Copernicus Webinar
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With J. Fumagalli, S. Garcia-Saenz, **S. Renaux-Petel**, J. Ronayne
Thanks also to **J. Martin**, T. Papanikolaou, Y. Tada, V. Vennin
Institut d'Astrophysique de Paris (IAP)

OBSERVATIONAL SIGNATURES OF MULTIFIELD INFLATION WITH CURVED FIELD SPACE

BACKGROUND, LINEAR FLUCTUATIONS AND NON-GAUSSIANITIES

Mainly based on [Fumagalli, Garcia-Saenz, LP, Renaux-Petel, Ronayne 2019] *Phys. Rev. Lett. 123, 201302*

[Garcia-Saenz, LP, Renaux-Petel 2020] *J. High Energ. Phys.* **2020,** 73 (2020)

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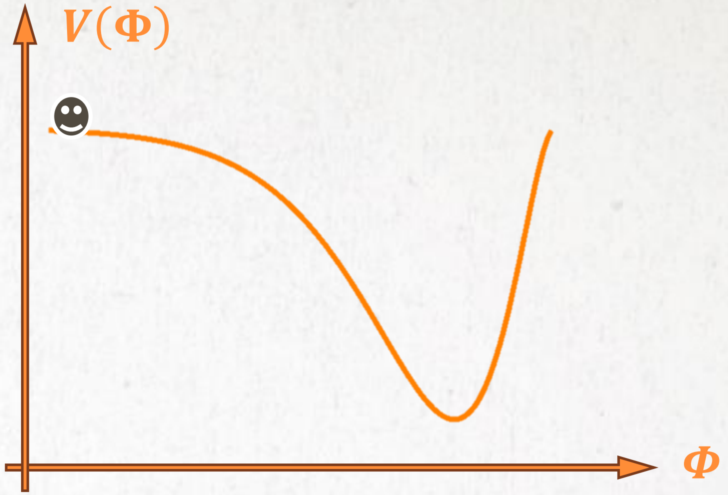
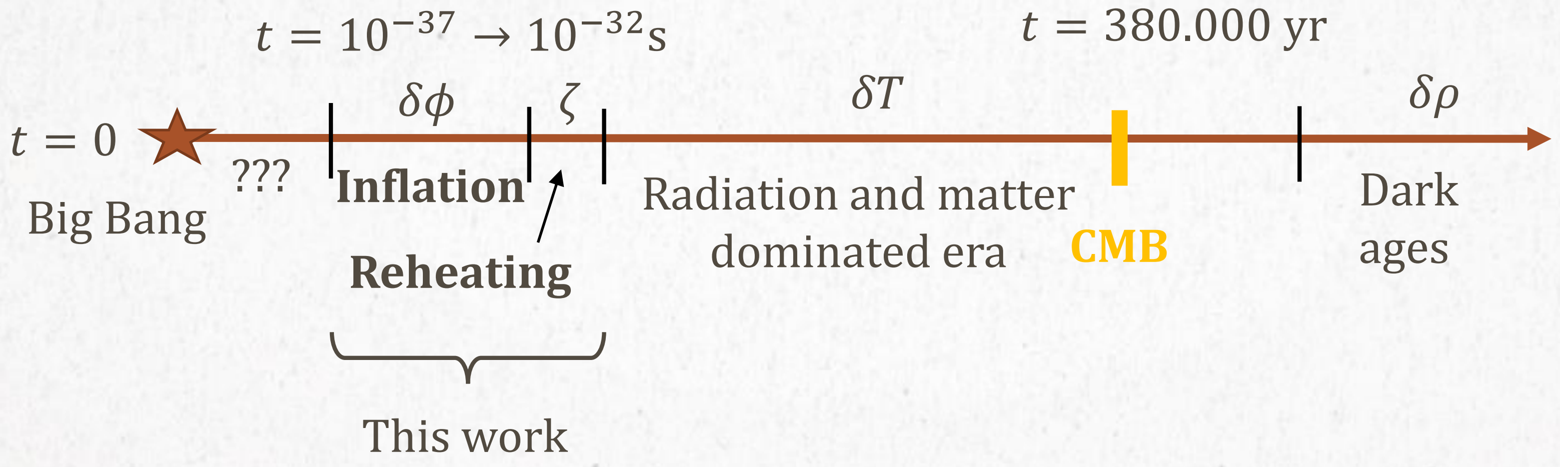
I. USUAL PICTURE OF INFLATION

A CONSISTENT COSMOLOGICAL STORY



VERY BROAD PICTURE

- Cosmology: history, content and laws of the Universe
- Early Universe: before emission of the CMB



CMB is the first observational consequence of the physics in the early universe

LINKS WITH HIGH-ENERGY THEORIES

$\log(E/GeV)$

Natural units: $\hbar = c = 1$ and the only dimension is energy (or mass)

19 — M_p: Planck mass

M_s: String theory, quantum loop gravity etc.

16 — M_{GUT} : Grand Unified Theories (GUT):

Unification of the fundamental non-grav. forces

? — H, inflation

The exact energy scale of inflation is not known

Detection of primordial GWs may tell us

3 — Standard Model, LHC

Accessible experiments on Earth

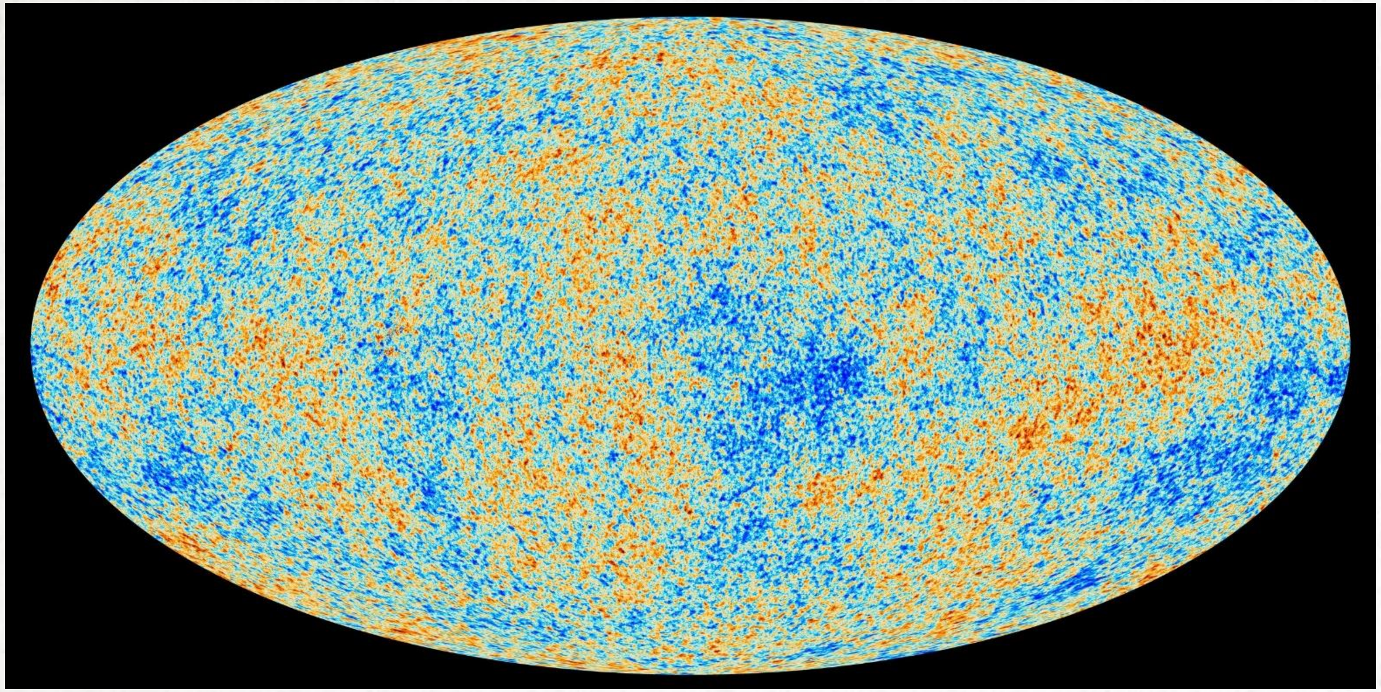
Inflation happens at high energies

It is sensitive to high-energy effects

We do not know physics at high energies so we can not say any thing about inflation

Let’s work on inflation and hopefully we learn about high-energy physics

CMB OBSERVATION MOTIVATES INFLATION



$$T \sim 2.73K \, ; \, \frac{\delta T}{T} \sim 10^{-5} \, ; \, |\Omega_k| \ll 1$$

- How is the universe so homogeneous?
  Horizon problem

- Why is the universe so spatially flat?
  Flatness problem

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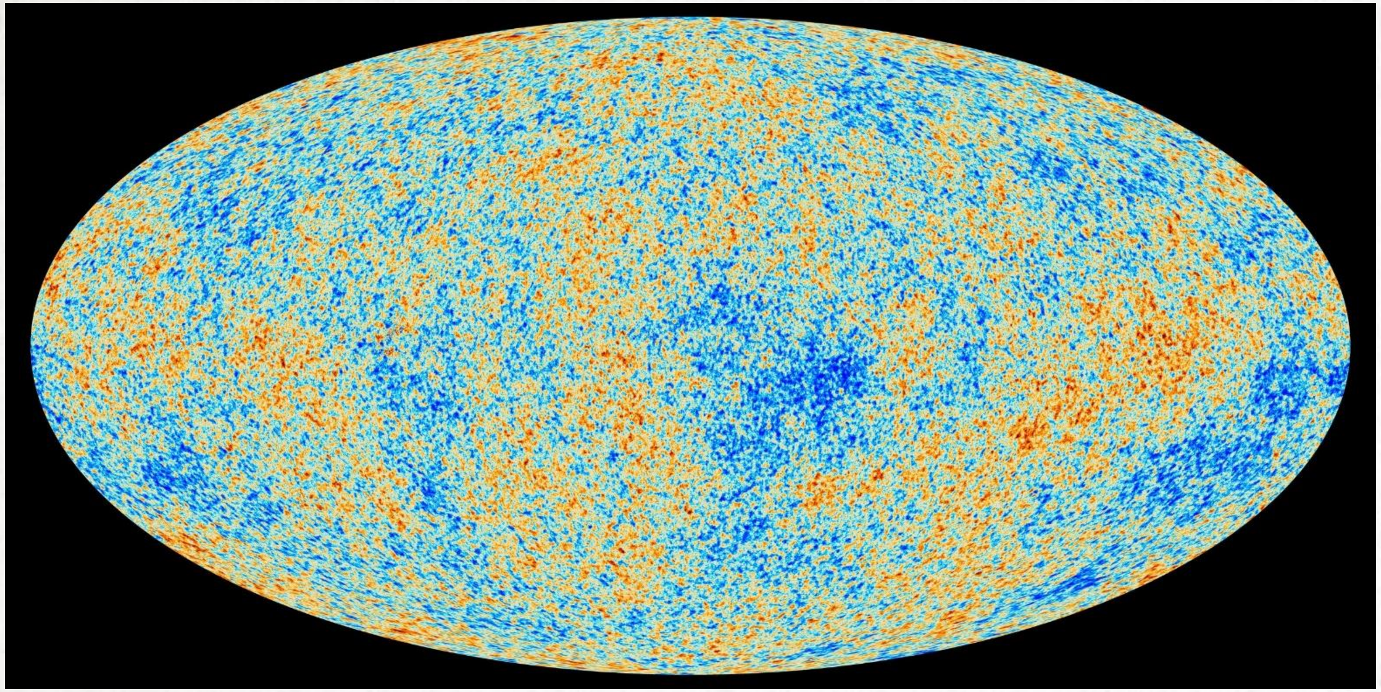

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- How is the universe so homogeneous?
  Horizon problem

- Why is the universe so spatially flat?
  Flatness problem

Inflation, an era of accelerated expansion of the Universe, solves both the horizon and flatness problems

$$N_{\mathrm{inf}} = \ln\left(\frac{a_{\mathrm{end}}}{a_{\mathrm{ini}}}\right) \gtrsim 55$$

OBSERVATIONAL CONSTRAINTS

Fluctuations **in the CMB** are mostly adiabatic

$\zeta(\tau, \vec{x})$ the adiabatic curvature perturbation

$\zeta_{\vec{k}}(\tau)$ its Fourier transform

dictates the statistics of the temperature anisotropies in the CMB

- 2-point (Gaussian) statistics:

$$< \zeta_{\vec{k}} \zeta_{\vec{k}'} > = (2\pi)^3 \ \delta^{(3)}(\vec{k} + \vec{k'}) \times P_\zeta(k)$$

Dimensionless power spectrum is

$$\mathcal{P}_\zeta(k) = \frac{2\,\pi^2}{k^3} P_\zeta(k) = \boldsymbol{A_s} \left(\frac{k}{k_*}\right)^{\boldsymbol{n_s}-1}$$

- 3-point (Non-Gaussian) statistics:

$$< \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} >= (2\pi)^3 \ \delta^{(3)}(\overrightarrow{k_1} + \overrightarrow{k_2} + \overrightarrow{k_3}) \times B_\zeta(\overrightarrow{k_1}, \overrightarrow{k_2}, \overrightarrow{k_3})$$

Dimensionless bispectrum is

$$\mathcal{B}_\zeta = \frac{2\pi^2}{k^3} B_\zeta = \boldsymbol{f_{NL}} \times \left(\mathcal{P}_\zeta\right)^2$$

- 4-point (Gaussian+Non-Gaussian)

Trispectrum $\mathcal{T}_\zeta = \boldsymbol{g_{NL}} \times \left(\mathcal{P}_\zeta\right)^3$

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A_s	n_s	r	$\lvert f_{NL} \rvert$	$\lvert g_{NL} \rvert$
1.6×10^{-9}	0.965 ± 0.004	< 0.10	< 50	$< 10^5$

$$\underbrace{A_s,\ n_s,\ r}_{\text{Power spectrum}}\quad \underbrace{\lvert f_{NL}\rvert}_{\text{Bispectrum}}\quad \underbrace{\lvert g_{NL}\rvert}_{\text{Trispectrum}}$$

Planck constraints from the CMB

$$\boldsymbol{r} = \frac{P_{GW}}{P_\zeta}$$

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Power spectrum — Bispectrum — Trispectrum

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$$\boldsymbol{r} = \frac{P_{GW}}{P_\zeta}$$

Planck constraints from the CMB

Large Scale Structure (LSS) experiments such as DESI or Euclid could constrain $|f_{NL}|$<10

- 2-point (Gaussian) statistics:

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Dimensionless power spectrum is

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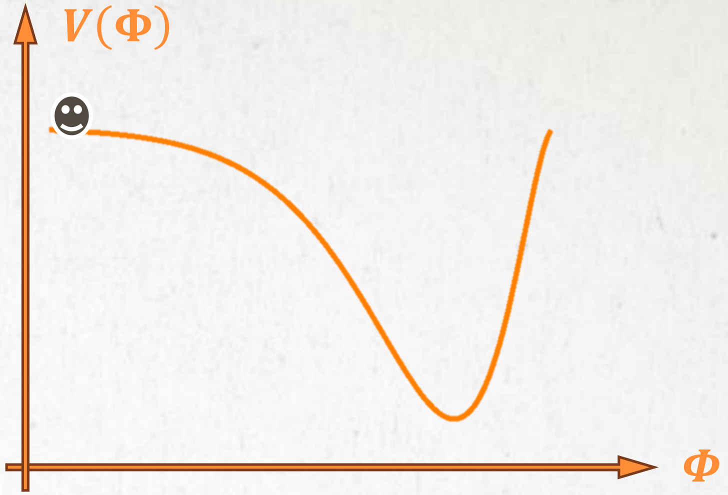
SLOW-ROLL SINGLE FIELD INFLATION

- Quasi de Sitter space: $\epsilon = -\frac{\dot{H}}{H^2} \ll 1\,; \eta = \frac{\dot{\epsilon}}{H\epsilon} \ll 1$

$$\Rightarrow \frac{M_p\, V'}{V} \ll 1\,; \frac{M_p^2 |V''|}{V} \ll 1$$

- Single-clock: only one scalar degree of freedom
- Canonical kinetic term



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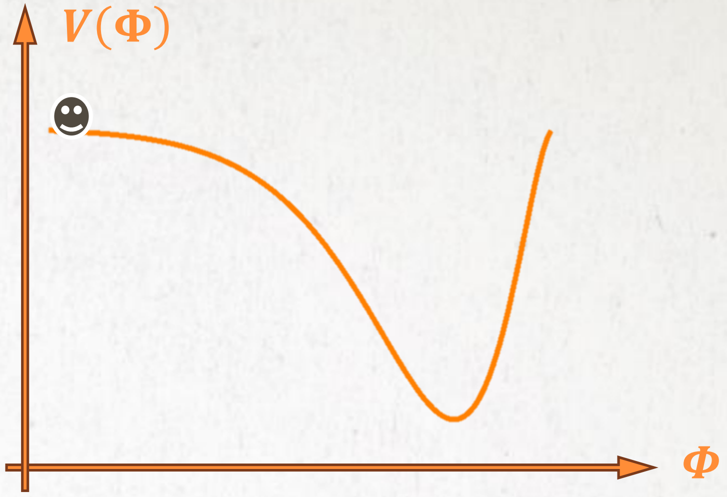


