Black Hole Information

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2021 April 8

Introduction

Black hole information is one of the greatest puzzles of theoretical physics from the 20th century that has persisted into the 21st century.

After Stephen Hawking discovered black hole evaporation in 1974, in 1976 he predicted that black hole formation and evaporation would cause a pure quantum state to change into a mixed state, effectively losing information from the universe.

In 1979 I questioned this conclusion, as many years later did many others, and in 2004 Hawking conceded that black hole evaporation does not lose information. However, a minority of gravitational theorists have not accepted Hawking's concession.

There do remain many puzzles about black hole information, such as how it gets out (if it indeed does), and whether there are firewalls at the surfaces of old black holes that would immediately destroy anything falling in (presumably not).

Black Hole Temperature and Entropy

In Planck units in which $\hbar = c = G = k = 4\pi\epsilon_0 = 1$, Hawking showed that a black hole of surface gravity κ and event horizon area A has temperature

$T = \frac{\kappa}{2\pi}$

and thermodynamic (Bekenstein-Hawking) entropy

$S_{BH} = \frac{A}{4}$.

For a static uncharged nonrotating black hole of mass M in vacuum asymptotically flat spacetime (Schwarzschild metric) in which the event horizon radius is $r_h = 2M$, the surface gravity is $\kappa = M/r_h^2 = 1/(4M)$, and the event horizon area is $A = 4\pi r_h^2 = 16\pi M^2$, the temperature is

$T = \frac{1}{8\pi M}$,

and the thermodynamic Bekenstein-Hawking entropy is

$S_{BH} = 4\pi M^2$.

Black Hole Evaporation Rates

D. N. Page, "Particle Emission Rates from a Black Hole: Massless Particles from an Uncharged, Nonrotating Hole," Phys. Rev. D **13**, 198 (1976).
D. N. Page, "Particle Emission Rates from a Black Hole. 2. Massless Particles from a Rotating Hole," Phys. Rev. D **14**, 3260 (1976).
D. N. Page, "Particle Emission Rates from a Black Hole. 3. Charged Leptons from a Nonrotating Hole," Phys. Rev. D **16**, 2402 (1977).

Photon and graviton emission from a Schwarzschild black hole:

- $dM/dt = -\alpha/M^2 \approx -0.000\,037\,474/M^2$.
- $d\tilde{S}_{\mathrm{BH}}/dt = -8\pi\alpha/M \approx -0.000\,941\,82/M$.
- $d\tilde{S}_{\mathrm{rad}}/dt \approx 0.001\,398\,4/M = -\beta d\tilde{S}_{\mathrm{BH}}/dt$.
- $\beta \equiv (d\tilde{S}_{\mathrm{rad}}/dt)/(-d\tilde{S}_{\mathrm{BH}}/dt) \approx 1.4847$.

Hawking's Argument for Information Loss

S. W. Hawking, "Breakdown of Predictability in Gravitational Collapse," Phys. Rev. D **14**, 2460 (1976), used quantum field theory in a classical dynamical black hole background to argue that information was lost into the absolute event horizon and could not get out, so that when the black hole evaporated away, the information was lost from the universe, resulting in the change from an initial pure quantum state to a mixed state of thermal Hawking radiation. This is certainly what one would get from local quantum field theory in a definite metric with an horizon out from which signals cannot escape (since they would have to travel faster than the speed of light, impossible in local quantum field theory), with the region behind the horizon collapsing into a spacetime singularity. One might have said the information is still inside the black hole, but if the black hole completely evaporates away, after it is gone the information would have completely disappeared from the universe.

My Objections to Hawking's Argument

D. N. Page, "Is Black Hole Evaporation Predictable?" Phys. Rev. Lett. **44**, 301 (1980), pointed out that Hawking's proposal violated CPT invariance, and that Hawking's "calculations have been made in the semiclassical approximation of a fixed background metric, which breaks down long before the final stages of evaporation," for example by the stochastic recoil of the black hole to the quantum fluctuations of the momentum of the Hawking radiation.

