Small scale signatures of non-trivial inflationary and post-inflationary dynamics

L. Sriramkumar

Centre for Strings, Gravitation and Cosmology, Department of Physics,
Indian Institute of Technology Madras, Chennai, India

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Plan of the talk

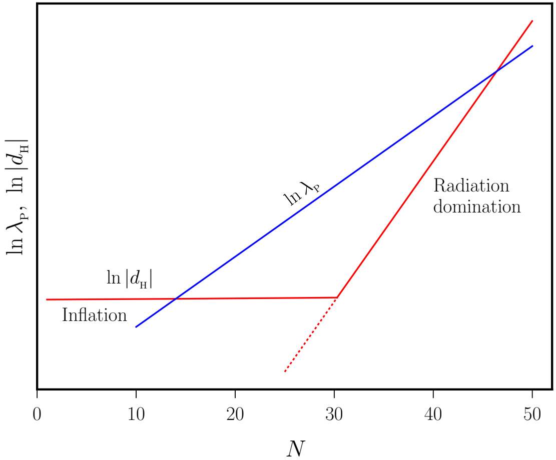
1. Constraints on inflation from Planck
2. Enhancing power on small scales
3. Implications for PBH formation and secondary GWs
4. Non-Gaussianities generated in ultra slow roll and punctuated inflation
5. Can enhanced power be generated from squeezed initial states?
6. Observational signatures of the epoch of reheating
7. Summary

This talk is based on. . .

- M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, *Generating primordial features at large scales in two field models of inflation*, JCAP **08**, 025 (2020) [arXiv:2004.00672 [astro-ph.CO]].
- M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, *Generating PBHs and small-scale GWs in two-field models of inflation* JCAP **08**, 001 (2020) [arXiv:2005.02895 [astro-ph.CO]].
- H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *PBHs and secondary GWs from ultra slow roll and punctuated inflation*, Phys. Rev. D **103**, 083510 (2021) [arXiv:2008.12202 [astro-ph.CO]].
- H. V. Ragavendra, L. Sriramkumar and J. Silk, *Could PBHs and secondary GWs have originated from squeezed initial states?*, JCAP **05**, 010 (2021) [arXiv:2011.09938 [astro-ph.CO]].
- Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, *Decoding the phases of early and late time reheating through imprints on primordial gravitational waves*, arXiv:2105.09242 [astro-ph.CO], to appear in Phys. Rev. D.

Plan of the talk

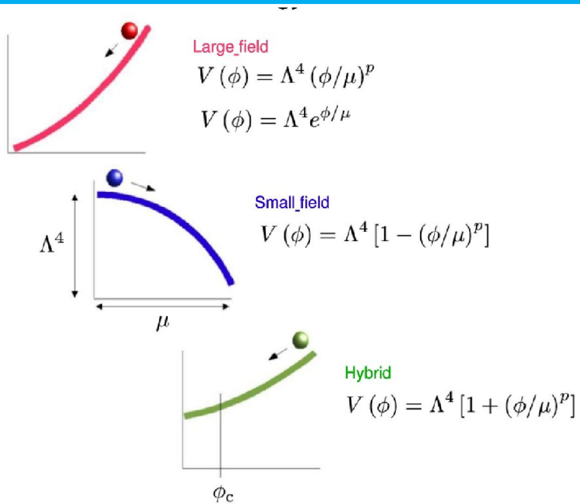
Bringing the modes inside the Hubble radius through inflation



The physical wavelength $\lambda_{\rm P} \propto a$ (in blue) and the Hubble radius $d_{\rm H} = H^{-1}$ (in red) in the inflationary scenario[1]. The scale factor is expressed in terms of e-folds N as $a(N) \propto {\rm e}^N$.

[1] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

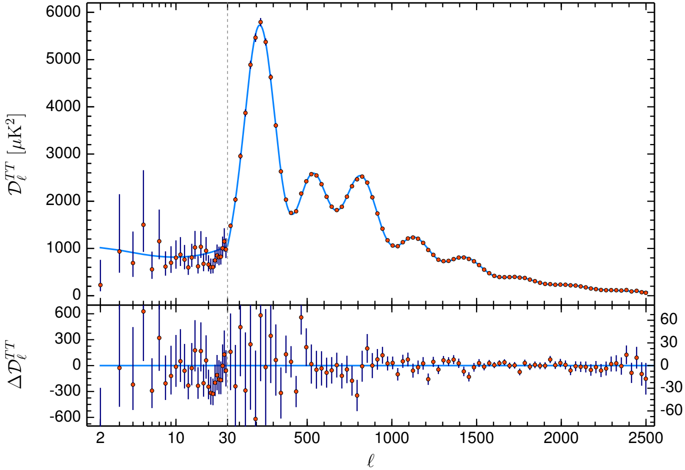
A variety of inflationary potentials to choose from



A variety of scalar field potentials have been considered to drive inflation[2]. Often, these potentials are classified as small field, large field and hybrid models.

[2] Image from W. Kinney, astro-ph/0301448.

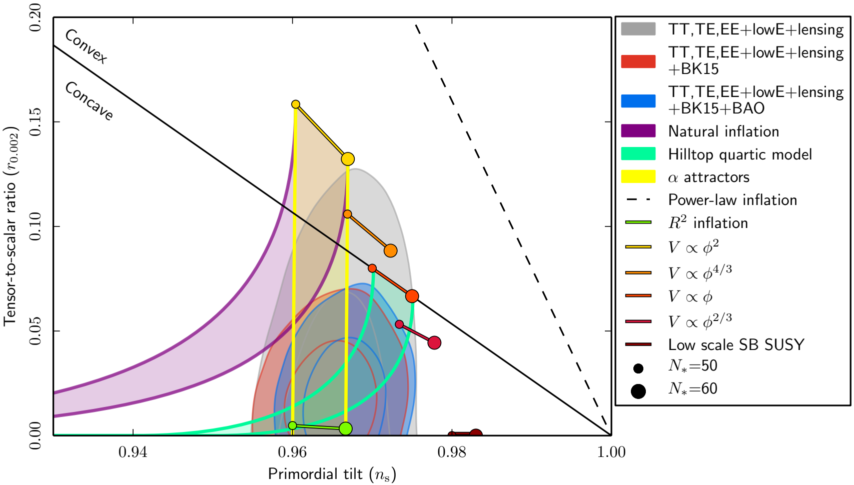
CMB angular power spectrum from Planck



The CMB TT angular power spectrum from the Planck 2018 data (red dots with error bars) and the best fit ΛCDM model with a power law primordial spectrum (solid blue curve)[3].

[3] Planck Collaboration (N. Aghanim *et al.*), Astron. Astrophys. **641**, A6 (2020).

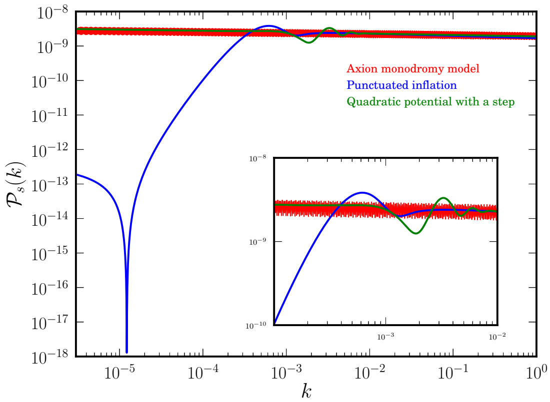
Performance of inflationary models in the n_s-r plane

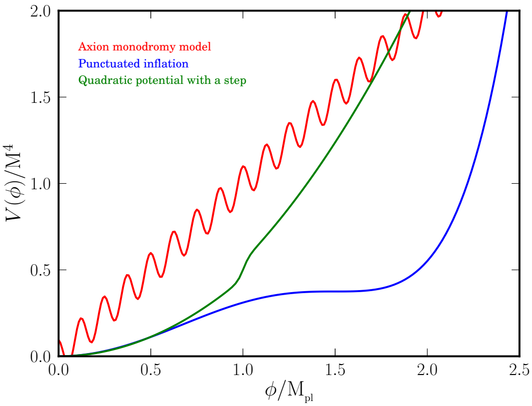


Joint constraints on n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models[4].