- Single-clock: only one scalar degree of freedom
- Canonical kinetic term

Success and failure

✓ Enough expansion to solve the horizon and flatness problems

✓ Nearly scale-invariant scalar power spectrum on large scales

✓ Small tensor-to-scalar ratio
Small non-Gaussianities

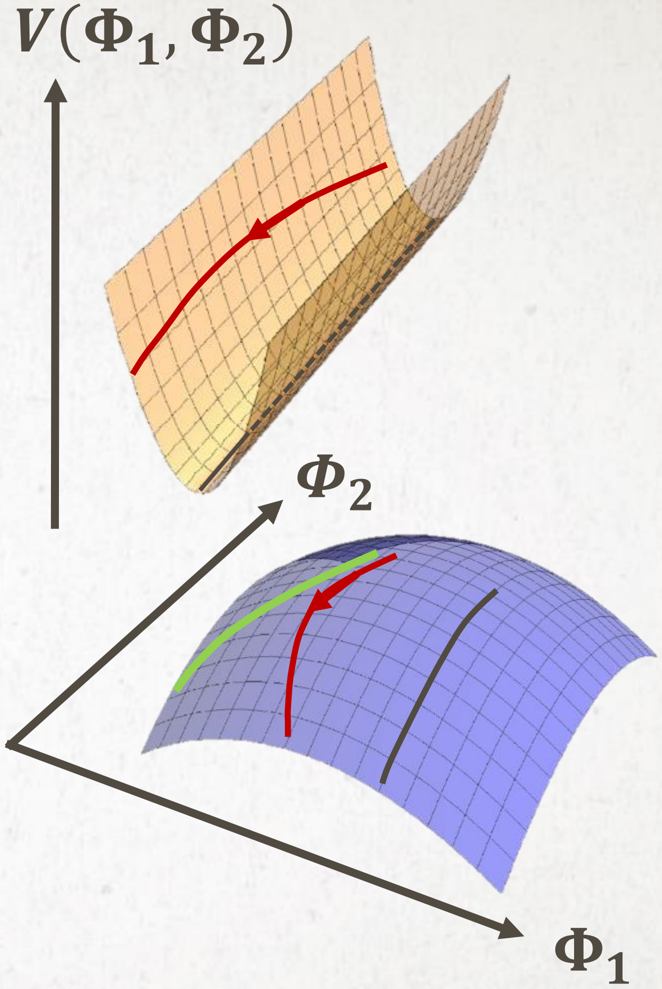
❖ Few theoretical motivation: realistic UV completions typically predict **several scalar fields with non-canonical kinetic terms**

❖ Sensitive to Planck scale suppressed operators, quantum loops renormalize the potential:

eta problem: $\boldsymbol{\frac{M_p^2|V''|}{V} > 1}$

II. MULTIFIELD INFLATION WITH CURVED FIELD SPACE

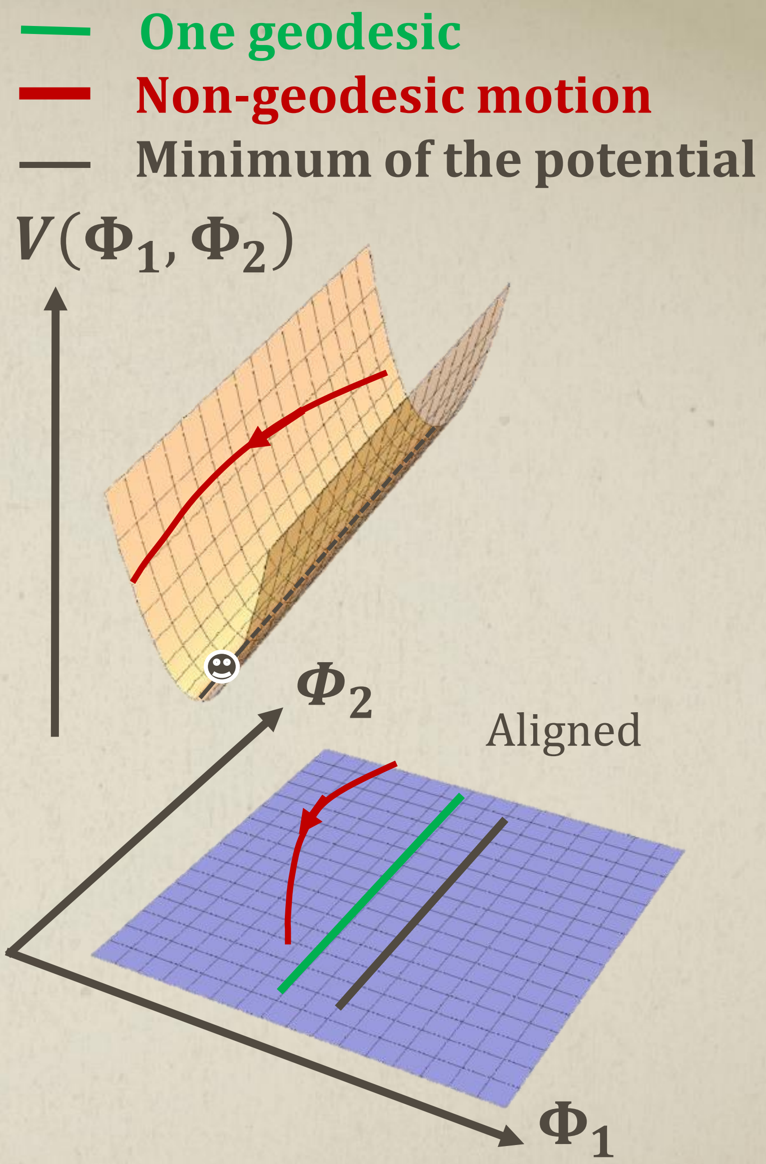
GEOMETRICAL EFFECTS UNVEILED



Curved field space

MULTIFIELD INFLATION

$$S = \int \sqrt{-g}\left(\frac{R}{2} - \sum_{a,b} g^{\mu\nu}\boldsymbol{\delta_{ab}}\partial_\mu\phi^a\partial_\nu\phi^b - \boldsymbol{V(\phi^c)}\right)$$



Flat field space

Vanishing curvature: $R_{fs} = 0$

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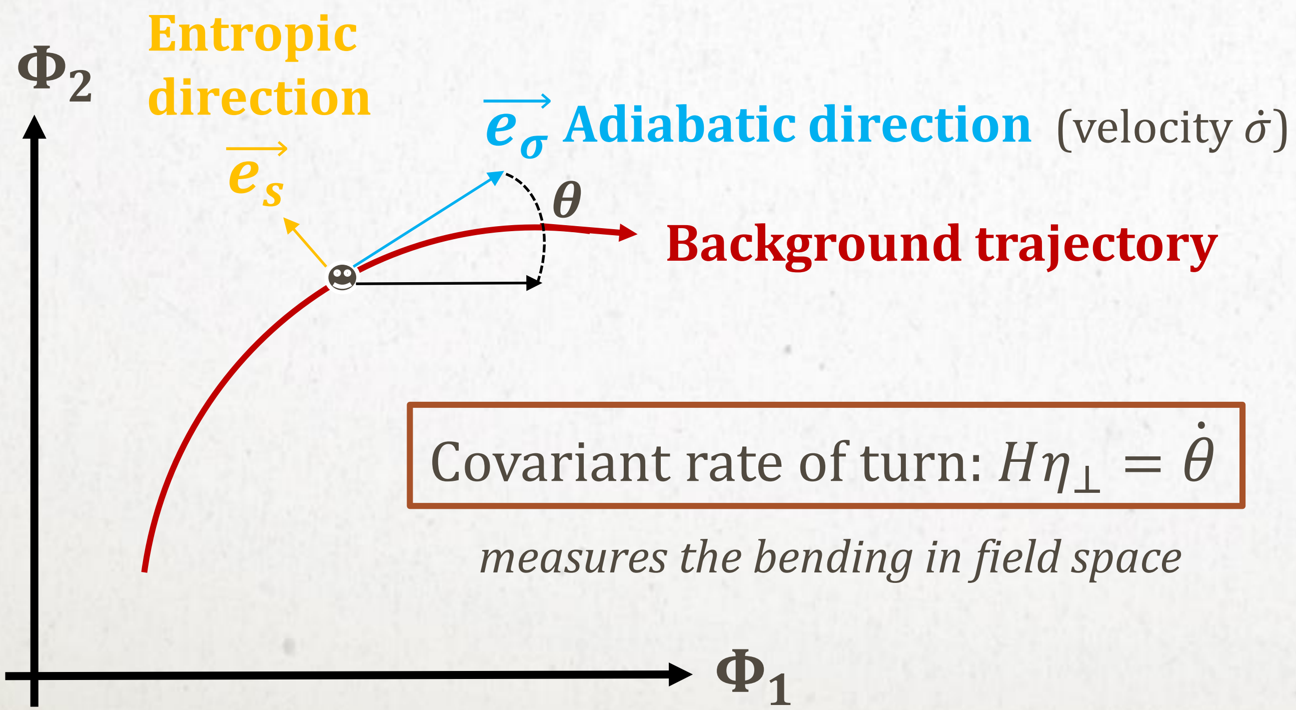
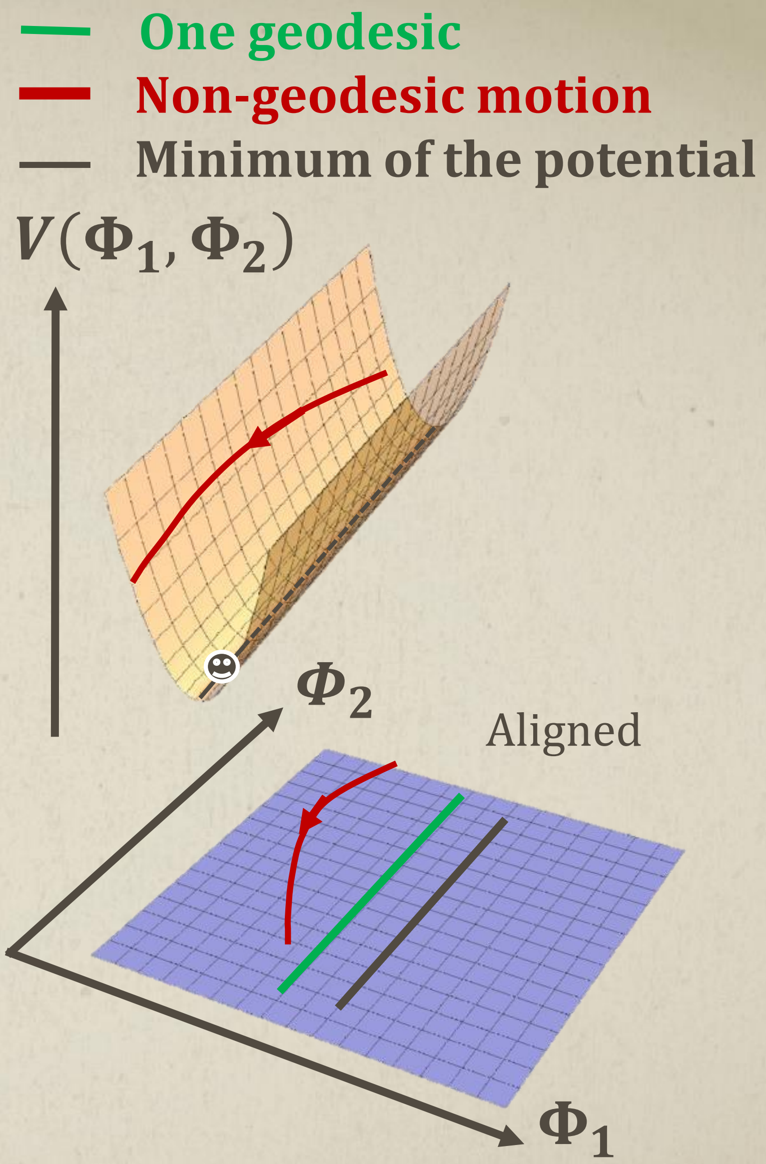


Covariant rate of turn: $H\eta_\perp = \dot{\theta}$

measures the bending in field space



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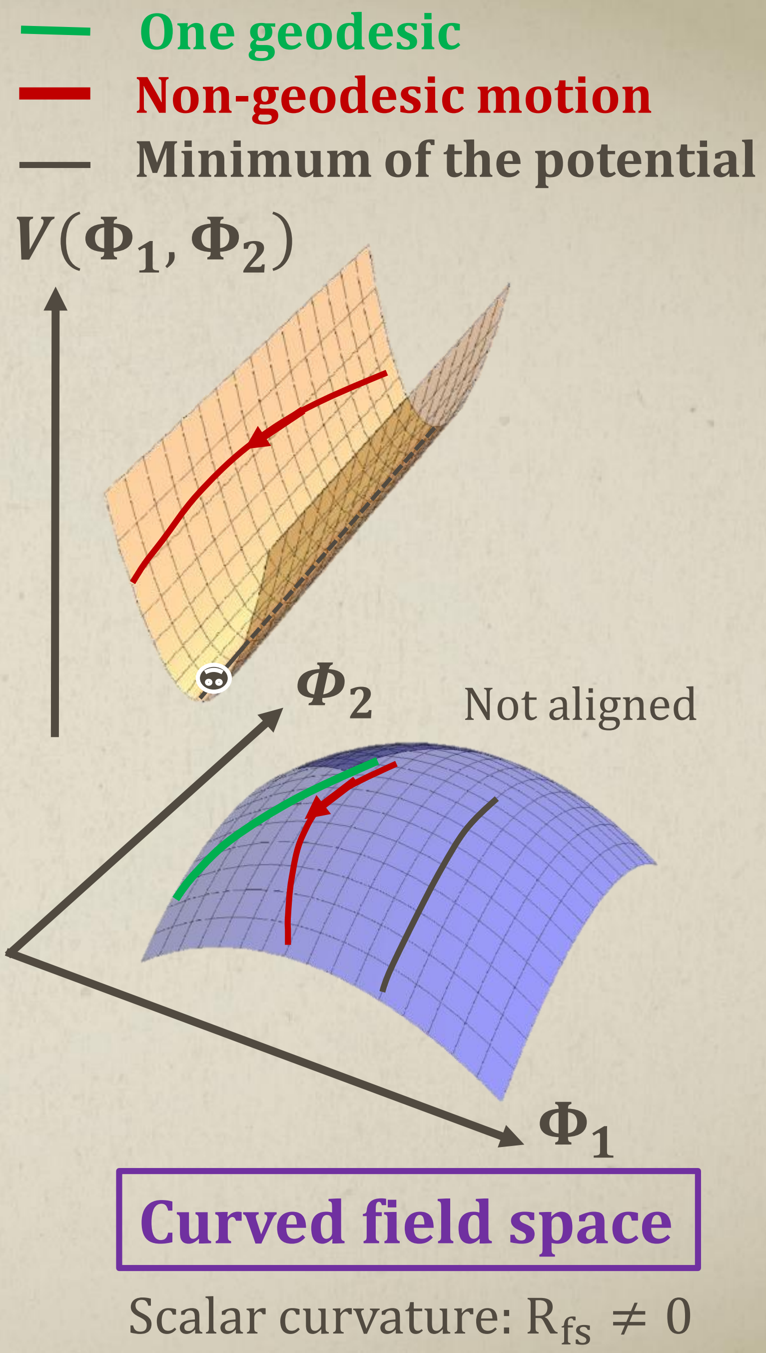


MULTIFIELD INFLATION WITH CURVED FIELD SPACE

$$S = \int \sqrt{-g}\left(\frac{R}{2} - \sum_{a,b} g^{\mu\nu} \boldsymbol{G_{ab}}(\boldsymbol{\phi^c})\partial_\mu \phi^a \partial_\nu \phi^b - \boldsymbol{V}(\boldsymbol{\phi^c})\right)$$

