Emergence of electromagnetic and gravitational wave from qubit ocean

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Copernicus Colloquium Series

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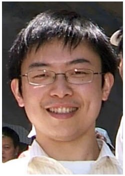

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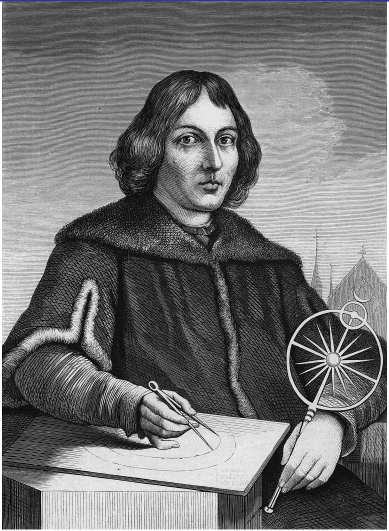
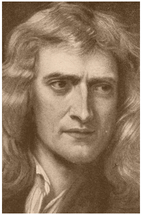
A mathematical model of a heliocentric system

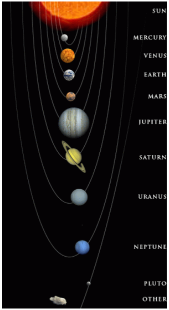


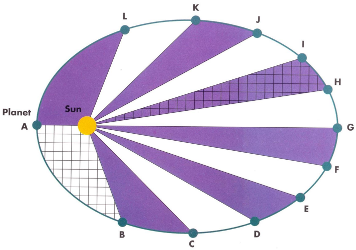
Dawn of physics: **Nicolaus Copernicus** (1473-1543)
Galilei (1564 - 1642); Kepler (1571 - 1630); ... ...

Mechanical revolution — Newton (1687)

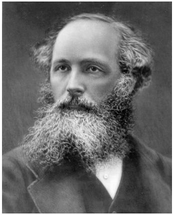


- **Unified** falling apples on earth and the planets motions in sky
- **New world view:** All matter are formed by collections of **particles**, and their motion is governed by the Newton's equation $F = ma$.
- **New math**: Calculus





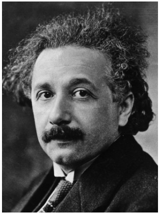
Electromagnetic revolution Maxwell (1861)



- **Unified** electricity, magnetism, and light
- **New world view:** There is a new form of matter – **wave-like** matter, which causes **electromagnetic interaction** between the particle-like matter. The motion of wave-like matter is governed by the Maxwell equation $\dot{\boldsymbol{E}} - c\partial \times \boldsymbol{B} = \dot{\boldsymbol{B}} + c\partial \times \boldsymbol{E} = \partial \cdot \boldsymbol{B} = \partial \cdot \boldsymbol{E} = 0$.
- **New math**: Fiber bundle (gauge theory)

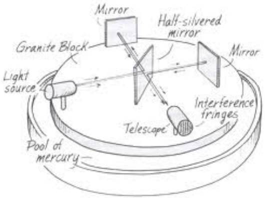
The first compass

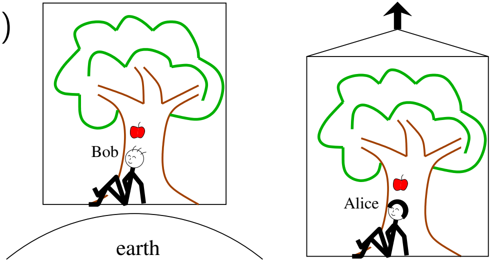
Relativity revolution Einstein (1905,1916)



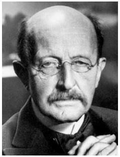
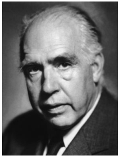
- **Unified** space, time, and gravity
- **New world view:** Even space-time is dynamical and its distortion is another wave-like matter. The new wave-like matter causes gravitational interaction between the particle-like matter, and satisfies the Einstein equation $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = -\frac{8\pi}{c^4}T_{\mu\nu}$
- **New math**: Riemannian geometry (curved space)

Michelson-Morley (1887)

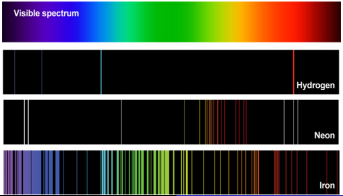


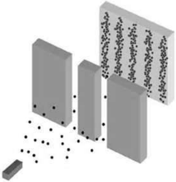


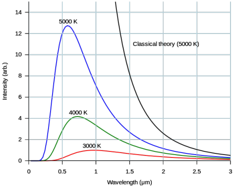
Quantum revolution

- **Unified**:Hydrogen spectra, blackbody radiation, interference
- **New world view:** particle-like matter and wave-like matter are unified. Matter is neither particle nor wave, and is both particle and wave. A new form of matter – **particle-wave-like** matter.
- **New math**: linear algebra & tensor product
  (algebra replaces calculus, discrete replaces contimum)