[4] Planck Collaboration (Y. Akrami *et al.*), Astron. Astrophys. **641**, A10 (2020).

Spectra leading to an improved fit to the CMB data



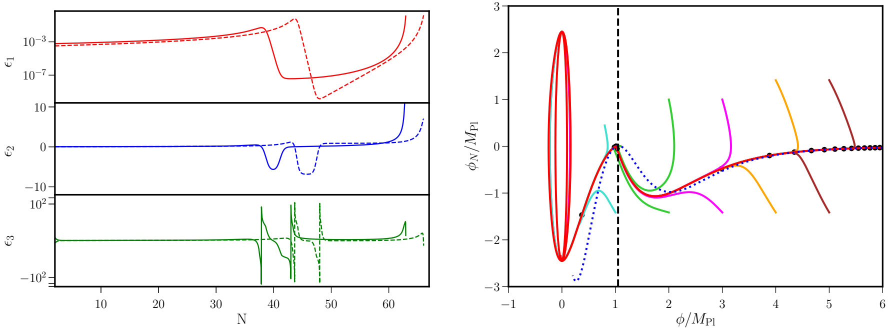


The scalar power spectra (on the left) arising in different inflationary models (on the right) that lead to a better fit to the CMB data than the conventional power law spectrum[5].

[5] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009); D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **10**, 008 (2010); M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, Phys. Rev. D **87**, 083526 (2013).

Plan of the talk

Potentials admitting ultra slow roll inflation



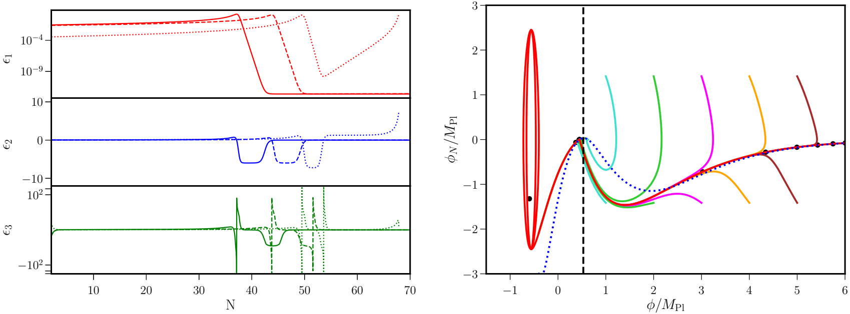
Potentials leading to ultra slow roll inflation (with $x = \phi/v$, v being a constant)[6]:

$$\mathrm{USR1}: V(\phi) = V_0\, \frac{6\,x^2 - 4\,\alpha\,x^3 + 3\,x^4}{(1+\beta\,x^2)^2},$$

$$\mathrm{USR2}: V(\phi) = V_0 \left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right) + A\,\sin\left[\frac{\tanh\left[\phi/\left(\sqrt{6}\,M_{_{\rm Pl}}\right)\right]}{f_\phi}\right]\right\}^2.$$

[6] J. Garcia-Bellido and E. R. Morales, Phys. Dark Univ. **18**, 47 (2017); I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

Potentials permitting punctuated inflation



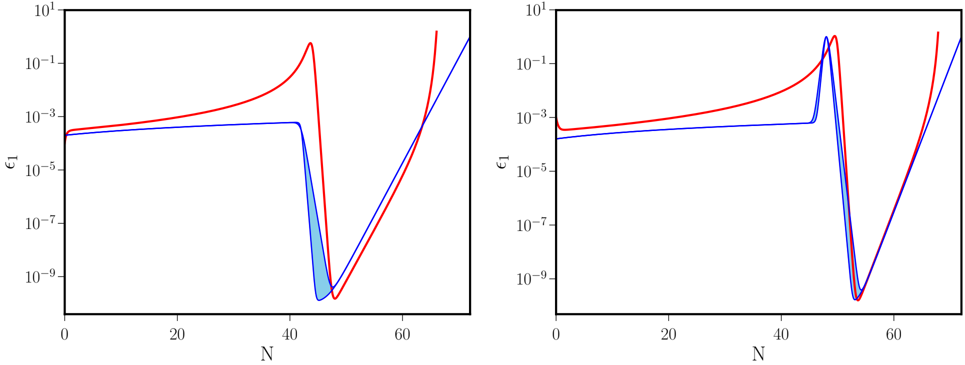
Potentials admitting punctuated inflation[7]:

$$\mathrm{PI1}: V(\phi) = V_0\,\left(1 + B\,\phi^4\right), \quad \mathrm{PI2}: V(\phi) = \frac{m^2}{2}\,\phi^2 - \frac{2\,m^2}{3\,\phi_0}\,\phi^3 + \frac{m^2}{4\,\phi_0^2}\,\phi^4,$$

$$\mathrm{PI3}: V(\phi) = V_0\,\left[c_0 + c_1\,\tanh\left(\frac{\phi}{\sqrt{6\,\alpha}\,M_{_{\rm Pl}}}\right) + c_2\,\tanh^2\left(\frac{\phi}{\sqrt{6\,\alpha}\,M_{_{\rm Pl}}}\right) + c_3\,\tanh^3\left(\frac{\phi}{\sqrt{6\,\alpha}\,M_{_{\rm Pl}}}\right)\right]^2.$$

[7] D. Roberts, A. R. Liddle and D. H. Lyth, Phys. Rev. D **51**, 4122 (1995);
R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009);
I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

Constructing scenarios of ultra slow roll and punctuated inflation



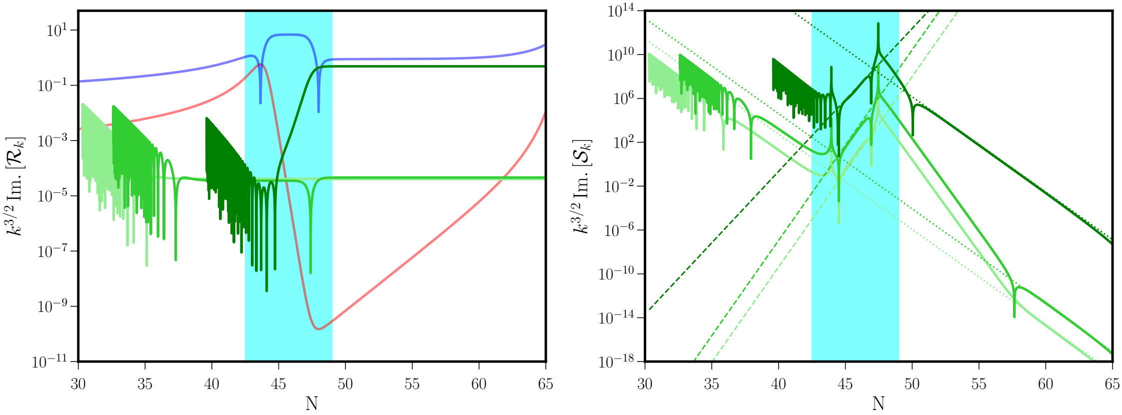
Behavior of the first slow roll parameter $\epsilon_1(N)$ leading to ultra slow and punctuated inflation[8]:

$$\mathrm{RSI}:\ \epsilon_1^{\mathrm{I}}(N) = [\epsilon_{1a}\ (1+\epsilon_{2a}\ N)]\left[1-\tanh\left(\frac{N-N_1}{\Delta N_1}\right)\right]+\epsilon_{1b}+\exp\left(\frac{N-N_2}{\Delta N_2}\right),$$

$$\mathrm{RSII}:\ \epsilon_1^{\mathrm{II}}(N) = \epsilon_1^{\mathrm{I}}(N)+\cosh^{-2}\left(\frac{N-N_1}{\Delta N_1}\right).$$