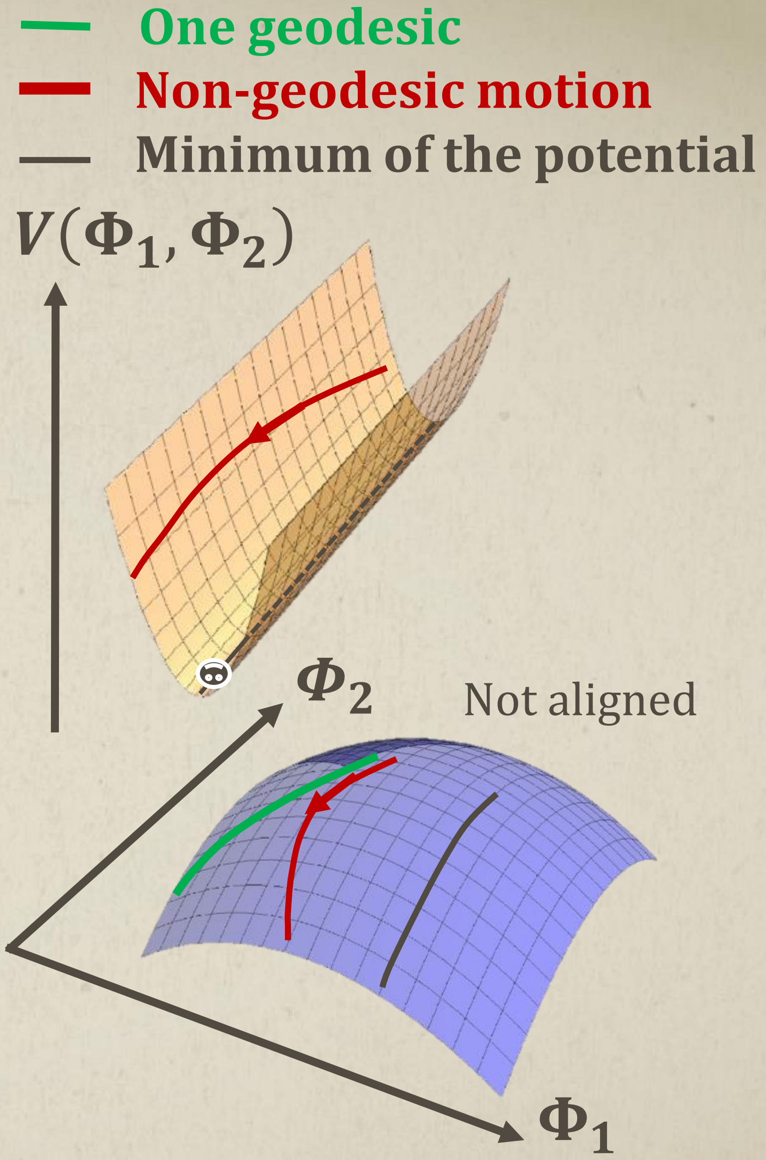
Covariant rate of turn: $H\eta_\perp = D_t\theta$

measures deviation from a geodesic in field space

Local curvature in field space

Ricci scalar R_{fs} constructed from G

Geometry	Flat	Spherical	Hyperbolic
R_{fs}	0	> 0	< 0



Curved field space

Scalar curvature: $R_{\mathrm{fs}} \neq 0$

INTERESTING MULTIFIELD FEATURES

SOME PREVIOUS WORKS

ζ S

- Super-Hubble evolution of adiabatic perturbations, sourced by entropic ones:
  - Single-field (SF) consistency relation is modified: $r = -8n_t \times \sin^2(\Delta)$, with
  - Correlated adiabatic-entropic perturbations: $\cos(\Delta) = P_{\zeta S}/\sqrt{P_{\zeta\zeta}P_{SS}}$

[Wands, Bartolo, Matarrese, Riotto 2002]

[Langlois 1999]

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- Features in the power spectrum (sudden turn, fastly-evolving entropic mass…):
  - Oscillations / step at scales that exit the horizon when the feature happens
  - Quantum clocks: minimal oscillations even without features in the background trajectory

[Lesgourgues 1999]

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  - Oscillations / step at scales that exit the horizon when the feature happens **[Lesgourgues 1999]**
  - Quantum clocks: minimal oscillations even without features in the background trajectory **[Chen, Namjoo, Wang 2015]**

- Non-Gaussianities are enhanced:
  - Maldacena's result $f_{\mathrm{nl}} = O(\epsilon, \eta)$ and SF consistency relation $f_{\mathrm{nl}}^{\mathrm{squeezed}} = n_s - 1$ are broken
  - An extra massive field affects the shape and amplitude of $f_{\mathrm{nl}}^{\mathrm{squeezed}}$ depending on its mass and spin

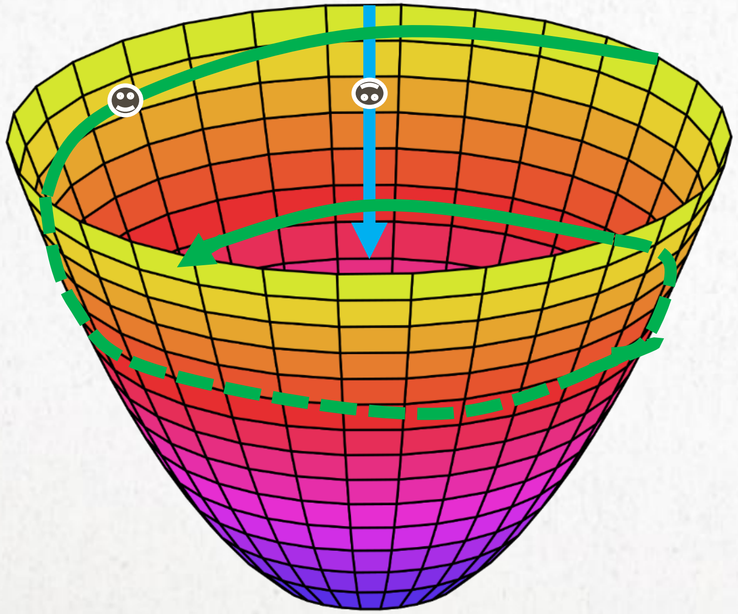
Quasi-Single Field: **[Chen, Wang 2009]**, Cosmological Collider: **[Arkani-Hamed, Maldacena 2015]**, Cosmological Bootstrap **[Arkani-Hamed, Baumann, Lee, Pimentel 2018]**

INTERESTING MULTIFIELD FEATURES

FURTHER DEVELOPMENTS

[Hetz, Palma 2016]
[Garcia-Saenz, Renaux-Petel, Ronayne 2018]
[Fumagalli, Garcia-Saenz, LP, Renaux-Petel, Ronayne 2019]

- Multifield enables to inflate along steep potentials: $\epsilon_V = \frac{V_{,\sigma}^2}{2V^2} \simeq \epsilon\left(1 + \frac{\eta_\perp^2}{9}\right) \geq 1$ if strong bending



Steep potential

No bending = too fast rolling to inflate

With bending = slow enough rolling to inflate

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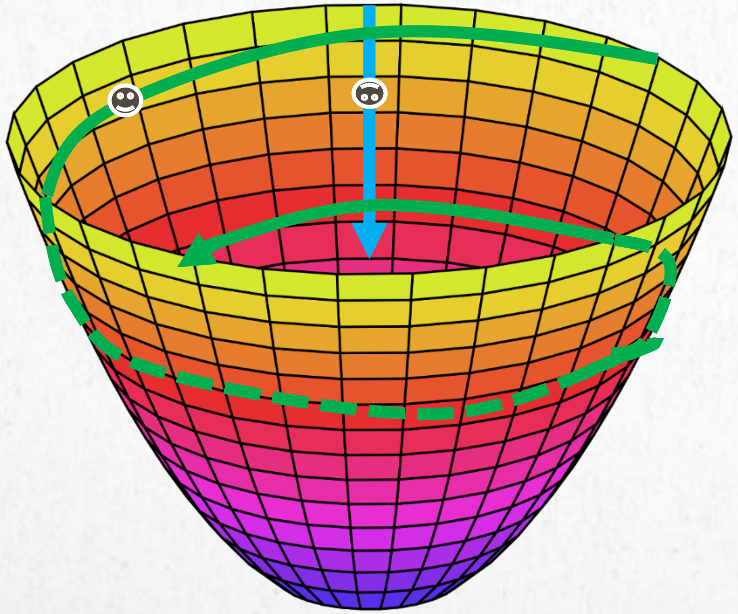
Multifield helps to satisfy the dS swampland conjectures

[Achucarro, Palma 2018]
[Bjorkmo, Marsh 2019]



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- Inflationary α-attractors: supersymmetric-inspired models with curved field space, match well Planck constraints

[Kallosh, Linde, Roest 2013]

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- Inflationary α-attractors: supersymmetric-inspired models with curved field space, match well Planck constraints [Kallosh, Linde, Roest 2013]

- Recent works about curved field space:

Geometrical destabilization of inflation [Renaux-Petel, Turzynski 2015]

Sidetracked inflation [Garcia-Saenz, Renaux-Petel, Ronayne 2018]

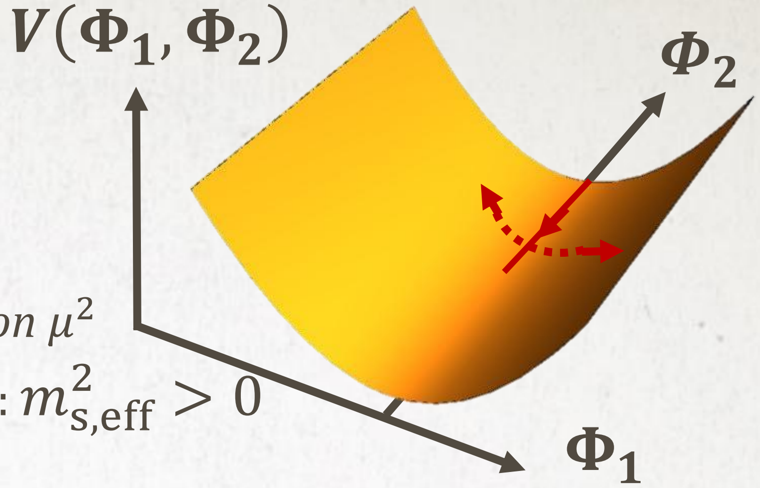
Multifield α-attractors [Achúcarro, Kallosh, Linde, Wang, Welling 2017]

Attractors and bifurcations in multifield inflation [Christodoulidis, Roest, Sfakianakis 2019]

Hyperinflation [Brown 2017], [Mizuno, Mukhoyama 2018]
[Fumagalli, Garcia-Saenz, LP, Renaux-Petel, Ronayne 2019]
[Bjorkmo, Marsh 2019]

STABILITY OF BACKGROUND TRAJECTORIES

GEOMETRICAL DESTABILIZATION OF INFLATION



Other notation μ^2

- A stable trajectory requires $\underbrace{\perp \textbf{ long wavelength} \text{ modes}}_{\textit{entropic perturbations on large scales}}$ to be stable: $m_{\mathrm{s,eff}}^2 > 0$

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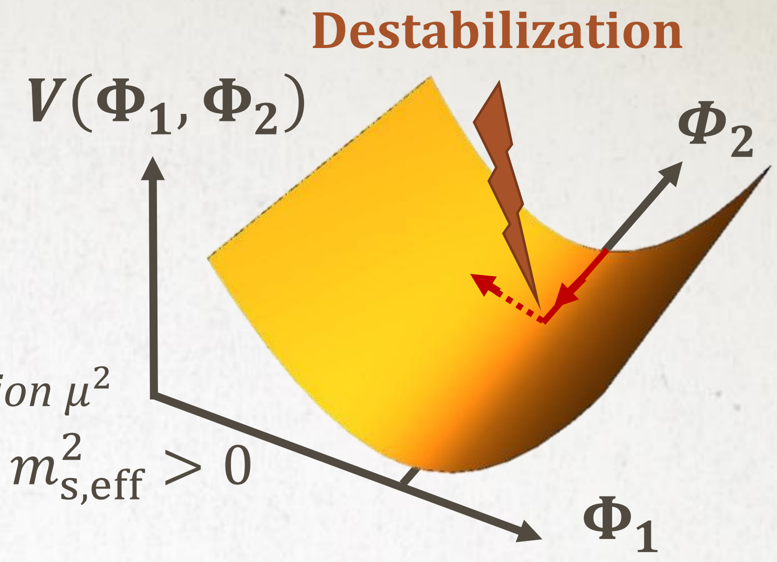

GEOMETRICAL DESTABILIZATION OF INFLATION

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Hessian of the potential　　Bending　　Geometry of field-space

- Geometrical destabilization of inflation: $\frac{m^2_{\mathrm{s,eff}}}{H^2} = \underbrace{\frac{V_{;ss}}{H^2} + 3\eta_\perp^2}_{>0} + \underbrace{\epsilon R_{\mathrm{fs}} M_p^2}_{<0 \text{ for hyperbolic field spaces}} < 0$

[S. Renaux-Petel, K. Turzynski 2015]

« bifurcation » in the language of dynamical systems

STABILITY OF BACKGROUND TRAJECTORIES

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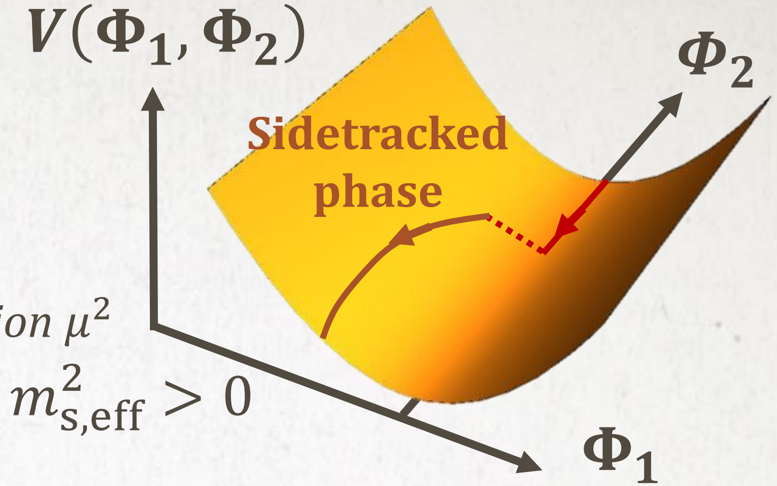

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- A stable trajectory requires ⊥ long wavelength modes to be stable: $m_{\mathrm{s,eff}}^2 > 0$

Hessian of the potential — Bending — Geometry of field-space

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[S. Renaux-Petel, K. Turzynski 2015]

→ Second, sidetracked phase of inflation

[O. Grocholski, M. Kalinowski, M. Kolanowski, S. Renaux-Petel, K.Turzynski, V. Vennin 2019]

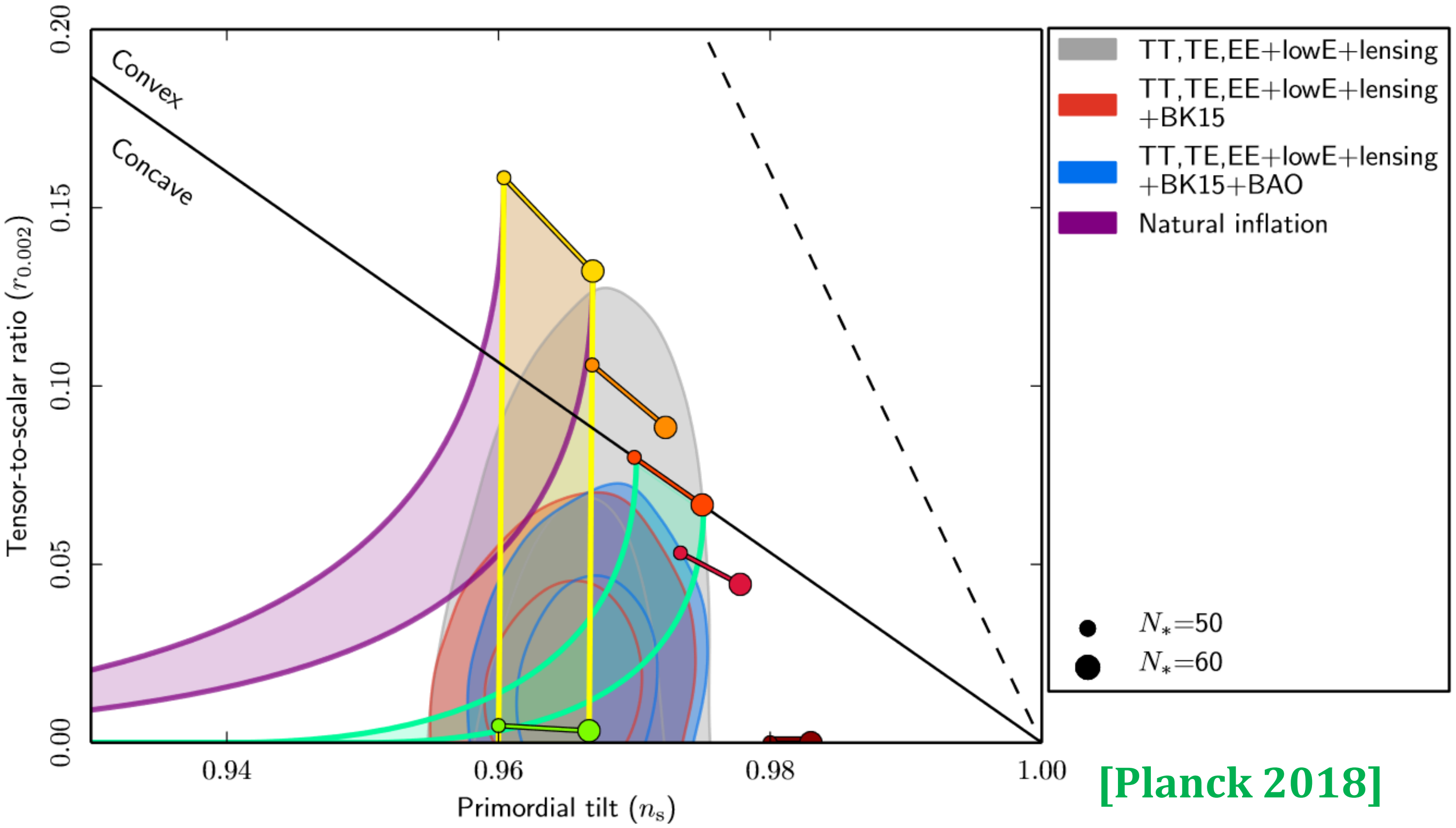
All observables ($N_{\mathrm{inflation}}, n_s, r, f_{\mathrm{nl}}$ …) affected