The deepest message from quantum theory

If you are brainwashed by quantum theory,
You may believe that

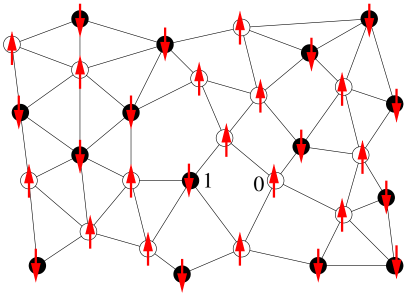
matter is **information** and
information is **matter**

- A change of information $\to$ frequency ω $\to$ energy $E = \hbar\omega$ $\to$ mass $m = \frac{E}{c^2} = \frac{\hbar\omega}{c^2}$
- Mass and energy = the amount of **matter** = change of **information** .
- I call the unification of matter and quantum information as **second quantum revolution**

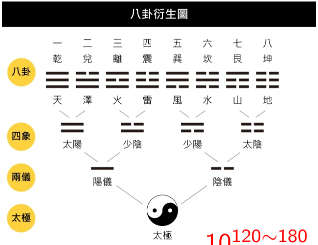
How matter and information get unified?

Photons, electrons, *etc* from quantum information

Space ∼ Qubit ocean; Elementary particle ∼ wave in ocean



qubit = 太極, 0, 1 = 兩儀



$10^{120\sim180}$ qubits

- Due to particle-wave duality in quantum theory, **the origin of a particle = the origin of a wave (a wave equation)**.
- **Entanglement between qubits → a sense of space**

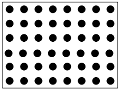
- Entangled qubits (connected by line) = neighboring qubits.
- The structures of neighborhood → dimensions of space.

- The deformation of the entanglement → waves → elem. particles

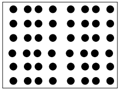
Principle of emergence: wave equation from order

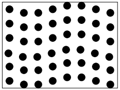
Can a wave in qubit ocean satisfies Maxwell equation?

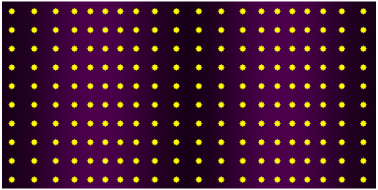
- Atoms in solid form a regular array $\rightarrow$ crystal
- Mechanical properties of crystal = properties of deformation
- Dynamics of deformation is described by a wave equation
- Wave equation in a crystal:

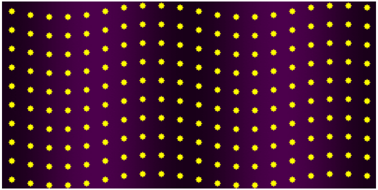


crystal order $\rightarrow$ elasticity equation $\partial_t^2 u^i - T_m^{ijk} \partial_j \partial_k u^m = 0$



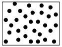


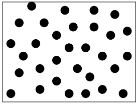




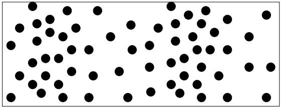
- One longitudinal mode (compression mode) and two transverse modes (shear modes), but light wave only has two transverse modes (two polarizations).

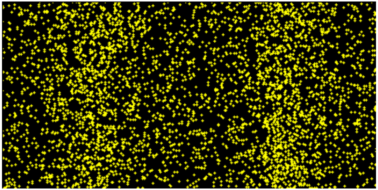
Principle of emergence: wave equation from order

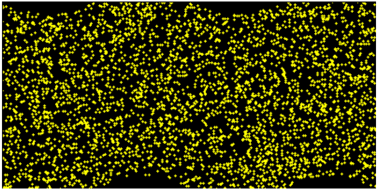
- Wave in superfluid $|\Phi_{SF}\rangle = \sum_{\text{all position conf.}}$ $\Big|$  $\Big\rangle$:



density fluctuations:
$\partial_t^2 \rho - \partial_x^2 \rho = 0$
$\rightarrow$ Longitudinal wave





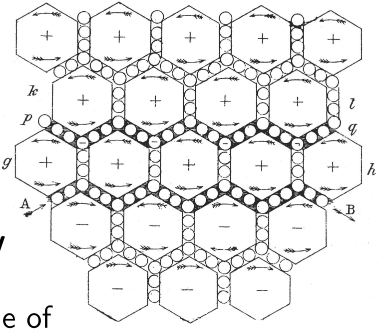


- Only one longitudinal mode (compression mode). Not light wave.
- **Principle of emergence: different order (organization) $\rightarrow$ different wave equation $\rightarrow$ different properties**

Condensed matter physicists = engineers of wave equations
= engineers of materials with desired properties

150 years search for the light-bearing aether

So, to understand the origin of elementary particles, such as **photons (light)**, from **qubit ocean** (*ie* quantum magnet), we just need to find an organization of qubits so that the deformation wave satisfy the Maxwell equation. Then, we can say *light (photons) come from quantum information*.