I listed 8 possible alternatives and suggested that it seemed most productive to pursue the most conservative possibility, a unitary S-matrix. I noted, "Hawking suggested that 'God not only plays dice, He sometimes throws the dice where they cannot be seen.' But it may be that 'if God throws dice where they cannot be seen, they cannot affect us.' ''

Turning the Tide

For several years little attention was given to black hole information. Relativists such as William Unruh and Robert Wald tended to support Hawking's position that information is lost, and particle physicists such as Edward Witten tended to support my position that information is not lost. Although he was not involved in the earliest days of the debate to cover them in his 2008 book, Leonard Susskind's book, *Black Hole War: My Battle with Stephen Hawking to Make the World Safe for Quantum Mechanics* is one entertaining perspective of the debate. I think it was mainly the AdS/CFT conjecture of Juan Maldacena that turned the tide toward the view that information is not lost. This conjecture was that a bulk gravitational theory in asymptotically anti-de Sitter (AdS) spacetime is dual to a conformal field theory (CFT) on the conformal boundary. Since the CFT is manifestly unitary, with no loss of information, so should be the bulk gravitational field.

Black Hole Firewalls

Interest in black hole information surged about eight years ago with A. Almheiri, D. Marolf, J. Polchinski and J. Sully, "Black Holes: Complementarity or Firewalls?," JHEP 1302 (2013) 062. They gave a provocative argument that suggested that an "infalling observer burns up at the horizon" of a sufficiently old black hole, so that the horizon would become what they called a "firewall."

Unitary evolution suggests that at late times the Hawking radiation is maximally entangled with the remaining black hole and neighborhood (including the modes just outside the horizon).
This further suggests that what is just outside cannot be significantly entangled with what is just inside.
But without this latter entanglement, an observer falling into the black hole should be burned up by high-energy radiation.

Time Dependence of Hawking Radiation Entropy

One cannot externally observe entanglement across the horizon. However, it should eventually be transferred to the radiation. Therefore, we would like to know the retarded time dependence of the von Neumann entropy of the Hawking radiation.

A. Strominger, "Five Problems in Quantum Gravity," Nucl. Phys. Proc. Suppl. **192-193**, 119 (2009) [arXiv:0906.1313 [hep-th]], emphasized this question and outlined five candidate answers:

- bad question
- information destruction
- long-lived remnant
- non-local remnant
- maximal information return

I shall assume without proof maximal information return.

Assumptions

- Unitary evolution (no loss of information)
- Initial approximately pure state (e.g., $S_{\mathrm{vN}}(0) \sim S(\mathrm{star}) \sim 10^{57} \ll \tilde{S}_{\mathrm{BH}}(0) \sim 10^{77}$)
- Nearly maximal entanglement between hole and radiation
- Complete evaporation into just final Hawking radiation
- Nonrotating uncharged (Schwarzschild) black hole
- Initial black hole mass large, $M_0 > M_\odot$
- Massless photons and gravitons; other particles $m > 10^{-10}$ eV
- Therefore, essentially just photons and gravitons emitted

Arguments for Nearly Maximal Entanglement

D. N. Page, "Average Entropy of a Subsystem," Phys. Rev. Lett. **71**, 1291 (1993) [gr-qc/9305007].
"There is less than one-half unit of information, on average, in the smaller subsystem of a total system in a random pure state."

D. N. Page, "Information in Black Hole Radiation," Phys. Rev. Lett. **71**, 3743 (1993) [hep-th/9306083].
"If all the information going into gravitational collapse escapes gradually from the apparent black hole, it would likely come at initially such a slow rate or be so spread out . . . that it could never be found or excluded by a perturbative analysis."

Y. Sekino and L. Susskind, "Fast Scramblers," JHEP **0810**, 065 (2008) [arXiv:0808.2096 [hep-th]], conjecture:

- The most rapid scramblers take a time logarithmic in the number of degrees of freedom.
- Black holes are the fastest scramblers in nature.

These conjectures support my results using an average over all pure states of the total system of black hole plus radiation.

von Neumann Entropies of the Radiation and Black Hole

Take the semiclassical entropies $\tilde{S}_{\mathrm{rad}}(t)$ and $\tilde{S}_{\mathrm{BH}}(t)$ to be approximate upper bounds on the von Neumann entropies of the corresponding subsystems with the same macroscopic parameters.

Therefore, the von Neumann entropy of the Hawking radiation, $S_{\mathrm{vN}}(t)$, which assuming a pure initial state and unitarity is the same as the von Neumann entropy of the black hole, should not be greater than either $\tilde{S}_{\mathrm{rad}}(t)$ or $\tilde{S}_{\mathrm{BH}}(t)$.

Take my 1993 results as suggestions for the *Conjectured Anorexic Triangle Hypothesis (CATH)*:

Entropy triangular inequalities are usually nearly saturated.