PHYSICS OF LINEAR FLUCTUATIONS

RESURRECTING NATURAL INFLATION?

$$V(\phi) = \Lambda^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right)$$

Discrete shift symmetry protecting potential from quantum corrections



[Planck 2018]

PHYSICS OF LINEAR FLUCTUATIONS

RESURRECTING NATURAL INFLATION?

$$V(\phi,\chi) = \Lambda^4\left(1+\cos\left(\frac{\phi}{f}\right)\right)+\frac{1}{2}m^2\chi^2$$

Negatively curved field spaces

Toy models (so far)

PHYSICS OF LINEAR FLUCTUATIONS

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Negatively curved field spaces

Toy models (so far)

[Garcia-Saenz, Renaux-Petel, Ronayne 2018]

- **Minimal metric:**

$$d\sigma^2 = \left(1+\frac{\boldsymbol{2\chi^2}}{\boldsymbol{M^2}}\right)d\phi^2 + d\chi^2$$

$$R_{\mathrm{fs}} = -\frac{4}{M^2(1+2\chi^2/M^2)^2}$$

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- **Hyperbolic metric:**

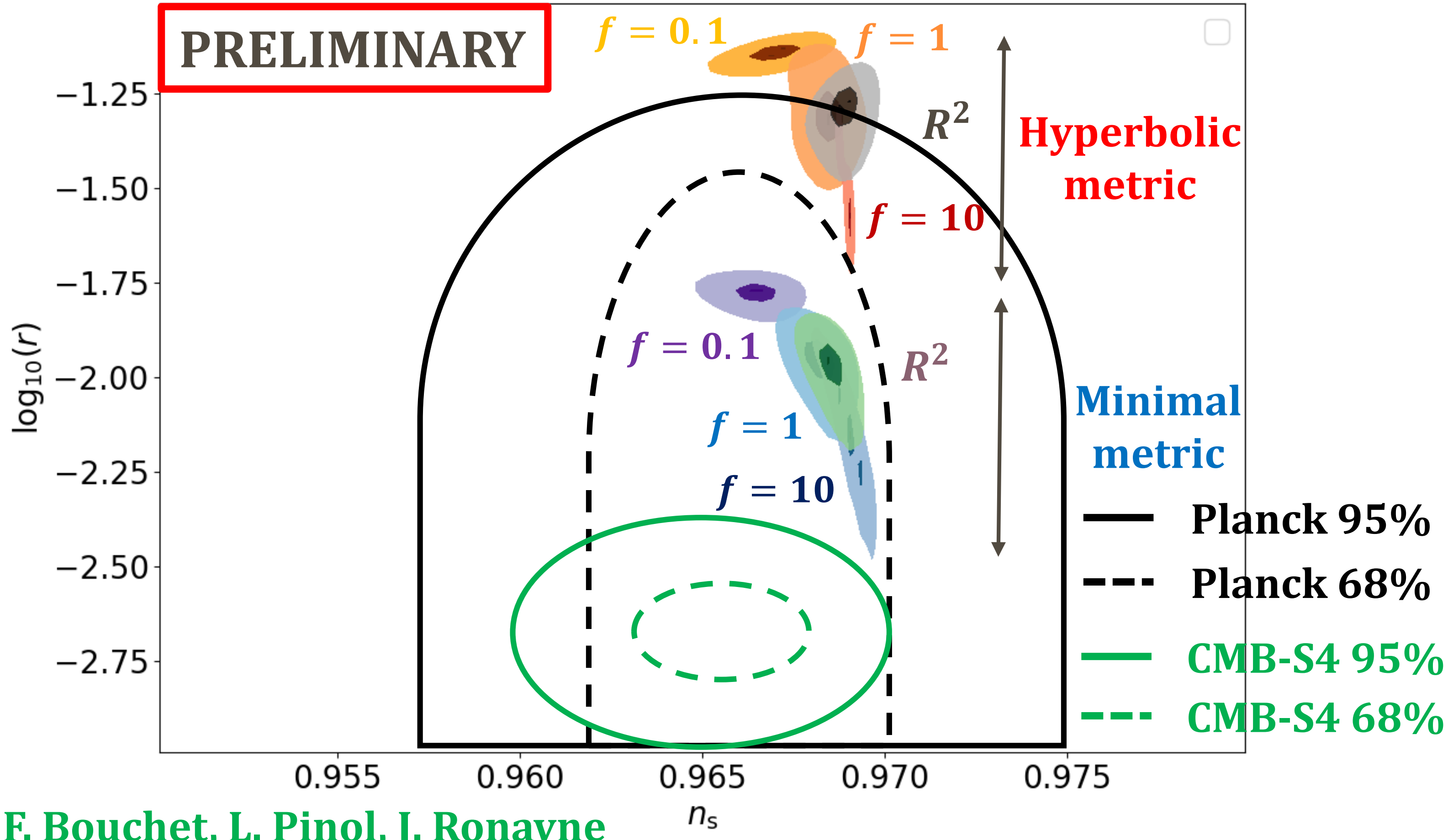
$$d\sigma^2 = \left(1+\frac{\boldsymbol{2\chi^2}}{\boldsymbol{M^2}}\right)d\phi^2 + \frac{\boldsymbol{2\sqrt{2}\,\chi}}{\boldsymbol{M}}\boldsymbol{d\phi d\chi} + d\chi^2$$

$$R_{\mathrm{fs}} = -\frac{4}{M^2}$$

PHYSICS OF LINEAR FLUCTUATIONS

RESURRECTING NATURAL INFLATION?

$$V(\phi,\chi) = \Lambda^4\left(1+\cos\left(\frac{\phi}{f}\right)\right)+\frac{1}{2}m^2\chi^2$$



F. Bouchet, L. Pinol, J. Ronayne

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➢ **Minimal metric:**

$$d\sigma^2 = \left(1+\frac{2\chi^2}{M^2}\right)d\phi^2 + d\chi^2$$

$$R_{\rm fs} = -\frac{4}{M^2(1+2\chi^2/M^2)^2}$$

➢ **Hyperbolic metric:**

$$d\sigma^2 = \left(1+\frac{2\chi^2}{M^2}\right)d\phi^2 + \frac{2\sqrt{2}\,\chi}{M}d\phi d\chi + d\chi^2$$

$$R_{\rm fs} = -\frac{4}{M^2}$$

NON-GAUSSIANITIES
HYPERINFLATION

[Fumagalli, Garcia-Saenz, Pinol, Renaux-Petel, Ronayne 2019]
Phys. Rev. Lett. 123, 201302

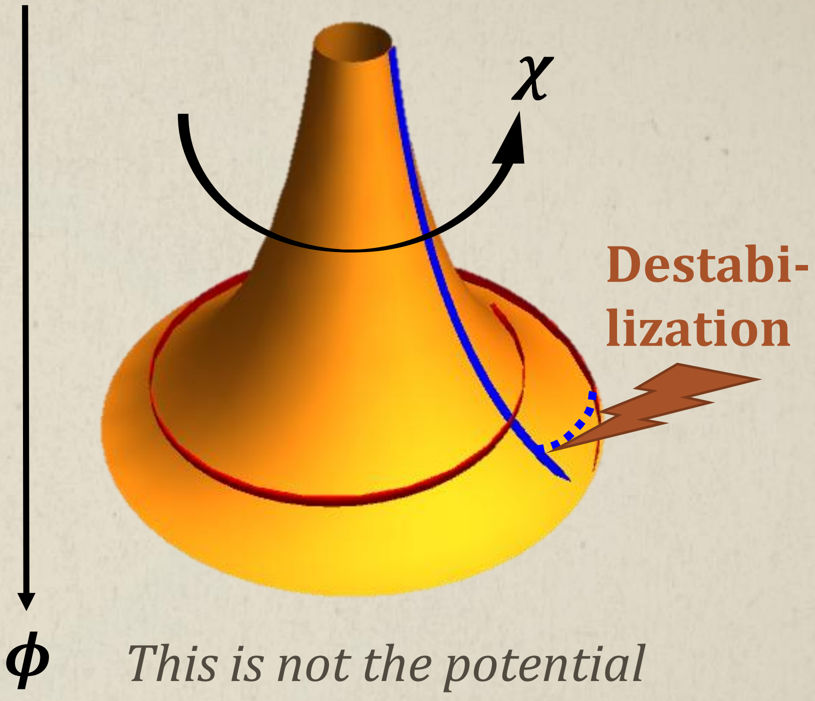
Setup radial angular

The scalar fields ϕ , χ live on an internal hyperbolic plane

Spiraling trajectory enables to inflate along steep potentials:

Interesting for eta problem and swampland conjectures!

Embedding of the hyperbolic plane in 3D
Radial trajectory
Hyperinflation trajectory



This is not the potential

Hyperbolic field space

$$R_{\rm fs} = -\frac{4}{M^2}, \qquad M \ll M_p$$

NON-GAUSSIANITIES

HYPERINFLATION

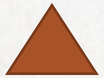
[Fumagalli, Garcia-Saenz, Pinol, Renaux-Petel, Ronayne 2019]
Phys. Rev. Lett. 123, 201302

Setup radial angular

The scalar fields ϕ , χ live on an internal hyperbolic plane

Interesting observational signatures:

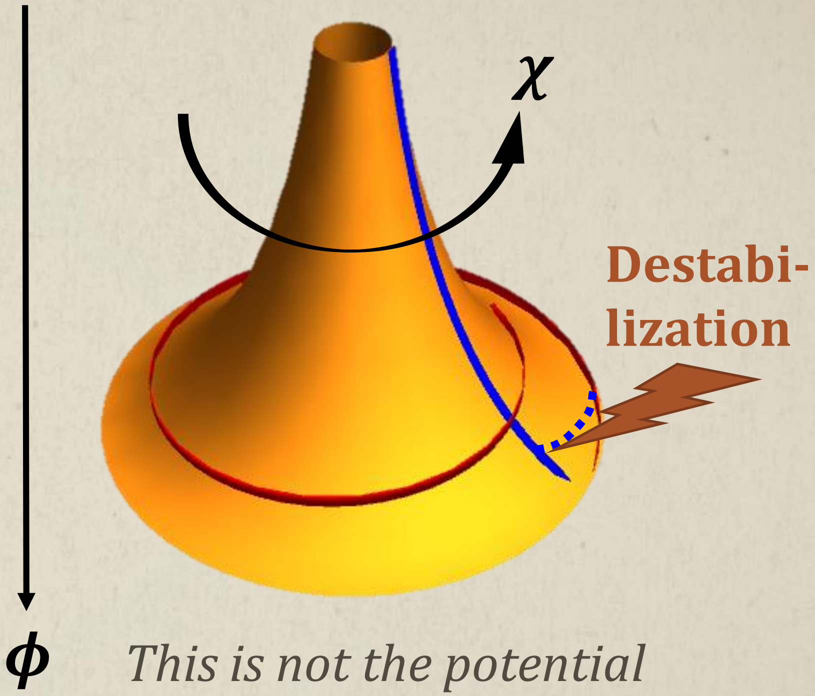
Large non-Gaussianities in exotic flattened configurations



$$\boxed{f_{\rm nl}^{\rm eq} = \mathcal{O}(1);\ f_{\rm nl}^{\rm flat} = \mathcal{O}(50)}$$

Target for upcoming LSS experiments

Embedding of the hyperbolic plane in 3D
Radial trajectory
Hyperinflation trajectory



This is not the potential

Hyperbolic field space

$$R_{\rm fs} = -\frac{4}{M^2}, \qquad M \ll M_p$$

HYPERINFLATION

BISPECTRUM USING PyTransport 2.0

[D. Mulryne, J. Ronayne 2016]

Transport approach to numerically evolve 2-pt and 3-pt correlation functions in multifield inflation with curved field space

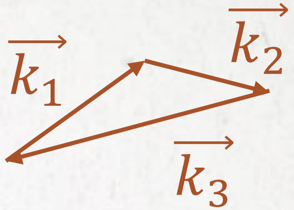
HYPERINFLATION

BISPECTRUM USING PyTransport 2.0

[D. Mulryne, J. Ronayne 2016]

equilateral local flat

3-point function: $< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} >= (2\pi)^3 \;\; \delta^{(3)}(\overrightarrow{k_1}+\overrightarrow{k_2}+\overrightarrow{k_3}) \times B_\zeta(\overrightarrow{k_1},\overrightarrow{k_2},\overrightarrow{k_3})$



With $k_1 = \frac{3k_*}{4}(1+\alpha+\beta),\;\; k_2 = \frac{3k_*}{4}(1-\alpha+\beta),\;\; k_3 = \frac{3k_*}{2}(1-\beta)$

HYPERINFLATION

BISPECTRUM USING PyTransport 2.0

[D. Mulryne, J. Ronayne 2016]

3-point function: $< \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} >= (2\pi)^3 \;\; \delta^{(3)}(\overrightarrow{k_1} + \overrightarrow{k_2} + \overrightarrow{k_3}) \times B_\zeta(\overrightarrow{k_1}, \overrightarrow{k_2}, \overrightarrow{k_3})$

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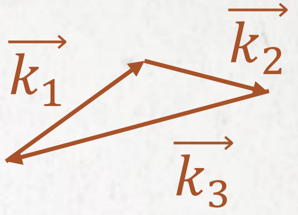
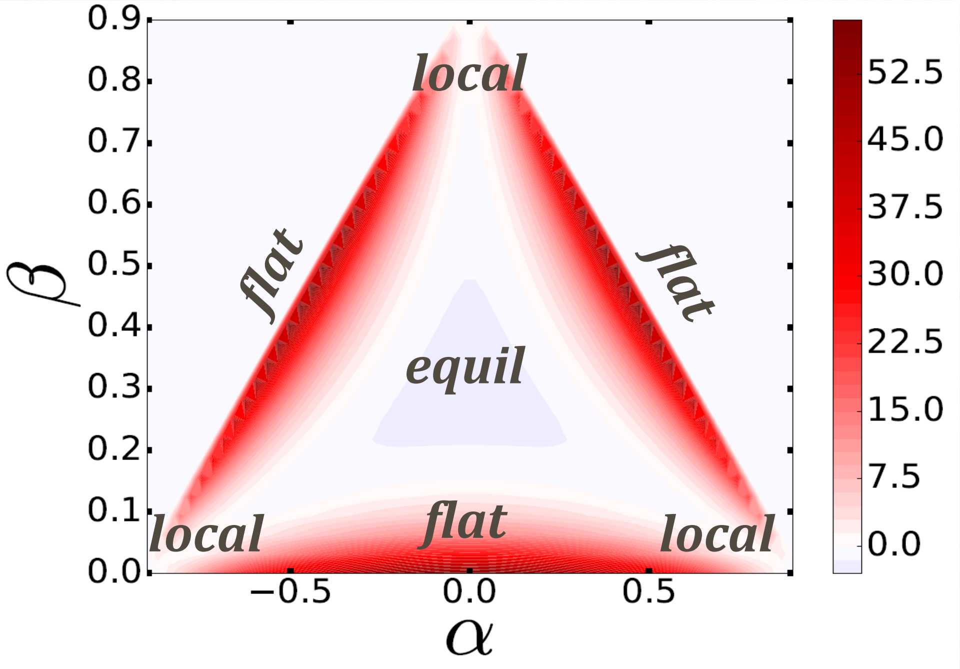
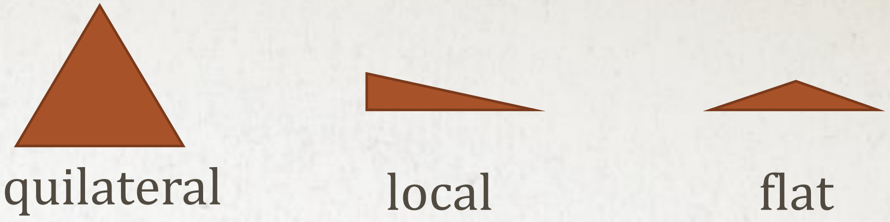