[8] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

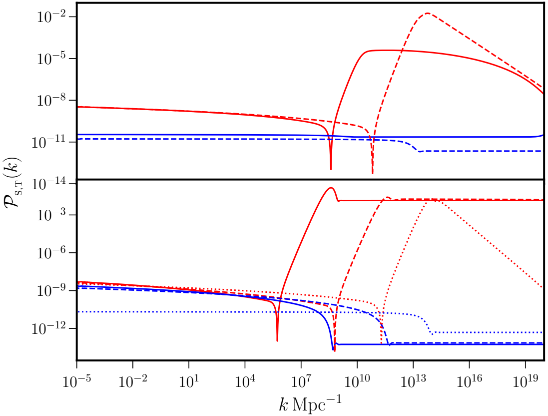
Role of the intrinsic entropy perturbation

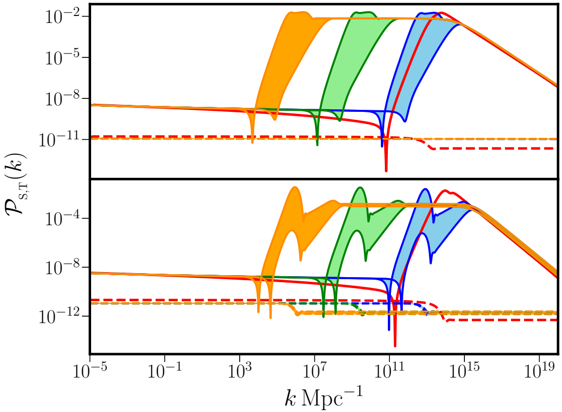


The evolution of the amplitudes of the imaginary parts of the curvature perturbation $\mathcal{R}_k$ (on the left) and the corresponding intrinsic entropy perturbation $\mathcal{S}_k$ (on the right) for the wave numbers $k = (10^{10}, 10^{11}, 10^{14})\,\mathrm{Mpc}^{-1}$ (in light, lime and dark green) in USR2[9]

[9] S. M. Leach, M. Sasaki, D. Wands, and A. R. Liddle, Phys. Rev. D **64**, 023512 (2001); R. K. Jain, P. Chingangbam, and L. Sriramkumar, JCAP **10** 003 (2007).

Power spectra in the inflationary models and reconstructed scenarios





The scalar and the tensor power spectra arising in the various inflationary models (in red and blue on the left) and the reconstructed scenarios (in blue, green and orange, on the right)[10].

[10] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

The two field model of interest

It has been noticed that two scalar fields ϕ and χ governed by the following action:

$$S[\phi, \chi] = \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{2}\,\partial^\mu\phi\,\partial_\mu\phi - \frac{f(\phi)}{2}\,\partial^\mu\chi\,\partial_\mu - V(\phi, \chi)\right]$$

described by the potential

$$V(\phi,\,\chi) = V_0\,\frac{\phi^2}{\phi_0^2 + \phi^2} + \frac{m_\chi^2}{2}\,\chi^2$$

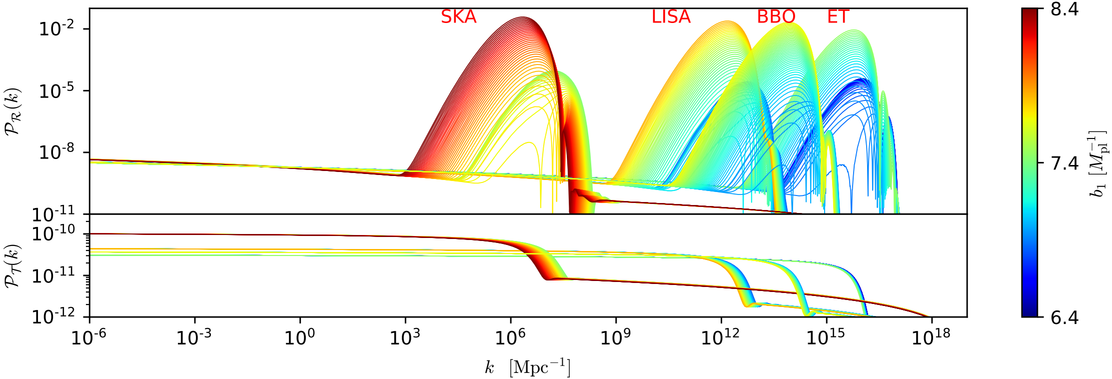
and the non-canonical coupling functions

$$f_1(\phi) = \mathrm{e}^{2\,b_1\,\phi} \quad \text{or} \quad f_2(\phi) = \mathrm{e}^{2\,b_2\,\phi^2}$$

can lead to features in the scalar power spectrum[11].

[11] M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08** 025 (2020).

Enhanced power on small scales in two field models



The scalar (on top) and the tensor (at the bottom) power spectra evaluated at the end of inflation have been plotted for a few different sets of initial conditions for the fields and a range of values of the parameter b_1[12].

[12] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

Plan of the talk

Formation of PBHs

The fraction of PBHs, say, $f_{_{\mathrm{PBH}}}(M)$, contributing to the dark matter density *today* can be expressed as[13]

$$f_{_{\mathrm{PBH}}}(M) = \left(\frac{\gamma_*}{0.2}\right)^{3/2} \left(\frac{\beta(M)}{1.46 \times 10^{-8}}\right) \left(\frac{g_{*,k}}{g_{*,\mathrm{eq}}}\right)^{-1/4} \left(\frac{M}{M_\odot}\right)^{-1/2},$$

where $g_{*,k}$ and $g_{*,\mathrm{eq}}$ are the number of effective relativistic degrees of freedom at the time of formation of the PBHs and at matter-radiation equality.

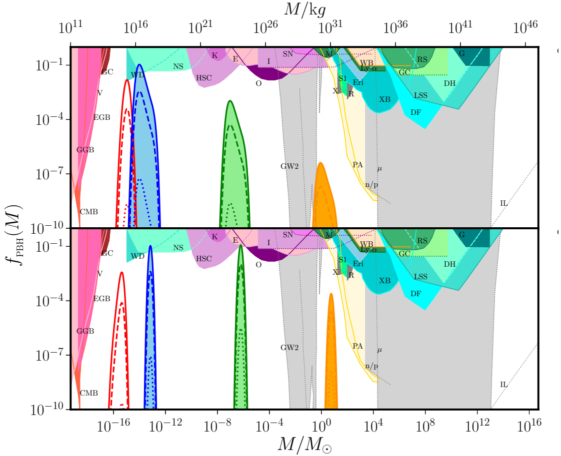
The quantity $\beta(M)$ denotes the fraction of the density fluctuations that collapse to form PBHs and is described by the integral

$$\beta(M) = \int_{\delta_\mathrm{c}}^{1} \mathrm{d}\delta\, P(\delta) = \frac{1}{\sqrt{2\,\pi\,\sigma^2}} \int_{\delta_\mathrm{c}}^{1} \mathrm{d}\delta\, \exp\left(-\frac{\delta^2}{2\,\sigma^2(M)}\right) \simeq \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\delta_\mathrm{c}}{\sqrt{2\,\sigma^2(M)}}\right)\right],$$

where $\mathrm{erf}(z)$ denotes the error function and δ_c represents the critical density contrast.

[13] See, for instance, B. Carr and J. Silk, Mon. Not. Roy. Astron. Soc. **478**, 3756 (2018); M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama, Class. Quant. Grav. **35**, 063001 (2018).

