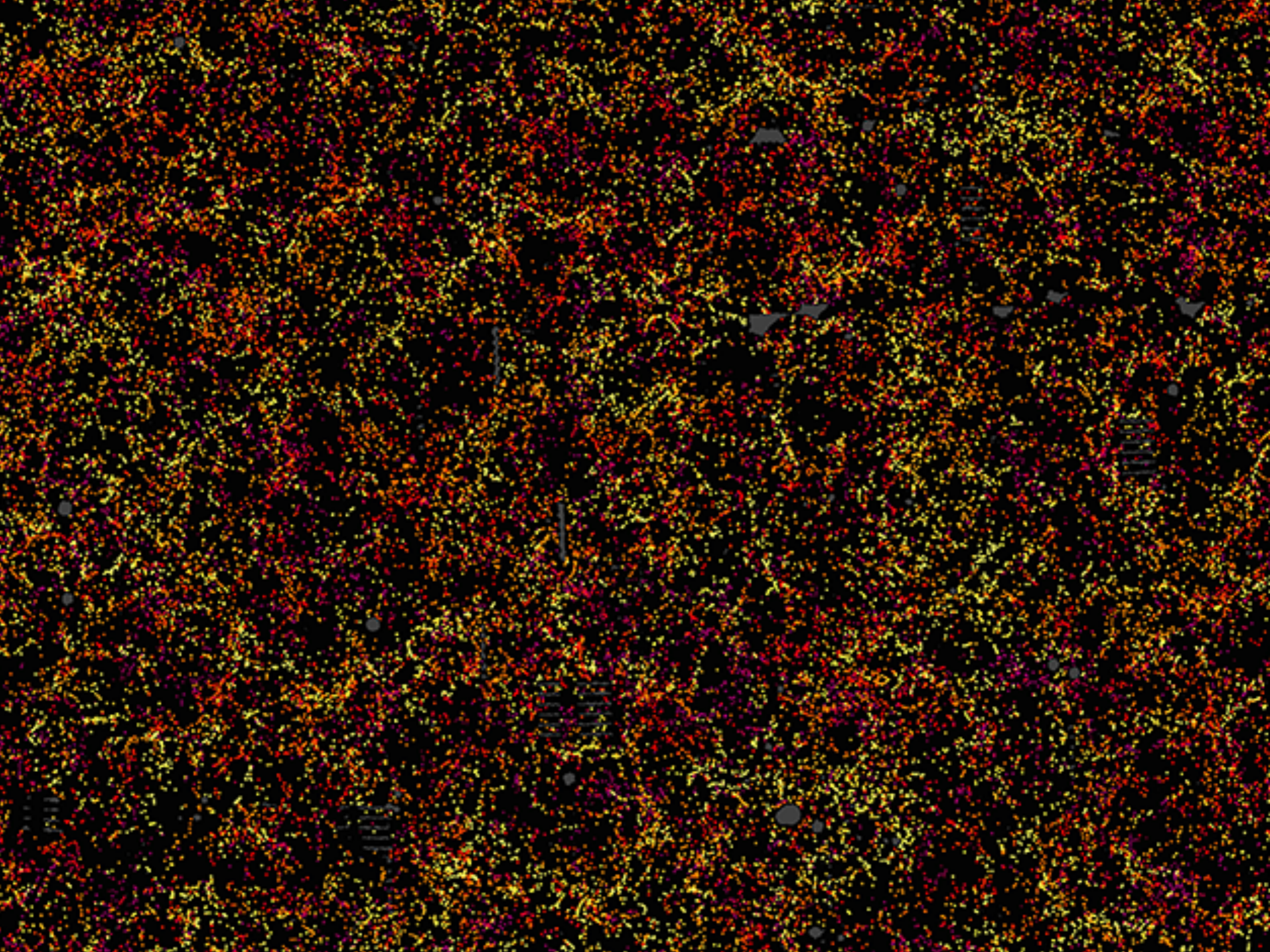
Signal of a Quantum Universe

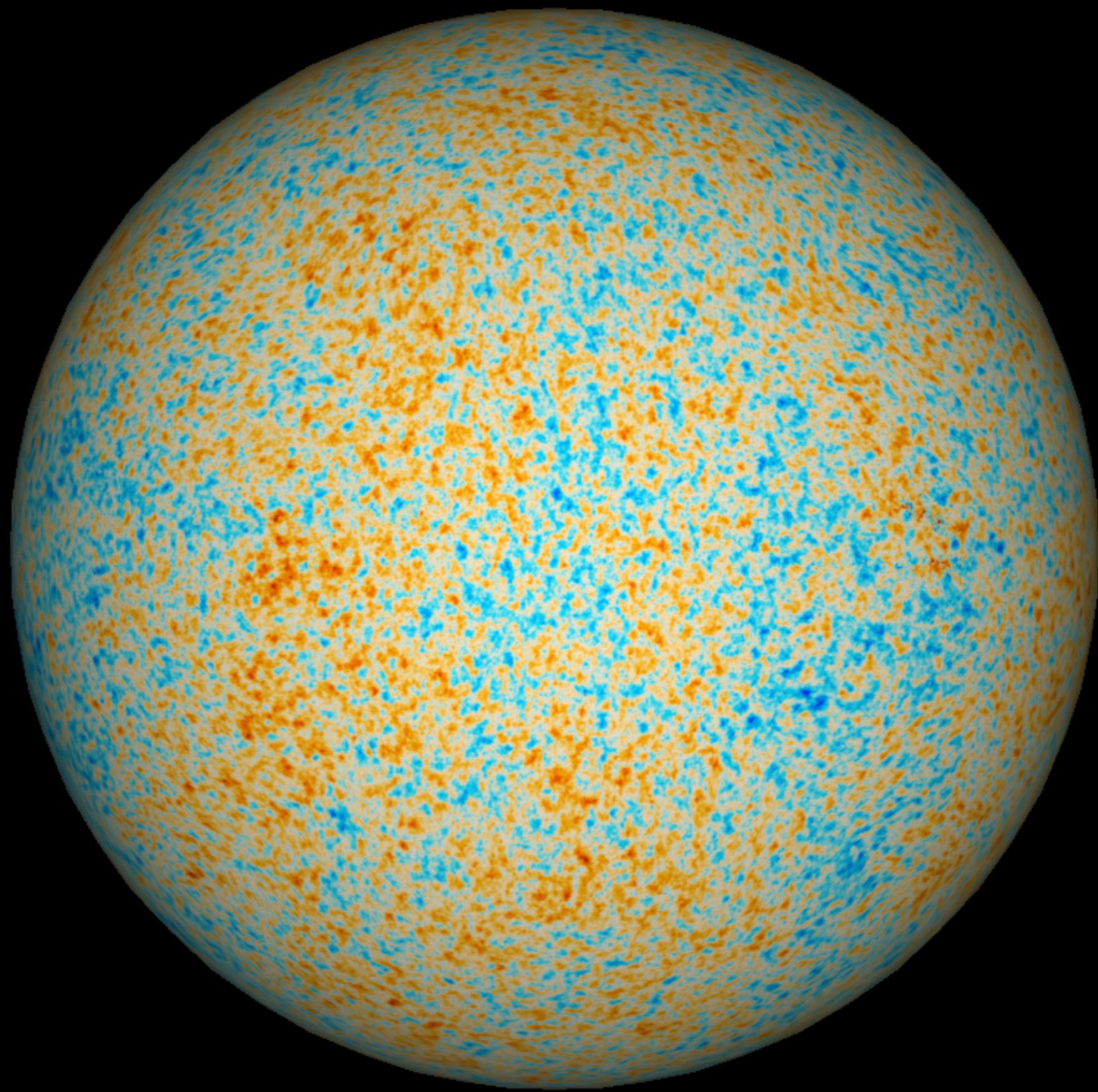
Observational Implications

# Structure in the Universe

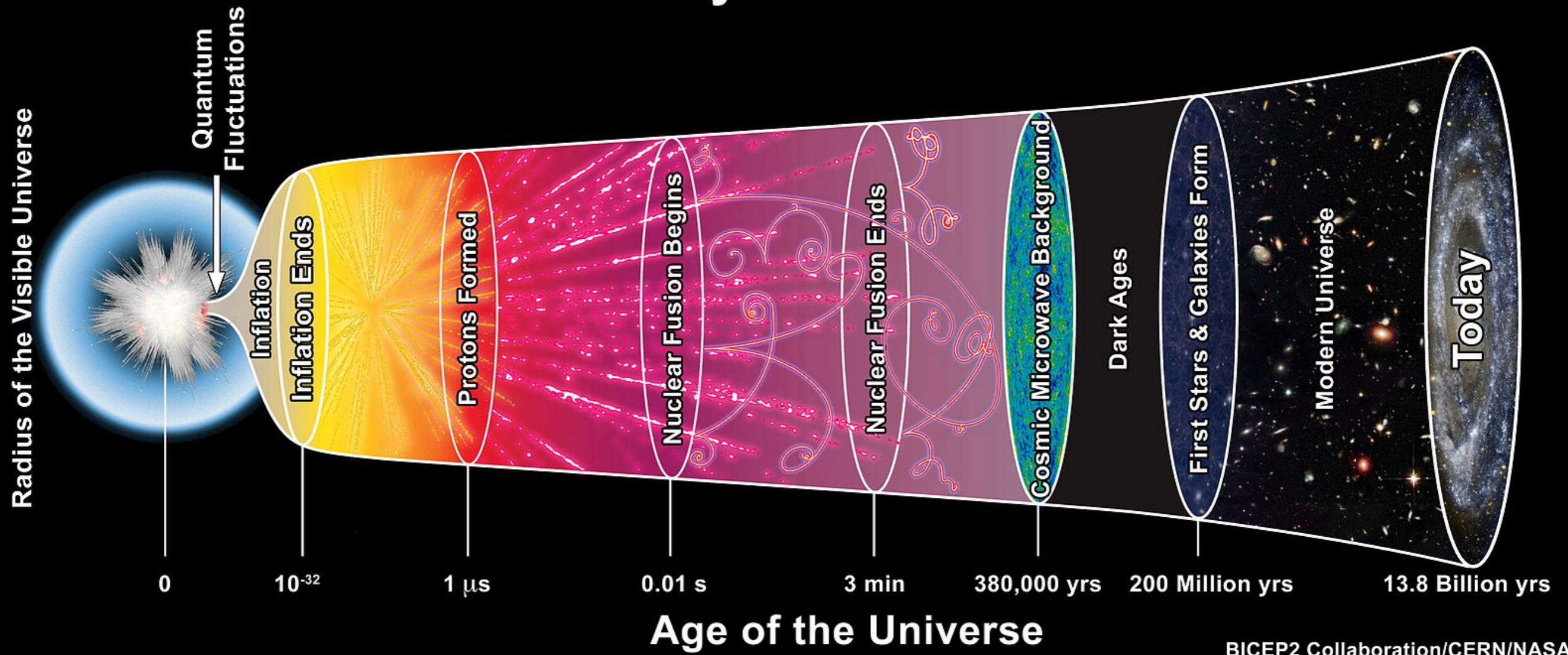




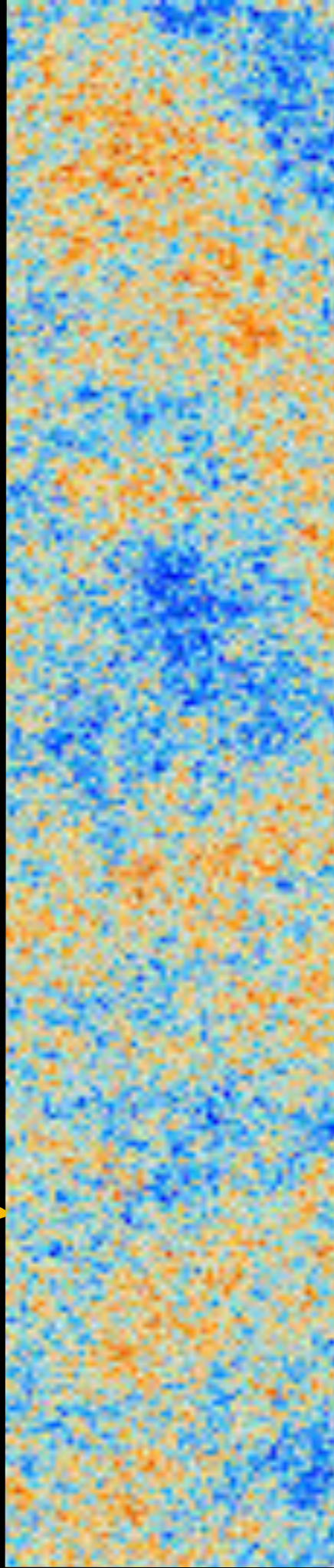
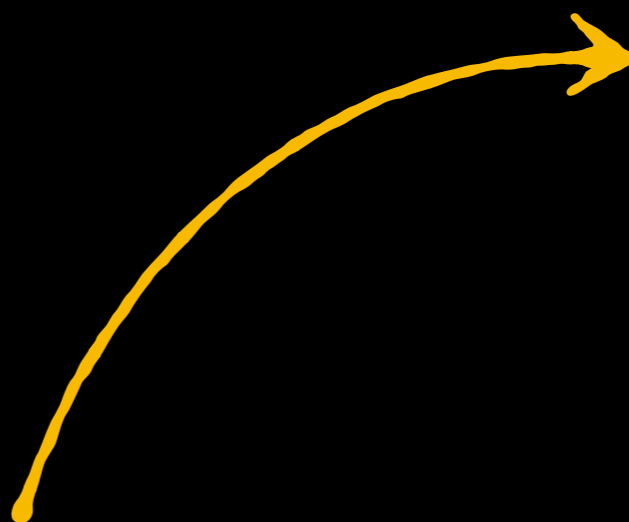
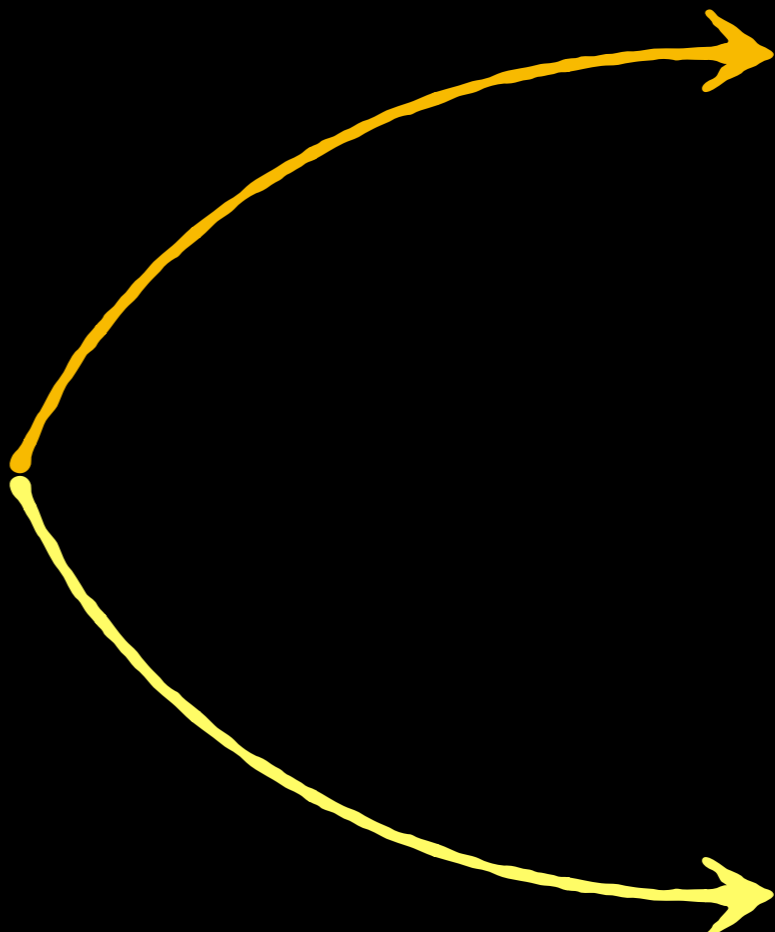
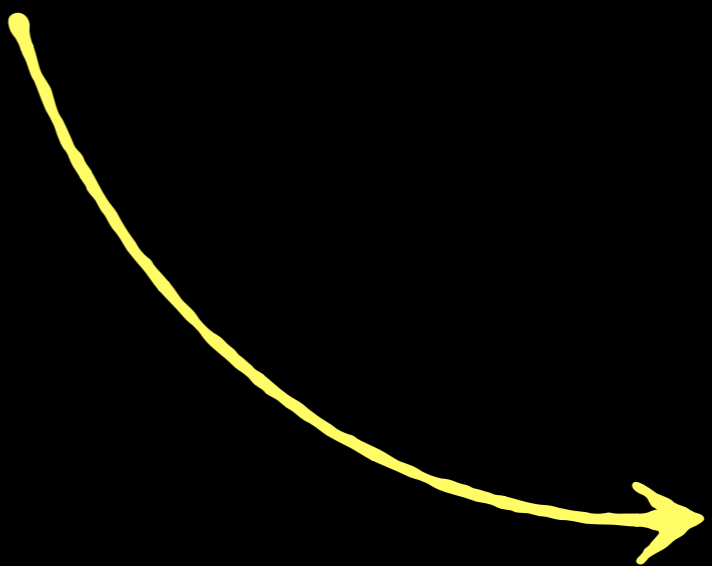
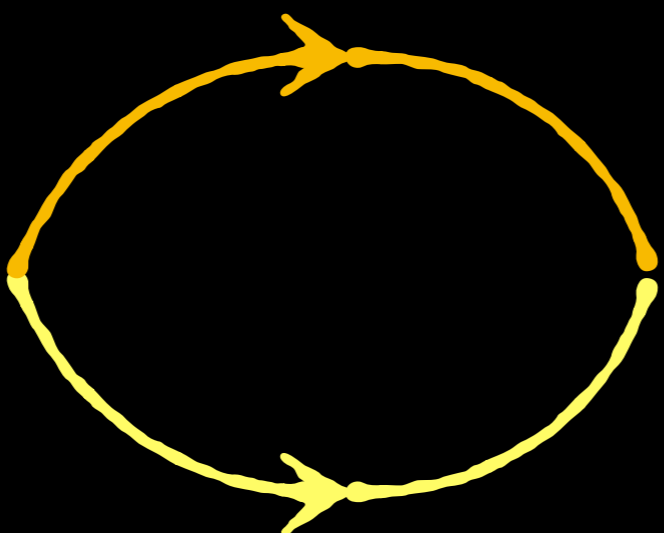
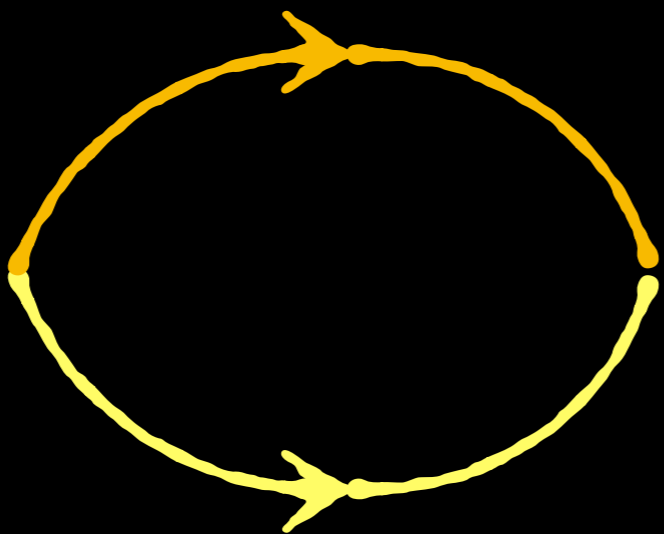
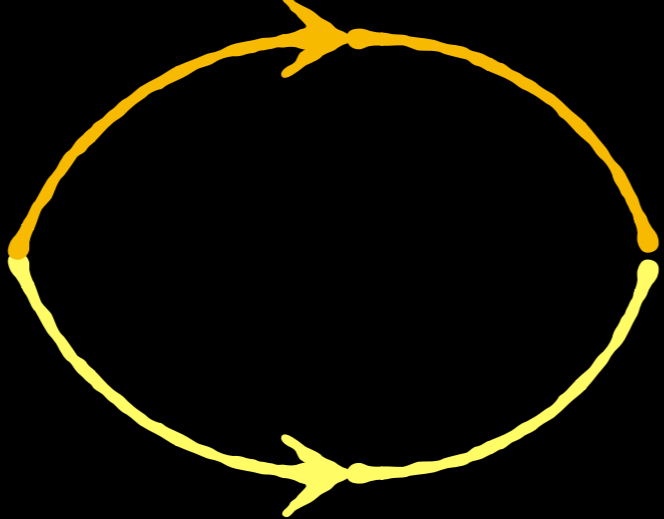
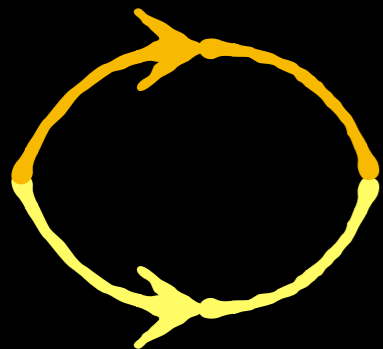
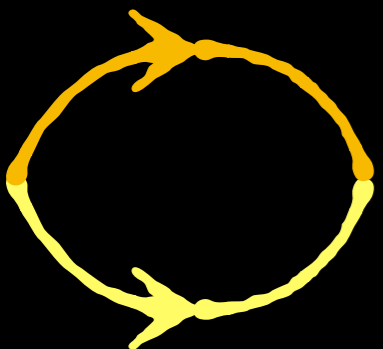
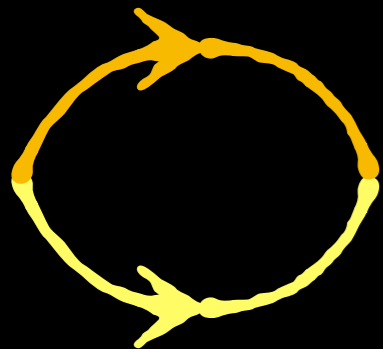