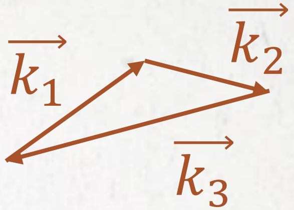
HYPERINFLATION

BISPECTRUM USING PyTransport 2.0

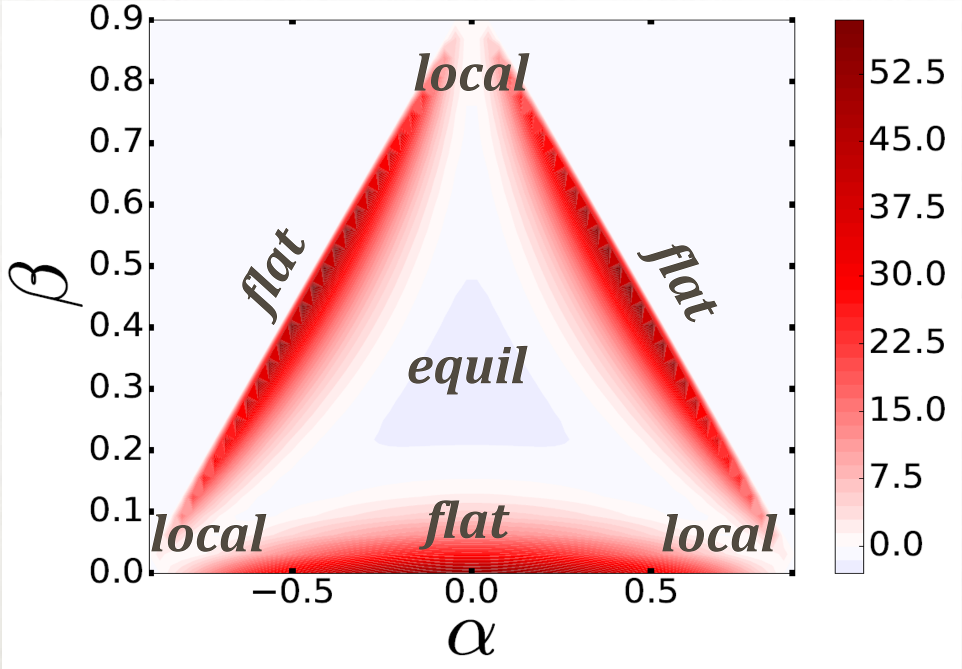
[D. Mulryne, J. Ronayne 2016]



3-point function: $< \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} >= (2\pi)^3 \ \delta^{(3)}(\overrightarrow{k_1} + \overrightarrow{k_2} + \overrightarrow{k_3}) \times B_\zeta(\overrightarrow{k_1}, \overrightarrow{k_2}, \overrightarrow{k_3})$



With $k_1 = \frac{3k_*}{4}(1+\alpha+\beta),\ k_2 = \frac{3k_*}{4}(1-\alpha+\beta),\ k_3 = \frac{3k_*}{2}(1-\beta)$



Characteristic large flattened bispectrum

f_{NL}^{equil}	f_{NL}^{flat}
-2.0	53.8

Single-clock inflation with Bunch-Davies initial states predicts equilateral non-Gaussianities

HYPERINFLATION

BISPECTRUM USING EFT

Effective single-field cubic action

$$S_{(3)}^{EFT}[\zeta] = \int d\tau d^3x \; a\frac{\epsilon}{H}M_p^2\left(\frac{1}{c_s^2}-1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

Here, $m_s^2<0$ and $c_s^2 = -1$, leading to exotic NGs

Justified because in this class of models, one has: $|m_s^2| \gg H^2$

One can « integrate out » entropic perturbations

$$\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2}$$

A unknown (so far)

Expected to be of order 1

HYPERINFLATION

BISPECTRUM USING EFT

Effective single-field cubic action

$$S_{(3)}^{EFT}[\zeta] = \int d\tau d^3x \;\; a\frac{\epsilon}{H}M_p^2\left(\frac{1}{c_s^2}-1\right)\left(\zeta'(\partial_i\zeta)^2+\frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

Here, $m_s^2<0$ *and* $c_s^2=-1$*,*
leading to exotic NGs

$$f_{\mathrm{NL}}^{\mathrm{flat}} \simeq 50\times(A+1)$$

$$f_{\mathrm{NL}}^{\mathrm{flat}} \sim O(50)$$

If $A\sim 1$

HYPERINFLATION

BISPECTRUM USING EFT

Effective single-field cubic action

$$S_{(3)}^{EFT}[\zeta] = \int d\tau d^3x \; a\frac{\epsilon}{H}M_p^2\left(\frac{1}{c_s^2}-1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{A}{c_s^2}\zeta'^3\right)$$

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$$f_{\rm NL}^{\rm flat} \sim O(50)$$

If $A \sim 1$

$$\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2}$$

***A* unknown (so far)**

III. REVISITING PRIMORDIAL NON-GAUSSIANITIES

GENERALIZING MALDACENA'S CALCULATION TO CURVED FIELD SPACE

[Garcia-Saenz, Pinol, Renaux-Petel]
J. High Energ. Phys. **2020,** 73 (2020)

$$\mathcal{L}(\zeta,\mathcal{F}) = \underbrace{\mathcal{L}^{(2)}(\zeta,\mathcal{F})}_{} + \underbrace{\mathcal{L}^{(3)}_{\text{Maldacena}}(\zeta) + \mathcal{L}^{(3)}_{\text{new}}(\zeta,\mathcal{F}) + \mathcal{D}^{(3)}}_{}$$

Dictating the power spectrum:
2-point function

Dictating the bispectrum:
3-point function

EXPANDING AND SIMPLIFYING THE ACTION

USING INTEGRATION BY PARTS AND LINEAR EQUATIONS OF MOTION

- We perform integrations by parts to make explicit the size of interactions
- Linear equations of motion $\frac{\delta S^{(2)}}{\delta \zeta} = 0$ and $\frac{\delta S^{(2)}}{\delta \mathcal{F}} = 0$ can be used at any time
- Resulting Lagrangian, after $O(40)$ integrations by parts and $O(10)$ uses of equations of motion:

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$$\mathcal{L}(\zeta, \mathcal{F}) = \mathcal{L}^{(2)}(\zeta, \mathcal{F}) + \mathcal{L}^{(3)}_{\mathrm{Maldacena}}(\zeta, \chi) + \mathcal{L}^{(3)}_{\mathrm{new}}(\zeta, \mathcal{F}) + \mathcal{D}$$

EXPANDING AND SIMPLIFYING THE ACTION

USING INTEGRATION BY PARTS AND LINEAR EQUATIONS OF MOTION

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$$\mathcal{L}(\zeta,\mathcal{F}) = \mathcal{L}^{(2)}(\zeta,\mathcal{F}) + \mathcal{L}^{(3)}_{\mathrm{Maldacena}}(\zeta,\chi) + \mathcal{L}^{(3)}_{\mathrm{new}}(\zeta,\mathcal{F}) + \mathcal{D}$$

$$\mathcal{L}^{(2)}(\zeta,\mathcal{F}) = \frac{a^3}{2}\left(2\epsilon M_p^2\left(\dot{\zeta}^2 - \frac{(\partial\zeta)^2}{a^2}\right) + \dot{\mathcal{F}}^2 - \frac{(\partial\mathcal{F})^2}{a^2} - \underbrace{m_s^2}\mathcal{F}^2 + 4\dot{\sigma}\eta_\perp\mathcal{F}\dot{\zeta}\right)$$

$$\boxed{m_s^2 = V_{;ss} - H^2\eta_\perp^2 + \epsilon R_{\mathrm{fs}} H^2 M_p^2}$$

Mixing via the bending

Hessian of the potential

Bending of the trajectory

Field-space curvature

$$\frac{\partial^2\chi}{a^2} = \epsilon\dot{\zeta} + \frac{\dot{\sigma}}{M_p^2}\eta_\perp\mathcal{F}$$

EXPANDING AND SIMPLIFYING THE ACTION

USING INTEGRATION BY PARTS AND LINEAR EQUATIONS OF MOTION

- We perform integrations by parts to make explicit the size of interactions
- Linear equations of motion $\frac{\delta S^{(2)}}{\delta\zeta} = 0$ and $\frac{\delta S^{(2)}}{\delta\mathcal{F}} = 0$ can be used at any time
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$$\mathcal{L}(\zeta,\mathcal{F}) = \mathcal{L}^{(2)}(\zeta,\mathcal{F}) + \mathcal{L}^{(3)}_{\text{Maldacena}}(\zeta,\chi) + \mathcal{L}^{(3)}_{\text{new}}(\zeta,\mathcal{F}) + \mathcal{D}$$

$$\mathcal{L}^{(3)}_{\text{Maldacena}}(\zeta,\chi) = a^3M_p^2\left[\epsilon(\epsilon-\eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2\right]$$

[J. Maldacena 2003]

EXPANDING AND SIMPLIFYING THE ACTION

USING INTEGRATION BY PARTS AND LINEAR EQUATIONS OF MOTION

- We perform integrations by parts to make explicit the size of interactions
- Linear equations of motion $\frac{\delta S^{(2)}}{\delta \zeta} = 0$ and $\frac{\delta S^{(2)}}{\delta \mathcal{F}} = 0$ can be used at any time
- Resulting Lagrangian, after $O(40)$ integrations by parts and $O(10)$ uses of equations of motion:

$$\boldsymbol{\mathcal{L}}(\boldsymbol{\zeta}, \boldsymbol{\mathcal{F}}) = \mathcal{L}^{(2)}(\zeta, \mathcal{F}) + \mathcal{L}^{(3)}_{\mathrm{Maldacena}}(\zeta, \chi) + \boldsymbol{\mathcal{L}^{(3)}_{\mathrm{new}}}(\boldsymbol{\zeta}, \boldsymbol{\mathcal{F}}) + \boldsymbol{\mathcal{D}}$$

New interactions

Boundary terms:
Total time derivatives
contribute to 3-pt functions

[C. Burrage, R. Ribeiro, D. Seery 2011]

[F. Arroja, T. Tanaka 2011]

NEW INTERACTIONS

$$\lambda_\perp = \frac{\dot{\eta_\perp}}{H\eta_\perp} \quad ; \quad \mu_s = \frac{\dot{m}_s}{Hm_s}$$

$$\frac{\partial^2\chi}{a^2} = \epsilon\dot{\zeta} + \frac{\dot{\sigma}}{M_p^2}\eta_\perp\mathcal{F}$$

$$\mathcal{L}^{(3)}_{\text{new}}(\zeta,\mathcal{F}) = \frac{1}{2}m_s^2\zeta\mathcal{F}\left((\epsilon+\mu_s)\mathcal{F} + (2\epsilon-\eta-2\lambda_\perp)\frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}\right) + \frac{\dot{\sigma}\eta_\perp}{a^2H}\mathcal{F}(\partial\zeta)^2$$
$$-\frac{\dot{\sigma}\eta_\perp}{H}\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon H^2M_p^2R_{fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{fs} + \epsilon H^2M_p^2R_{fs,s}\right)\mathcal{F}^3$$
$$+\frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}(\partial\mathcal{F})(\partial\chi)$$

Check: ζ is well massless at any order as it should (Weinberg adiabatic mode)

NEW INTERACTIONS

Applications: *quasi-single field, cosmological collider physics, single-field effective theory*

$$\mathcal{L}^{(3)}_{\mathbf{new}}(\zeta,\mathcal{F}) = \frac{1}{2}m_s^2\zeta\mathcal{F}\left((\epsilon+\mu_s)\mathcal{F} + (2\epsilon-\eta-2\lambda_\perp)\frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}\right) + \frac{\dot{\sigma}\eta_\perp}{a^2H}\mathcal{F}(\partial\zeta)^2$$

$$-\frac{\dot{\sigma}\eta_\perp}{H}\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon H^2 M_p^2 R_{fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{fs} + \epsilon H^2 M_p^2 R_{fs,s}\right)\mathcal{F}^3$$

$$+\frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}(\partial\mathcal{F})(\partial\chi)$$

NEW INTERACTIONS

Applications: *quasi-single field, cosmological collider physics,* ***single-field effective theory***

$$\mathcal{L}^{(3)}_{\mathbf{new}}(\zeta,\mathcal{F}) = \frac{1}{2}m_s^2\zeta\mathcal{F}\left((\epsilon+\mu_s)\mathcal{F} + (2\epsilon-\eta-2\lambda_\perp)\frac{2\dot\sigma\eta_\perp}{m_s^2}\dot\zeta\right) + \frac{\dot\sigma\eta_\perp}{a^2 H}\mathcal{F}(\partial\zeta)^2$$

$$-\frac{\dot\sigma\eta_\perp}{H}\dot\zeta^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon H^2 M_p^2 R_{fs}\right)\dot\zeta\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot\sigma H\eta_\perp R_{fs} + \epsilon H^2 M_p^2 R_{fs,s}\right)\mathcal{F}^3$$

$$+\frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}(\partial\mathcal{F})(\partial\chi)$$

Useful form of the action for integrating out $\mathcal{F}$ when it is heavy

IV. INTEGRATING OUT HEAVY ENTROPIC FLUCTUATIONS

AN EFFECTIVE THEORY FOR THE OBSERVABLE CURVATURE PERTURBATION

$$S[\zeta, \mathcal{F}] \xrightarrow{\mathcal{F}_{\text{heavy}}(\zeta)} S_{\text{EFT}}[\zeta] = S[\zeta, \mathcal{F}_{\text{heavy}}(\zeta)]$$

[Garcia-Saenz, Pinol, Renaux-Petel]
J. High Energ. Phys. **2020,** 73 (2020)

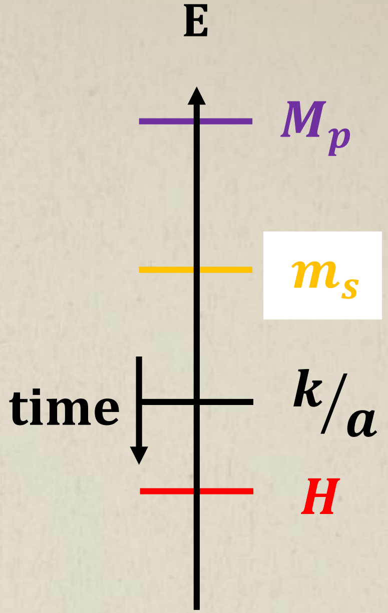
A HIERARCHY OF SCALES

WHEN ENTROPIC FLUCTUATIONS ARE HEAVY

- Equation of motion for $\mathcal{F}$:

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \left(m_s^2 + \frac{k^2}{a^2}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

Hierarchy of scales



Energy of the "experiment"

$$H \ll m_s$$

Integrate out the heavy perturbation

Like in the Fermi theory:
Integrate out the heavy W, Z bosons and consider contact interactions for fermions

A HIERARCHY OF SCALES

WHEN ENTROPIC FLUCTUATIONS ARE HEAVY

- Equation of motion for $\mathcal{F}$:

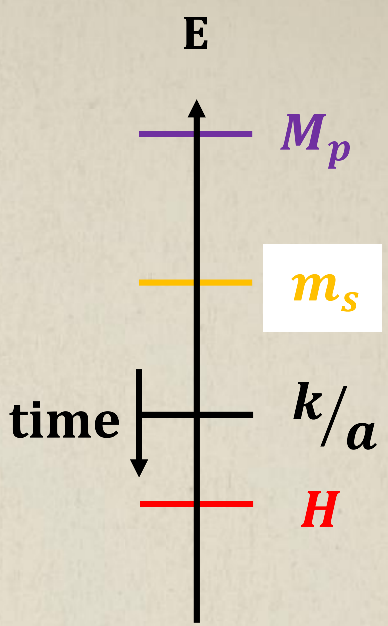
$$\cancel{\ddot{\mathcal{F}}} + \cancel{3H\dot{\mathcal{F}}} + \left(m_s^2 + \cancel{\frac{k^2}{a^2}}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

When $\mathcal{F}$ is heavy

$$\boxed{\mathcal{F}_{\rm heavy}^{\rm LO} = \frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}}$$

$$\boxed{\omega^2, \omega H, \frac{k^2}{a^2} \ll m_s^2}$$

Hierarchy of scales



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A HIERARCHY OF SCALES

THE QUADRATIC EFFECTIVE ACTION

- Equation of motion for $\mathcal{F}$:

$$\cancel{\ddot{\mathcal{F}}} + 3\cancel{H\dot{\mathcal{F}}} + \left(m_s^2 + \cancel{\frac{k^2}{a^2}}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

When $\mathcal{F}$ is heavy

$$\boxed{\mathcal{F}^{\mathrm{LO}}_{\mathrm{heavy}} = \frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}}$$

$$\Downarrow$$

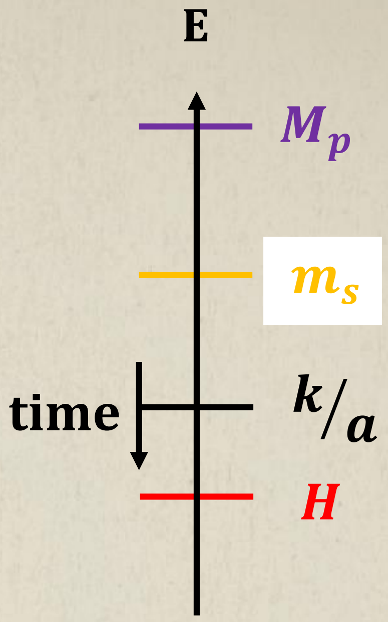
Effective single-field action for the curvature perturbation

$$S_2^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2\epsilon\left(\frac{\zeta'^2}{c_s^2} - (\partial_i\zeta)^2\right)$$

With a speed of sound c_s:

$$\boxed{\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2}}$$

Hierarchy of scales



Energy of the "experiment"

$$H \ll m_s$$

Integrate out the heavy perturbation

Like in the Fermi theory:
Integrate out the heavy W, Z bosons and consider contact interactions for fermions

$s = \dot{c}_s / H c_s$

THE CUBIC EFFECTIVE ACTION

FULL RESULT

P(X) cubic lagrangian:

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{c_s^2} \begin{pmatrix} \frac{g_1}{\mathcal{H}} \zeta'^3 + \\ g_2 \zeta'^2 \zeta + \\ g_3 c_s^2 \zeta (\partial_i \zeta)^2 + \\ \frac{\tilde{g}_3 c_s^2}{\mathcal{H}} \zeta' (\partial_i \zeta)^2 + \\ g_4 \zeta' \partial_i \partial^{-2} \zeta' \partial_i \zeta + \\ g_5 \partial^2 \zeta (\partial_i \partial^{-2} \zeta')^2 \end{pmatrix} \text{ with } \begin{cases} g_1 = \left(\frac{1}{c_s^2} - 1\right) \boldsymbol{A} \\ g_2 = \epsilon - \eta + 2s \\ g_3 = \epsilon + \eta \\ \tilde{g}_3 = \frac{1}{c_s^2} - 1 \\ g_4 = \frac{-2\epsilon}{c_s^2}\left(1 - \frac{\epsilon}{4}\right) \\ g_5 = \frac{\epsilon^2}{4 c_s^2} \end{cases}$$

The only new parameter is **A**, and depends on the UV physics

THE CUBIC EFFECTIVE ACTION

RECOVERING THE EFT OF INFLATION

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{c_s^2} \left(\begin{array}{c} \frac{g_1}{\mathcal{H}}\zeta'^3 + \\ \cancel{g_2\zeta'^2\zeta} + \\ \cancel{g_3 c_s^2 \zeta(\partial_i\zeta)^2} + \\ \frac{\tilde{g}_3 c_s^2}{\mathcal{H}}\zeta'(\partial_i\zeta)^2 + \\ \cancel{g_4\zeta'\partial_i\partial^{-2}\zeta'\partial_i\zeta} + \\ \cancel{g_5\partial^2\zeta(\partial_i\partial^{-2}\zeta')^2} \end{array} \right) \text{ with } \left\{ \begin{array}{l} g_1 = \left(\frac{1}{c_s^2} - 1\right) \boldsymbol{A} \\ \tilde{g}_3 = \frac{1}{c_s^2} - 1 \end{array} \right.$$

$\epsilon, \eta, s \to 0$

Slow-varying result:

Non-Gaussianities $\sim \frac{1}{c_s^2} - 1$

The only new parameter is **A**, and depends on the UV physics

THE EFT OF INFLATION

REVISITED...