$f_{\rm PBH}(M)$ in ultra slow roll and punctuated inflation



The fraction of PBHs contributing to the dark matter density today $f_{\rm PBH}(M)$ has been plotted for the various models and scenarios of interest, viz. USR2 and RS1 (on top, in red and blue) and PI3 and RS2 (at the bottom, in red and blue)[14].

[14] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

GWs sourced by second order scalar perturbations

At the second order in the perturbations, one finds that the equation governing the tensor modes, say, $h_{\boldsymbol{k}}$, can be written as[15]

$$h_{\boldsymbol{k}}^{\lambda\,\prime\prime} + 2\,\mathcal{H}\,h_{\boldsymbol{k}}^{\lambda\,\prime} + k^2\,h_{\boldsymbol{k}}^{\lambda} = S_{\boldsymbol{k}}^{\lambda}$$

with the source term $S_{\boldsymbol{k}}^{\lambda}$ being given by

$$S_{\boldsymbol{k}}^{\lambda}(\eta) = 4\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\,\pi)^{3/2}}\, e^{\lambda}(\boldsymbol{k},\boldsymbol{p}) \biggl\{2\,\Psi_{\boldsymbol{p}}(\eta)\,\Psi_{\boldsymbol{k}-\boldsymbol{p}}(\eta) + \frac{4}{3\,(1+w)\,\mathcal{H}^2}\,\left[\Psi_{\boldsymbol{p}}'(\eta) + \mathcal{H}\,\Psi_{\boldsymbol{p}}(\eta)\right]\,\left[\Psi_{\boldsymbol{k}-\boldsymbol{p}}'(\eta) + \mathcal{H}\,\Psi_{\boldsymbol{k}-\boldsymbol{p}}(\eta)\right]\biggr\},$$

where $\Psi_{\boldsymbol{k}}$ represents the Fourier modes of the Bardeen potential, while $\mathcal{H}$ and w denote the conformal Hubble parameter and the equation of state parameter describing the universe at the conformal time η. Also, $e^{\lambda}(\boldsymbol{k},\boldsymbol{p}) = e_{ij}^{\lambda}(\boldsymbol{k})\,p^i\,p^j$, with $e_{ij}^{\lambda}(\boldsymbol{k})$ representing the polarization of the tensor perturbations.

[15] K. N. Ananda, C. Clarkson and D. Wands, Phys. Rev. D **75**, 123518 (2007); D. Baumann, P. J. Steinhardt, K. Takahashi and K. Ichiki, Phys. Rev. D **76**, 084019 (2007).

The spectrum of secondary GWs today

The dimensionless parameter $\Omega_{\mathrm{GW}}(k,\eta)$ describing the energy density of GWs, when evaluated at late times during the radiation dominated epoch, can be expressed as[16]

$$\Omega_{\mathrm{GW}}(k,\eta) = \frac{\rho_{\mathrm{GW}}(k,\eta)}{\rho_{\mathrm{cr}}(\eta)} = \frac{1}{972}\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left[\frac{4\,v^2-(1+v^2-u^2)^2}{4\,u\,v}\right]^2 \mathcal{P}_{\mathrm{S}}(k\,v)\,\mathcal{P}_{\mathrm{S}}(k\,u) \times \left[\mathcal{I}_c^2(u,v)+\mathcal{I}_s^2(u,v)\right]$$

where the quantities $\mathcal{I}_c(u,v)$ and $\mathcal{I}_s(u,v)$ are determined by the transfer function $\mathcal{T}(k,\eta)$ for the scalar perturbations.

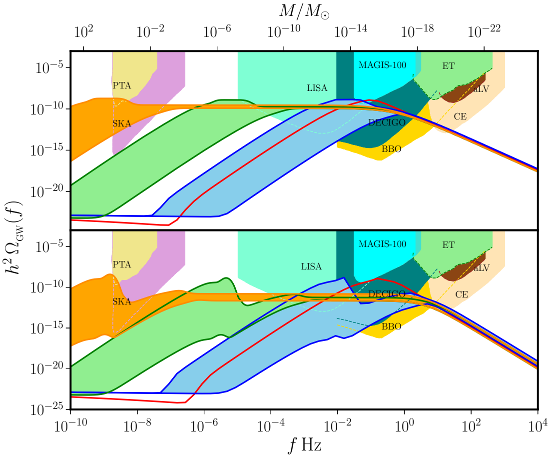
We can express $\Omega_{\mathrm{GW}}(k)$ today in terms of the above $\Omega_{\mathrm{GW}}(k,\eta)$ as follows:

$$h^2\,\Omega_{\mathrm{GW}}(k) \simeq 1.38\times 10^{-5}\left(\frac{g_{*,k}}{106.75}\right)^{-1/3}\left(\frac{\Omega_r\,h^2}{4.16\times 10^{-5}}\right)\Omega_{\mathrm{GW}}(k,\eta),$$

where Ω_{r} denotes the dimensionless energy density of radiation today, while $g_{*,k}$ and $g_{*,0}$ represent the number of relativistic degrees of freedom at reentry and today, respectively.

[16] K. Kohri and T. Terada, Phys. Rev. D **97**, 123532 (2018);
J. R. Espinosa, D. Racco and A. Riotto, JCAP **09**, 012 (2018).

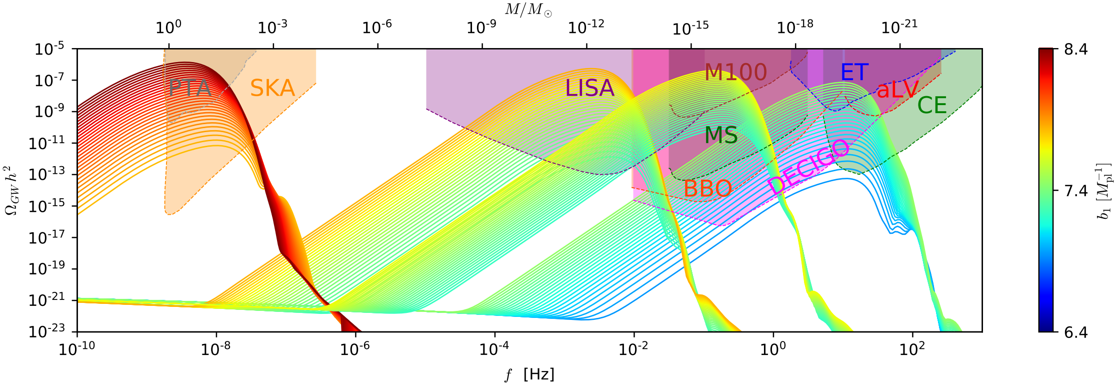
$\Omega_{\rm GW}(f)$ in ultra slow roll and punctuated inflation



The dimensionless density parameter $\Omega_{\rm GW}$ arising in the models and reconstructed scenarios of USR2 and RS1 (in red and blue, on top) as well as PI3 and RS2 (in red and blue, at the bottom) have been plotted as a function of the frequency f[17].

[17] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

$\Omega_{\rm GW}(f)$ in the two field model



The dimensionless density parameter $\Omega_{\rm GW}$ arising in the two field model has been plotted as function of frequency for a set of initial conditions for the background fields as well as a range of values of the parameter b_1[18].

[18] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

Plan of the talk

Constraints on the scalar non-Gaussianity parameters

The constraints on the primordial values of the non-Gaussianity parameters from the Planck data are as follows[19]:

$$\begin{aligned} f_{\rm NL}^{\rm loc} &= -0.9 \pm 5.1, \\ f_{\rm NL}^{\rm eq} &= -26 \pm 47, \\ f_{\rm NL}^{\rm ortho} &= -38 \pm 24. \end{aligned}$$

These constraints imply that slowly rolling single field models involving the canonical scalar field which are favored by the data at the level of power spectra are also consistent with the data at the level of non-Gaussianities.