# History of the Universe

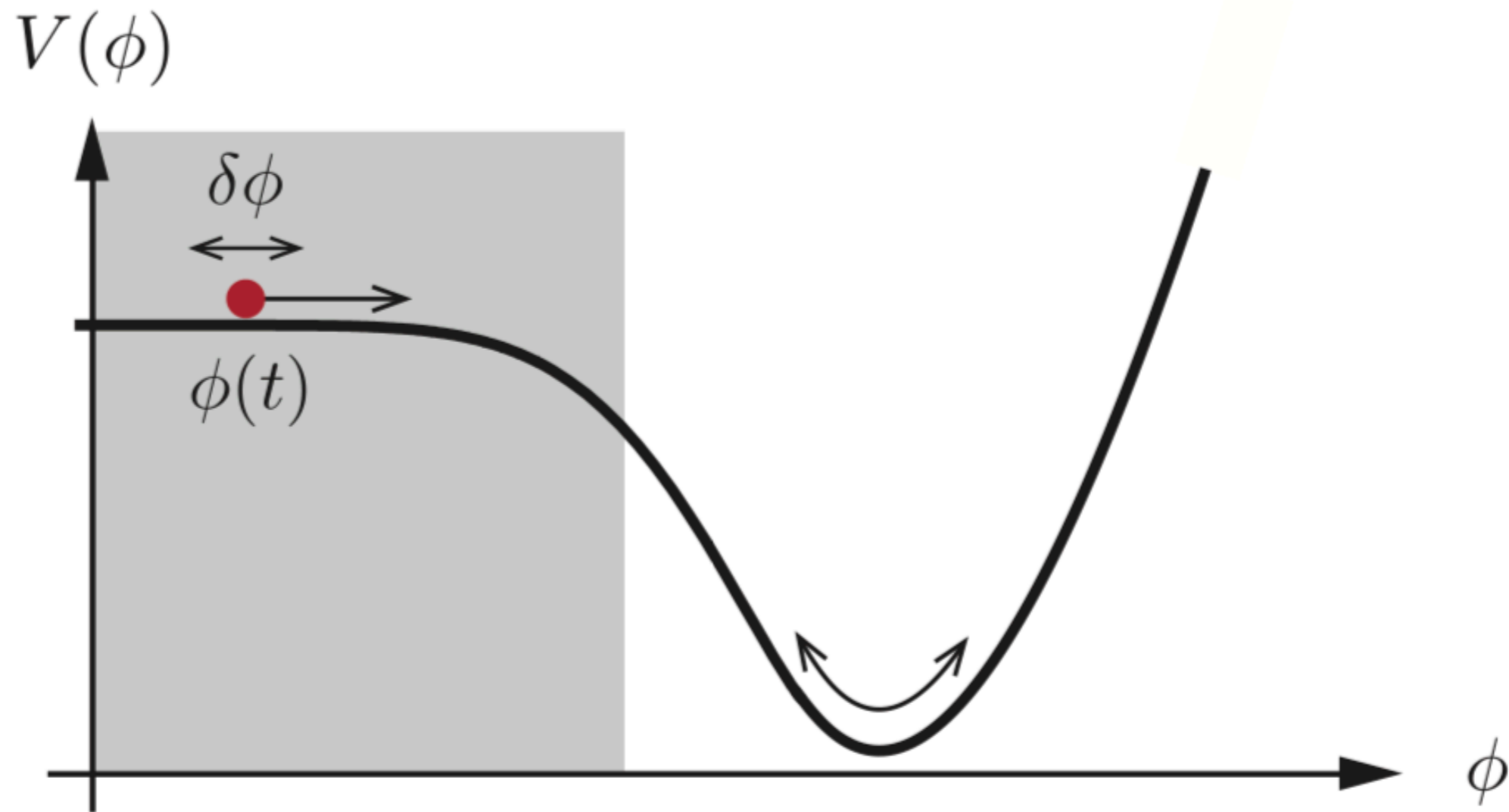


BICEP2 Collaboration/CERN/NASA





# Quantum Origin of Structure



From Baumann & McAllister

# Quantum Origin of Structure



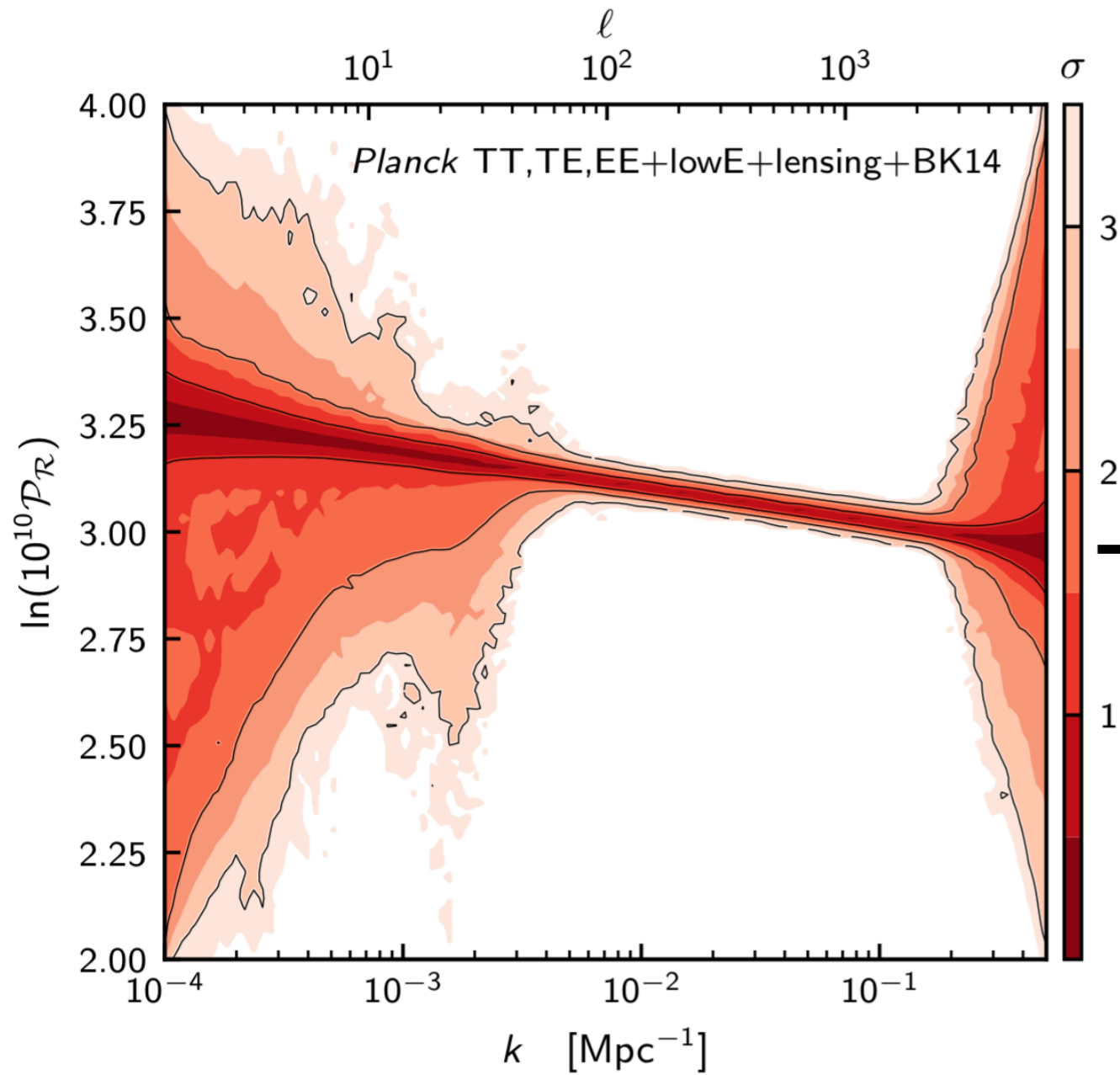
Vacuum fluctuations can explain origin of structure

This explanation captures the imagination:

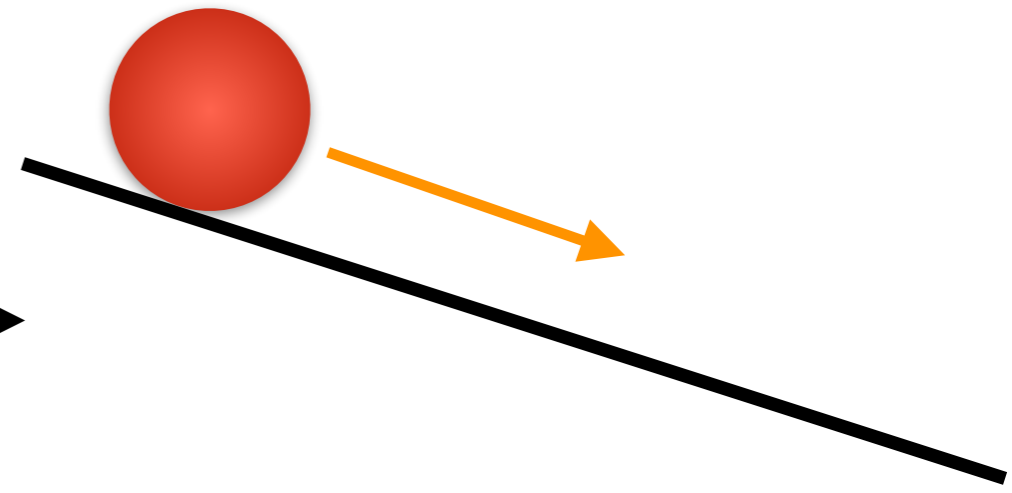
The largest objects in the universe are explained by  
the unusual phenomena of the smallest scales

But is this how the universe works?