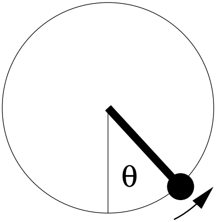
- In fact, after introducing his equation, Maxwell himself designed a **mechanical rotor** model. Now we design **quantum rotor** (qubit) model.
- As engineers for all kinds of wave equations and being guided by **symmetry breaking theory**, we checked all possible qubit orders, and unfortunately found none of them can produce wave that satisfy Maxwell equation.
- Not knowing waving of what give rise to light, we declare that **light is fundamental** and do not ask where it comes from.

A quantum rotor and quantum freeze

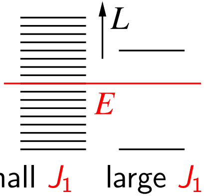
Let us do not give up and try again

Sound wave in solid has one longitudinal mode (helicity 0) and two transverse modes (helicity ± 1).

How to get rid off the helicity 0 mode, so to have only helicity ± 1 modes and convert sound wave to light wave?



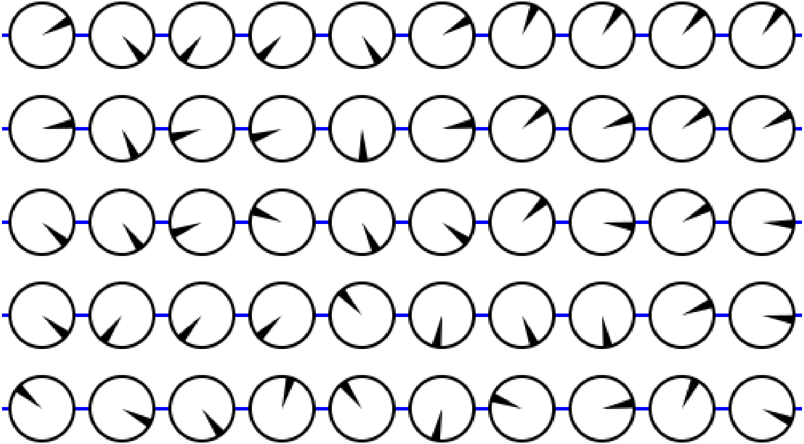
- A rotor described by Hamiltonian $H = J_1 L^2$
  $\to$ a particle on a ring with mass $\frac{1}{2}J_1^{-1}$
  (θ, L) coordinate-momentum pair.
  At energy E, $E \sim J_1 L^2$, $\delta L \sim \sqrt{E/J_1}$
- Quantum fluctuations: $\delta\theta \sim 1/\delta L \sim \sqrt{J_1/E}$
- Quantum freeze: large J_1 (small mass $\frac{1}{J_1}$), $\delta\theta > 2\pi$
  Angular momentum L is quantized as integer,
  is pinned at $L = 0$.
  (Non-zero L costs a large energy $J_1 \gg E$.)



Quantum freeze can gap out a gapless mode

- Coupled rotors $H = \sum_I J_1 L_I^2 - g\cos(\theta_I - \theta_{I+1})$

\- Small $J_1 \to$ gapless modes $\theta_I(t)$ with dispersion $\omega_k \sim \sqrt{J_1 g}\, k$.



\- Large $J_1 \to$ gapped modes $\theta_I(t) + \delta\theta_I$ (gap $\Delta E \sim J_1$ when $\frac{J_1}{g} \gg 1$)

Two quantum rotors and partial quantum freeze

- Hamiltonian

$$H = J_1(L_1)^2 + J_1(L_2)^2 + 2J_2L_1L_2$$
$$= \frac{1}{2}(J_1+J_2)(L_1+L_2)^2 + \frac{1}{2}(J_1-J_2)(L_1-L_2)^2$$

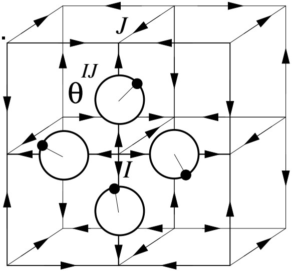
- $(\theta_1+\theta_2, L_1+L_2)$ particle with small mass $1/(J_1+J_2)$
  $(\theta_1-\theta_2, L_1-L_2)$ particle with large mass $1/(J_1-J_2)$
- Energy gap: for L_1+L_2: $\sim J_1+J_2$, for L_1-L_2: $\sim J_1-J_2$
- Partial quantum freeze when $J_1 \sim J_2$ for $J_1-J_2 \ll E \ll J_1+J_2$:

