Null Surface Thermodynamics

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Based on my recent and upcoming papers with
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■ Gravity & Thermodynamics

- Distinctive feature of gravity is its universality.

- Thermodynamics has a similar universality.

- These two universal theories seem to be deeply related:

  - Black holes [Carter-Bardeen-Hawking & Hawking, Bekenstein (early 1970's)], [Wald (1993,4)];

  - Accelerated observers see a thermal bath [Unruh (1976)];

  - Einstein equations from thermodynamics [Jacobson (1995)];

  - Gravity as entropic force [E. Verlide (2010)];

  - Holographic principle & AdS/CFT.

Boundary symmetries and d.o.f.

- Presence of boundaries in spacetime brings in boundary d.o.f.

- It may be an asymptotic boundary or any arbitrary codimension one surface in spacetime separating spacetime in two parts.

- For gauge or diffeomorphism inv. theories boundary d.o.f. may be labeled by surface charges associated with non-trivial gauge/diff. transf.

- We focus on the boundary instead of the usual bulk viewpoint.

- We show boundary d.o.f for gravity theories follow a local thermodynamic description regardless of the details of the boundary dynamics.

Outline

- Einstein GR and equivalence principle in presence of boundaries
- Null surfaces and boundaries as models for BH horizons
- Null boundary symmetries and charges, D dimensional example
- Null Surface Thermodynamics
- Summary and Outlook

Einstein GR and its local (gauge) symmetry

- Einstein GR is a generally (in/co)variant theory.

- Physical observables in the Einstein GR are all defined through local diffeomorphism invariant quantities.

- In particular, any two metric tensors related by diffeomorphisms are physically equivalent:

$$x^\mu \to x^\mu + \xi^\mu(x), \quad g_{\eta\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}, \ \delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

- We partially fix diffeomorphisms through choice of observers.

■ Einstein GR, generic structure of d.o.f & EoM

- In a D dimensional spacetime, metric has $D(D+1)/2$ components:

  $D(D-3)/2$ propagating gravitons,
  D diffeos.

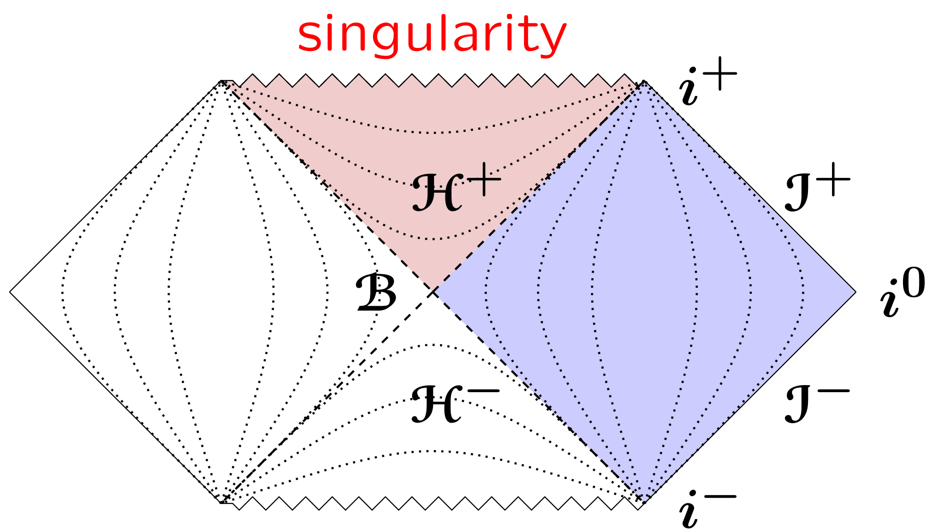
- Out of $D(D+1)/2$ field equations, $G_{\mu\nu} = 8\pi G T_{\mu\nu}$,

  $D(D-3)/2$ are second order diff.eq.,
  D constraints ($\nabla^\mu G_{\mu\nu} = 0$) and D first order equations.

- Solutions are fully specified by boundary and/or initial data and in the most general case involve $2D$ functions over codimension one boundary.