# Quantum Origin of Structure

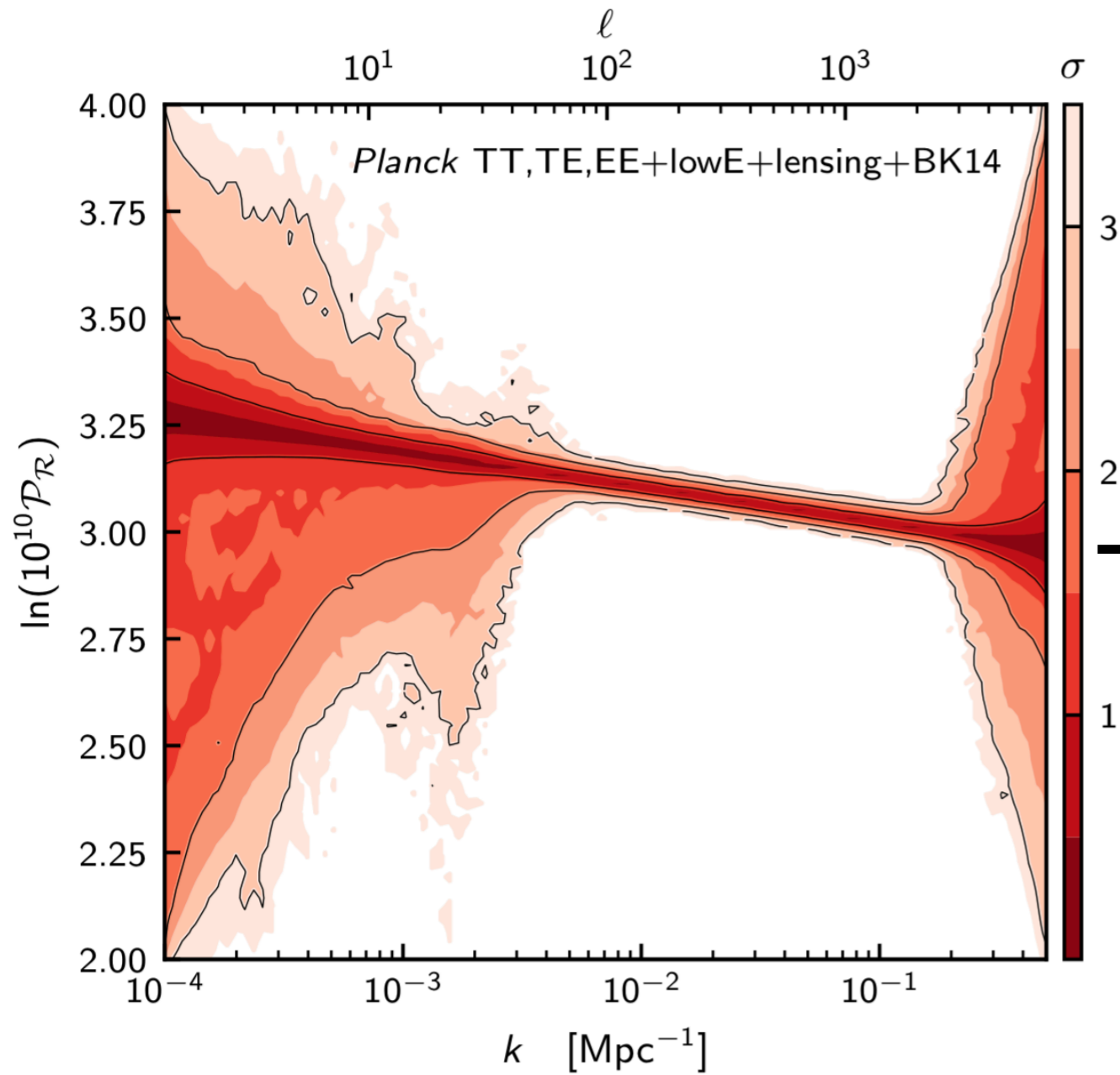


?



Planck 2018

# Quantum Origin of Structure



?



Planck 2018



# Quantum Origin of Structure

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Classical physics gives rise to structures too!

[e.g. thermal fluctuations]

Bell gives a definitive way to tell a state IS quantum

Unfortunately, the universe we observe is classical

We need a way to tell is WAS in a quantum

# Cosmic Bell's Inequalities

The background of the slide is a dark, textured field filled with intricate white patterns. These patterns consist of numerous overlapping, concentric circles and spirals of varying sizes, interspersed with long, thin, and sometimes curved lines that crisscross the space. The overall effect is reminiscent of a complex web or a map of gravitational wells and cosmic filaments, suggesting a deep connection to the physics of gravity and spacetime curvature.

# Review of Bell's Inequalities

Really just a reflections of two kinds of probability

Quantum: given  $\mathcal{A}_{if} \in \mathbb{C}$  for each path

$$P_f = \left| \sum_i \mathcal{A}_{if} \right|^2 \quad \sum_f P_f = 1$$

Classical: given  $p_{if} \in [0, 1]$  for each path

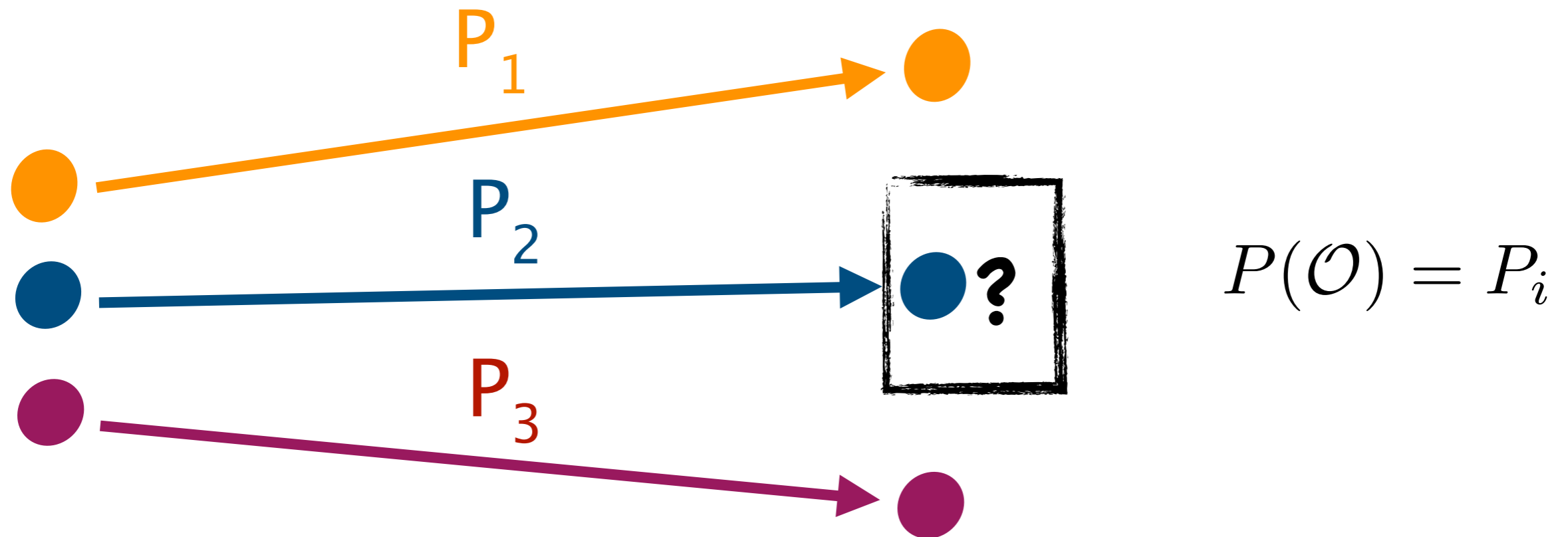
$$P_f = \sum_i p_{if} \quad \sum_f P_f = 1$$



# Review of Bell's Inequalities

Classical Probability = lack of knowledge

time

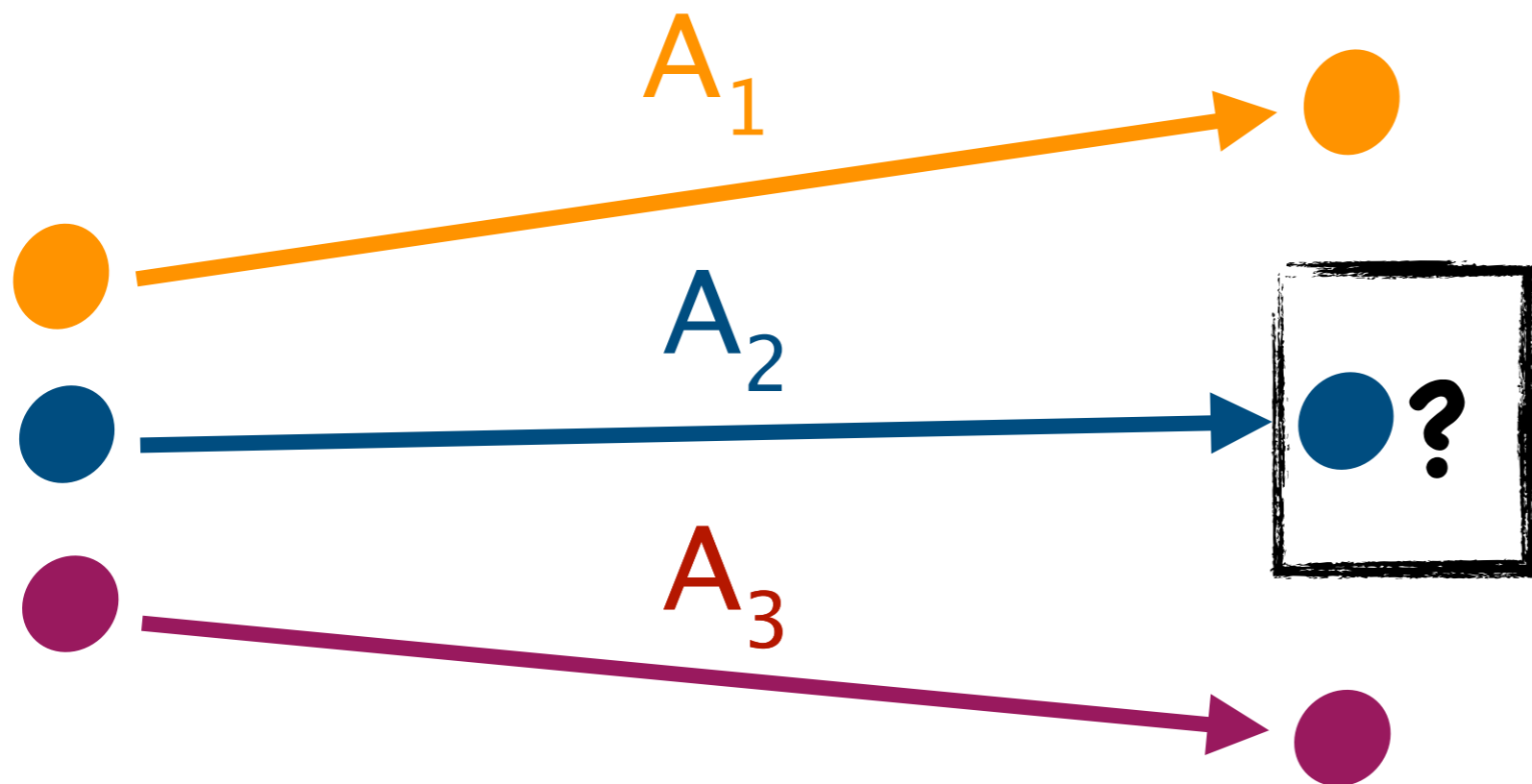


The universe picks one trajectory

# Review of Bell's Inequalities

Quantum Probably = true uncertainty

time



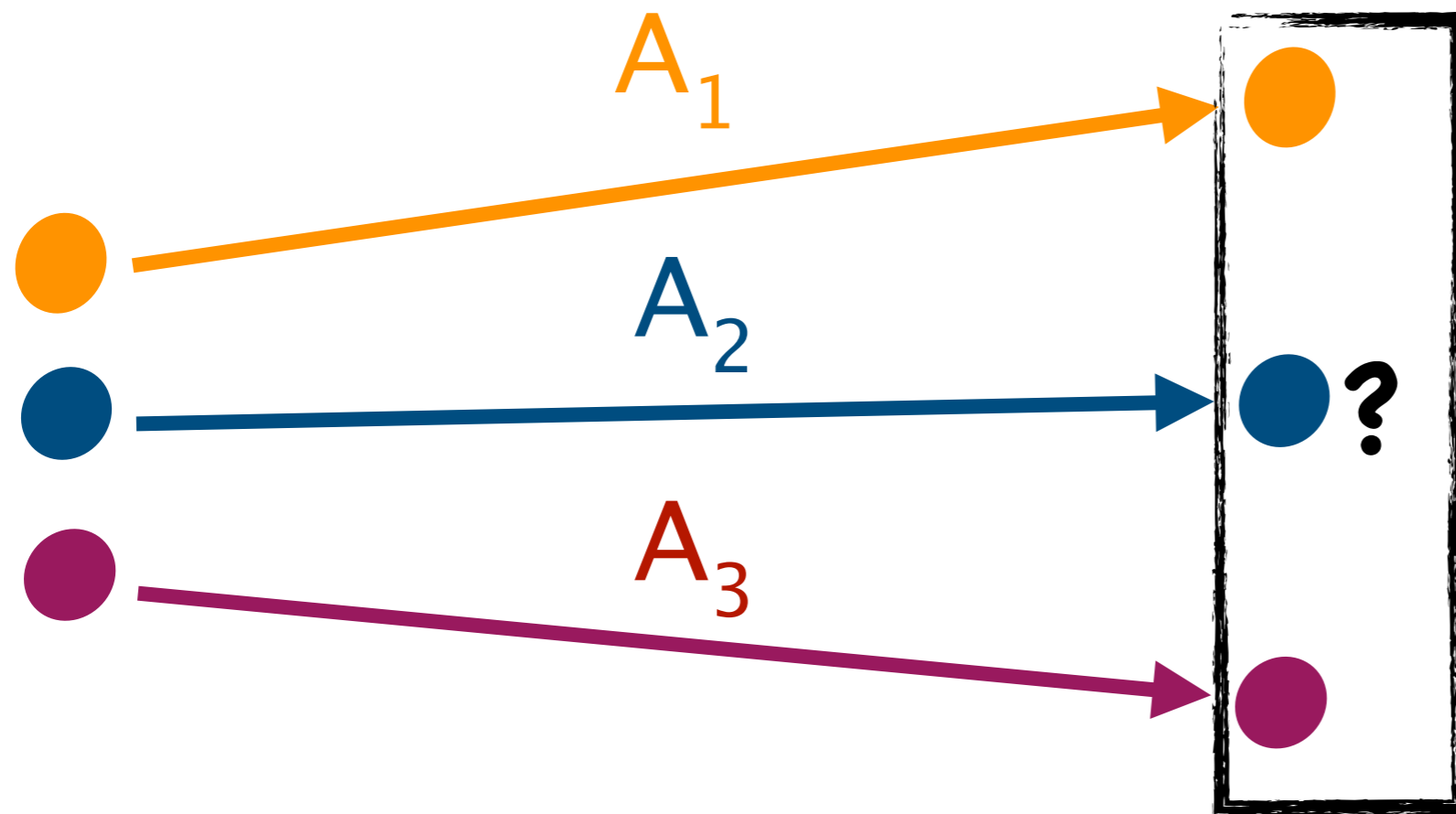
$$P(\mathcal{O}) = |\mathcal{A}_i|^2$$

It might even look classical if  $[H, \mathcal{O}] = 0$

# Review of Bell's Inequalities

Quantum Probably = true uncertainty

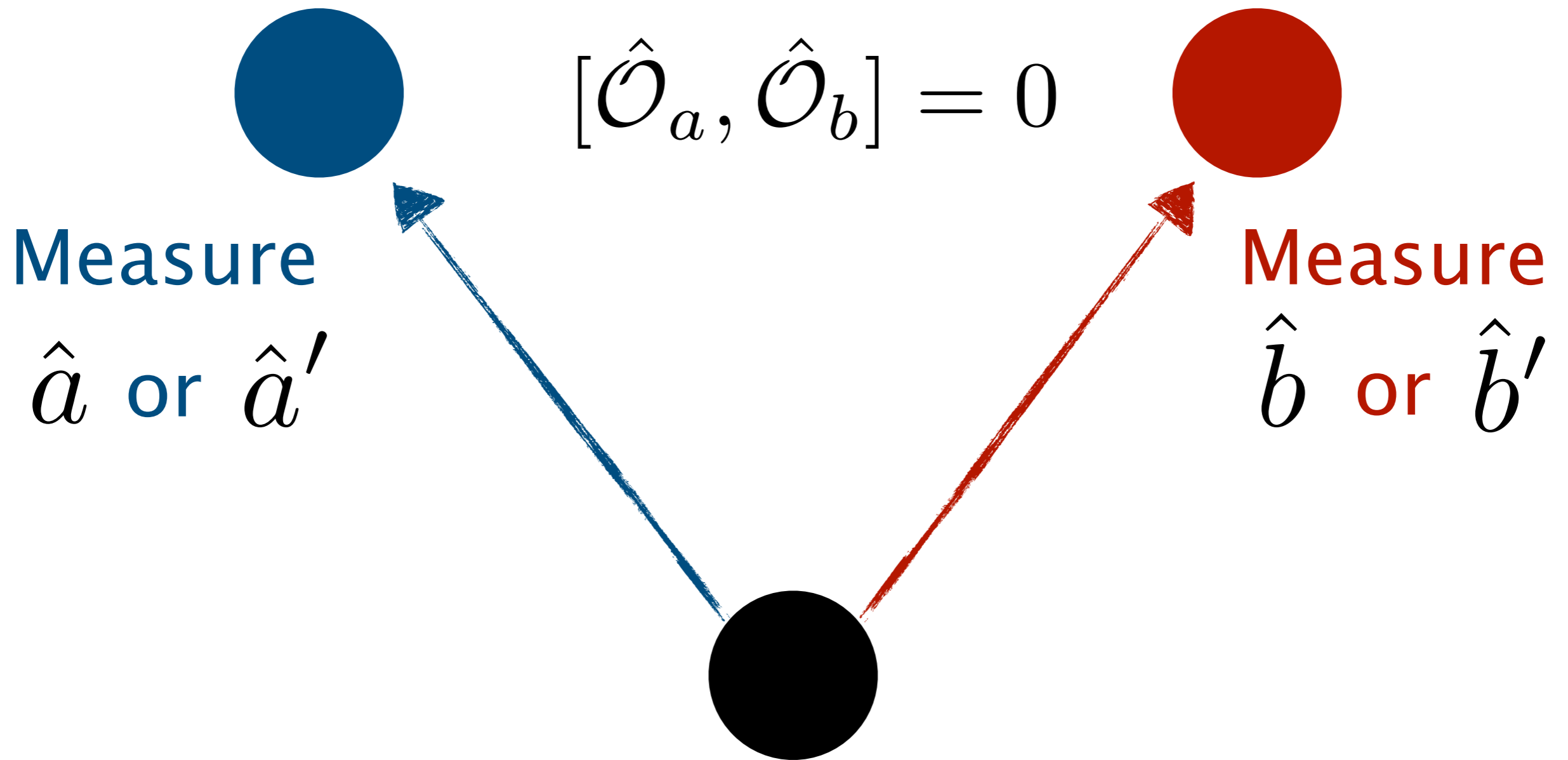
time



$$P(\mathcal{O}') = \left| \sum_j \mathcal{A}_{i',j} \right|^2$$

But not if we measure in basis  $[\mathcal{O}', \mathcal{O}] \neq 0$

# Review of Bell's Inequalities



$$\hat{a}^2 = \hat{a}'^2 = \hat{b}^2 = \hat{b}'^2 = 1$$

# Review of Bell's Inequalities

Key idea: I can choose what I measure locally

$$\hat{a}|\psi_a\rangle \quad \bullet \quad \hat{a}'|\psi_a\rangle$$

[pure or mixed state]