[19] Planck Collaboration (Y. Akrami *et al.*), Astron. Astrophys. **641**, A9 (2020).

The cubic order action governing the perturbations

At the third order, the action describing the curvature perturbation $\mathcal{R}$ can be obtained to be[20]

$$\delta S_3[\mathcal{R}] = M_{\rm Pl}^2 \int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta \int {\rm d}^3\boldsymbol{x} \biggl[a^2\,\epsilon_1^2\,\mathcal{R}\,\mathcal{R}'^2 + a^2\,\epsilon_1^2\,\mathcal{R}\,(\partial\mathcal{R})^2 - 2\,a\,\epsilon_1\,\mathcal{R}'\,(\partial\mathcal{R})\,(\partial\chi) + \frac{a^2}{2}\,\epsilon_1\,\epsilon_2'\,\mathcal{R}^2\,\mathcal{R}' + \frac{\epsilon_1}{2}\,(\partial\mathcal{R})\,(\partial\chi)\,\partial^2\chi + \frac{\epsilon_1}{4}\,\partial^2\mathcal{R}\,(\partial\chi)^2 + 2\,\mathcal{F}(\mathcal{R})\,\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\biggr],$$

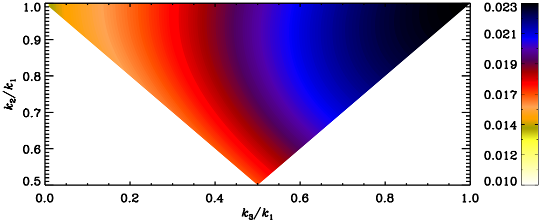
where $\mathcal{L}_2$ denotes the Lagrangian density at the second order, while $\partial^2\chi = a\,\epsilon_1\,\mathcal{R}'$, and these bulk terms are supplemented by the following temporal boundary terms[21]:

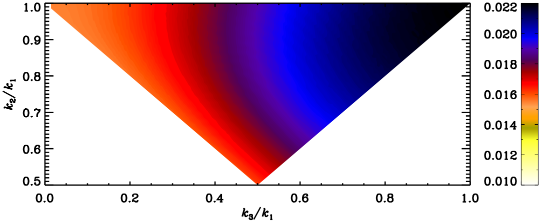
$$\delta S_3^{\rm B}[\mathcal{R}] = M_{\rm Pl}^2 \int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta \int {\rm d}^3\boldsymbol{x}\, \frac{\rm d}{{\rm d}\eta}\biggl\{-9\,a^3\,H\,\mathcal{R}^3 + \frac{a}{H}\,(1-\epsilon_1)\,\mathcal{R}\,(\partial\mathcal{R})^2 - \frac{1}{4\,a\,H^3}\,(\partial\mathcal{R})^2\,\partial^2\mathcal{R} - \frac{a\,\epsilon_1}{H}\,\mathcal{R}\,\mathcal{R}'^2 - \frac{a\,\epsilon_2}{2}\,\mathcal{R}^2\,\partial^2\chi + \frac{1}{2\,a\,H^2}\,\mathcal{R}\,\left(\partial_i\partial_j\mathcal{R}\,\partial_i\partial_j\chi - \partial^2\mathcal{R}\,\partial^2\chi\right) - \frac{1}{2\,aH}\,\mathcal{R}\,\left[\partial_i\partial_j\chi\,\partial_i\partial_j\chi - (\partial^2\chi)^2\right]\biggr\}.$$

[20] J. Maldacena, JHEP **0305**, 013 (2003);
D. Seery and J. E. Lidsey, JCAP **06**, 003 (2005);
X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **01**, 002 (2007).

[21] F. Arroja and T. Tanaka, JCAP **05**, 005 (2011).

The shape of the slow roll bispectrum

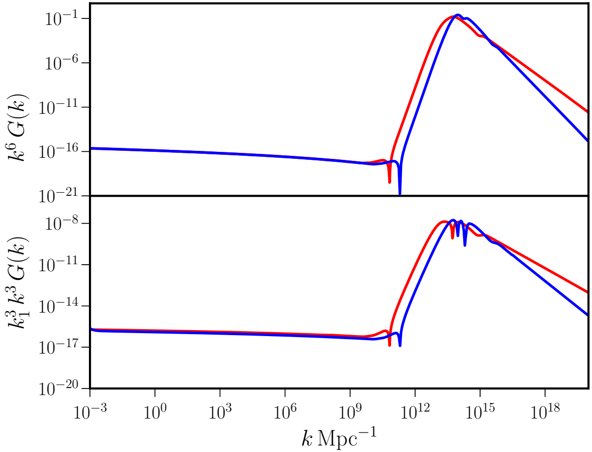




The non-Gaussianity parameter f_{NL}, evaluated in the slow roll approximation (analytically on the left and numerically on the right), has been plotted as a function of k_3/k_1 and k_2/k_1 for the case of the popular quadratic potential[22]. Note that the non-Gaussianity parameter peaks in the equilateral limit wherein $k_1 = k_2 = k_3$. In slow roll scenarios involving the canonical scalar field, the largest value of f_{NL} is found to be of the order of the first slow roll parameter ϵ_1.

[22] D. K. Hazra, L. Sriramkumar and J. Martin, JCAP **05**, 026, (2013).

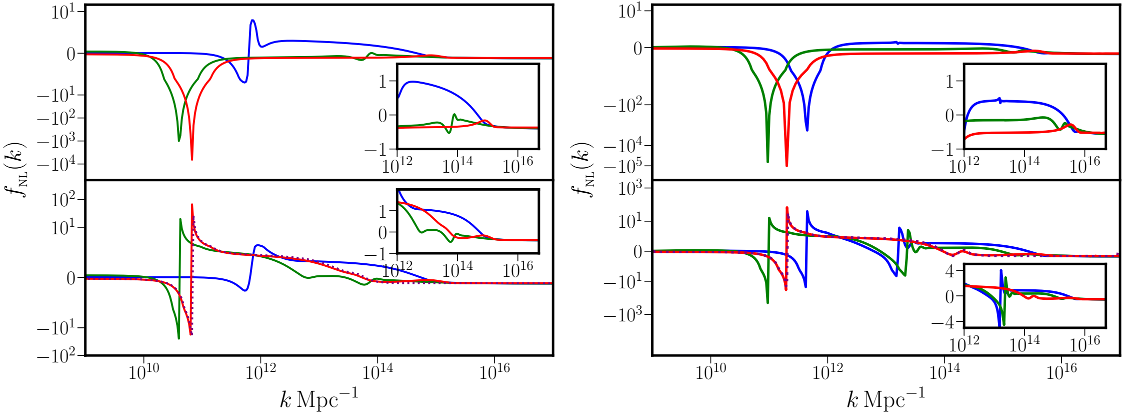
The scalar bispectrum in ultra slow roll and punctuated inflation



The amplitude of the dimensionless scalar bispectra has been plotted in the equilateral (on top) and squeezed limits (at the bottom) for the models USR2 (in red) and PI3 (in blue). The bispectra have approximately the same shape as the corresponding power spectra[23].