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2} - 1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = \underbrace{-\frac{1}{2}(1 + c_s^2)}_{\text{Previously known}} + \cdots$$

THE EFT OF INFLATION
REVISITED…

Bending radius of the trajectory: $\kappa = \frac{\sqrt{2\epsilon}M_p}{\eta_\perp}$

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2}-1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = -\frac{1}{2}(1+c_s^2) \underbrace{-\frac{1}{6}(1-c_s^2)\frac{\kappa V_{;sss}}{m_s^2}}_{} + \cdots$$

3^{rd} derivative of the potential (expected)

Self-coupling of entropic fluctuations

THE EFT OF INFLATION

REVISITED...

Bending radius of the trajectory: $\kappa = \dfrac{\sqrt{2\epsilon}M_p}{\eta_\perp}$

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$$\text{with } \boldsymbol{A} = -\frac{1}{2}(1+c_s^2) - \frac{1}{6}(1-c_s^2)\frac{\kappa V_{;sss}}{m_s^2} + \underbrace{\frac{2}{3}(1+2c_s^2)\frac{\epsilon R_{\mathrm{fs}} H^2 M_p^2}{m_s^2}}_{\text{Scalar curvature of the field space}} + \cdots$$

THE EFT OF INFLATION

REVISITED…

Bending radius of the trajectory: $\kappa = \dfrac{\sqrt{2\epsilon}M_p}{\eta_\perp}$

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2} - 1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = -\frac{1}{2}(1+c_s^2) + \frac{2}{3}(1+2c_s^2)\frac{\epsilon R_{\text{fs}} H^2 M_p^2}{m_s^2} - \frac{1}{6}(1-c_s^2)\left(\frac{\kappa V_{;sss}}{m_s^2} + \underbrace{\frac{\kappa\epsilon H^2 M_p^2 R_{\text{fs},s}}{m_s^2}}_{\text{Derivative of the scalar curvature}}\right)$$

THE EFT OF INFLATION

REVISITED...

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Then you can compute f_{nl} in a slow-varying approximation

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$$\boxed{f_{\rm nl}^{\rm eq} \simeq \left(\frac{1}{c_s^2}-1\right)\left(-\frac{85}{324}+\frac{15}{243}\boldsymbol{A}\right)}$$

All contributions matter, none is a priori negligible

THE EFT OF INFLATION

HYPERINFLATION

Conditions to integrate out entropic perturbations are fulfilled

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HYPERINFLATION

Conditions to integrate out entropic perturbations are fulfilled

- Our new formula enables to **compute**

0 without the geometric $\propto R_{\mathrm{fs}}$ contribution

$$\boldsymbol{A} \simeq -0.33$$

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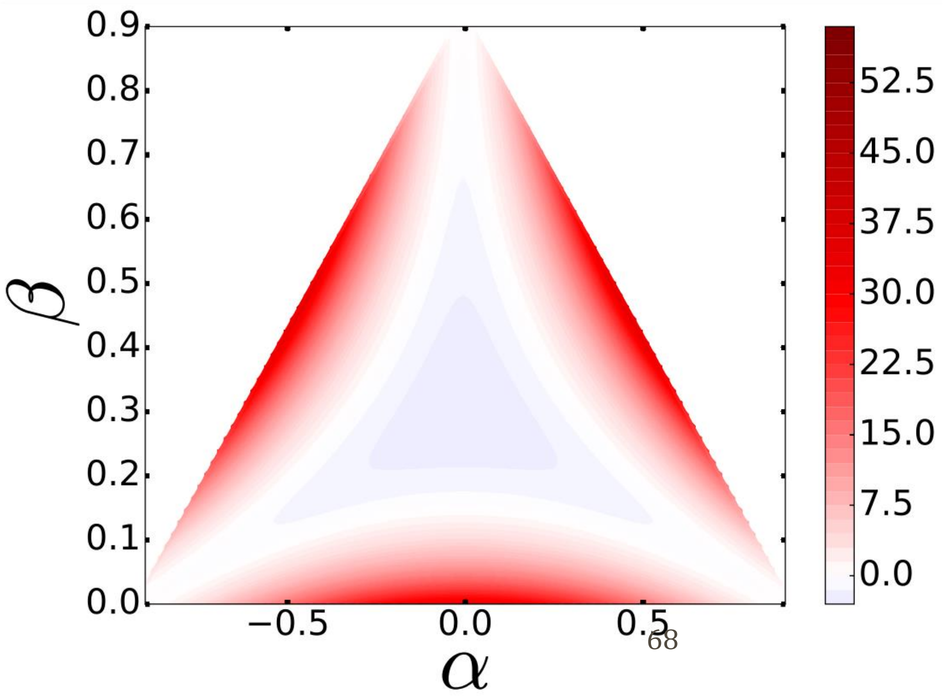
➢ Analytical prediction for the whole shape of the bispectrum:

Vs. Numerics?



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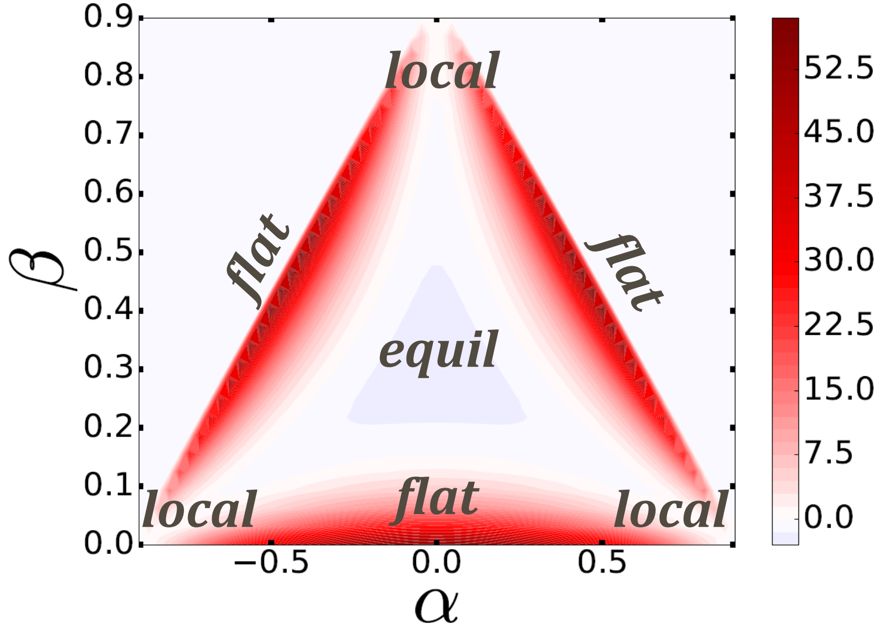

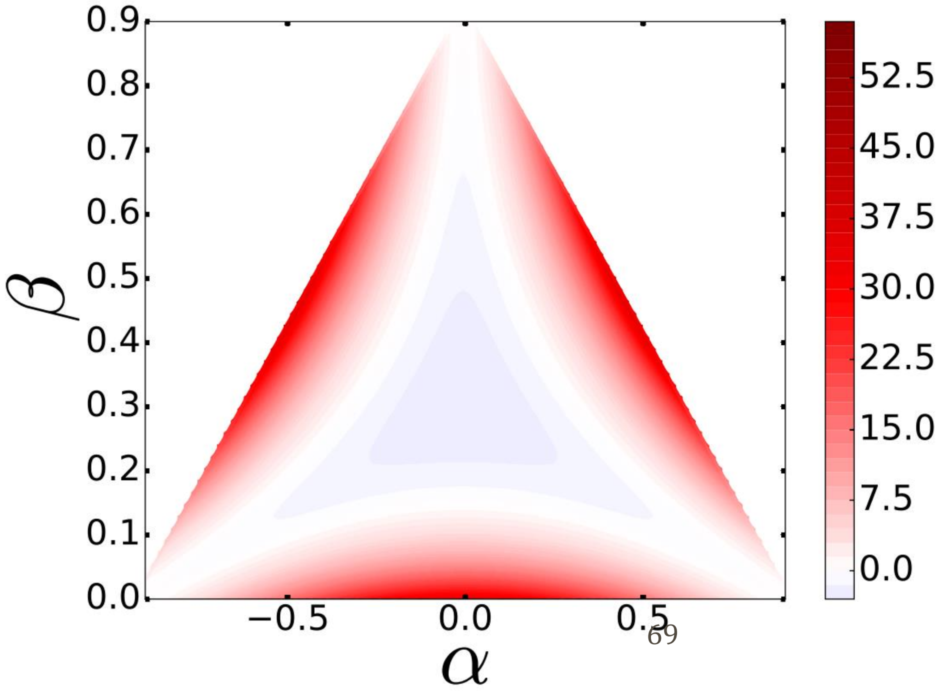

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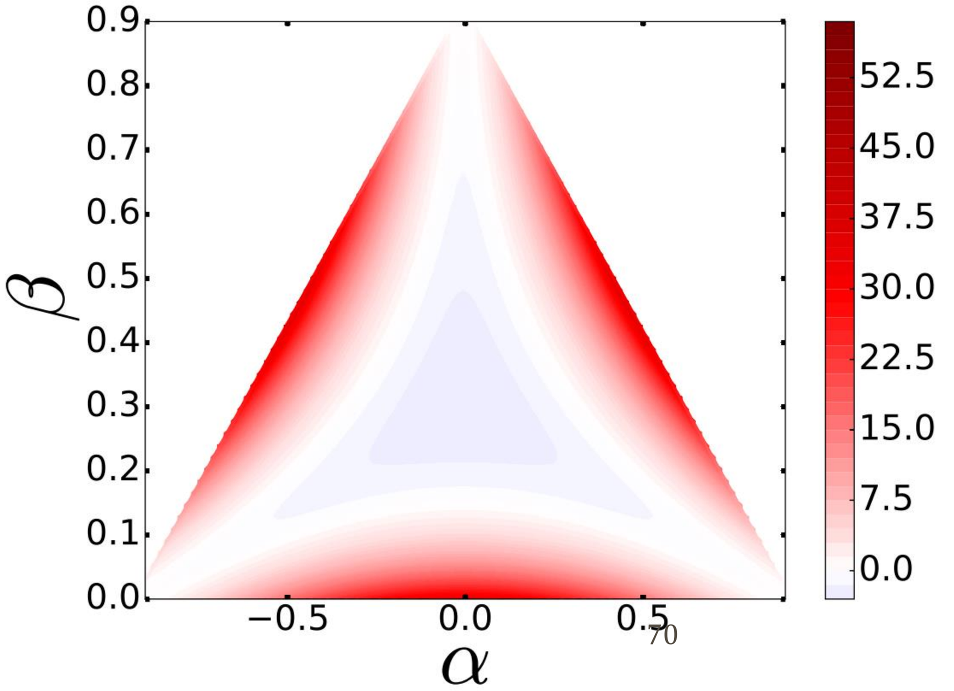

RECAP OF THIS PART

Generic 2-field inflationary model with curved field space

$$\boxed{S=\int\sqrt{-g}\left(\frac{R}{2}-\sum_{a,b}g^{\mu\nu}G_{ab}(\phi^c)\partial_\mu\phi^a\partial_\nu\phi^b-V(\phi^c)\right)}$$

Expanding the action to 3rd order | Choice of comoving gauge

$$\boxed{\mathcal{L}(\zeta,\mathcal{F})=\mathcal{L}^{(2)}(\zeta,\mathcal{F})+\mathcal{L}^{(3)}_{\text{not simplified}}(\zeta,\mathcal{F})}$$

Integrations by parts
Uses of e.o.m.

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P(X)

$c_s \to 1$

Canonical single-field

$\epsilon, \eta, s \to 0$

Decoupling limit in EFT of inflation: predictions for c_s^2, A and thus f_{nl}

$$A \sim O(V_{;sss}) + O(R_{fs}) + O(R_{fs,s})$$

OUTLOOK 1

- Other signatures of multifield models

  - Production of Primordial Black Holes (PBHs) in models with transient turns
    [G. Palma, S. Sypsas, C. Zenteno 2020]
    [J. Fumagalli, S. Renaux-Petel, J. Ronayne, L. Witkowski 2020]

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- Extending Maldacena's calculation from 2 to N scalar fields, integrating out N-1 entropic perturbations **[LP 2020]** *soon!*

OUTLOOK 2

- Post-inflationary dynamics is relevant in multifields scenarios: $\dot{\zeta} = -\frac{\dot{\sigma}\eta_\perp}{\epsilon M_p^2}\mathcal{F} + O\left(\frac{k^2}{a^2}\right)$

Observable adiabatic perturbation evolves on super-horizon scales, fed by isocurvature perturbations

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Necessary step: study the coupling of scalar fields to cosmological fluids (radiation, dark matter) during reheating, to derive reliable observable predictions!

- ➢ General formalism + study of isocurvature perturbations to be released soon: **[J. Martin, LP 2020]** *soon!*

- ➢ Generic single-field instability at small scales -> copious production of PBHs **[J. Martin, T. Papanikolaou, LP, V. Vennin 2020]** *JCAP 05(2020)003*

OUTLOOK 3

- Non-perturbative results during multifield inflation:

Standard Perturbation Theory (classical background + quantum perturbations) breaks down for very light fields

Stochastic inflation

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- ➢ Many interesting applications to come (would require another 1hr): solving Fokker-Planck, Langevin and non-Markovian dynamics, numerical simulations, etc.

CONCLUSION

- Slow-roll single-field inflation challenged: theory or model?

- Multifield inflation with curved field space is more generic and motivated by UV completions (string theory compactifications, supergravity…)

- Internal geometry plays a role already at the background level: GEOmetrical DEStabilization of Inflation (ERC working group « GEODESI » led by S. Renaux-Petel at IAP)

- It crucially affects the physics of linear fluctuations and can shift (n_s, r) predictions by a lot

- Non-Gaussianities can be enhanced, thus providing exotic detectable signatures

- Step towards the general understanding of Non-Gaussianities of such models:

    Extending Maldacena's calculation

    Single-field effective theory: explicit geometry-dependent f_{nl}

+ interesting prospects

THANKS FOR YOUR ATTENTION!