Do not have to commute  $[\hat{a}, \hat{a}'] \neq 0$

But both will only return  $\pm 1$

Q vs C distinguished by  $\hat{C} = \hat{a}\hat{b} + \hat{a}'\hat{b} + \hat{a}'\hat{b}' - \hat{a}\hat{b}'$

# Challenge for Cosmology

The central challenge in cosmology is that

$$[\zeta(\vec{x}, t), \dot{\zeta}(\vec{x}', t)] \propto \frac{1}{\sqrt{-g}} \delta(\vec{x} - \vec{x}') = \frac{1}{a^3} \delta(\vec{x} - \vec{x}')$$

e.g. Grishchuk & Sidorov

Suppressed by the volume of the universe

$$\langle \hat{\zeta}(\vec{x}, t) \dots \rangle = \int d\zeta (\zeta(\vec{x}, t) \dots) |\Psi[\zeta]|^2$$

This is completely equivalent to classical statistics

$$|\Psi[\zeta]|^2 \rightarrow P[\zeta]$$



A Signal of a  
Quantum Universe

# Key Ideas

Our only hope is to use statistics. Since

$$\langle \hat{\zeta}(\vec{x}, t) \dots \rangle = \int d\zeta (\zeta(\vec{x}, t) \dots) |\Psi[\zeta]|^2$$

We must ask whether

$$|\Psi[\zeta]|^2 \rightarrow P[\zeta] \neq P_{\text{classical}}[\zeta]$$

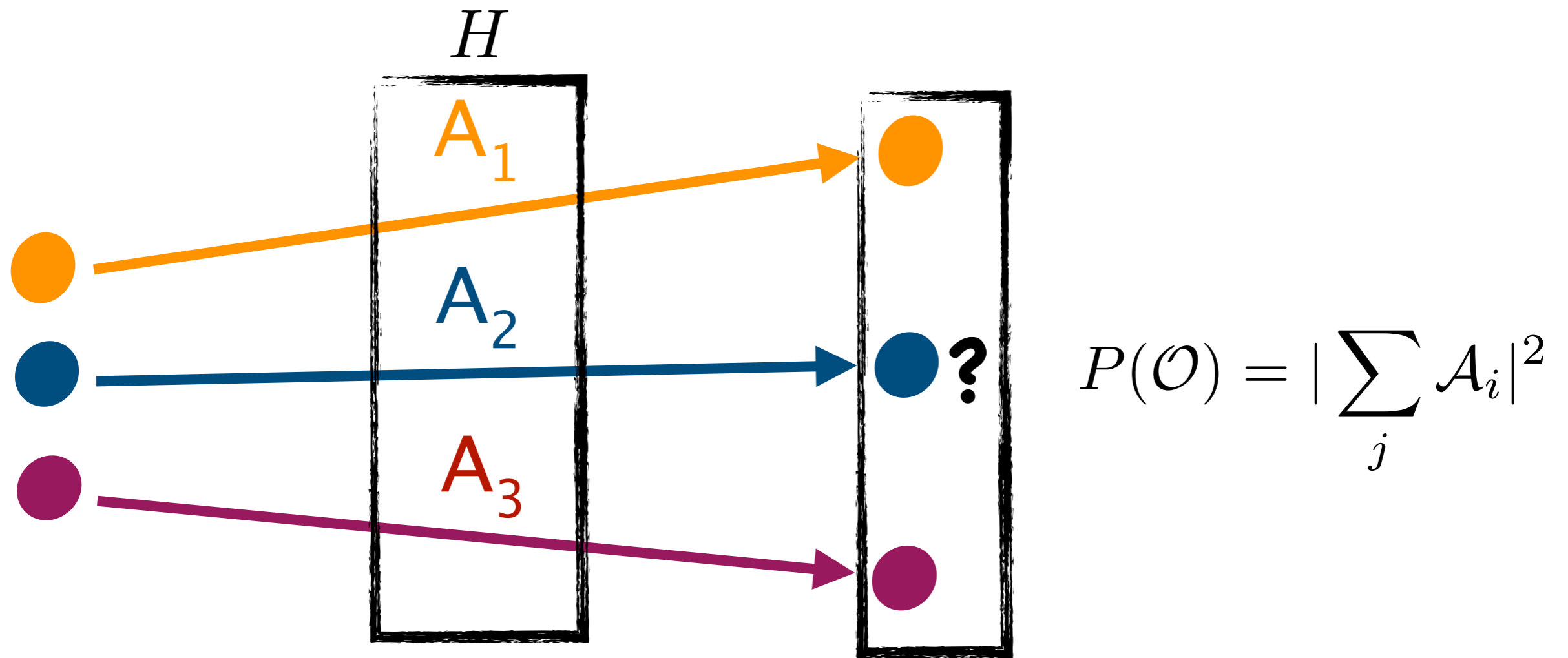
I.e. possible final probability distribution different

Need to understand of how we generate  $P[\zeta]$



# Key Ideas

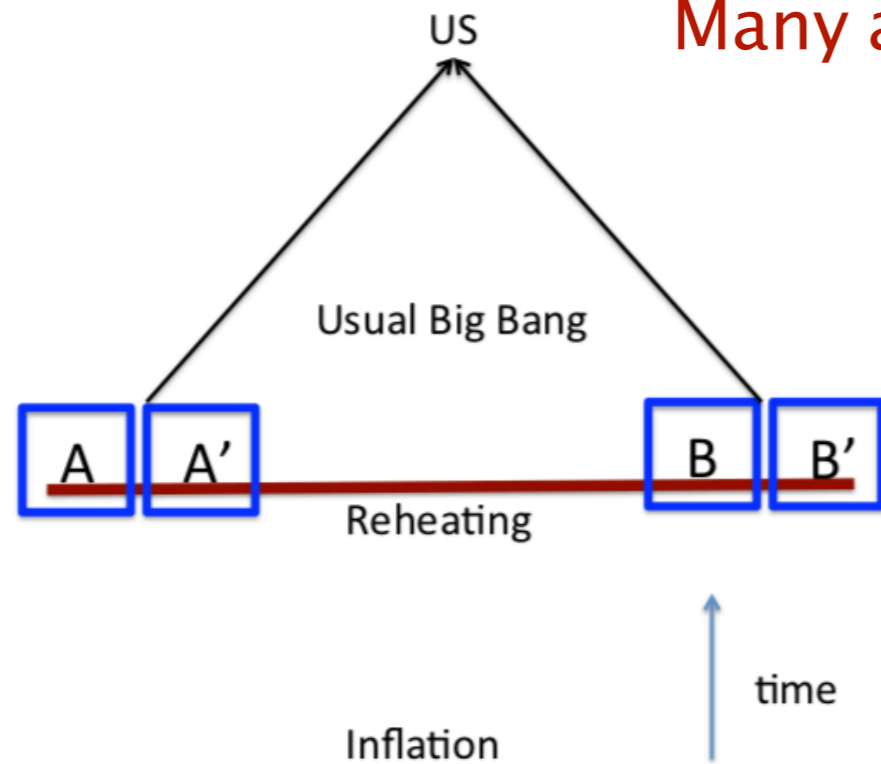
Can the evolution itself act as a Bell measurement



Not obviously classical when  $[H, \mathcal{O}] = \mathcal{O}' \neq 0$

## Bell measurement

Many authors

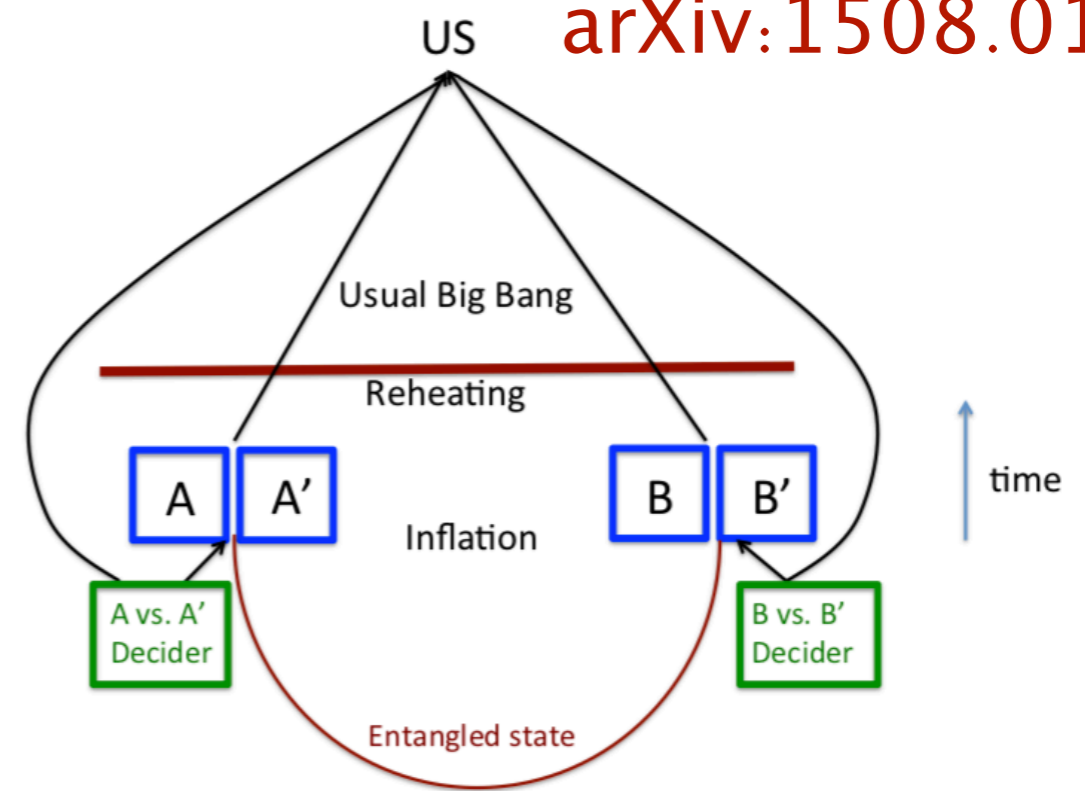


Suppressed by  $10^{-115}$

E.g. Martin & Vennin, arXiv:1706.05001  
de Putter & Doré, arXiv:1905.01394

## Maldacena's Model

arXiv:1508.01082



This works, but only in this highly contrived model

We want something more generic

# Key Ideas

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Classical fluctuations are always real/physical

Classical statistics just represents our uncertainty

**Classical modes are sines and cosines**

Quantum (vacuum) fluctuations become physical

**Quantum modes are positive / negative frequencies**

Direct reflect of quantum vs classical statistics

# Gaussian Fluctuations

Gaussian modes described as

$$ds^2 = \frac{1}{H^2 \tau^2} (-d\tau^2 + d\mathbf{x}^2)$$

$$\zeta(\mathbf{x}, \tau) = \int \frac{d^3 k}{(2\pi)^3} \frac{\Delta\zeta}{\sqrt{k^3}} e^{i\mathbf{k}\cdot\mathbf{x}} \left[ a_{\mathbf{k}}^\dagger (1 - ik\tau) e^{ik\tau} + a_{-\mathbf{k}} (1 + ik\tau) e^{-ik\tau} \right]$$

**Quantum:**  $\langle 0 | a_{\mathbf{k}'} a_{\mathbf{k}}^\dagger | 0 \rangle = \delta(\mathbf{k} - \mathbf{k}'), \quad \langle 0 | a_{\mathbf{k}}^\dagger a_{\mathbf{k}'} | 0 \rangle = 0$

**Classical:**  $\langle a_{\mathbf{k}}^\dagger a_{\mathbf{k}'} \rangle_c = \frac{1}{2} \delta(\mathbf{k} - \mathbf{k}') = \langle a_{\mathbf{k}'} a_{\mathbf{k}}^\dagger \rangle_c$

# Gaussian Fluctuations

Equal-time correlators are identical

$$\langle \zeta_{\mathbf{k}}(\tau) \zeta_{\mathbf{k}'}(\tau) \rangle_{q,c} = \frac{\Delta_{\zeta}^2}{k^3} (1 + k^2 \tau^2) (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

Non-equal times differ, reflecting  $[\zeta, \dot{\zeta}]_q \neq 0$

$$\langle \zeta_{\vec{k}}(0) \zeta_{\vec{k}'}(\tau) \rangle'_q = \frac{\Delta_{\zeta}^2}{k^3} e^{ik\tau} (1 - ik\tau)$$

$$\langle \zeta_{\vec{k}}(0) \zeta_{\vec{k}'}(\tau) \rangle'_c = \frac{\Delta_{\zeta}^2}{k^3} (\cos(k\tau) + k\tau \sin(k\tau))$$

# Non-Gaussian Fluctuations



What about non-Gaussian correlators?

We will assume NG arises from nonlinear evolution  
I.e. no non-local correlations in the initial state

We do not want to assume special interactions  
E.g. not allowed to arrange a Bell measurement

Key difference between cosmology and computing

Let us work out a simple example

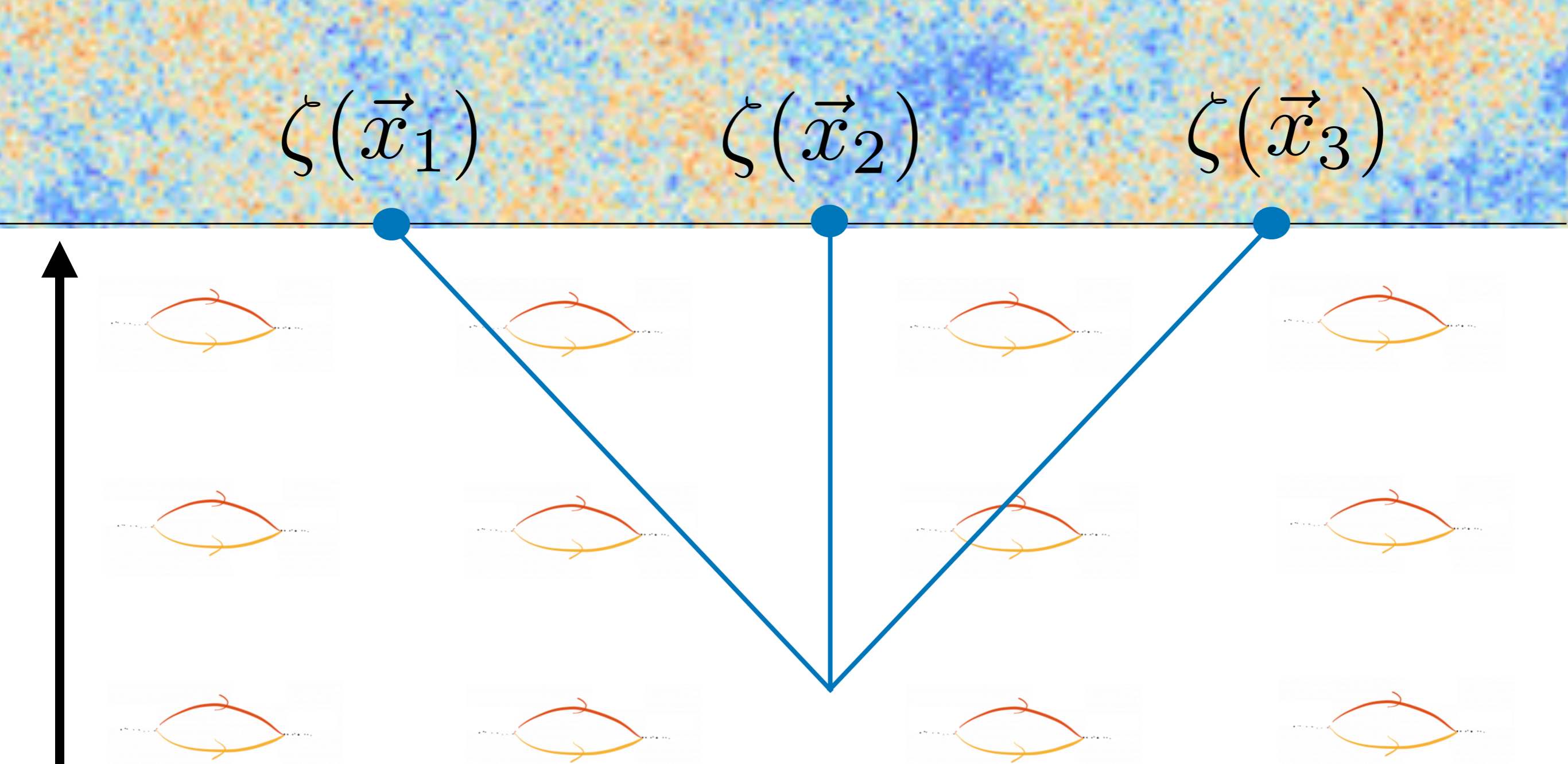
$$H_{\text{int}} = -\frac{\lambda}{3!} \zeta^3$$