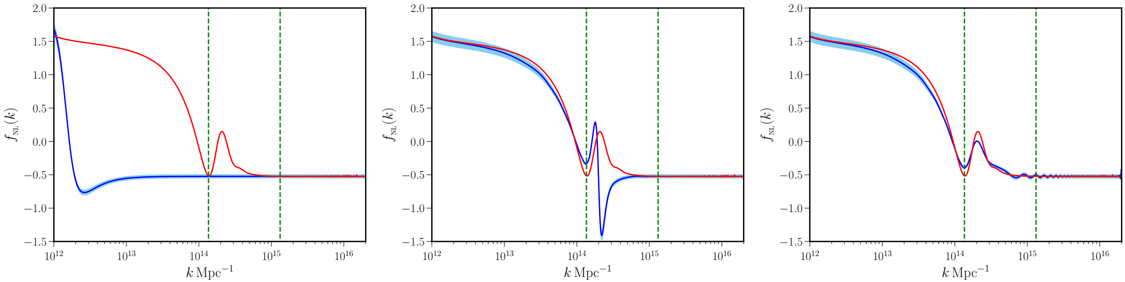
[23] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

$f_{\rm NL}$ in ultra slow roll and punctuated inflation



The scalar non-Gaussianity parameter $f_{\rm NL}$ has been plotted in the equilateral (on top) and the squeezed (at the bottom) limits for the models of USR2 and PI3 (in red, on the left and the right) and the reconstructed scenarios RS1 and RS2 (in blue and green, on the left and the right).

A closer examination of the consistency relation



The non-Gaussianity parameter $f_{\rm NL}$ in the squeezed limit (in blue) and the consistency condition $f_{\rm NL}^{\rm CR}$ (in red) have been plotted for the model PI3 over wave numbers around the peak in the scalar power spectrum. We have set the squeezed mode to be $k_1 = 10^{-1}\,k$ (on the left), $k_1 = 10^{-3}\,k$ (in the middle) and $k_1 = 10^{-5}\,k$ (on the right) in plotting these figures. We have also indicated the 5% uncertainty in our numerical estimate as bands (in blue).

Modifications to the scalar power spectrum due to non-Gaussianities

The scalar non-Gaussianity parameter $f_{_{\rm NL}}$ is usually introduced through the relation[24]

$$\mathcal{R}(\eta, \boldsymbol{x}) = \mathcal{R}^{\rm G}(\eta, \boldsymbol{x}) - \frac{3}{5}\, f_{_{\rm NL}}\, \left[\mathcal{R}^{\rm G}(\eta, \boldsymbol{x})\right]^2,$$

where $\mathcal{R}^{\rm G}$ denotes the Gaussian contribution. Upon using this expression and evaluating the corresponding two-point correlation function in Fourier space, one obtains that

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}}\, \hat{\mathcal{R}}_{\boldsymbol{k}'} \rangle = \frac{2\,\pi^2}{k^3}\, \delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}')\, \left[\mathcal{P}_{_{\rm S}}(k) + \left(\frac{3}{5}\right)^2 \frac{k^3}{2\,\pi}\, f_{_{\rm NL}}^2 \int {\rm d}^3\boldsymbol{p}\, \frac{\mathcal{P}_{_{\rm S}}(p)}{p^3}\, \frac{\mathcal{P}_{_{\rm S}}(\vert\boldsymbol{k}-\boldsymbol{p}\vert)}{\vert\boldsymbol{k}-\boldsymbol{p}\vert^3}\right],$$

where $\mathcal{P}_{_{\rm S}}(k)$ is the original scalar power spectrum defined in the Gaussian limit, while the second term represents the leading non-Gaussian correction. The non-Gaussian correction to the scalar power spectrum, say, $\mathcal{P}_{_{\rm C}}(k)$, can be expressed as follows[25]:

$$\mathcal{P}_{_{\rm C}}(k) = \left(\frac{12}{5}\right)^2 f_{_{\rm NL}}^2 \int_0^\infty {\rm d}s \int_0^1 \frac{{\rm d}d}{(s^2-d^2)^2}\, \mathcal{P}_{_{\rm S}}[k\,(s+d)/2]\, \mathcal{P}_{_{\rm S}}[k\,(s-d)/2].$$

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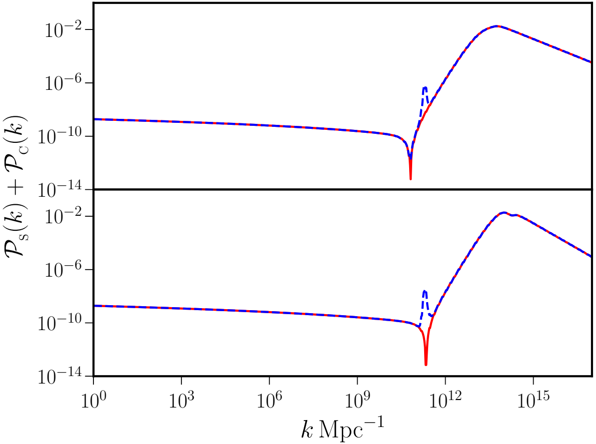
[24] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).

[25] R.-g. Cai, S. Pi and M. Sasaki, Phys. Rev. Lett. **122**, 201101 (2019); C. Unal, Phys. Rev. D **99**, 041301 (2019).

The modified scalar power spectrum



The original scalar power spectrum $\mathcal{P}_{\mathrm{S}}(k)$ (in solid red) and the modified spectrum $\mathcal{P}_{\mathrm{S}}(k) + \mathcal{P}_{\mathrm{C}}(k)$ (in dashed blue) arrived at upon including the non-Gaussian corrections, have been plotted for the models of USR2 (on top) and PI3 (at the bottom)[26].

[26] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

Accounting for the complete scale dependence of $f_{\rm NL}$

To account for a scale dependent $f_{\rm NL}$, one can write

$$\mathcal{R}_{\boldsymbol{k}}(\eta) = \mathcal{R}^{\rm G}_{\boldsymbol{k}}(\eta) - \frac{3}{5}\int \frac{{\rm d}^3\boldsymbol{k}_1}{(2\,\pi)^{3/2}} \mathcal{R}^{\rm G}_{\boldsymbol{k_1}}(\eta)\mathcal{R}^{\rm G}_{\boldsymbol{k}-\boldsymbol{k_1}}(\eta)\, f_{\rm NL}[\boldsymbol{k}, (\boldsymbol{k}_1-\boldsymbol{k}), -\boldsymbol{k}_1],$$

where $\mathcal{R}_{\boldsymbol{k}}$ is the mode function corresponding to the curvature perturbation $\mathcal{R}$ and $\mathcal{R}^{\rm G}_{\boldsymbol{k}}$ denotes the Gaussian part of $\mathcal{R}_{\boldsymbol{k}}$[27]. In real space, this corresponds to the relation

$$\mathcal{R}(\eta,\boldsymbol{x}) = \mathcal{R}^{\rm G}(\eta,\boldsymbol{x}) - \frac{3}{5}\int \frac{{\rm d}^3\boldsymbol{k}}{(2\,\pi)^{3}} \int {\rm d}^3\boldsymbol{k}_1\, \mathcal{R}^{\rm G}_{\boldsymbol{k_1}}(\eta)\,\mathcal{R}^{\rm G}_{\boldsymbol{k}-\boldsymbol{k_1}}(\eta)\, f_{\rm NL}[\boldsymbol{k}, (\boldsymbol{k}_1-\boldsymbol{k}), -\boldsymbol{k}_1]\,{\rm e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}}.$$

In such a case, the correction $\mathcal{P}_{\rm C}(k)$ to the original spectrum is given by[28]

$$\mathcal{P}_{\rm C}(k) = \frac{9}{50\pi}k^3\int {\rm d}^3\boldsymbol{k}_1\, \frac{\mathcal{P}^{\rm G}_{\rm S}(k_1)}{k_1^3}\,\frac{\mathcal{P}^{\rm G}_{\rm S}(|\boldsymbol{k}-\boldsymbol{k}_1|)}{|\boldsymbol{k}-\boldsymbol{k}_1|^3}\, f_{\rm NL}^2[k, |\boldsymbol{k}_1-\boldsymbol{k}|, k_1].$$

[27] F. Schmidt and M. Kamionkowski, Phys. Rev. D **82**, 103002 (2010); I. Agullo, D. Kranas and V. Sreenath, arXiv:2105.12993 [gr-qc].