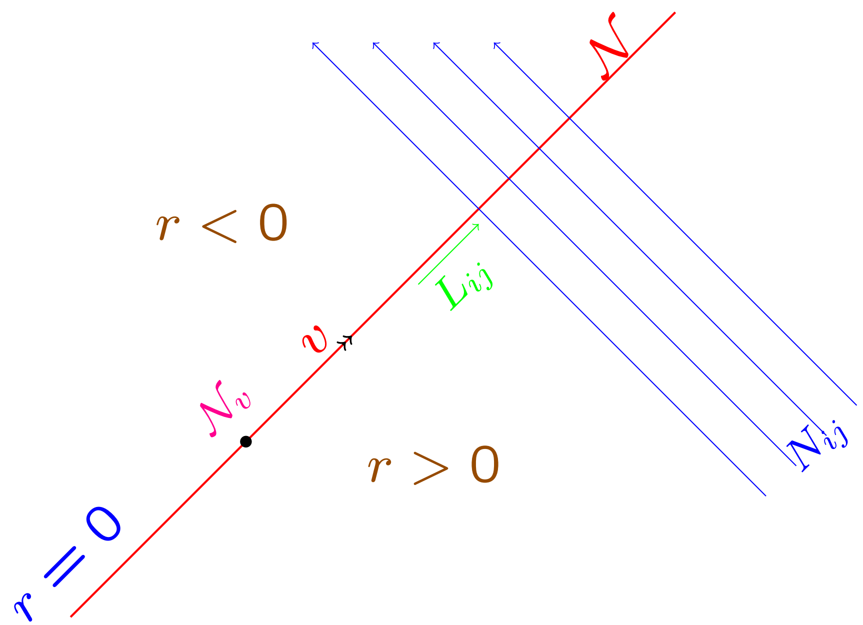
Null boundaries as models of horizons

- In a stationary black hole horizon is boundary of outside observers.



- Horizons are typically one way surfaces.

Depiction of a null surface



b.d.o.f. are residing on $\mathcal{N}$.

bulk d.o.f. N_{ij} & L_{ij}.

N is boundary of locally accelerated observers.

Nothing passes through $\mathcal{N}$ to $r > 0$ region.

- Let $\mathcal{N}$ be a null surface, sitting at $r = 0$:

$$ds^2 = -F dv^2 + 2\eta dr dv + h_{ij}(dx^i + g^i dv)(dx^j + g^j dv) \qquad (1)$$

F, g^i, h_{ij} are functions of r, v, x^i, $i = 1, 2, \cdots, D-2$ and $\eta = \eta(v, x^i)$,

$$g^{rr}\Big|_{r=0} = 0 \quad \Longrightarrow \quad (Fh + g^2)\Big|_{r=0} = 0,$$

where $h := \det h_{ij}, g^2 := h^{ij} g_i g_j$.

- We choose $r = 0$ to be the boundary of our spacetime and restrict ourselves to $r \geq 0$.

- The role of the excises $r < 0$ region is played by the boundary theory.

Solution space

- Metric (1) has $1 + 1 + (D-2) + (D-1)(D-2)/2$ functions in it.

- These may be decomposed into

  - three scalars $(F, h; \eta)$,

  - one vector g_i and

  - one symmetric-traceless tensor $H_{ij} := h_{ij}/h^{1/(D-2)}$,

  from the viewpoint of codimension two surface $\mathcal{N}_v$, (constant v slice on $\mathcal{N}$).

- These functions are subject to field equations, here, Einstein vacuum equations, which determine their r dependence.

- r dependence of the tensor mode H_{ij} is determined through

$$\gamma_{ij}(v, x^i) := H_{ij}(r = 0; v, x^i), \qquad L_{ij}(v, x^i) := \partial_r H_{ij}(r = 0; v, x^i).$$

- r dependence of the vector mode obeys first order eq. in r and is completely specified by $\mathcal{G}_i(v, x^i) := g_i(r = 0; v, x^i)$.

- Raychaudhuri equation + the condition that $\mathcal{N}$ is null, allows for solving F in terms of $\mathcal{G}_i, h, \eta$.

- r dependence of the other two are determined in terms of $\eta := \eta(v, x^i)$, $\Omega(v, x^i) := \sqrt{h(r = 0; v, x^i)}$.

- Null surface solution phase space is determined by
  - "Tensor modes" (gravitons) γ_{ij}, L_{ij},
  - Vector mode $\mathcal{G}_i$,
  - Scalars modes Ω, η,

- These are respectively, $D(D-3)$, $D-2, 2$ functions of v, x^i.

- We have only assumed smoothness of metric at $r=0$,

- but no particular behavior (falloff condition), around $r=0$.

- The boundary $r = 0$ is not a special place in spacetime and can be any given (null) $D-1$ dimensional hypersurface.

- By construction all solution geometries which are smooth around $r = 0$, are of the form (1) and

$$\begin{aligned} F &= \eta\left(\Gamma + \frac{2}{D-2}\frac{\mathcal{D}_v\Omega}{\Omega} - \frac{\mathcal{D}_v\eta}{\eta}\right) r + \mathcal{O}(r^2) \\ g^i &= \mathcal{G}^i - r\frac{\eta}{\Omega}\mathcal{J}^i + \mathcal{O}(r^2) \\ h_{ij} &= \Omega_{ij} + \mathcal{O}(r) \end{aligned} \tag{2}$$

where all the fields are functions of v, x^i and Γ is acceleration (non-affinity parameter) & $\mathcal{G}^i$ is the angular velocity of null rays generating $\mathcal{N}$.

$$\Omega_{ij} := \Omega^{2/(D-2)}\gamma_{ij}, \qquad \Omega := \sqrt{\det \Omega_{ij}}, \qquad \det \gamma_{ij} = 1.$$

Ω^{ij} and Ω_{ij} raise and lower capital Latin indices.

$$\mathcal{D}_v := \partial_v - \mathcal{L}_{\mathcal{G}},$$

where $\mathcal{L}_{\mathcal{G}}$ is the Lie derivative along $\mathcal{G}^i$ direction.