Standard in-in perturbation theory

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle'_q = i \int d\tau' \langle [H_{\text{int}}(\tau'), \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3}(0)] \rangle$$

$$= \frac{4\lambda H^{-1} \Delta_{\zeta}^6}{(k_1 + k_2 + k_3)^3 k_1 k_2 k_3}$$

“total energy pole”



Creation of three particles from vacuum

$$\delta(E_1 + E_2 + E_3) \rightarrow \frac{1}{(E_1 + E_2 + E_3)^n}$$

Scattering

Uncertainty Principle



Residue is related to the S-matrix

Maldacena & Pimentel; Raju

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle'_q = \frac{4\lambda H^{-1} \Delta_\zeta^6}{(k_1 + k_2 + k_3)^3 k_1 k_2 k_3}$$

“total energy pole”

$$\propto \frac{\mathcal{A}_{\text{on-shell}}(\vec{k}_1, \vec{k}_2, \vec{k}_3)}{(k_1 + k_2 + k_3)^3 k_1^2 k_2^2 k_3^2}$$

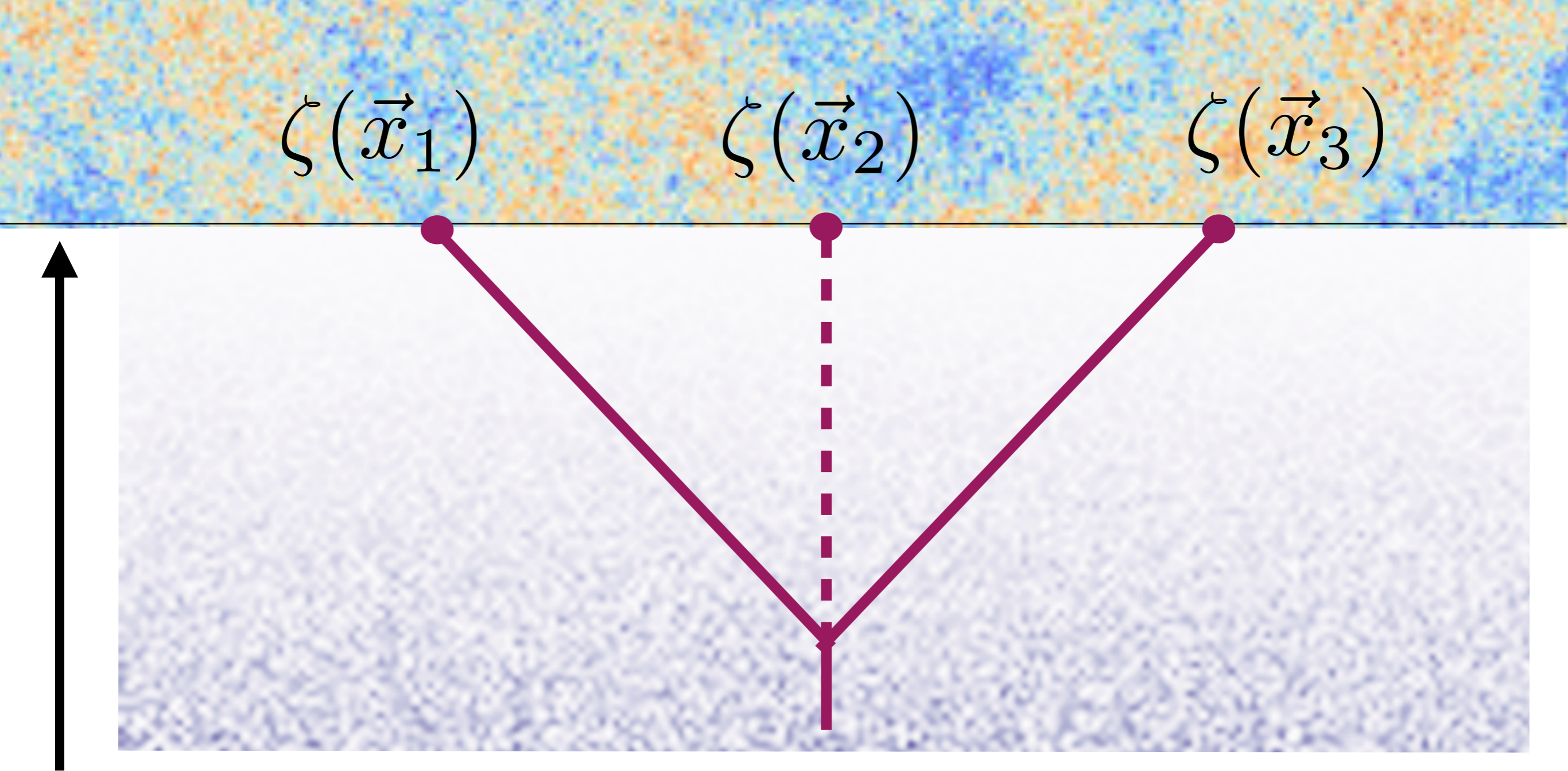
The pole itself is just signaling energy conservation

Classical analogue: solve EoM perturbatively

$$\zeta_{\mathbf{k}}^{(2)}(\tau) = \lambda \int \frac{d\tau' d^3p}{(2\pi)^3} (-H\tau')^{-1} (\partial_{\tau'} G_{\mathbf{k}}(\tau, \tau')) \partial_{\tau'} \zeta_{\mathbf{p}}^{(1)}(\tau') \partial_{\tau'} \zeta_{\mathbf{k}-\mathbf{p}}^{(1)}(\tau')$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle'_c = \frac{\lambda H^{-1} \Delta_{\zeta}^6}{3k_1 k_2 k_3} \left[ \frac{3}{(k_1 + k_2 + k_3)^3} \text{ “total energy pole”} \right. \\ \left. + \frac{1}{(k_1 + k_2 - k_3)^3} + \frac{1}{(k_1 - k_2 + k_3)^3} + \frac{1}{(k_2 - k_1 + k_3)^3} \right]$$

On-shell pole or “Folded Shape”



Creation of three particles from vacuum

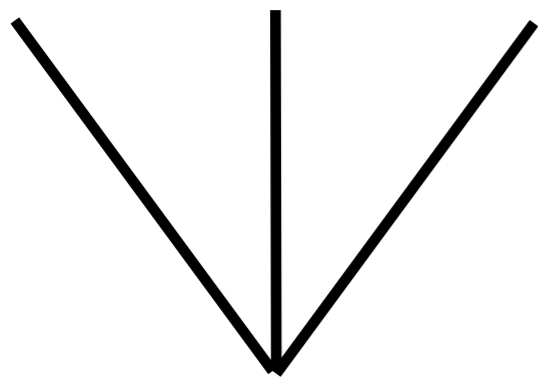
$$\delta(E_1 - E_2 - E_3) \rightarrow \frac{1}{(E_1 - E_2 - E_3)^n}$$

Scattering

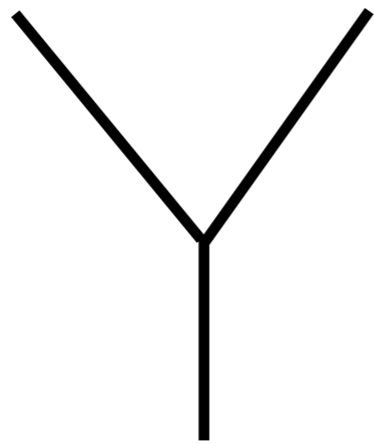
On-Shell Decay

A result of crossing symmetry of the S-matrix

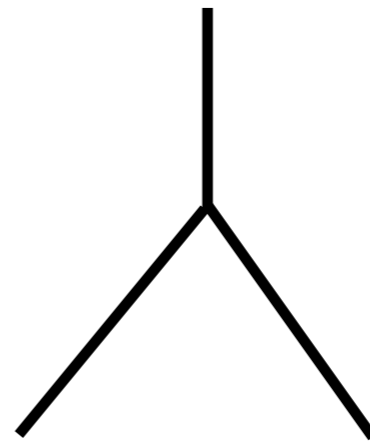
LSZ formula shows that you must have all processes



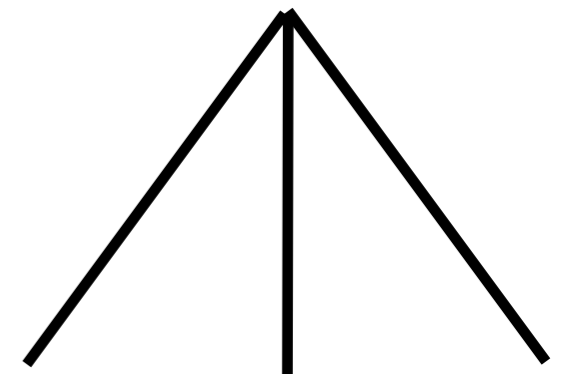
$0 \rightarrow 3$



$1 \rightarrow 2$



$2 \rightarrow 1$



$3 \rightarrow 0$

Only differ by number of particles in the initial state

Classical fluctuations = particles in the initial state

Fluctuations always includes negative frequencies

$$\zeta(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^3} \frac{\Delta\zeta}{\sqrt{k^3}} e^{i\mathbf{k}\cdot\mathbf{x}} [a_{R,\mathbf{k}}(\cos(k\tau) + k\tau \sin(k\tau)) + a_{I,\mathbf{k}}(\sin(k\tau) - k\tau \cos(k\tau))]$$

$$a_{\vec{k}} = \frac{1}{2} [a_{R,\vec{k}} + ia_{I,\vec{k}}]$$

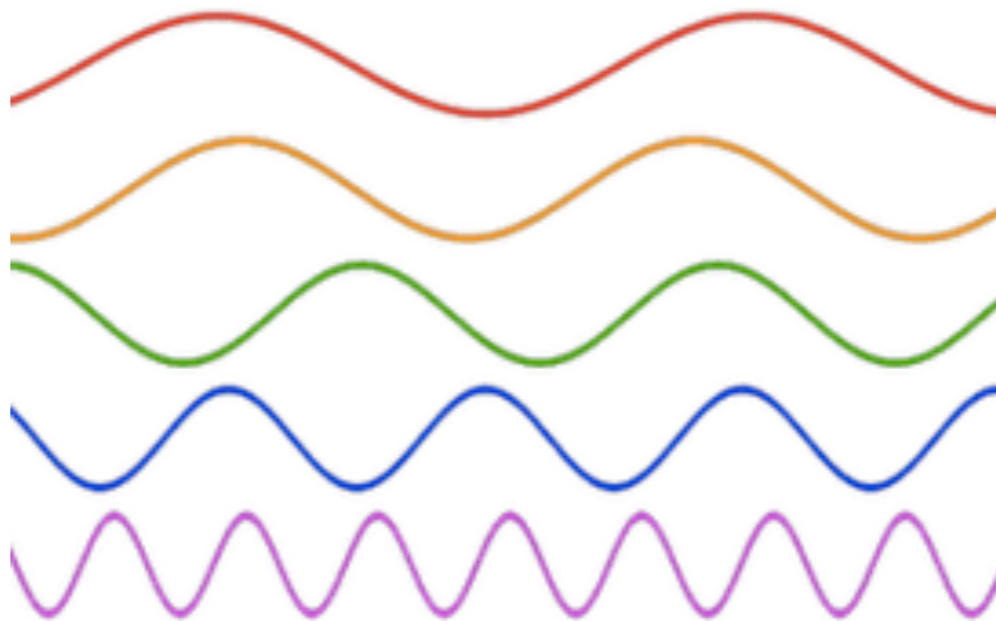
$$\langle a_{R,\mathbf{k}} a_{R,\mathbf{k}'} \rangle_c = \delta(\mathbf{k} + \mathbf{k}') = \langle a_{I,\mathbf{k}} a_{I,\mathbf{k}'} \rangle_c$$

Classical probability is just lack of knowledge

Classical fluctuations = particles in the initial state

Observed fluctuations exist in the far past

Observed



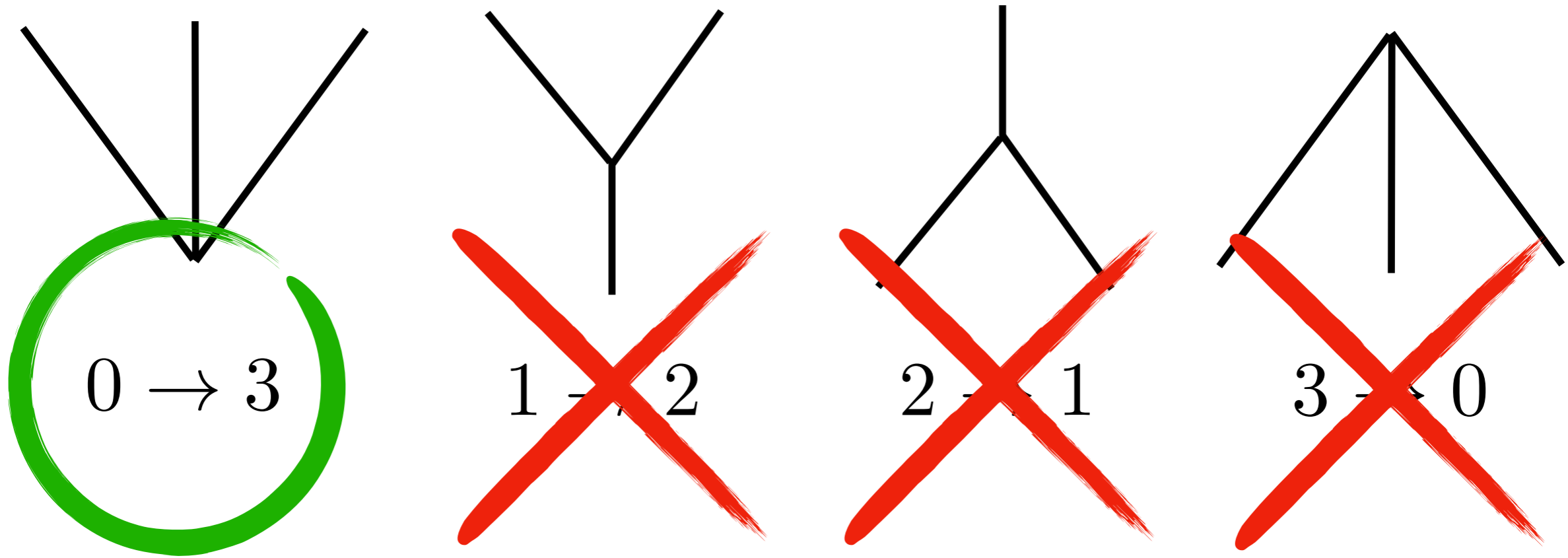
Created

Classical probability is just lack of knowledge

# Quantum vs Classical

No classical analogue of vacuum fluctuations

Quantum vacuum has no initial particles



# Quantum vs Classical

In detail: compare quantum In-In to classical EoM

$$\begin{aligned} \langle \zeta(\mathbf{x}_1, \tau) \zeta(\mathbf{x}_2, \tau) \zeta(\mathbf{x}_3, \tau) \rangle_q &= -i \int_{-\infty}^{\tau} d\tau' d^3\mathbf{x}' a^4(\tau') [\langle [H_{\text{int}}, \zeta(\mathbf{x}_1, \tau)] \zeta(\mathbf{x}_2, \tau) \zeta(\mathbf{x}_3, \tau) \rangle \\ &+ \langle \zeta(\mathbf{x}_1, \tau) [H_{\text{int}}(\mathbf{x}', \tau'), \zeta(\mathbf{x}_2, \tau)] \zeta(\mathbf{x}_3, \tau) \rangle + \langle \zeta(\mathbf{x}_1, \tau) \zeta(\mathbf{x}_2, \tau) [H_{\text{int}}(\mathbf{x}', \tau'), \zeta(\mathbf{x}_3, \tau)] \rangle] \\ &= \lambda \int_{-\infty}^t dt' d^3x' a^3(t') \left[ \dot{G}(t, t'; \mathbf{x}_1 - \mathbf{x}') \langle \dot{\zeta}^2(\mathbf{x}', t') \zeta(\mathbf{x}_2, t) \zeta(\mathbf{x}_3, t) \rangle \right. \\ &+ \dot{G}(t, t'; \mathbf{x}_2 - \mathbf{x}') \langle \zeta(\mathbf{x}_1, t) \dot{\zeta}^2(\mathbf{x}', t') \zeta(\mathbf{x}_3, t) \rangle + \dot{G}(t, t'; \mathbf{x}_3 - \mathbf{x}') \langle \zeta(\mathbf{x}_1, t) \zeta(\mathbf{x}_2, t) \dot{\zeta}^2(\mathbf{x}', t') \rangle \end{aligned}$$