[28] H. V. Ragavendra, arXiv:2108.04193 [astro-ph.CO].

Plan of the talk

Reconstructing power spectra from squeezed initial states

In slow roll inflation, the modes $f_k(\eta)$ describing the scalar perturbations that emerge from initial conditions corresponding to squeezed states can be expressed as

$$f_k(\eta) = \frac{i\,H_{\mathrm{I}}}{2\,M_{\mathrm{Pl}}\,\sqrt{k^3\,\epsilon_1}}\,\left[\alpha(k)\,\left(1 + i\,k\,\eta\right)\,\mathrm{e}^{-i\,k\,\eta} - \beta(k)\,\left(1 - i\,k\,\eta\right)\,\mathrm{e}^{i\,k\,\eta}\right],$$

where $\alpha(k)$ and $\beta(k)$ are the so-called Bogoliubov coefficients.

Consider a power spectrum with a localized feature over a certain range of wave numbers, say, $g(k)$, so that $\mathcal{P}_{_{\mathrm{S}}}(k)$ is given by

$$\mathcal{P}_{_{\mathrm{S}}}(k) = \mathcal{P}_{_{\mathrm{S}}}^{0}(k)\,\left[1 + g(k)\right],$$

where $\mathcal{P}_{_{\mathrm{S}}}^{0}(k)$ represents the standard power spectrum arising in slow roll inflation.

Such a power spectrum can be generated with the following choice of the Bogoliubov coefficients[29]:

$$\alpha(k) = \frac{2 + g(k)}{2\,\sqrt{1 + g(k)}}, \quad \beta(k) = \frac{-g(k)}{2\,\sqrt{1 + g(k)}}.$$

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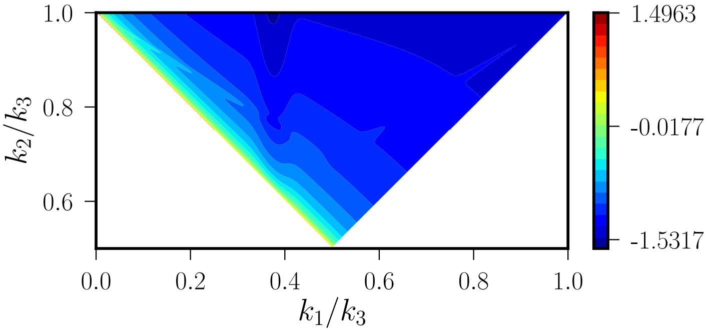
[29] See, for example, L. Sriramkumar and T. Padmanabhan, Phys. Rev. D **71**, 103512 (2005).

The non-Gaussianity parameter $f_{\rm NL}$ in squeezed initial states



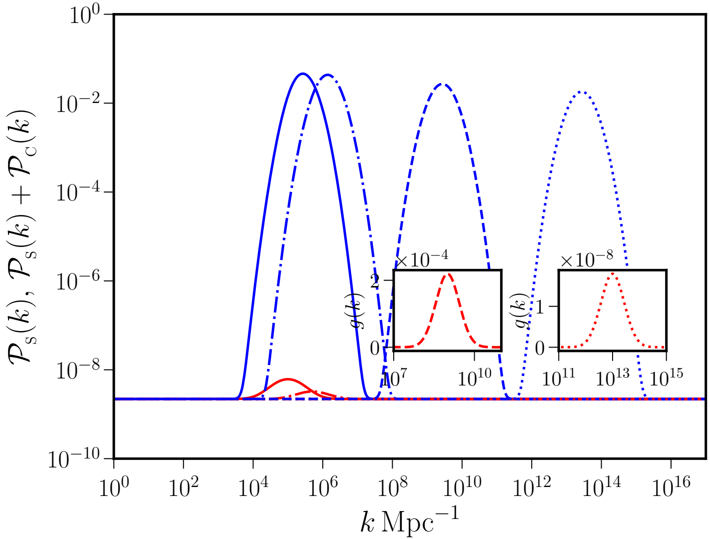
The non-Gaussianity parameter $\log|f_{\rm NL}|$ has been plotted as a density plot in the k_1/k_3–k_2/k_3 plane for

$$g(k) = \left(\gamma/\sqrt{2\,\pi\,\Delta_k^2}\right)\,\exp\,\left[-\ln^2\,(k/k_{\rm f})\,/\,\left(2\,\Delta_k^2\right)\right]$$

with $\gamma = 4.5$, $k_{\rm f} = 10^5\,{\rm Mpc}^{-1}$ and $\Delta_k = 1$. We have set $k_3 = k_{\rm f}$ and have varied k_1/k_3 over the range $[5\times 10^{-4}, 1]$ in arriving at this figure[30].

[30] H. V. Ragavendra, L. Sriramkumar and J. Silk, JCAP **05**, 010 (2021).

Are large non-Gaussian corrections to power spectrum possible?



The original (in red) and the modified (in blue) scalar power spectra corresponding to our choice of $g(k)$.

The issue of backreaction

It can be shown that the energy density associated with the curvature perturbation can be expressed as follows:

$$\rho_{\mathcal{R}} \simeq \frac{1}{2\,\pi^2\,a^4} \int_{-\eta^{-1}}^{\infty} \mathrm{d}k\, k^3\, |\beta(k)|^2,$$

where $\beta(k)$ is the Bogoliubov coefficient which indicates the extent of deviation from the Bunch-Davies vacuum.

For the backreaction due to the non-vacuum initial state to be negligible, we require that $\rho_{\mathcal{R}} \ll \rho_{\mathrm{I}} = 3\,H_{\mathrm{I}}^2\,M_{\mathrm{Pl}}^2$, which leads to the condition[31]

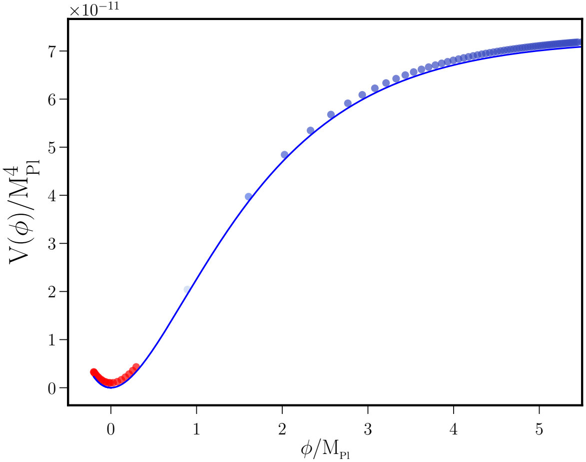
$$\frac{\gamma^2\,\mathrm{e}^4\,\Delta_k^2}{\Delta_k} \left(\frac{k_{\mathrm{f}}}{a\,H_{\mathrm{I}}}\right)^4 \ll 48\,\pi^{5/2} \left(\frac{M_{\mathrm{Pl}}}{H_{\mathrm{I}}}\right)^2.$$

If we choose $k_{\mathrm{f}} = 10^5\,\mathrm{Mpc}^{-1}$, this condition leads to $\gamma \ll 10^{-16.5}/\sqrt{r}$, where r denotes the inflationary tensor-to-scalar ratio. In other words, for $r \simeq 10^{-3}$, we require $\gamma < 10^{-15}$.

[31] H. V. Ragavendra, L. Sriramkumar and J. Silk, JCAP **05**, 010 (2021).