OF RELEVANT ENTROPIC MASS SCALES

- Equation of motion for $\mathcal{F}$:

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \left(m_s^2 + \frac{k^2}{a^2}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

Dynamics dictated by:

- m_s^2
- $\eta_\perp$ and $\dot{\zeta}$

- Equation of motion for ζ on large scales:

$$\dot{\zeta} = -\frac{\dot{\sigma}\eta_\perp}{\epsilon M_p^2}\mathcal{F} + O\left(\frac{k^2}{a^2}\right)$$

- Effective equation of motion for $\mathcal{F}$ on large scales:

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \underbrace{(m_s^2 + 4H^2\eta_\perp^2)}_{m_{s,\mathrm{eff}}^2}\mathcal{F} = O\left(\frac{k^2}{a^2}\right)$$

Dynamics dictated by:

- $m_{s,\mathrm{eff}}^2$

GAUGE FIXING

2 constrained parameters
4 dynamical scalar d.o.f.
2 can be removed

- Two scalar degrees of freedom can be fixed by a choice of gauge

- ADM formalism $ds^2 = -\boldsymbol{N}^2 dt^2 + g_{ij}\left(dx^i + \boldsymbol{N^i} dt\right)\left(dx^j + \boldsymbol{N^j} dt\right)$

  with $g_{ij}(t,\vec{x}) = a^2(t) e^{2\boldsymbol{\psi}(t,\vec{x})}\left(\delta_{ij} + \partial_i\partial_j \boldsymbol{E}(t,\vec{x})\right)$

- $\boldsymbol{Q_\sigma}(t,\vec{x}) = e_a^\sigma(t) Q^a(t,\vec{x})$ and $\boldsymbol{Q_s}(t,\vec{x}) = e_a^s(t) Q^a(t,\vec{x})$, adiabatic and entropic perturbations

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Flat gauge: $E = \psi = 0 \ \rightarrow Q_\sigma = Q_\sigma^{\text{flat}}$ and $Q_s = Q_s^{\text{flat}}$	**Comoving gauge: $E = Q_\sigma = 0 \ \rightarrow \ \psi^{\text{com}} = \zeta$ and $Q_s^{\text{com}} = \mathcal{F}$**
✓ $\mathcal{L}^{(3)}(Q_\sigma^{\text{flat}}, Q_s^{\text{flat}})$: already known **[Elliston, Seery, Tavakol 2012]** ❖ Correlation functions of observable perturbation ζ computed numerically from the ones of Q's **[Mulryne, Ronayne 2016]**	❖ $\mathcal{L}^{(3)}(\zeta, \mathcal{F})$: not known before this work: **[Garcia-Saenz, Pinol, Renaux-Petel]** *J. High Energ. Phys.* **2020,** 73 (2020) ✓ Correlation functions of observable perturbation ζ computed numerically directly ✓ Analytical studies possible

$$X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

THE CUBIC EFFECTIVE ACTION

RECOVERING P(X) THEORY

P(X) cubic lagrangian:

Redundancy of operators

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{c_s^2} \left(\begin{array}{c} \frac{g_1}{\mathcal{H}}\zeta'^3 + \\ g_2\zeta'^2\zeta + \\ g_3 c_s^2 \zeta(\partial_i\zeta)^2 + \\ \cancel{\frac{\tilde{g}_3 c_s^2}{\mathcal{H}}\zeta'(\partial_i\zeta)^2} + \\ g_4\zeta'\partial_i\partial^{-2}\zeta'\partial_i\zeta + \\ g_5\partial^2\zeta(\partial_i\partial^{-2}\zeta')^2 \end{array} \right) \text{ with } \left\{ \begin{array}{l} g_1 = \left(\frac{1}{c_s^2} - 1\right)(1 + 2A) \\ g_2 = \frac{1}{c_s^2}\left(3(c_s^2 - 1) + \epsilon - \eta\right) \\ g_3 = \frac{1}{c_s^2}\left(-(c_s^2 - 1) + \epsilon + \eta - 2s\right) \\ g_4 = \frac{-2\epsilon}{c_s^2}\left(1 - \frac{\epsilon}{4}\right) \\ g_5 = \frac{\epsilon^2}{4c_s^2} \end{array} \right.$$

Direct mapping with P(X):

$$\frac{2\lambda}{\Sigma} = -\left(\frac{1}{c_s^2} - 1\right)A \quad \text{with}$$

$$\Sigma = XP_{,X} + 2X^2P_{,XX}$$

$$\lambda = X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX}$$

[X. Chen, M. Huang, S. Kachru, G. Shiu 2008]

[C. Burrage, R. Ribeiro, D. Seery 2011]

THE CUBIC EFFECTIVE ACTION

RECOVERING CANONICAL SINGLE-FIELD LIMIT

Maldacena's result:
Non-Gaussianities $\sim \boldsymbol{O(\epsilon,\eta)}$

$\boldsymbol{c_s^2 \rightarrow 1}$

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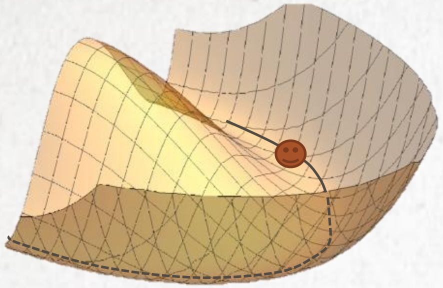
~~The only new parameter is **A**, and depends on the UV physics~~

THE GELATON CHECK

The gelaton scenario



- 2 fields (ϕ, ψ), curved field-space
- ψ is very heavy and adiabatically follows the min of its effective potential
- The full field ψ can be integrated out, giving a single-field P(X) theory

Our procedure

- Keeping $\bar{\psi}$ at the level of the background
- Integrating out heavy entropic fluctuations
- Get P(X)-like cubic Lagrangian

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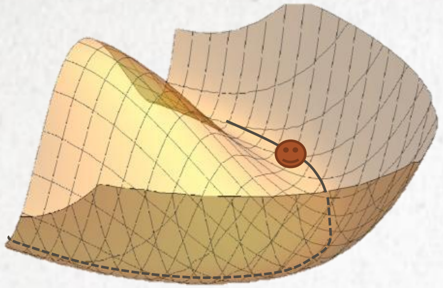

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Same P(X) theory!

REGIME OF VALIDITY OF THE EFT

MAKING ASSUMPTIONS MORE PRECISE

$$\mathcal{F}^{\mathrm{LO}}_{\mathrm{heavy}} = \frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}$$

- A more formal solution to $(m_s^2 - \Box)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$ is $\mathcal{F}_{\mathrm{heavy}} = \frac{1}{m_s^2}\sum_{i=0}^{\infty}\left(\frac{\Box}{m_s^2}\right)^i 2\dot{\sigma}\eta_\perp\dot{\zeta}$

For consistency, NLO (i=1) correction must be neglible compared to LO (i=0) in the expansion

$$\Box\left(2\dot{\sigma}\eta_\perp\dot{\zeta}\right) \ll m_s^2\left(2\dot{\sigma}\eta_\perp\dot{\zeta}\right)$$

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[S. Céspedes, V. Atal, G. Palma 2012]

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<u>Adiabaticy conditions</u>

$$\left(\frac{\dot{\eta}_\perp}{\eta_\perp m_s}\right)^2 \ll 1 \quad ; \quad \left(\frac{\dot{c}_s}{c_s m_s}\right)^2 \ll 1$$

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- The EFT is useful only if it is well valid at sound

horizon crossing: $\boxed{\frac{H^2}{m_s^2 c_s^2} \ll 1}$

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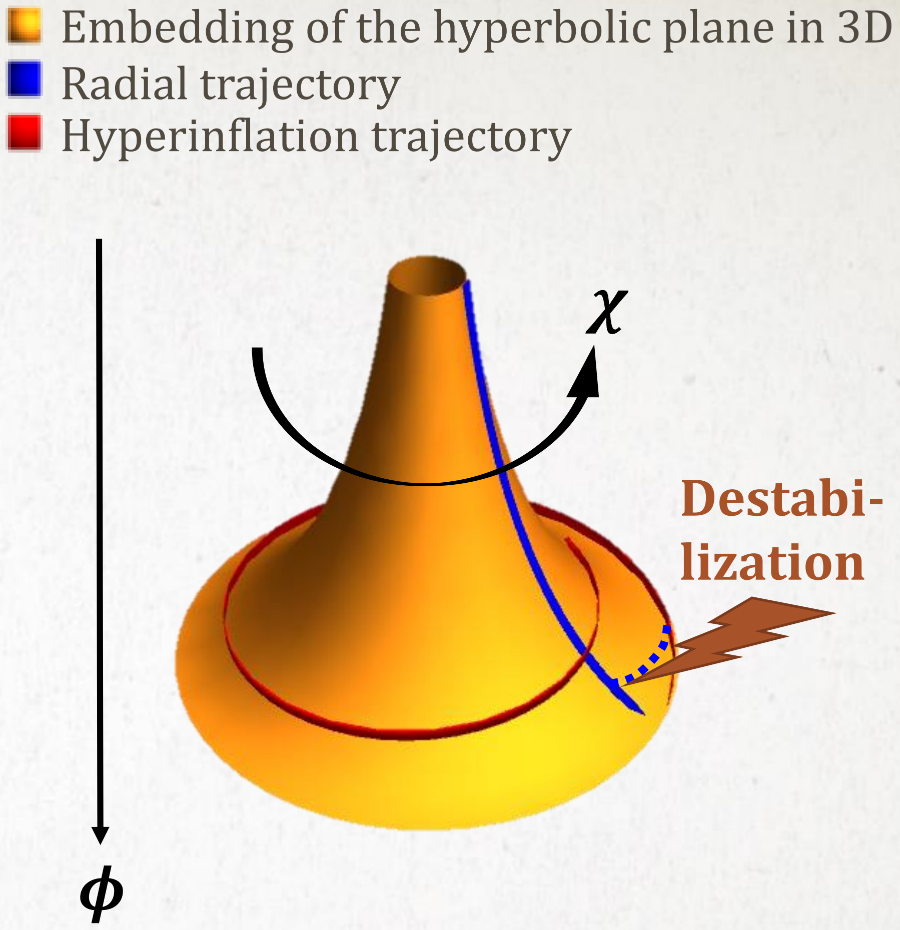
V. HYPERINFLATION



MULTIFIELD INSTABILITY AND SINGLE-FIELD EFFECTIVE THEORY

[Fumagalli, Garcia-Saenz, Pinol, Renaux-Petel, Ronayne 2019]
Phys. Rev. Lett. 123, 201302

Hyperbolic field space

$$R_{\text{fs}} = -\frac{4}{M^2}, \qquad M \ll M_p$$

HYPERINFLATION

BACKGROUND ANALYSIS

Angular momentum $J = a^3 M^2 \sinh^2\left(\frac{\phi}{M}\right)\dot{\chi}$

- $J = 0$ radial trajectory: geodesic, effectively single-field

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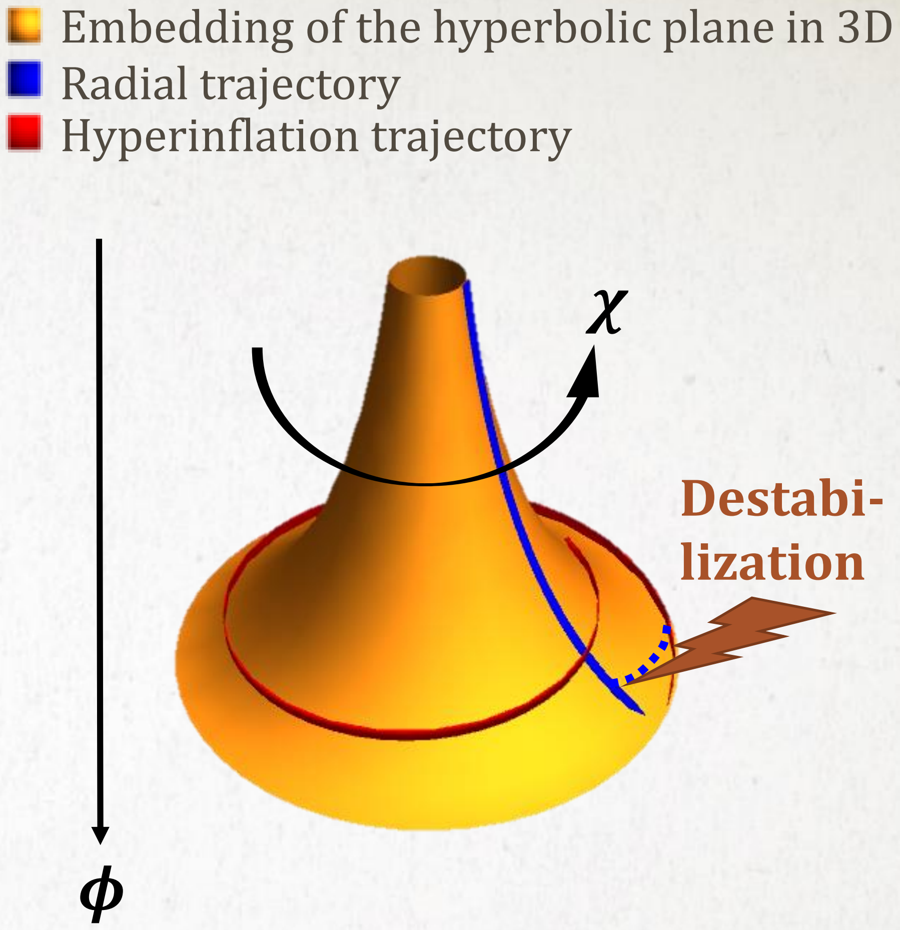
With steep potentials, geometrical destabilisation



Hyperbolic field space

$$R_{\mathrm{fs}} = -\frac{4}{M^2}, \qquad M \ll M_p$$

HYPERINFLATION

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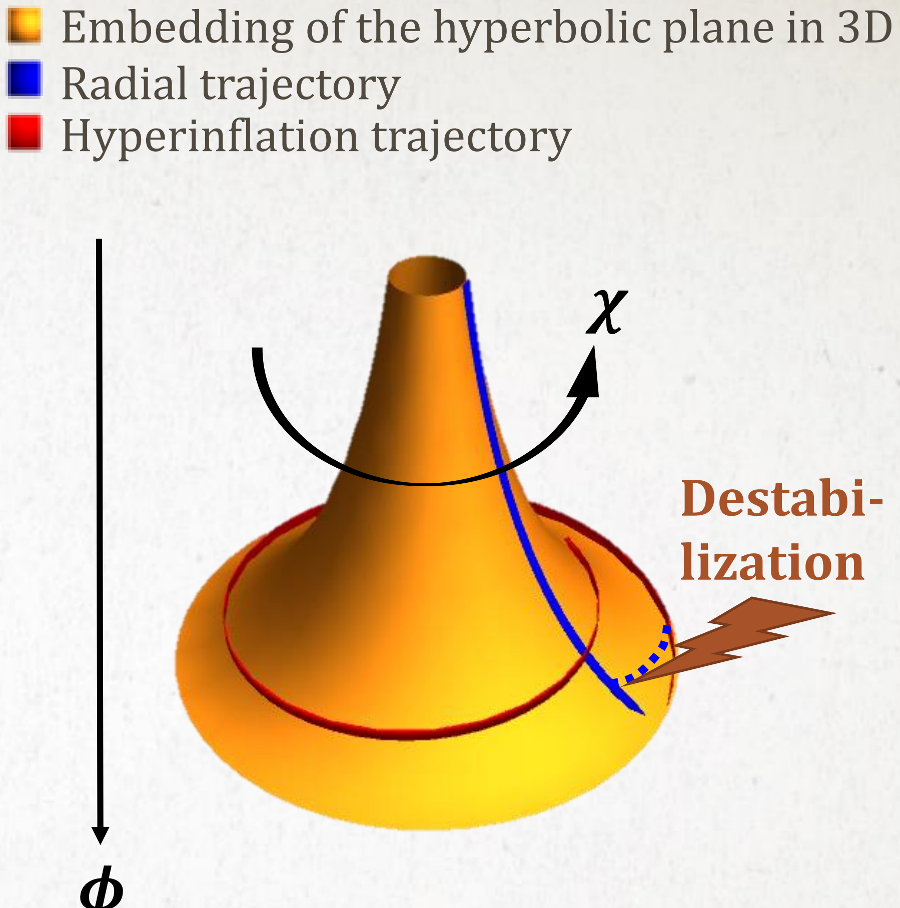
With steep potentials, the sidetracked phase is the attractor



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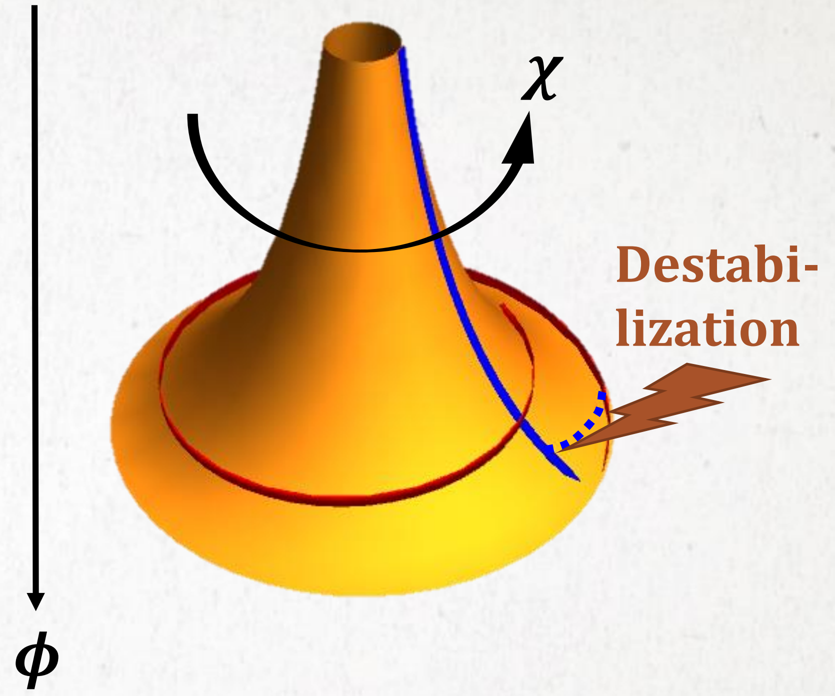
swampland conjectures, naturalness, eta problem