- Low energy states satisfy the **constraint** $L_1+L_2=0$, or
  $\mathrm{e}^{\mathrm{i}\phi(L_1+L_2)}|\text{low}\rangle = \mathrm{e}^{\phi(\frac{\partial}{\partial\theta_1}+\frac{\partial}{\partial\theta_2})}|\text{low}\rangle = |\text{low}\rangle$.
  Gauge invariance: Invariance under transformation generated by the constraint $\mathrm{e}^{\mathrm{i}\phi(L_1+L_2)}$: $(\theta_1,\theta_2)\to(\theta_1+\phi,\theta_2+\phi)$.
- In-phase motion has strong fluctuations (generated by gauge transformation $\mathrm{e}^{\mathrm{i}\phi(L_1+L_2)}$): $\delta\theta_1=\delta\theta_2=\phi\sim 2\pi$ gapped (frozen)
- Out-phase has weak fluctuations $\delta\theta_1=-\delta\theta_2\sim 0$: low energy

Emergence of photons via partial quantum freeze

Design a lattice model to produce waves (low energy modes) satisfying Maxwell equation

Each link: one rotor $(\theta^{IJ}, L_{IJ}) = (-\theta^{JI}, -L_{JI})$.
Three sets of rotors on x, y, z links
$\rightarrow$ three modes



$$\mathcal{L} = \sum_{\text{links}} L_{IJ}\dot{\theta}^{IJ} - U \sum_{I} (\overbrace{\sum_{J \text{ next to } I} L_{IJ}}^{\text{constraint to 0}})^2 - J \sum_{\text{links}} (L_{IJ})^2 - g \sum_{\text{all-sq}} \prod_{\text{1-sq}} e^{i\theta^{IJ}}$$

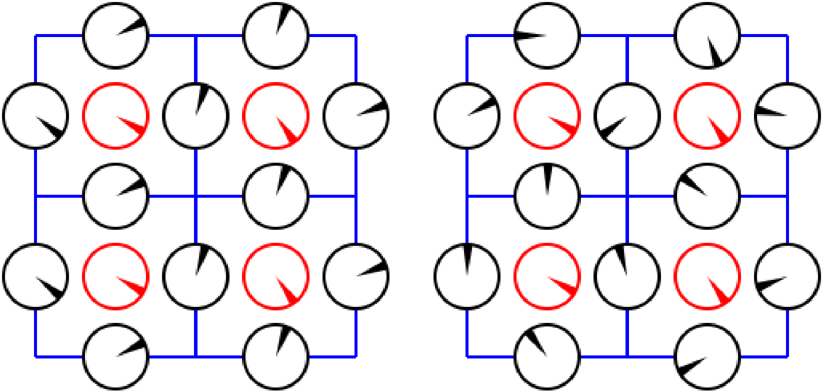
- Large U $\rightarrow$ partial quantum freeze (gap one mode out of three)

- Low energy states satisfy $e^{i\phi_I \sum_j L_{IJ}}|\text{low}\rangle = |\text{low}\rangle$ $\rightarrow$ Strong fluctuations are generated by $e^{i\phi_I \sum_J L_{IJ}}$ (gauge transformations): $\theta_{IJ} + \phi_I - \phi_J$, $\phi_I \sim 2\pi$. The corresponding mode is gapped.

Motrunich Senthil cond-mat/0205170; Wen cond-mat/0210040

Emergence of photons via partial quantum freeze

Small U (classical) Large U (partial quantum freeze)



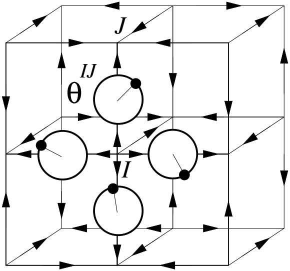
Strong fluctuating mode (frozen mode) ϕ_I: $\theta_{IJ} \to \theta_{IJ} + \phi_I - \phi_J$

Weak fluctuating modes (gapless modes that do not depend on ϕ_i, *ie* gauge invariant) $\phi_{IJKL} = \theta_{IJ} + \theta_{JK} + \theta_{KL} + \theta_{LI}$

A closer look at the lattice model

- In large U limit $\to$ Low energy states are invariant under $e^{i\phi_I \sum_J L_{IJ}}$
  The fluctuations ϕ_I generated by $e^{i\phi_I \sum_j L_{IJ}}$ are $\gtrsim 2\pi$ and gapped
- J-term and g-term commute with $e^{i\phi_I \sum_J L_{IJ}}$ (gauge invariance), acting within the low energy subspace $\to$ dynamics of other modes