This leads to the assumption of nearly maximal entanglement between hole and radiation, so $S_{\mathrm{vN}}(t)$ should be near the minimum of $\tilde{S}_{\mathrm{rad}}(t)$ and $\tilde{S}_{\mathrm{BH}}(t)$.

Time of Maximum von Neumann Entropy

Since the semiclassical entropy $\tilde{S}_{\rm rad}(t)$ is monotonically increasing with time, and since the semiclassical entropy $\tilde{S}_{\rm BH}(t)$ is monotonically decreasing with time, the maximum von Neumann entropy is at the crossover point, at time

$$t_* = \epsilon t_{\rm decay} \approx 0.5381\, t_{\rm decay} \approx 4786\, M_0^3 \approx 6.236 \times 10^{66} (M_0/M_\odot)^3 {\rm yr},$$

with $\beta \equiv (d\tilde{S}_{\rm rad}/dt)/(-d\tilde{S}_{\rm BH}/dt) \approx 1.4847$ and hence

$$\epsilon \equiv 1 - [\beta/(\beta+1)]^{3/2} \approx 0.5381,$$

at which time the mass of the black hole is

$$M_* = [\beta/(\beta+1)]^{1/2} M_0 \approx 0.7730\, M_0,$$

and its semiclassical Bekenstein-Hawking entropy $4\pi M^2$ is

$$\tilde{S}_{{\rm BH}*} = [\beta/(\beta+1)]\tilde{S}_{\rm BH}(0) \approx 0.5975\, \tilde{S}_{\rm BH}(0).$$

Maximum von Neumann Entropy of the Hawking Radiation

At the time t_* when $\tilde{S}_{\mathrm{rad}}(t) = \tilde{S}_{\mathrm{BH}}(t)$, the von Neumann entropy of the radiation and of the black hole is maximized and has the value

$$S_* \equiv S_{\mathrm{vN}}(t_*) = \tilde{S}_{\mathrm{rad}}(t_*) = \tilde{S}_{\mathrm{BH}}(t_*) = \left(\frac{\beta}{\beta+1}\right) 4\pi M_0^2 \approx 0.5975\, \tilde{S}_{\mathrm{BH}}(0)$$
$$= 0.5975(4\pi M_0^2) \approx 7.509 M_0^2 \approx 6.268 \times 10^{76} (M_0/M_\odot)^2.$$

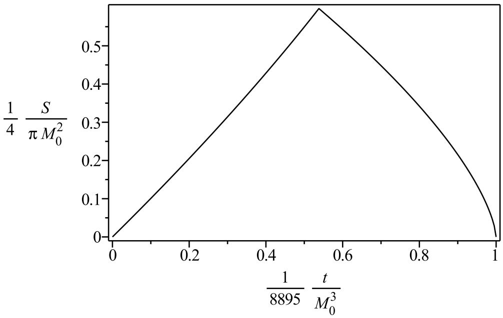
Note that this maximum of the von Neumann entropy is about 19.5% greater than half the original semiclassical Bekenstein-Hawking entropy of the black hole. The time t_* for the maximum von Neumann entropy is about 0.8324 times the time $t_{1/2} = (1 - 2^{-3/2}) t_{\mathrm{decay}} \approx 0.6464\, t_{\mathrm{decay}} \approx 1.201\, t_*$ for the black hole to lose half its area and semiclassical Bekenstein-Hawking entropy.

Time Dependence of the Entropy of the Hawking Radiation

The von Neumann entropy of the Hawking radiation $S_{\mathrm{vN}}(t)$ from a large nonrotating uncharged black hole is very nearly the semiclassical radiation entropy $\tilde{S}_{\mathrm{rad}}(t)$ for $t < t_*$ and is very nearly the Bekenstein-Hawking semiclassical black hole entropy $\tilde{S}_{\mathrm{BH}}(t)$ for $t > t_*$, or, using the Heaviside step function $\theta(x)$,

$$\begin{aligned}
S_{\mathrm{vN}}(t) &\approx 4\pi\beta M_0^2 \left[1 - \left(1 - \frac{t}{t_{\mathrm{decay}}}\right)^{2/3}\right] \theta(t_* - t) \\
&+ 4\pi M_0^2 \left(1 - \frac{t}{t_{\mathrm{decay}}}\right)^{2/3} \theta(t - t_*) \\
&\approx 4\pi(1.4847) M_0^2 \left[1 - \left(1 - \frac{t}{8895 M_0^3}\right)^{2/3}\right] \theta(4786 M_0^3 - t) \\
&+ 4\pi M_0^2 \left(1 - \frac{t}{8895 M_0^3}\right)^{2/3} \theta(t - 4786 M_0^3).
\end{aligned}$$