$\epsilon, \eta \ll 1 \Rightarrow \frac{3M^2}{M_p^2} < \frac{MV'}{V} \ll 1$ and $\frac{M|V''|}{V'} \ll 1$

Embedding of the hyperbolic plane in 3D
Radial trajectory
Hyperinflation trajectory



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HYPERINFLATION

$$h^2 = \frac{V'}{MH^2} - 9 \gg 1$$

LINEAR PERTURBATIONS

We compute $\begin{cases} \eta_\perp^2 \approx h^2 \\ \epsilon R_{fs} M_p^2 \approx -h^2 \\ V_{;ss}/H^2 \ll 1 \end{cases} \Rightarrow \begin{cases} \dfrac{m_s^2}{H^2} \approx -2h^2 < 0 \\ \dfrac{m_{s,\mathrm{eff}}^2}{H^2} \approx 2h^2 > 0 \end{cases}$

$\frac{m_s^2}{H^2} < 0$ → Unstable, growing sub-Hubble perturbations

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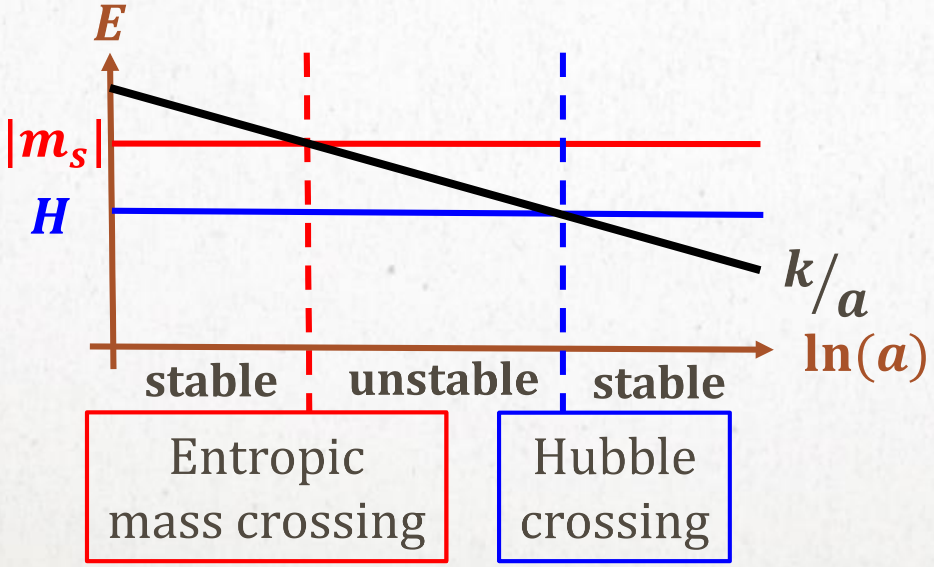
This tachyonic instability is only transient for each k-mode

Remember in the e.o.m. for Q_s, the mass term is $\left(\frac{k^2}{a^2} + m_s^2\right) > 0$ *deep under the horizon*

$m_{s,\mathrm{eff}}^2 > 0$ *on super-horizon scales*



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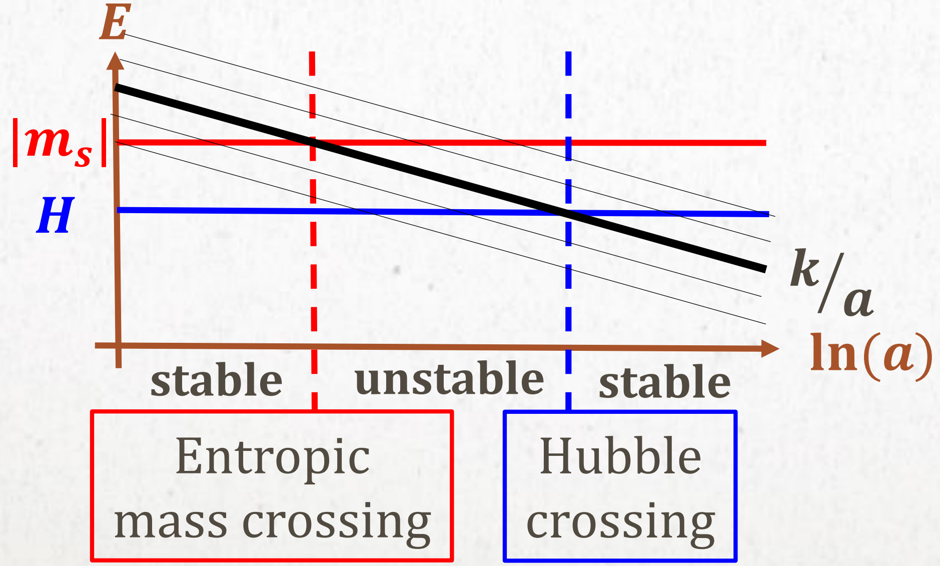

HYPERINFLATION

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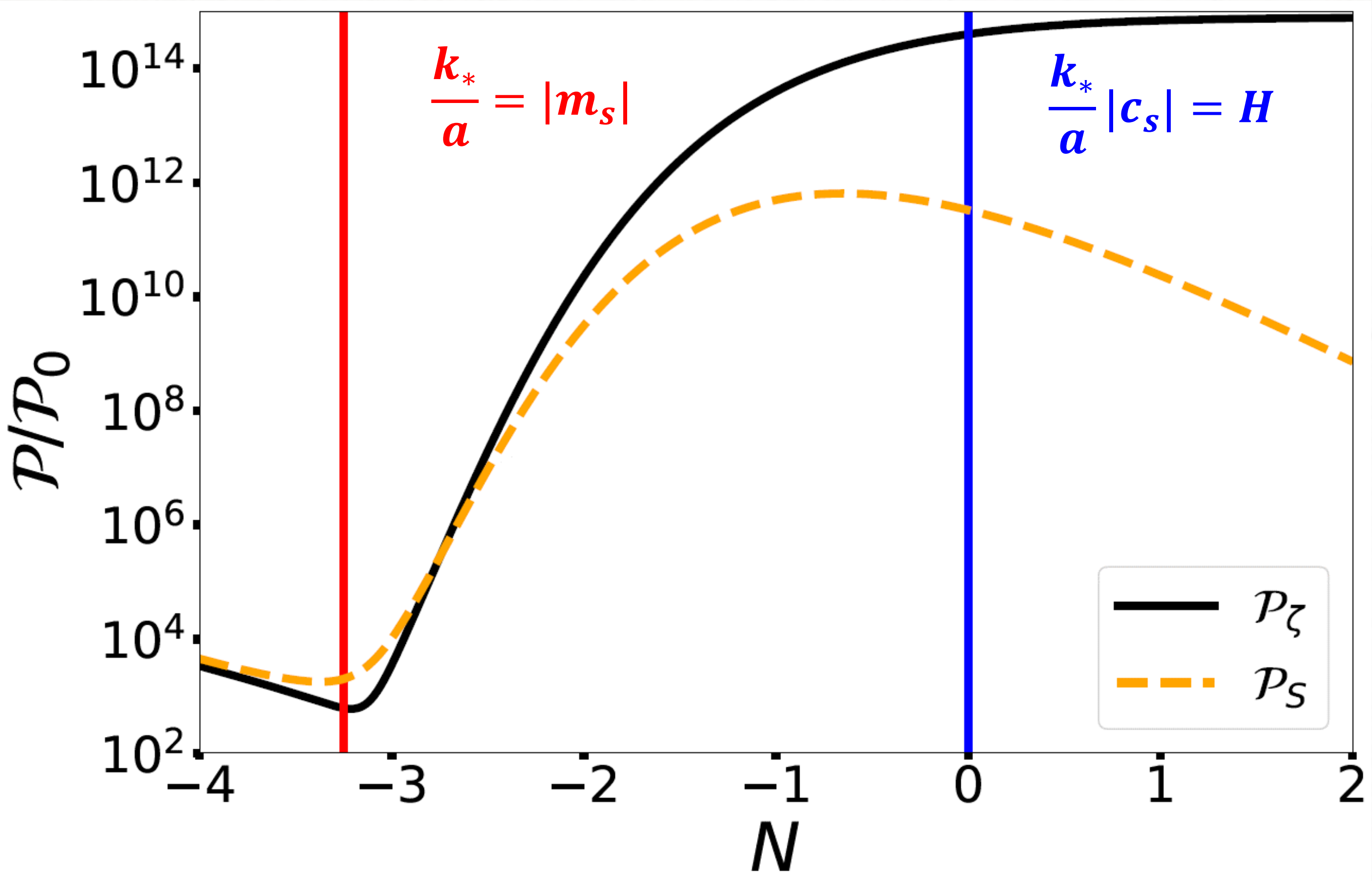


$r \ll 0.01,$
$n_s > 1:$

Excluded by CMB

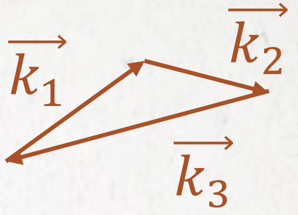
$$V = \frac{1}{2}m^2\phi^2 \text{ with } m = M = 10^{-2}M_p$$

HYPERINFLATION

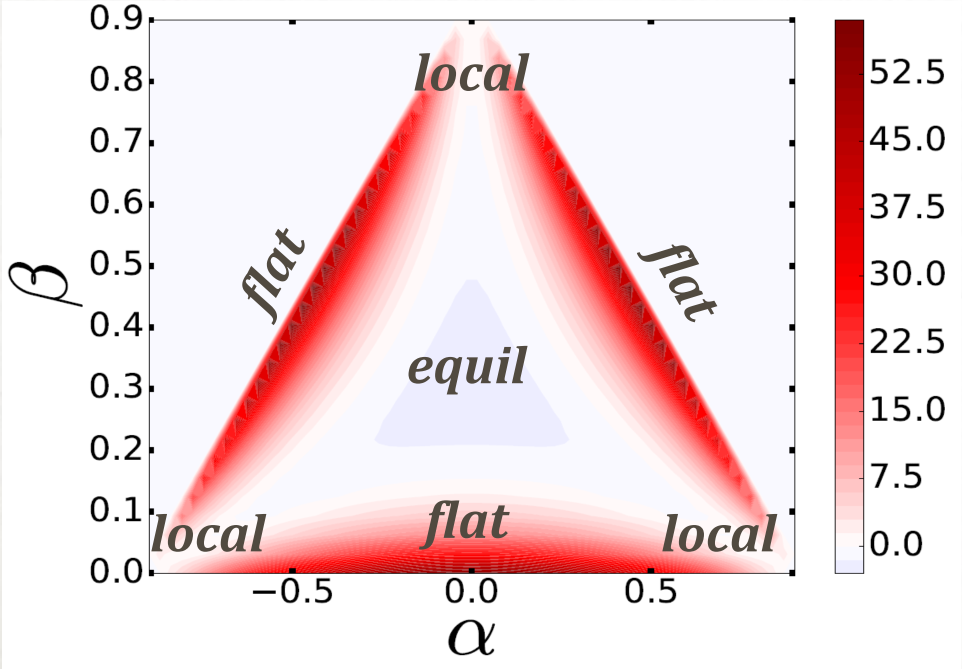
BISPECTRUM USING PyTransport 2.0

[D. Mulryne, J. Ronayne 2016]

3-point function: $< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} >= (2\pi)^3\ \delta^{(3)}(\overrightarrow{k_1}+\overrightarrow{k_2}+\overrightarrow{k_3}) \times B_\zeta(\overrightarrow{k_1},\overrightarrow{k_2},\overrightarrow{k_3})$



With $k_1 = \frac{3k_*}{4}(1+\alpha+\beta),\ k_2 = \frac{3k_*}{4}(1-\alpha+\beta),\ k_3 = \frac{3k_*}{2}(1-\beta)$



Characteristic large flattened bispectrum

f_{NL}^{equil}	f_{NL}^{flat}
-2.0	53.8

Single-clock inflation with Bunch-Davies initial states predicts equilateral non-Gaussianities

HYPERINFLATION

SINGLE-FIELD EFFECTIVE THEORY

- Equation of motion for $\mathcal{F}$:

$$\cancel{\ddot{\mathcal{F}}} + \cancel{3H\dot{\mathcal{F}}} + \left(m_s^2 + \cancel{\frac{k^2}{a^2}}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

$\mathcal{F}$ is 'heavy' and tachyonic

$$\boxed{\mathcal{F}^{\mathrm{LO}}_{\mathrm{heavy}} = -\frac{2\dot{\sigma}\eta_\perp}{|m_s^2|}\dot{\zeta}}$$

$$\boldsymbol{\omega^2, \omega H, \frac{k^2}{a^2} \ll |m_s^2|}$$

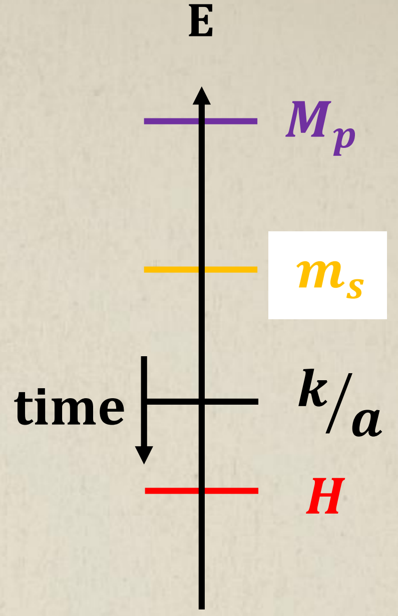
$$\Downarrow$$

Effective single-field action for the curvature perturbation

$$S_2^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 \epsilon \left(\frac{\zeta'^2}{c_s^2} - (\partial_i\zeta)^2\right)$$

$$\boxed{\frac{1}{c_s^2} = 1 - \frac{4H^2\eta_\perp^2}{|m_s^2|} \simeq -1}$$

Hierarchy of scales



Energy of the "experiment"

$H \ll m_s$

Integrate out the heavy perturbation

Like in the Fermi theory: Integrate out the heavy W, Z bosons and consider contact interactions for fermions

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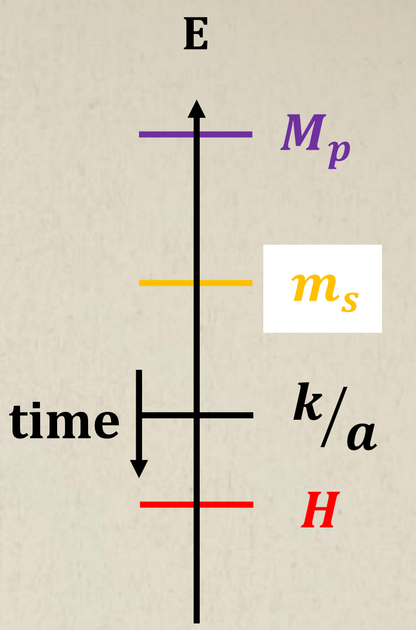
$\longrightarrow$

$$\zeta_{\mathrm{growing}}(\tau) \sim \alpha e^{k|c_s|\tau + x}$$

$$\zeta_{\mathrm{decaying}}(\tau) \sim i\alpha e^{-(k|c_s|\tau + x)}$$

Hierarchy of scales



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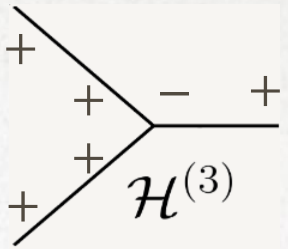
HYPERINFLATION

BISPECTRUM USING EFT

$\zeta_{\text{grown}} \sim \alpha e^{x}$



$\sim \alpha^4 e^{4x}$

Effective single-field cubic action

$$S_{(3)}^{EFT}[\zeta] = \int d\tau d^3x \;\, a\frac{\epsilon}{H} M_p^2 \left(\frac{1}{c_s^2} - 1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

Need to contract one decaying mode

Growing modes are purely real

$\longrightarrow$ No exponential enhancement of $f_{NL} \sim \dfrac{<\zeta^3>}{<\zeta^2>^2}$

$$f_{\text{NL}}^{\text{flat}} = \frac{5}{576}\left(\frac{1}{|c_s^2|} + 1\right)\left(39(A-1) + 12x^2 + 4(A+1)x^3\right)$$

$f_{\text{NL}}^{\text{flat}} \sim O(50)$ $\Leftarrow$ Cubic polynomial in x with

If $A \sim 1$ $\quad$ $x \sim 10$ in hyperinflation

[S. Garcia-Saenz, S. Renaux-Petel 2018]

HYPERINFLATION

OTHER PHENOMENOLOGICAL ASPECTS

- Estimation of higher n-point functions with the EFT of inflation

We find enhanced flattened configurations: $\dfrac{< \zeta^n >}{< \zeta^2 >^{n-1}} \propto \left[\left(\dfrac{1}{|c_s|^2} + 1\right) x^3\right]^{n-2}$

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- If the large bending is only transient, then only scales in the instability band *at that time* are enhanced

If that happens after CMB scales have exited horizon, could produce PBHs without affecting CMB

[Fumagalli, Renaux-Petel, Ronayne, Witkowski 2020]