Θ expansion of vector field generating the null surface $\mathcal{N}$:

$$\Theta := \mathcal{D}_v \ln \Omega,$$

N_{ij} the *news tensor* associated with flux of gravitons through $\mathcal{N}$:

$$N_{ij} := \tfrac{1}{2}\Omega^{2/(D-2)}\mathcal{D}_v\gamma_{ij}$$

N_{ij} as defined above is a symmetric-traceless tensor.

■ Einstein Field Equations at $r = 0$

$$\mathcal{D}_v \Omega = \Theta\ \Omega, \tag{3a}$$

$$\mathcal{D}_v \mathcal{P} = -\Gamma + \frac{2}{\Theta} N_{ij} N^{ij}, \tag{3b}$$

$$\mathcal{D}_v \mathcal{J}_i + \Theta \Omega \partial_i \mathcal{P} - \Omega \partial_i \Gamma + 2\Omega \bar{\nabla}^j N_{ij} = 0. \tag{3c}$$

where

$$\mathcal{P} := \ln \frac{\eta}{\Theta^2},$$

and $\bar{\nabla}_i$ is covariant derivative w.r.t Ω_{ij}.

So the solution space may be parametrized by

Boundary modes: $\Omega, \mathcal{P}, \mathcal{J}_i$ and

Bulk modes: N_{ij}, L_{ij}.

■ Symplectic form over the solution phase space

$$\Omega = \int_{\mathcal{N}} \left[\delta\Gamma \wedge \delta\Omega + \delta(\Theta\Omega) \wedge \delta\mathcal{P} + \delta\mathcal{G}^i \wedge \delta\mathcal{J}_i + \delta(\Omega N_{ij}) \wedge \delta\Omega^{ij}\right]. \tag{4}$$

- L_{ij} do not appear in the symplectic form.

- Einstein equations (3) may be solved for $\Gamma, \mathcal{G}^i$ in terms of the charges.

- Ω^{ij} is canonical conjugate to $N_{ij} \sim \partial_v\Omega_{ij}$, as in any usual field theory,

- canonical conjugates to the boundary modes $\Omega, \mathcal{P}, \mathcal{J}_i$ are respectively $\Gamma, \mathcal{D}_v\Omega, \mathcal{G}^i$. We will see these consitute a thermodynamical phase space.

Residual diffeos over the null surface $\mathcal{N}$

- We have partially used diffeos to fix the null surface $\mathcal{N}$ at $r = 0$.

- The measure zero subset of residual diffeos keep $r = 0$ intact:

$$\begin{aligned} v &\to v + T(v, x^i) + \mathcal{O}(r) \\ r &\to \Big(\partial_v T(v, x^i) - W(v, x^i)\Big) r + \mathcal{O}(r^2) \\ x^i &\to x^i + Y^i(v, x^i) + \mathcal{O}(r) \end{aligned} \tag{5}$$

  T, Y^i are supertranslations in v, x^i and W is the superboost on $\mathcal{N}$ (superscaling in r).

- Subleading terms in r may be fixed order-by-order requiring that (5) keep the form of metric within solution space (1).

- Residual diffeos are specified by two scalar functions $T(v, x^i), W(v, x^i)$ and one vector $Y^i(v, x^i)$ over $r = 0$ null surface.

Symmetries of the solution space

- Upon (5), metric (1) keeps its form but with transformed functions:

$$\mathcal{G}_i \to \mathcal{G}_i + \delta\mathcal{G}_i, \qquad \eta \to \eta + \delta\eta, \qquad \Omega \to \Omega + \delta\Omega,$$
$$N_{ij} \to N_{ij} + \delta N_{ij}, \qquad L_{ij} \to L_{ij} + \delta L_{ij}, \tag{6}$$

  where δX are linear in residual diffeo functions T, W, Y^i.

- Besides dynamical, propagating gravitons, there are $2+(D-2)$ boundary modes in our solution space.

- There are $2+(D-2)$ functions over $\mathcal{N}$ in our residual diffeos.

- Residual diffeos rotate us within the solution space. They are hence symmetry generators in the usual classic(al) Noether sense.