Plot of Hawking Radiation Entropy vs. Time



Arguments Against Firewalls

A firewall at the surface of a black hole would be, as seen by an infalling observer, a shock wave giving a sudden huge increase in the energy density and curvature. Although this is not directly ruled out by general relativity and is not strictly speaking a violation of the equivalence principle, it would be very surprising. Similarly it would be consistent with general relativity and with all our past observations for a strong shock wave to be coming to destroy us without warning within the next few minutes, but we do not expect the state of the universe to include such a surprising shock wave. (Well, actually we could be destroyed so fast there would be no time to feel any surprise.)

So I think most gravitational physicists are highly sceptical about the existence of firewalls, at least for most black holes in the actual state of the universe, but there is the challenge of finding good arguments against them.

Here I wish to summarize one general objection to the argument for firewalls and three arguments I have originated against firewalls.

General Objection to the Firewall Argument

The firewall argument asserts that the following four assumptions cannot all be true:

(1) Black hole evaporation is unitary (no loss of information).
(2) Black holes have a number of states $\sim \exp(A/4)$.
(3) Local quantum field theory applies outside the horizon.
(4) There are no firewalls (drama at or near the horizon).

Then the firewall argument is that Assumption (4) is the most plausible to give up, thus leading to the conclusion of firewalls.

However, almost certainly quantum gravity is nonlocal, so Assumption (3) appears more plausible to give up. In particular, the energy and angular momentum of a quantum state is recorded in the gravitational field at infinity, so that if the energy and angular momentum eigenstates are nondegenerate, one cannot change the state at any location without changing it at infinity, unlike the case with local quantum field theories.

Extreme Cosmic Censorship Argument Against Firewalls

D. N. Page, "Excluding Black Hole Firewalls with Extreme Cosmic Censorship," JCAP **1406**, 051 (2014).

Firewalls are shock waves moving along the horizon, with intensity exponentially decreasing to the future and exponentially growing to the past, so that in the past they would effectively become singular. Therefore, firewalls would be excluded if there are no singularities to the past, which would be the case if the **Extreme Cosmic Censorship Hypothesis** were true:

The universe is entirely nonsingular (except for singularities inside black holes which go away when the black holes evaporate).

Without such an exclusion, the number of quantum states for a black hole could be unbounded, greatly exceeding the Bekenstein-Hawking $\exp(A/4)$.

I suggest that it is impossible to form firewall states from nonsingular initial conditions (or from sending in regular data from a boundary of AdS in AdS/CFT).

Naked Firewalls Argument

P. Chen, Y. C. Ong, D. N. Page, M. Sasaki and D. H. Yeom, "Naked Black Hole Firewalls," Phys. Rev. Lett. **116**, no. 16, 161304 (2016).

If a firewall exists at a location determined by its causal past that is a close estimate of where the event horizon would be expected to be, future quantum fluctuations can change the location of the teleological event horizon in such a way that the firewall is outside the event horizon, making the firewall naked, visible from the outside.

This is analogous to a *reductio ad absurdum* argument, making a firewall appear even less plausible than it otherwise might be thought to be.

Argument from a Qubit Transfer Model

K. Osuga and D. N. Page, "Qubit Transport Model for Unitary Black Hole Evaporation without Firewalls," Phys. Rev. D**97**, no. 6, 066023 (2018).

To combat the impression that avoiding a firewall by invoking the nonlocality of quantum gravity would require unobserved nonlocal behavior, my former Ph.D. student Kento Osuga and I proposed a qubit transport model. In this, local matter quantum fields interact with nonlocal quantum gravity degrees of freedom so that the matter fields start out just outside the horizon strongly correlated with the matter fields inside the horizon in the right way to avoid a firewall there. But then as the matter field modes propagate outward through the nonlocal gravitational field, they lose their entanglement with the matter field modes just inside the horizon and instead pick up entanglement with the quantum gravitational degrees of freedom of the black hole. Therefore, when the matter field modes get a long way from the black hole and become the Hawking radiation, they are then entangled with the black hole.