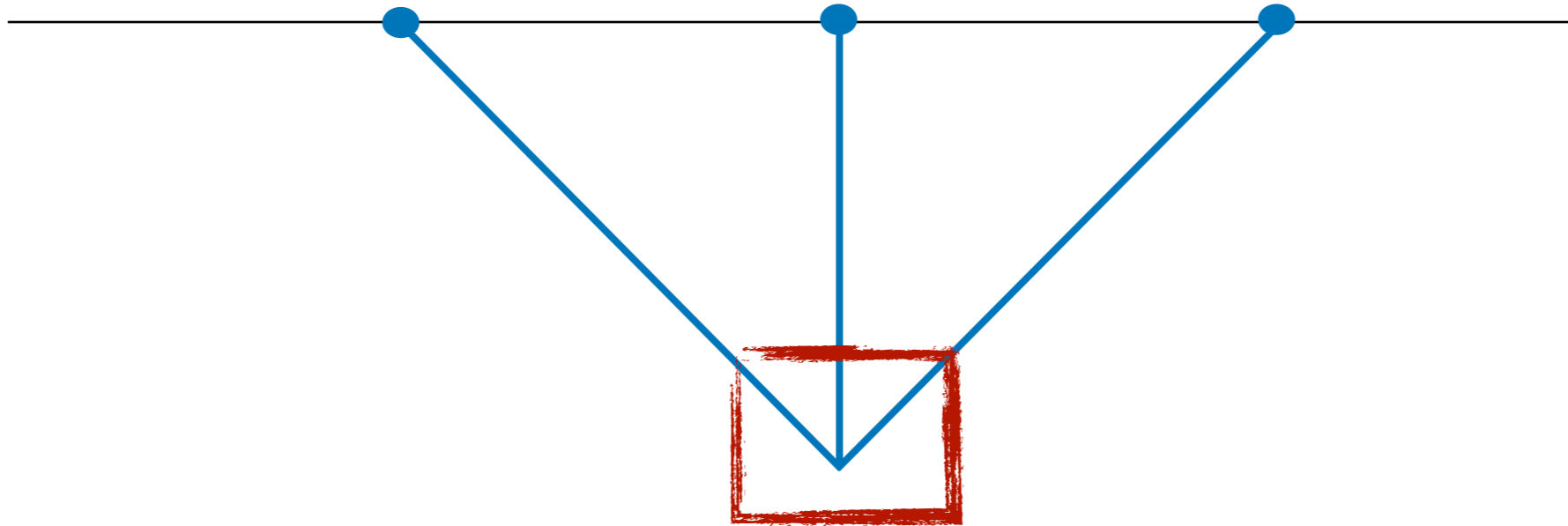
Terms differ by operator ordering



# Quantum vs Classical

## Removing ordering with commutators

$$\langle \zeta(\mathbf{x}_1, \tau) \zeta(\mathbf{x}_2, \tau) \zeta(\mathbf{x}_3, \tau) \rangle_q - \langle \zeta(\mathbf{x}_1, \tau) \zeta(\mathbf{x}_2, \tau) \zeta(\mathbf{x}_3, \tau) \rangle_c =$$
$$+ \frac{i\lambda}{4} \int_{-\infty}^{\tau} d\tau' d^3x' a^4(\tau') \left[ \zeta(\mathbf{x}_1, \tau), \dot{\zeta}(\mathbf{x}', \tau') \right] \left[ \zeta(\mathbf{x}_2, \tau), \dot{\zeta}(\mathbf{x}', \tau') \right] \left[ \zeta(\mathbf{x}_3, \tau), \dot{\zeta}(\mathbf{x}', \tau') \right]$$



Commutator non-zero at the  
intersection of past light-cones



# Non-Local Hidden Variables

# Hidden Variables

We can evade Bell's inequality if we give up locality  
Is there a similar result in cosmological signature?

Idea: Classical theory prefers positive frequencies

Use a complex scalar

$$\phi_{\mathbf{k}}(\tau) = \frac{\Delta\phi}{k^{3/2}} \left[ a_{\mathbf{k}}^\dagger (1 - ik\tau) e^{ik\tau} + b_{-\mathbf{k}} (1 + ik\tau) e^{-ik\tau} \right]$$

Only excite the positive frequencies

$$\left\langle a_{\mathbf{k}}^\dagger a_{\mathbf{k}'} \right\rangle_c = \left\langle a_{\mathbf{k}'} a_{\mathbf{k}}^\dagger \right\rangle_c = \delta(\mathbf{k} - \mathbf{k}'), \quad \left\langle b_{\mathbf{k}} b_{\mathbf{k}'}^\dagger \right\rangle_c = 0$$

# Hidden Variables

Now we will simply invent a “Green’s function”

$$\dot{G}_{\mathbf{k}} \rightarrow G_{\mathbf{k}}^{\text{eff}} (\tau \rightarrow 0, \tau') = \frac{1}{k} e^{-ik\tau'}$$

By analogy with an interaction  $\mathcal{L}_{\text{int}} = \lambda_{\phi} \dot{\phi}^3 + \lambda_{\phi}^* \dot{\phi}^{*3}$

$$\phi_{\mathbf{k}}(\tau \rightarrow 0) = \frac{i}{3} \lambda_{\phi} \int \frac{d^3 p}{(2\pi)^3} \int d\tau' \frac{1}{k} e^{-ik\tau'} \dot{\phi}_{\mathbf{p}}^*(\tau') \dot{\phi}_{\mathbf{k}-\mathbf{p}}^*(\tau')$$

Describes 1 particle decaying to 2 anti-particles

# Hidden Variables

Using this to compute the bispectrum gives

$$\begin{aligned}\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \rangle' &= i\lambda_\phi H^{-1} \frac{\Delta_\phi^6}{k_1 k_2 k_3} \int_{-\infty}^0 d\tau' \tau'^2 e^{-i(k_1 + k_2 + k_3)\tau'} \\ &= \frac{2\lambda_\phi H^{-1} \Delta_\phi^6}{(k_1 + k_2 + k_3)^3 k_1 k_2 k_3}\end{aligned}$$

Only net creation of three antiparticles

Only a total energy pole

# Hidden Variables

This theory is non-local

$$G_{\mathbf{k}}^{\text{eff}} (\tau \rightarrow 0, \tau' \rightarrow 0) = \frac{1}{k}$$

Non-zero at space-like separations

$$G^{\text{eff}} (\vec{x}, \tau \rightarrow 0, \tau' \rightarrow 0) \propto \frac{1}{x^2}$$

In a local theory, must vanish outside the light-cone

$$G_{\mathbf{k}}^{\text{eff}} \sim \frac{1}{k} \sin(k\tau) \rightarrow G^{\text{eff}} (\vec{x}) \propto \delta(\vec{x})$$

# Hidden Variables

In this local theory

$$\phi_{\mathbf{k}}(\tau \rightarrow 0) = \frac{i}{3} \lambda_{\phi} \int \frac{d^3 p}{(2\pi)^3} \int d\tau' \frac{1}{k} \sin(k\tau') \dot{\phi}_{\mathbf{p}}^*(\tau') \dot{\phi}_{\mathbf{k}-\mathbf{p}}^*(\tau')$$

and

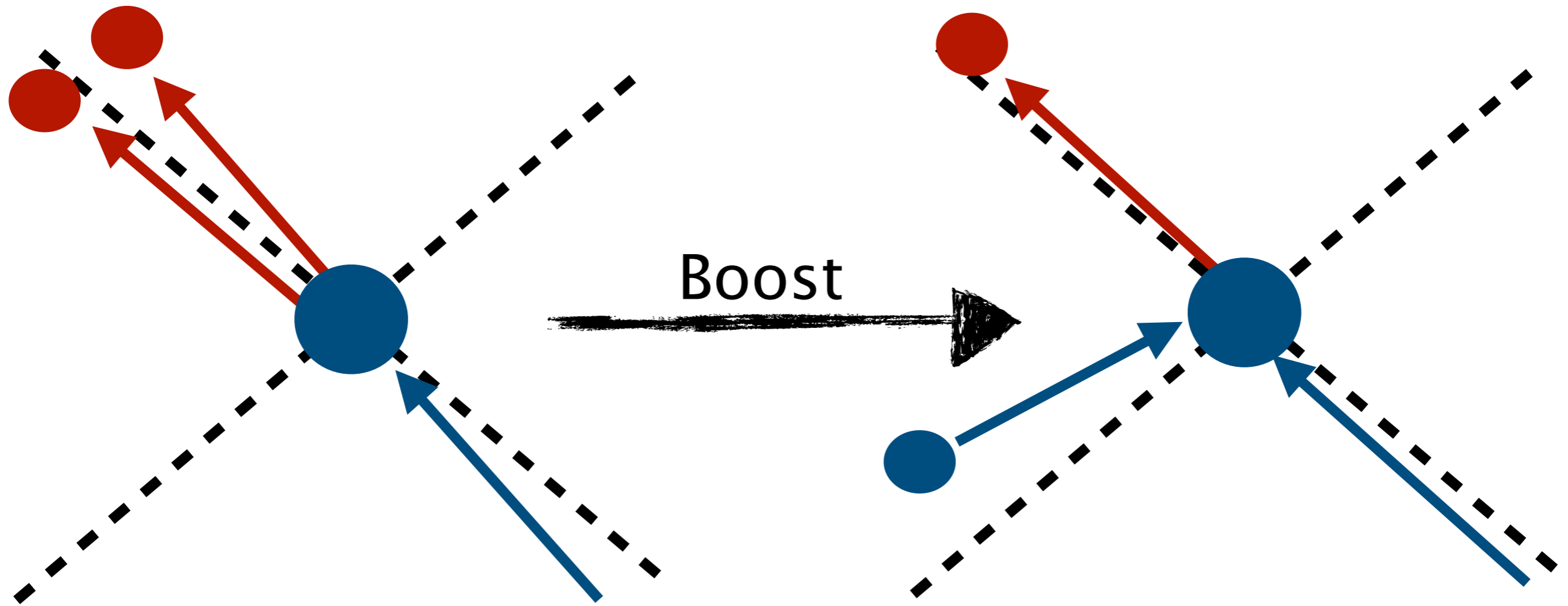
$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \rangle' \supset \frac{\lambda_{\phi} H^{-1} \Delta_{\phi}^6}{(k_1 - k_2 + k_3)^3 k_1 k_2 k_3}$$

Include 2 particles annihilating to 1 anti-particle

Non-trivial relation between crossing and causality

# Hidden Variables

This is the same as the need for antiparticles



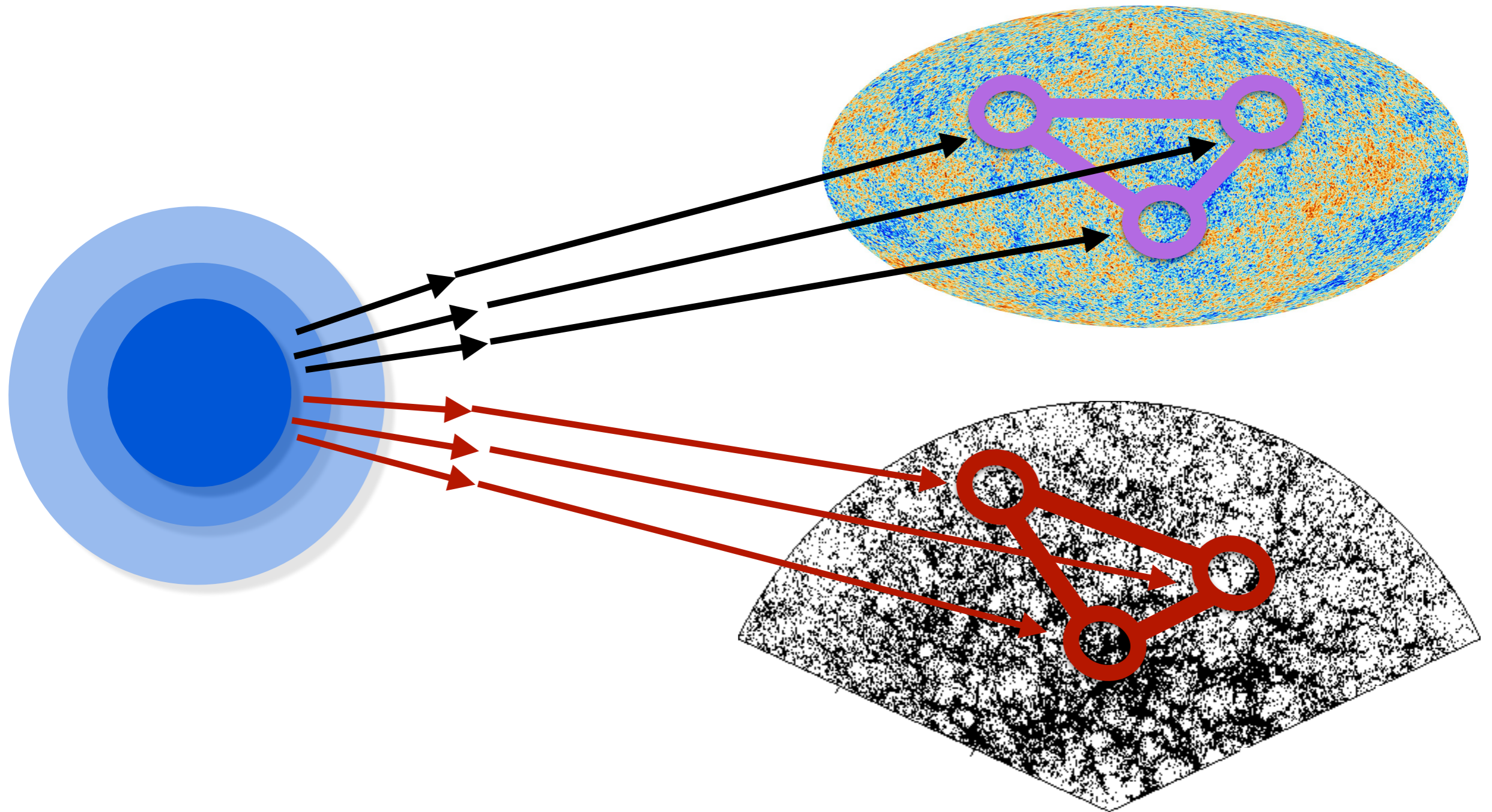
Lorentz invariance requires they come together





# Observational Implications

# Primordial Non-Gaussianity

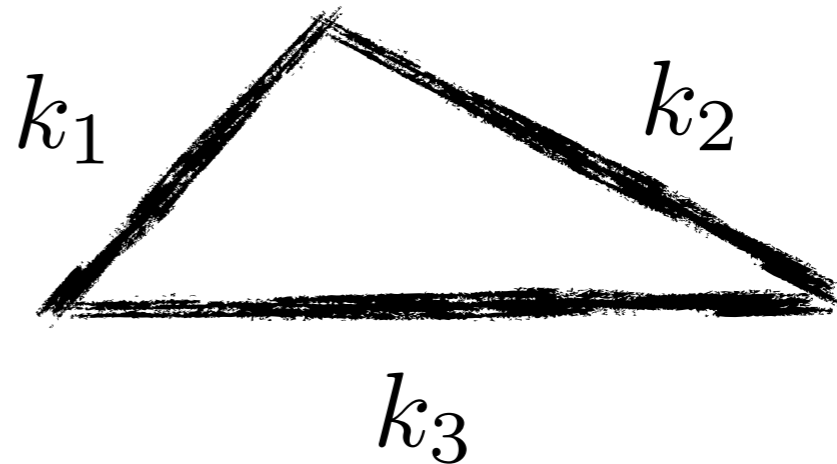


# Primordial Non-Gaussianity

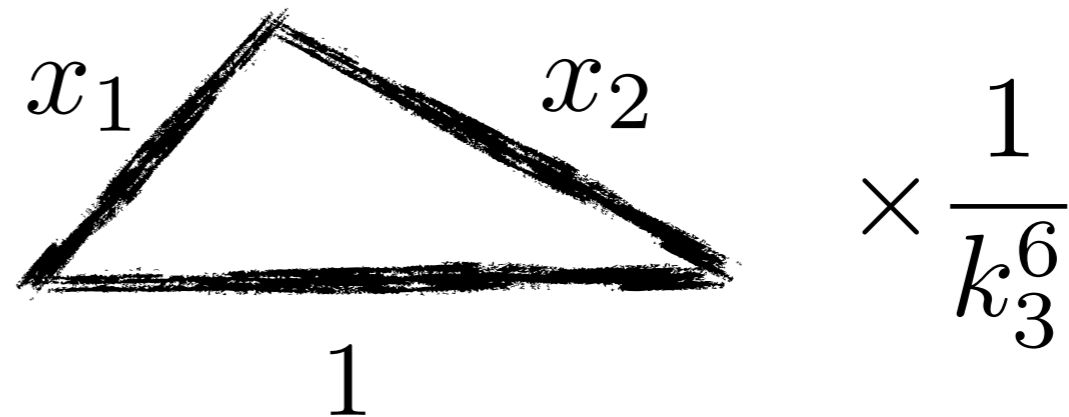
On general groups, bispectra take the form