  Why the large U gaps out the longitudinal mode and leave two transverse modes to have low energy? **Take continuum limit**
- Each link: one rotor $(\theta^{IJ}, L_{IJ}) = -(\theta^{JI}, L_{JI})$. Three sets of rotors on x, y, z links $\to$ three modes $\to$ a vector field $\theta^{IJ} \to \boldsymbol{A}$, $L_{IJ} \to \boldsymbol{E}$,



$$\mathcal{L} = \sum_{\text{links}} L_{IJ}\dot\theta^{IJ} - U\sum_I \Big(\sum_J L_{IJ}\Big)^2 - J\sum_{\text{links}}(L_{IJ})^2 - g\sum_{\text{all-sq}} e^{i(\theta^{IJ}+\theta^{JK}+\theta^{KL}+\theta^{LI})} + h.c.$$

$$\sim \boldsymbol{E}\cdot\dot{\boldsymbol{A}} - U(\boldsymbol{\partial}\cdot\boldsymbol{E})^2 - J\boldsymbol{E}^2 - g(\partial_i A_j - \partial_j A_i)^2 + \boldsymbol{A}^4 + \cdots$$

Gapping out $h = 0$ mode: partial quantum freeze

- Two transverse $h = \pm 1$ modes $\boldsymbol{E}, \boldsymbol{A} \perp \boldsymbol{k}$ (propagating direction):
$$\mathcal{L} \sim E_\perp \dot{A}_\perp - JE_\perp^2 - gk^2 A_\perp^2 \quad \to \quad \mathcal{L} \sim J^{-1} \dot{A}_\perp^2 - gk^2 A_\perp^2$$
Classical gapless mode with dispersion $\omega_k \sim \sqrt{gJ}k$
- Quantum fluctuations at cut-off scale $k \sim 1$ in $J \ll g$ limit
$$\delta E_\perp \delta A_\perp \sim 1, \quad \delta A_\perp \sim \sqrt{J/g} \ll 2\pi, \quad \delta E_\perp \sim \sqrt{g/J} \gg 1$$
Classical picture is valid ($E_\perp \sim$ int.) and the modes remain gapless

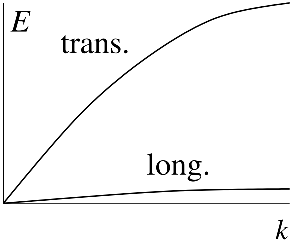
- A longitudinal $h = 0$ mode $\boldsymbol{E}, \boldsymbol{A} \parallel \boldsymbol{k}$ (propagating direction):
$$\mathcal{L} \sim E_\parallel \dot{A}_\parallel - (J + k^2 U) E_\parallel^2 + 0 A_\parallel^2 \quad \to \quad \mathcal{L} \sim \frac{1}{J + k^2 U} \dot{A}_\parallel^2 + 0 A_\parallel^2$$
Classical gapless mode with dispersion $\omega_k \sim 0$ (zero velocity)
- Quantum fluctuations at cut-off scale $k \sim 1$ in $J \ll g \ll U$ limit:
$$\delta E_\parallel \delta A_\parallel \sim 1, \quad \delta A_\parallel \sim \sqrt{U/0} \gtrsim 2\pi, \quad \delta E_\parallel \sim \sqrt{0/U} \to E_\parallel = 0$$
The classical picture is invalid $\to$ quantum freeze, gapped ($\sim U$).

Gapping out $h = 0$ mode: partial quantum freeze

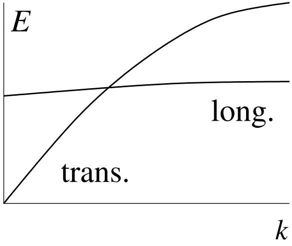
For lattice model with compact variables:

Hard classical modes $\rightarrow$ $\delta\theta_I \ll 2\pi$ $\rightarrow$ semi-classical $\rightarrow$ gapless

Soft classical modes $\rightarrow$ $\delta\theta_I \gg 2\pi$ $\rightarrow$ quantum freeze $\rightarrow$ gap



Classical



Quantum

- Quantum freeze makes $\boldsymbol{\partial} \cdot \boldsymbol{E} = 0$. $\boldsymbol{E} \sim L_{IJ} =$ int. and gaps out the longitudinal mode. The low energy states are invariant under $\mathrm{e}^{\mathrm{i} \int \mathrm{d}^3\boldsymbol{x} \phi(\boldsymbol{x}) \boldsymbol{\partial}\cdot\boldsymbol{E}} |\text{low}\rangle = |\text{low}\rangle$, invariant under the gauge transformation $\mathrm{e}^{\mathrm{i} \int \mathrm{d}^3\boldsymbol{x} \phi(\boldsymbol{x}) \boldsymbol{\partial}\cdot\boldsymbol{E}}$: $\boldsymbol{A} \rightarrow \boldsymbol{A} + \boldsymbol{\partial}\phi$.