Net Effect of the Qubit Transfer Model

The net effect of the qubit transfer model is that the emission of one outgoing radiation qubit gives the transfer of the information in one black hole qubit to one Hawking radiation qubit. But rather than simply saying that this transfer is nonlocal, from the inside of the black hole to the outside, we are saying that the black hole qubit itself is always nonlocal, and that the outgoing radiation qubit picks up the information in the black hole qubit locally, as it travels outward through the nonlocal gravitational field of the black hole. Therefore, in this picture in which we have separated the quantum field theory qubits of the radiation from the black hole qubits of the gravitational field, we do not need to require any nonlocality for the quantum field theory modes, but only for the gravitational field. In this way the nonlocality of quantum gravity might not have much observable effect on experiments in the laboratory focussing mainly on local quantum field theory modes.

Giddings' Physical Conditions

S. B. Giddings, "Black Holes, Quantum Information, and Unitary Evolution," Phys. Rev. D **85**, 124063 (2012), has proposed a list of physical constraints on models of black hole evaporation:

(i) *Evolution is unitary.* Our model is explicitly unitary.

(ii) *Energy is conserved.* Our model is consistent with this.

(iii) *The evolution should appear innocuous to an infalling observer crossing the horizon; in this sense the horizon is preserved.* We explicitly assume that the radiation modes are in their vacuum states when they are near the horizon, so there is no firewall.

(iv) *Information escapes the black hole at a rate* $dS/dt \sim 1/R$*.* If one radiation qubit comes out during a time $\sim R$, $dS/dt \sim 1/R$.

(v) *The coarse-grained features of the outgoing radiation are still well-approximated as thermal.* Scrambling of the black hole qubits would be expected to achieve this.

(vi) *Evolution of a system* $\mathcal{H}_A \otimes \mathcal{H}_B$ *saturates the subadditivity inequality* $S_A + S_B \geq S_{AB}$*.* This criterion would only be expected to apply for $n_A + n_B < n/2$, but then it should be true for our model.

Summary of the Qubit Transfer Model

Kento Osuga and I have given a toy qubit model for black hole evaporation that is unitary and does not have firewalls. It does have nonlocal degrees of freedom for the black hole gravitational field, but the quantum field theory radiation modes interact purely locally with the gravitational field, so in some sense the nonlocality is confined to the gravitational sector. The model satisfies all of the constraints that Giddings has proposed, though further details would need to be added to give the detailed spectrum of Hawking radiation. The model is in many ways *ad hoc*, such as in the details of the qubit transfer, so one would like a more realistic interaction of the radiation modes with the gravitational field than the simple model sketched here. One would also like to extend the model to include possible ingoing radiation from outside the black hole.

Recent Work Deriving the Curve for von Neumann Entropy

Recently there has been a lot of work giving derivations of the curve I calculated from a simpler toy model, such as

G. Penington, "Entanglement wedge reconstruction and the information paradox," arXiv:1905.08255 [hep-th] (2019).

A. Almheiri, N. Engelhardt, D. Marolf, and H. Maxfield, "The entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole," arXiv:1905.08762 [hep-th] (2019).

A. Almheiri, R. Mahajan, J. Maldacena, and Y. Zhao, "The Page curve of Hawking radiation from semiclassical geometry," Journal of High Energy Physics, 2020, 149 (2020). https://doi.org/10.1007/JHEP03(2020)149.

A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, and A. Tajdini, "The entropy of Hawking radiation," arXiv:2006.06872 [hep-th] (2020).

I don't understand this well enough to comment intelligently, but it looks very exciting to see such progress toward understanding the black hole information puzzle and favoring unitary evolution.

Conclusions

Whether or not black hole formation and evaporation is a unitary quantum process that preserves information is still a somewhat open question, though the majority of gravitational theorists now seem to believe that information is not lost, and very recent work appears to add considerable support to this view.

Assuming unitarity, the Bekenstein-Hawking density of states, and local quantum field theory outside the horizon appeared for a time to lead to the unpleasant prediction of firewalls at the surface of certain black holes, a prediction that would seem to be even more unpleasant if quantum fluctuations in the event horizon can make the firewall naked. How to avoid this prediction without grossly giving up local quantum field theory is not completely known, but recent work (including that by at least two of the AMPS authors) suggests that there may be no firewalls after all. Perhaps Extreme Cosmic Censorship and/or qubit transfer of information with nonlocal gravitational degrees of freedom of the black hole can also help in the understanding of the black hole information puzzle.