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = B(k_1, k_2, k_3) (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

Momentum conservation:

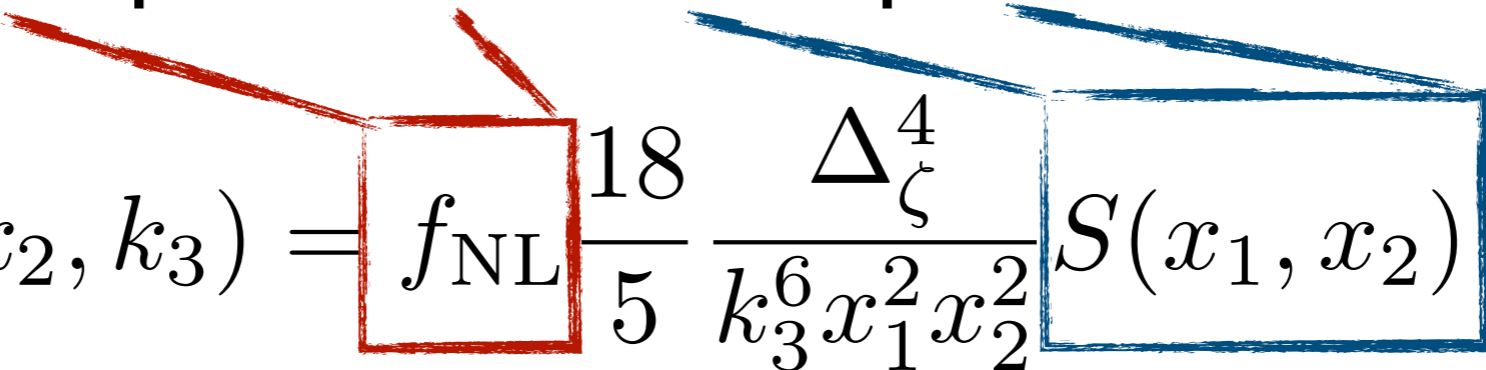


Scale invariance:



# Primordial Non-Gaussianity

Defined by amplitude and “shape”

$$B(k_1, k_2, k_3) = f_{\text{NL}} \frac{18}{5} \frac{\Delta_{\zeta}^4}{k_3^6 x_1^2 x_2^2} S(x_1, x_2)$$


The shapes live in a basis of orthogonal functions

$$\int dx_1 dx_2 S_1(x_1, x_2) S_2(x_1, x_2) = S_1 \cdot S_2 = \cos_{12}$$

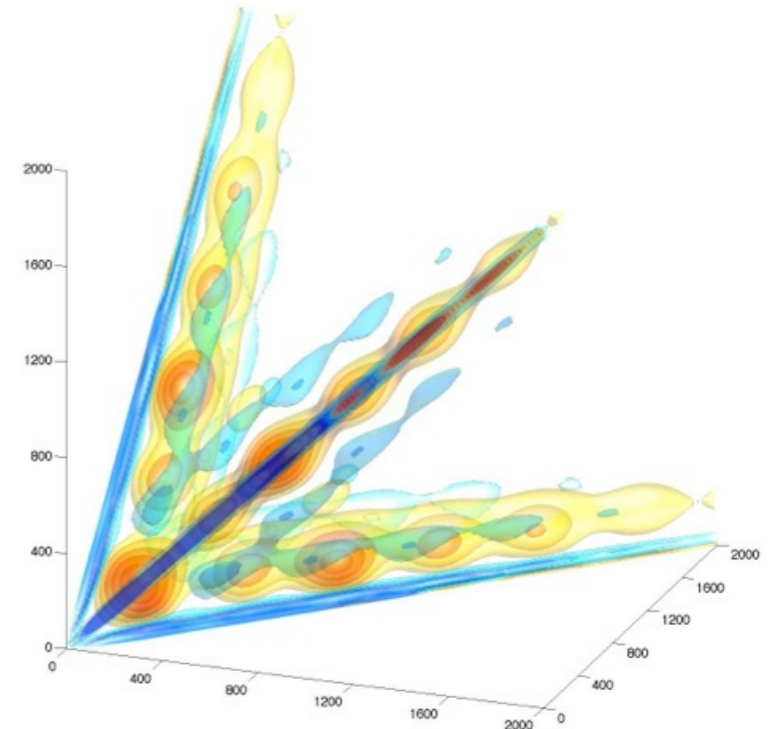
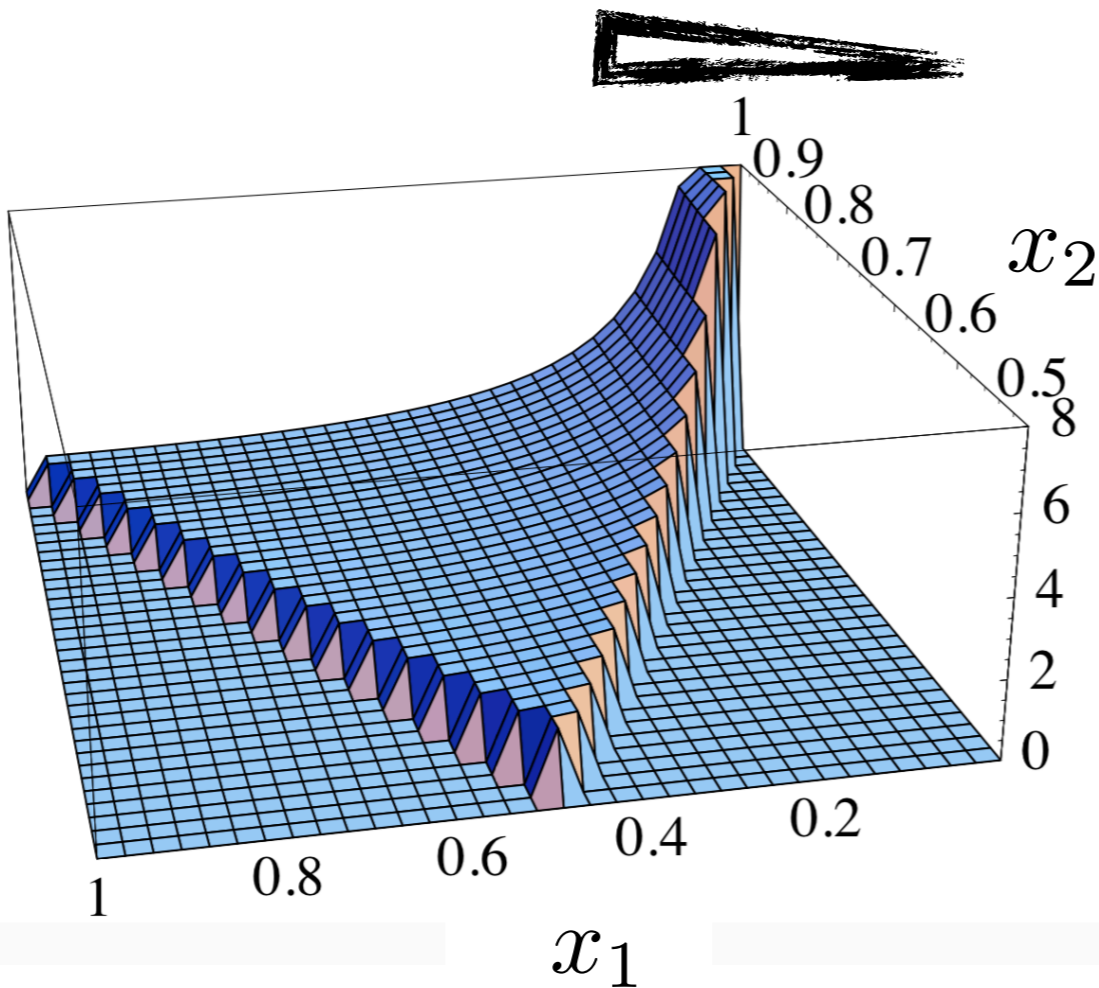
Cosine is how easily they are distinguish (in 3pt)

# Current Limits

## The “Local Shape”

$$f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1$$

Planck 2018



Courtesy of Fergusson & Shellard

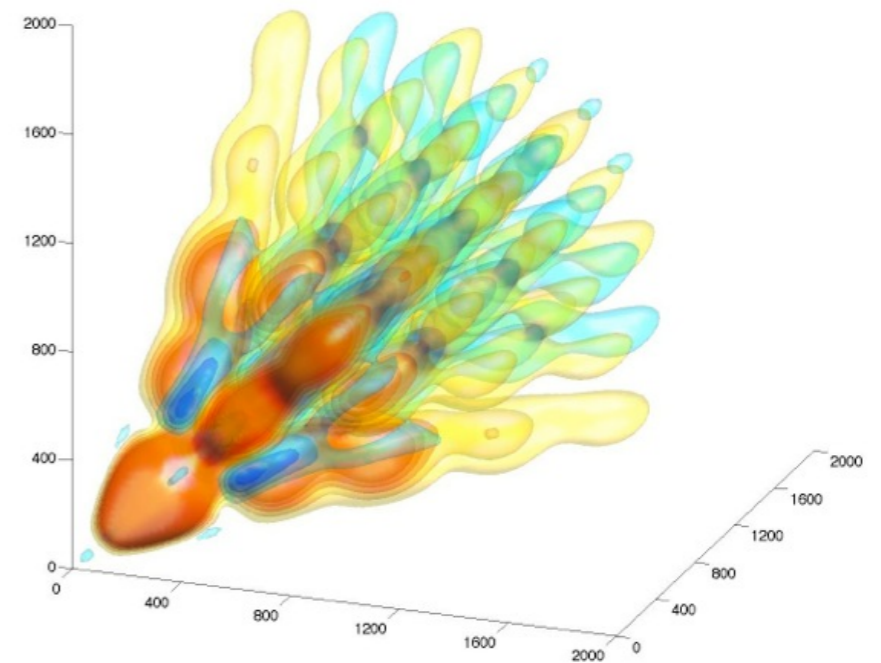
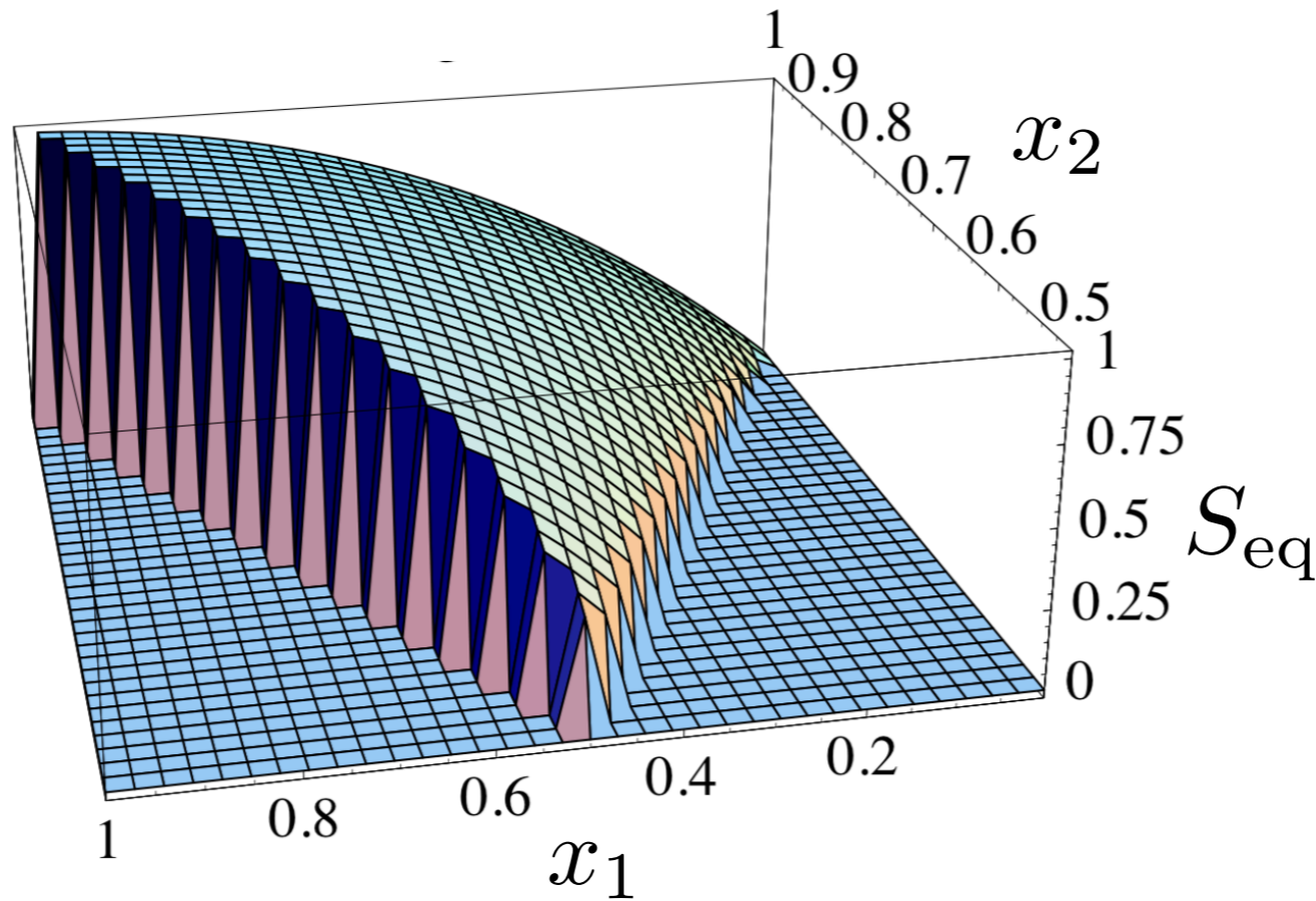
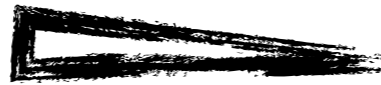
Babich et al.

# Current Limits

## The “Equilateral Shape”

$$f_{\text{NL}}^{\text{equil}} = -26 \pm 47$$

Planck 2018



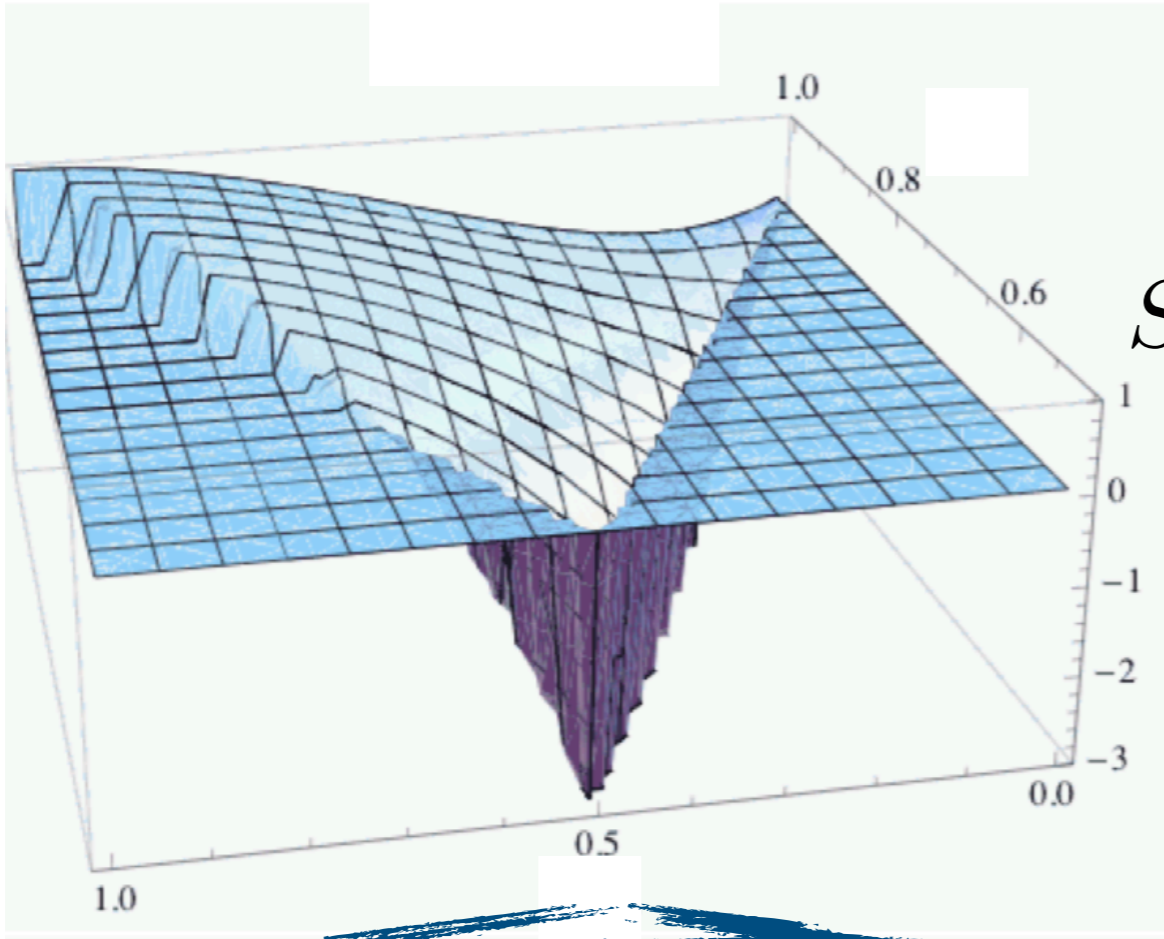
Courtesy of Fergusson & Shellard

Babich et al.