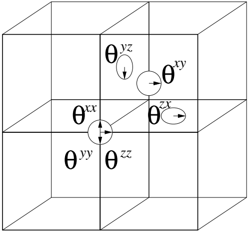
Quantum freeze $\rightarrow$ emergent "gauge symmetry"

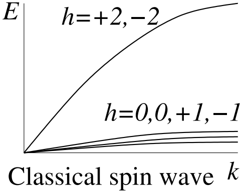
Emergence of gravitons

- Each vertex: three rotors $(\theta_{ii}, L^{ii}), ii = xx, yy, zz$.
  Each face: one rotor (θ_{ij}, L^{ij}), $ij = xy, yz, zx$.
  $\mathcal{L} = \sum L^{ij}\dot{\theta}_{ij} - \text{Complicated } H$
  Total six modes (spin waves) with helicity $0, 0, \pm 1, \pm 2$. $\to$ Continuum theory with symmetric tensor fields $(\theta_{ij}, L^{ij}) \to (a_{ij}, \mathcal{E}^{ij})$

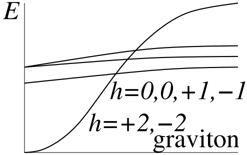
$$L = \int \mathrm{d}^3\boldsymbol{x}\ \mathcal{E}^{ij}\dot{a}_{ij} - H(a_{ij}, \mathcal{E}^{ij})$$

- Design the lattice Hamiltonian $\to$ helicity ± 2 modes have a finite velocity, helicity $0, 0, \pm 1$ modes have a nearly zero velocity (correspond to gauge fluctuations).
- We start with continuum field theory first, then try to put the continuum field theory on lattice.





Classical spin wave



Partial quantum freeze

Vector constraint to gap out $h = 0, \pm 1$ modes

The **vector constraint**: large $U(\partial_i \mathcal{E}^{ij})^2$-term makes

$$\partial_i \mathcal{E}^{ij} = 0$$

generates gauge transformations (strongly fluctuating mode)

$$\delta a_{ij} = \{a_{ij}(\boldsymbol{x}), \int \mathrm{d}^3 \boldsymbol{y} f_j(\boldsymbol{y}) \partial_i \mathcal{E}^{ij}(\boldsymbol{y})\}_{PB} = \partial_i f_j(\boldsymbol{x}) + \partial_j f_i(\boldsymbol{x})$$

The gauge invariant combinations are symmetric tensors

$$R^{ij} = R^{ji} = \epsilon^{imk} \epsilon^{jln} \partial_m \partial_l a_{nk}, \qquad \mathcal{E}^{ij}$$

$\to$ continuum Lagrangian

Cenke Xu, cond-mat/0602443

$$\mathcal{L} = \mathcal{E}^{ij} \partial_0 a_{ij} \overbrace{-\alpha(\mathcal{E}^{ij})^2 - \beta(\mathcal{E}^{ii})^2 - \gamma(R^{ij})^2 - \lambda(R^{ii})^2}^{\text{gauge invariant terms}} \overbrace{-U(\partial_i \mathcal{E}^{ij})^2}^{\partial_i \mathcal{E}^{ij}=0 \text{ constraint}}$$

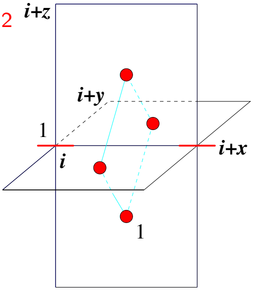
The constraints remove $h = 0, \pm 1$ modes.
$h = 0, \pm 2$ has $\omega \sim k^2$. Pseudo gravitons

Put the field theory on lattice (UV completion)

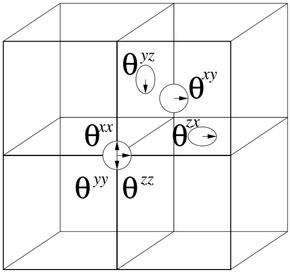
- Vector constrain $U(\partial_i \mathcal{E}^{ij})^2 \to U\sum_{\boldsymbol{I},\boldsymbol{\mu}}[V(\boldsymbol{I},\boldsymbol{I}+\boldsymbol{\mu})]^2$

$$\partial_i \mathcal{E}^{ij} \;\to\; V(\boldsymbol{I},\boldsymbol{I}+\boldsymbol{x}) = \sum_{\boldsymbol{r},ij} d^{ij}_{\boldsymbol{I},\boldsymbol{I}+\boldsymbol{x},\boldsymbol{r}} L^{ij}(\boldsymbol{r})$$

  The short solid lines mark the non-zero $d^{ii}(\boldsymbol{I},\boldsymbol{I}+\boldsymbol{x},\boldsymbol{J})$ which are equal to 1.
  The filled circles mark the non-zero $d^{xy}(\boldsymbol{I},\boldsymbol{I}+\boldsymbol{x},\boldsymbol{I}+\frac{\boldsymbol{x}}{2}+\frac{\boldsymbol{y}}{2})$ *etc* , also equal to 1



- The low energy space has $V(\boldsymbol{I},\boldsymbol{I}+\boldsymbol{x})=0$ on every link.
- The lattice Hamiltonian $U\sum_{\boldsymbol{I},\boldsymbol{\mu}} V(\boldsymbol{I},\boldsymbol{I}+\boldsymbol{\mu})+H_{\text{low}}$, where H_{low} commutes with $V(\boldsymbol{I},\boldsymbol{I}+\boldsymbol{x}) \to$ "gauge symmetry", acts with low energy subspace, decoupling of the strongly fluctuating mode.