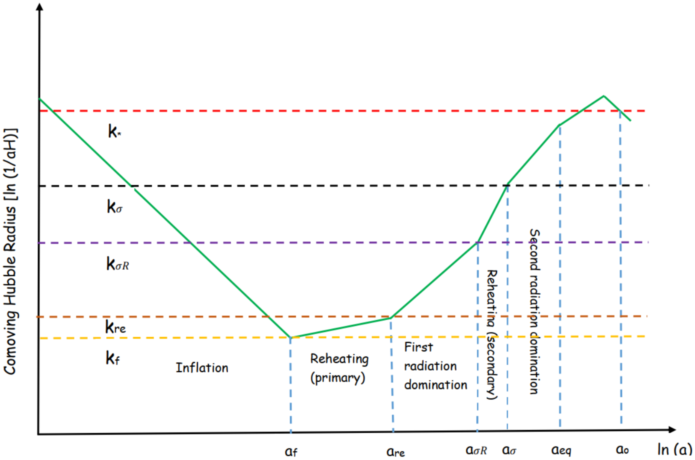
Plan of the talk

Evolution of the scalar field in an inflationary potential



The evolution of the scalar field in the so-called Starobinsky model has been indicated (as circles, in blue and red) at regular intervals of time. Inflation is terminated as the field approaches the bottom of the potential (near the light blue dot). Thereafter, the field oscillates at the bottom of the potential (indicated by the red dots).

Behavior of the comoving wave number and Hubble radius



Behavior of the comoving wave number k (horizontal lines in different colors) and the comoving Hubble radius $d_{\mathrm{H}}/a = (a\,H)^{-1}$ (in green) across different epochs[32].

[32] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, arXiv:2105.09242 [astro-ph.CO].

Duration of reheating and the reheating temperature

The duration of the epoch of reheating $N_{\rm re}$ and the reheating temperature $T_{\rm re}$ can be expressed in terms of the equation of state parameter w_ϕ during reheating and the inflationary parameters as follows[33]:

$$N_{\rm re} = \frac{4}{(3\, w_\phi - 1)} \left[N_\ast + \frac{1}{4}\, \ln\left(\frac{30}{\pi^2\, g_{\ast,{\rm re}}}\right) + \frac{1}{3}\, \ln\left(\frac{11\, g_{s,{\rm re}}}{43}\right) + \ln\left(\frac{k_\ast}{a_0\, T_0}\right) + \ln\left(\frac{\rho_{\rm f}^{1/4}}{H_{_{\rm I}}}\right)\right],$$

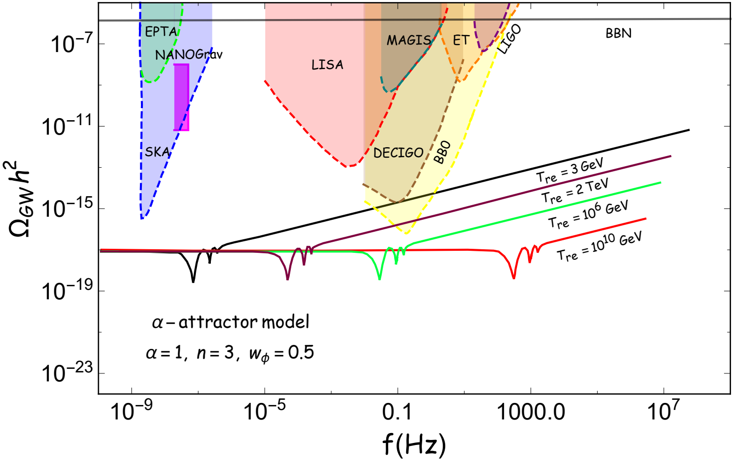
$$T_{\rm re} = \left(\frac{43}{11\, g_{s,{\rm re}}}\right)^{1/3} \left(\frac{a_0\, H_{_{\rm I}}}{k_\ast}\right) {\rm e}^{-(N_\ast+N_{\rm re})}\, T_0,$$

where $H_{_{\rm I}}$ is the Hubble parameter during inflation, $T_0 = 2.725\, {\rm K}$ is the present temperature of the CMB, and H_0 denotes the current value of the Hubble parameter.

Note that $k_\ast/a_0 \simeq 0.05\, {\rm Mpc}^{-1}$ represents the CMB pivot scale and $N_\ast$ denotes the number of e-folds *prior to the end of inflation* when the pivot scale leaves the Hubble radius.

[33] J. Martin and C. Ringeval, Phys. Rev. D **82**, 023511 (2010);
L. Dai, M. Kamionkowski and J. Wang, Phys. Rev. Lett. **113**, 041302 (2014);
J. L. Cook, E. Dimastrogiovanni, D. A. Easson and L. M. Krauss, JCAP **04**, 047 (2015).

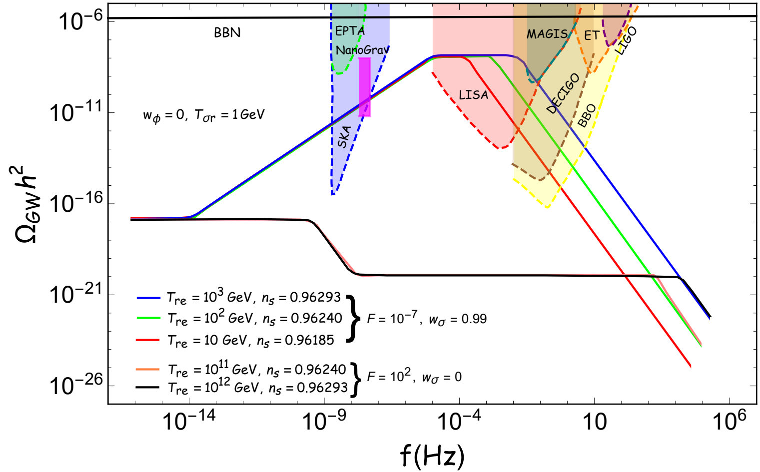
Effects on $\Omega_{\rm GW}$ due to reheating



The behavior of the dimensionless spectral energy density of primordial GWs today, viz. $\Omega_{\rm GW}(f)$ has been plotted over a wide range of frequencies (in red, green, brown and black) for different reheating temperatures[34].

[34] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, arXiv:2105.09242 [astro-ph.CO].

Effects on $\Omega_{\rm GW}$ due to late time entropy production



The dimensionless spectral energy density of primordial GWs observed today $\Omega_{\rm GW}(f)$ has been plotted in a scenario involving late time production of entropy.

Plan of the talk

Summary

- Models of ultra slow roll and punctuated inflation lead to enhanced power on small scales resulting in significant production of PBHs and increased strength of secondary GWs, possibly of detectable amplitudes.
- Despite the significant levels of deviation from slow roll, the non-Gaussianities generated in such single field models are relatively small with $f_{_{\rm NL}} \simeq \mathcal{O}(1)$ near the peaks in the power spectrum. These non-Gaussianities lead to insignificant corrections to $\Omega_{_{\rm GW}}$ today.
- The two field models require less amount of fine tuning to generate features in the primordial spectrum and hence seem better motivated[35].
- The backreaction due to squeezed initial states severely limits significant deviations from the Bunch-Davies vacuum and therefore the possibility of generating features at small scales from such non-vacuum initial states.
- A secondary phase of reheating with a suitable equation of state parameter leads to primary GWs with significantly high amplitudes that could be detected by the ongoing or forthcoming GW observatories[36].

[35] G. A. Palma, S. Sypsas, C. Zenteno, Phys. Rev. Lett. **125**, 121301, (2020);
J. Fumagalli, S. Renaux-Petel, J. W. Ronayne, L. T. Witkowski, arXiv:2004.08369 [hep-th].

[36] Y. Gouttenoire, G. Servant, and P. Simakachorn, arXiv:2108.10328 [hep-ph];
R. T. Co, D. Dunsky, N. Fernandez, A. Ghalsasi, L. J. Hall, K. Harigaya and J. Shelton, arXiv:2108.09299 [hep-ph].

Thank you for your attention