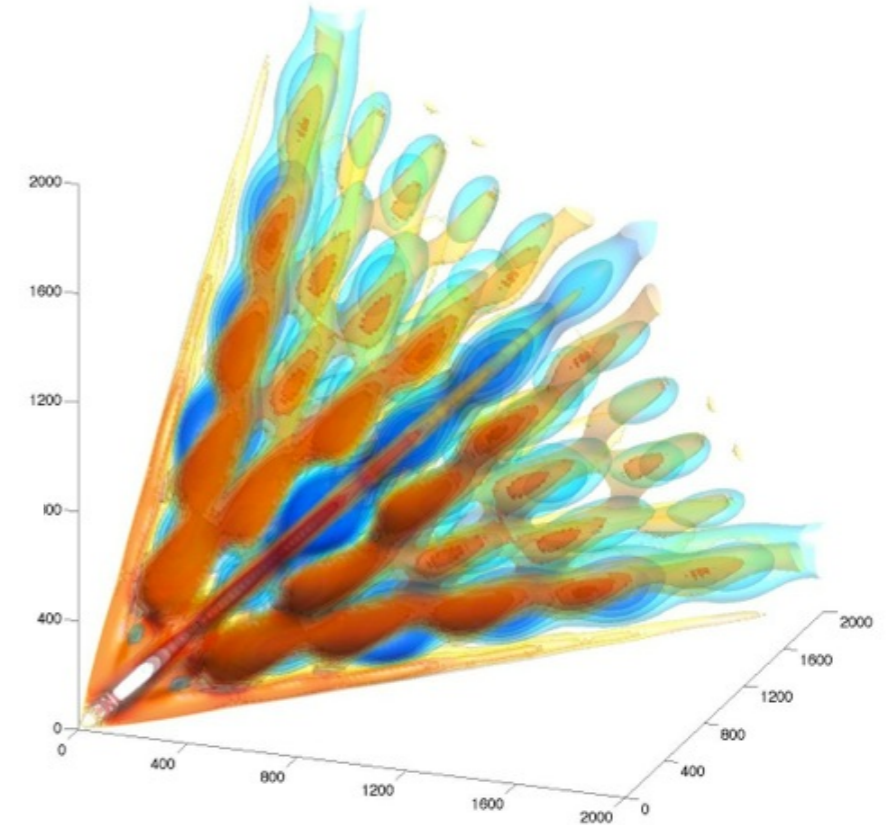
## The “Orthogonal Shape”

$$f_{\text{NL}}^{\text{ortho}} = -38 \pm 24$$

Planck 2018



Smith et al.

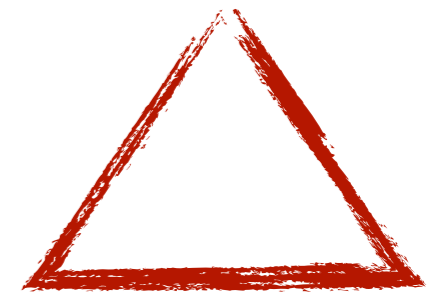


Courtesy of Fergusson & Shellard

# Observational Implications

Quantum NG = Equilateral NG

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle'_q = \frac{4\lambda H^{-1} \Delta_\zeta^6}{(k_1 + k_2 + k_3)^3 k_1 k_2 k_3}$$



Classical NG = Pole in folded limit

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_1} \zeta_{\vec{k}_1} \rangle'_c \supset \frac{\lambda H^{-1} \Delta_\zeta^6}{(k_1 - k_2 + k_3)^3 k_1 k_2 k_3}$$





# Observational Implications

As written, classical shape has  $S/N \rightarrow \infty$

Pole resolved into a bump (finite  $S/N$ ) by loops

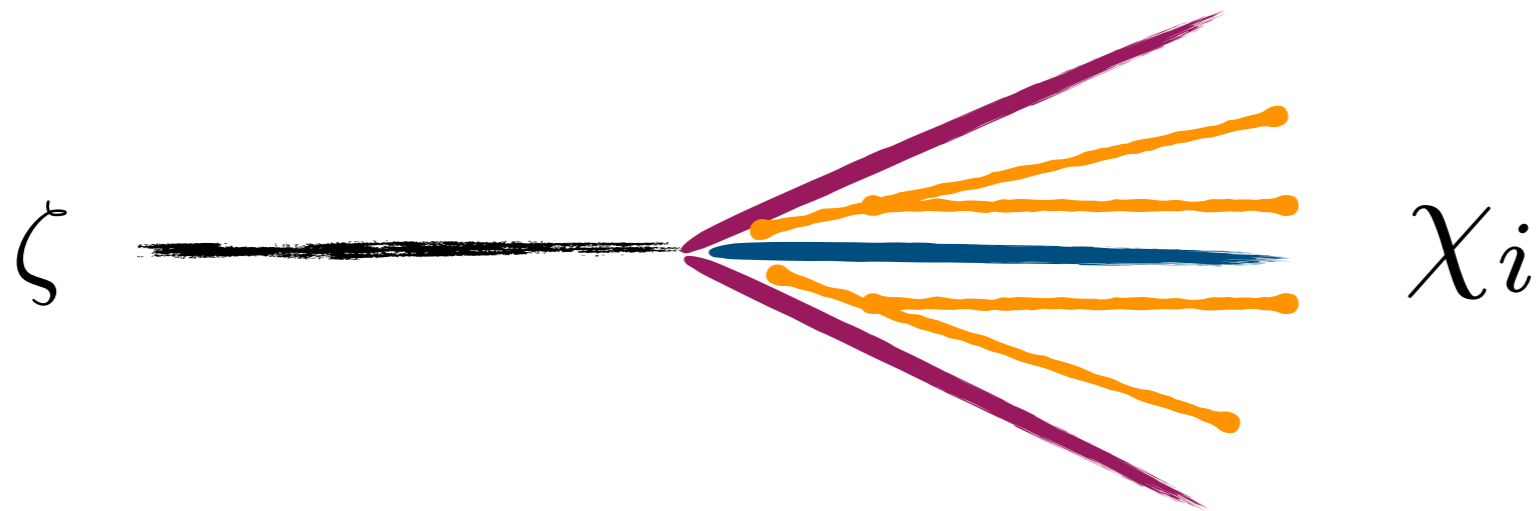
$$\frac{1}{(\kappa_1 - \kappa_2 + \kappa_3)^3} \rightarrow \frac{1}{(\kappa_1 - \kappa_2 + \kappa_3 + \kappa_\Gamma)^3}$$

If perturbative,  $S/N$  is still enhanced

$$(S/N)_{\text{class}} \approx \frac{1}{\lambda} (S/N)_{\text{quantum}} \gg (S/N)_{\text{quantum}}$$

# Observational Implications

Suppressed S/N with non-perturbative dissipation



Increasing decay rate is tied to amplitude of NG


$$k_{\Gamma} \propto \log \Gamma$$

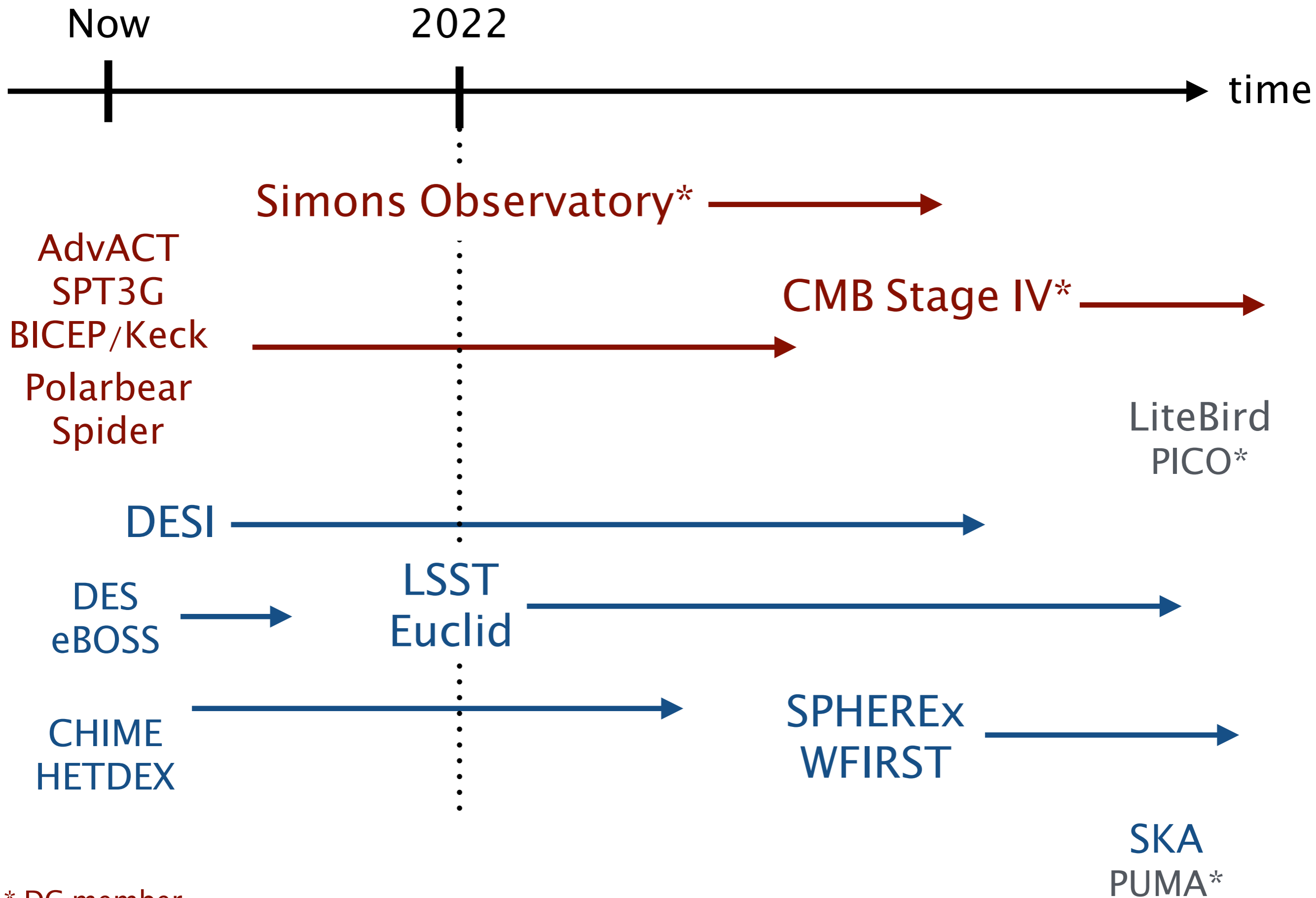
$$f_{\text{NL}} \propto \Gamma$$

Hard to hide the classical signal given current limits

# Observational Implications

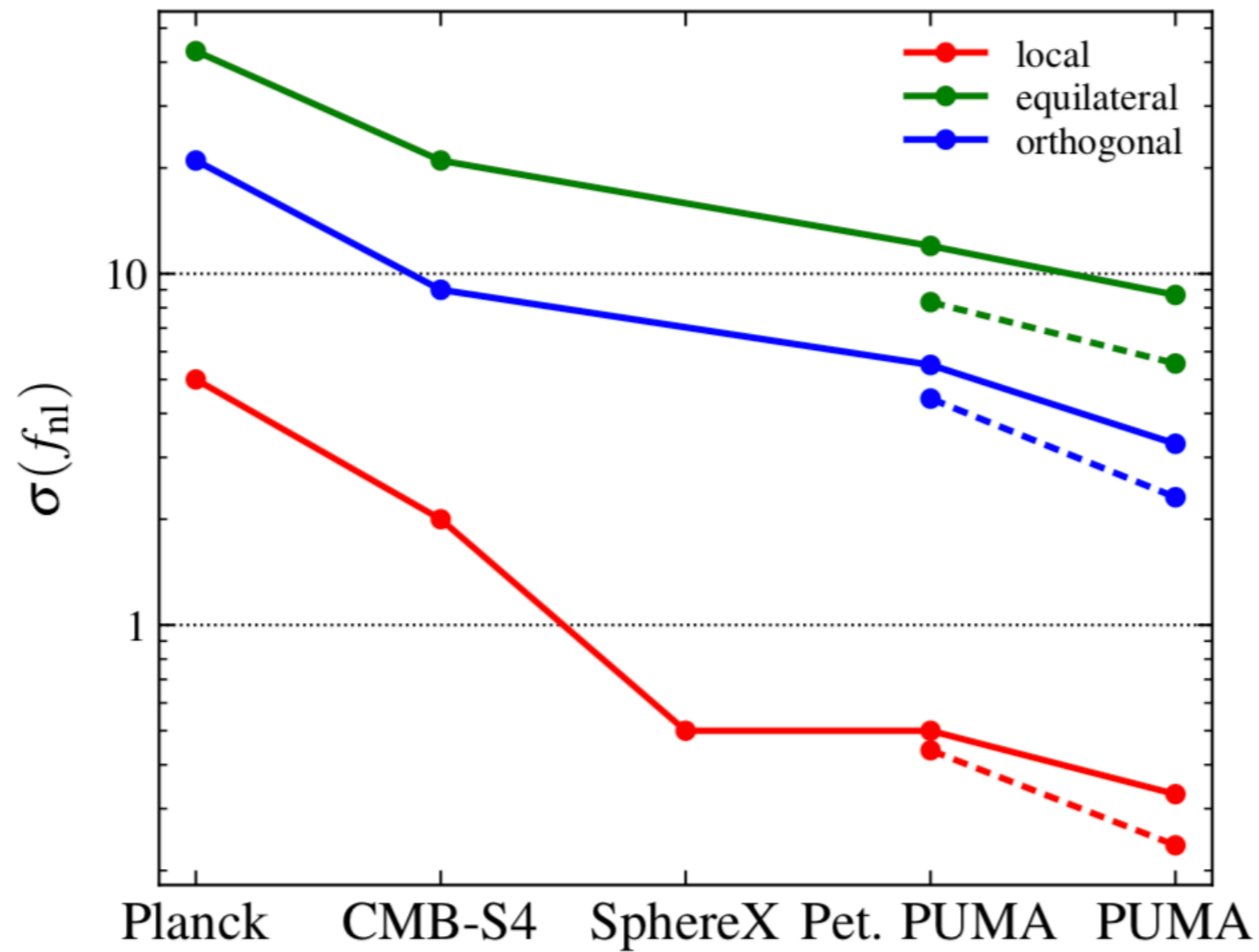
## Conclusions:

- (1) We need to detect equilateral NG
- (2) Check for enhanced folded pole
  - i. Most classical models would have large  $S/N$
  - ii. Extreme dissipation might (just) be excluded
- (3) Check squeezed/soft limit (local shape)
  - i. Test of single / multi-field 
  - ii. Sensitive to excited states / extra particles



\* DG member

Forecasted improvements on current limits:



# Summary



The “total energy pole” is a generic feature of NG

- The cosmological analogue of energy conservation
- Quantum vacuum fluctuations only have these
- Classical fluctuations also have physical poles

This result is a consequence of locality

Can only be avoided with non-local hidden variables



Thank you