A scaler constraint to gap out $h = 0$ mode

A **scaler constraint**: large $U(\mathcal{E}^{ii})^2$-term makes $\mathcal{E}^{ii} = 0$.
$\mathcal{E}^{ii}$ generates gauge transformations (strong fluctuating mode)

$$\delta a_{ij} = \{a_{ij}(\boldsymbol{x}), \int \mathrm{d}^3\boldsymbol{y} f(\boldsymbol{y})\mathcal{E}^{ii}(\boldsymbol{y})\}_{PB} = \delta_{ij} f(\boldsymbol{x})$$

The gauge invariant combinations for both $a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i$ and $a_{ij} \to a_{ij} + \delta_{ij} f$ are   Gu Wen arXiv:0907.1203; Xu Horava arXiv:1003.0009

$$F^i = \partial_j R^{ji} = \partial_j \epsilon^{imk} \epsilon^{jln} \partial_m \partial_l a_{nk}, \quad \mathcal{E}^{ij}$$

$\to$ conitnuum Lagrangian (Can also be put on lattice)

$$\mathcal{L} = \mathcal{E}^{ij}\partial_0 a_{ij} \overbrace{-\alpha(\mathcal{E}^{ij})^2 - \beta(\mathcal{E}^{ii})^2 - \gamma(F^i)^2}^{\text{gauge invariant terms}} \overbrace{-U(\partial_i\mathcal{E}^{ij})^2 - U(\mathcal{E}^{ii})^2}^{\partial_i\mathcal{E}^{ij}=0,\ \mathcal{E}^{ii}=0 \text{ constraints}}$$

Only $h = \pm 2$ modes with $\omega \sim k^3$ $\to$ Soft gravitons

- But the scaler charge $\mathcal{E}^{ii} = q\delta(\boldsymbol{x})$ is not topological (can be created by local operators), and is not a source of the gravitational field. **There is no scalar mass**

Another scaler constraint to gap out $h = 0$ mode

Large $U(R^{ii})^2$-term to make $R^{ii} = (\delta_{ij}\partial^2 - \partial_i\partial_j)a_{ij} = 0$.
R^{ii} ganerates gauge transformation (strongly fluctuating mode)

$$\mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j)f, \quad \text{(and } \overbrace{a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i}^{\text{from the vector constraint}}\text{)}$$

The corresponding gauge invariant combination is given by

$$C^i_j = \epsilon^{imn}\partial_m \left(\mathcal{E}^{nj} - \frac{1}{2}\delta_{nj}\mathcal{E}^{ll} \right)$$

$\to$ Lagrangian density

Gu Wen arXiv:0907.1203

$$\mathcal{L} = \mathcal{E}^{ij}\partial_0 a_{ij} \overbrace{-\frac{J}{2}C^i_j C^i_j - \frac{g}{2}R^{ij}R^{ij}}^{\text{gauge invariant}} \overbrace{-U(\partial_i\mathcal{E}^{ij})^2 - U(R^{ii})^2}^{\partial_i\mathcal{E}^{ij}=0,\ R^{ii}=0 \text{ constraints}}$$

Only $h = \pm 2$ modes with $\omega \sim k^3 \to$ Soft gravitons (linearised Lifshitz gravity). **Quantized** topological scaler charge $(\delta_{ij}\partial^2 - \partial_i\partial_j)a_{ij} = M\delta(\boldsymbol{x}) \to$ Quantized mass produces gravity.

The scaler and vector constraints commute

- $\{R^{ii}(\boldsymbol{x}), \partial_j \mathcal{E}^{jk}(\boldsymbol{y})\}_{PB} = 0$.
  The two constrains act independently and consistently.

- The strong fluctuations of $\mathcal{E}^{ij}$ from the scaler constraint

$$R^{ii} = 0 \quad \to \quad \delta\mathcal{E}^{ij} = (\delta_{ij}\partial^2 - \partial_i\partial_j)f$$

is consistent with the vector constrain $\partial_i \mathcal{E}^{ij} = 0$

Put the second field theory on lattice

Gu Wen arXiv:0907.1203

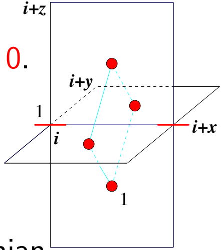
$$\mathcal{L} = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}C^i_j C^i_j - \frac{g}{2}R^{ij}R^{ij} \overbrace{-U(\partial_i\mathcal{E}^{ij})^2 - U(R^{ii})^2}^{\partial_i\mathcal{E}^{ij}=0,\ R^{ii}=0 \text{ constraints}}$$

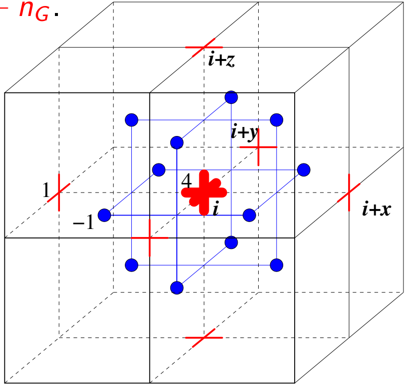
To impliment $R^{ii} = 0$ via energy penalty $\delta H = U(R^{ii})^2$, we need to discretize $\theta^{ij} = \frac{2\pi}{n_G} \times \text{int.}$. So $L^{ij} \sim L^{ij} + n_G$.

$V(\boldsymbol{I}, \boldsymbol{I}+\boldsymbol{x}) = \mathrm{e}^{\mathrm{i}\sum_{\boldsymbol{r}} d^{ij}_{\boldsymbol{I},\boldsymbol{I}+\boldsymbol{x},\boldsymbol{r}} \frac{2\pi}{n_G} L^{ij}(\boldsymbol{r})}$

$S(\boldsymbol{I}) = \mathrm{e}^{\mathrm{i}\sum_{\boldsymbol{r}} c^{ij}_{\boldsymbol{I},\boldsymbol{r}}\theta_{ij}(\boldsymbol{r})}$,

- We can check $[V(\boldsymbol{I}, \boldsymbol{I}+\boldsymbol{x}), S(\boldsymbol{J})] = 0$.
- Constructed H_{low} for $\frac{J}{2}C^i_j C^i_j + \frac{g}{2}R^{ij}R^{ij}$ that commute with $V(\boldsymbol{I}, \boldsymbol{I}+\boldsymbol{x}), S(\boldsymbol{J})$.
- Total lattice Hamiltonian





$$H_{\text{low}} - U\sum[V(\boldsymbol{I}, \boldsymbol{I}+\boldsymbol{\mu}) + V^\dagger(\boldsymbol{I}, \boldsymbol{I}+\boldsymbol{\mu})] - U\sum[S(\boldsymbol{I}) + S^\dagger(\boldsymbol{I})]$$

Low energy dynamics of the lattice model

The constraint Hilbert space $V(\boldsymbol{I}, \boldsymbol{I}+\boldsymbol{x}) = 1,\ S(\boldsymbol{J}) = 1$ still have many states
$\to$ Low energy dynamics of the lattice model:

There are semi-classical $h = \pm 2$ modes (within the constraint Hilbert space).
There are strong fluctuating quantum $h = 0, 0, \pm 1$ modes (outside the constraint Hilbert space, gapped).

They decouple completely at lattice level.

Linearized Einstein gravity

Same constraints, but different Hamiltonian ($\omega \propto k^3 \to \omega \propto k$):

$$\mathcal{L} = \mathcal{E}^{ij}\partial_0 a_{ij} \overbrace{- \frac{J}{2}[\mathcal{E}^{ij}\mathcal{E}^{ij} - \frac{1}{2}(\mathcal{E}^{ii})^2] - \frac{g}{2} a_{ij} R^{ij}}^{\text{quasi-gauge-invariant}} \overbrace{- U(\partial_i \mathcal{E}^{ij})^2 - U(R^{ii})^2}^{\partial_i\mathcal{E}^{ij}=0,\ R^{ii}=0 \text{ constraints}}$$

- $\mathcal{E}^{ij}\mathcal{E}^{ij} - \frac{1}{2}(\mathcal{E}^{ii})^2$ is invariant under $\mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j)f$, provided that $\partial_i \mathcal{E}^{ij} = 0$
- $a_{ij}R^{ij}$ is invariant under $a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i$ up to a total derivative term.

Gu Wen arXiv:0907.1203

- Only $h = \pm 2$ modes with $\omega \sim k$ at low energies $\to$ **linearized Einstein gravity with** $a_{ij} \sim g^{ij} - \delta^{ij}$ (the fulctuation of metrics)
- We can put the above continuum theory on lattice, and **under quadraric approximation**, the low energy weakly fluctuating $h = \pm 2$ modes decouple from the gapped strongly fluctuating $h = 0, 0, \pm 1$ modes. **But they do not decouple beyond the quadratic approxmation.**

Summary
– attempts to get linearized quantum gravity

- We constructed a lattice model, that produces light waves at low energies that satisfy Maxwell equation (with helicity $h = \pm 1$ and dispersion $\omega_k \propto k$).
- We constructed a lattice model, that produces soft-gravity waves at low energies, with helicity $h = \pm 2$ and dispersion $\omega_k \propto k^3$
- We constructed a lattice model, that produces gravity waves at low energies, with helicity $h = \pm 2$ and dispersion $\omega_k \propto k$, under an **uncontralled approaximation**. We do not have a controlled approximation to obtain the low energy properties of the the lattice model.