■ Symmetries of the solution phase space

- One may use Covariant Phase Space Formalism (CPSF) to show solution space is a phase space and there is a charge (Hamiltonian generator) associated with the boundary symmetries.

- These surface charges are given by integrals over codimension-2 compact spacelike surfaces, constant v slices $\mathcal{N}_v$.

- Surface charges are linear in symmetry generators $T(v, x^i), W(v, x^i)$ and $Y^i(v, x^j)$, but may have different field/states dependence, i.e.

- integrands of the surface charge integrals may have different functional dependence on $\Omega, \mathcal{P}, \mathcal{J}_i$ as well as N_{ij}.

Surface charges and their algebra

- Standard computations yields the following surface charge variations associated with the symmetry generators ξ

$$\delta\!\!\!/ Q_\xi = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x \Big[(W + \Gamma T)\,\delta\Omega + (Y^i + \mathcal{G}^i T)\delta\mathcal{J}_i + T\Omega\Theta\delta\mathcal{P} - T\Omega\Omega^{ij}\delta N_{ij}\Big], \tag{7}$$

- Charge variation is an integral over $\sum_{A=1}^{4} \mathcal{C}_A\ \delta\boldsymbol{\mathcal{Q}}_A$,

- $\boldsymbol{\mathcal{Q}}_A \in \{\Omega, \mathcal{J}_i, \mathcal{P}; N_{ij}\}$ parameterize the solution phase space.

- $\mathcal{C}_A$ are linear combination of symmetry generators W, T, Y^i and the canonical conjugate variables $\Gamma, \mathcal{G}^i, \Omega_{ij}$.

- Charge variation may be split into integrable part Q^{N} and the 'flux' part F, using the Barnich-Troessaert method:

$$\not\delta Q_\xi = \delta Q^{\mathsf{N}}{}_\xi + F_\xi(\delta g; g).$$

- One may show that the integrable part may be equated with the Noether charge, using the W-freedom/ambiguity. [see also recent papers of Freidel et al & Leigh et al].

- Q^{N} may be computed for the Einstein-Hilbert action using the standard Noether procedure, yielding

$$\boxed{Q^{\mathsf{N}}{}_\xi = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x \left[W\,\Omega + Y^i\,\mathcal{J}_i + T\,\left(\Gamma\Omega + \mathcal{G}^i\mathcal{J}_i\right)\right]} \tag{8}$$

$$\boxed{F_\xi(\delta g; g) = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\, T\left(-\Omega\delta\Gamma - \mathcal{J}_i\delta\mathcal{G}^i + \Omega\Theta\delta\mathcal{P} - \Omega\Omega^{ij}\delta N_{ij}\right)} \tag{9}$$

- Symmetry generators T, W, Y^i are assumed to be field-independent, i.e. $\delta T = \delta W = 0 = \delta Y^i$.

- $\mathcal{P}$ and N_{ij} only appear in the flux and not in the Noether charges.

- The zero mode Noether charges,

$$\boxed{Q^{\mathrm{N}}{}_{-r\partial_r} = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\ \Omega}$$
$$\boxed{Q^{\mathrm{N}}{}_{\partial_i} = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\ \mathcal{J}_i} \tag{10}$$
$$\boxed{Q^{\mathrm{N}}{}_{\partial_v} := \mathbf{E} = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\ (\Gamma\Omega + \mathcal{G}^i\mathcal{J}_i)}$$

- Note that the charge variation associated with ∂_v is

$$\boxed{\not{\delta} Q_{\partial_v} := \not{\delta}\mathbf{H} = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\ \left(\Gamma\delta\Omega + \mathcal{G}^i\delta\mathcal{J}_i + \Omega\Theta\delta\mathcal{P} - \Omega\Omega^{ij}\delta N_{ij}\right)}$$

- Balance equation

$$\boxed{\frac{\mathsf{d}}{\mathsf{d}v} Q^{\mathsf{N}}{}_{\xi} \approx -F_{\partial_v}(\delta_\xi g; g)} \tag{11}$$

where $\approx$ denotes on-shell equality.

- The abov Eq. is
  - a manifestation of the EoM projected at the boundary written in terms of charges;
  - a generalized charge conservation equation relating time dependence, or non-conservation, of the charge (as viewed by the null boundary observer) to the flux passing through the boundary;
  - and shows how passage of flux through the null boundary is ‘balanced’ by the rearrangements in the charges.

Review of Thermodynamics

- Consider a thermodynamical system with
  - chemical potentials μ_a $(a = 1, 2, \cdots, N)$ and temperature T,
  - charges Q_a, the entropy S and the energy E;
- There are $N + 2$ charges and $N + 1$ chemical potentials.
- In microcanonical ensemble (which we assume), the first law takes the form

$$\mathrm{d}E = T\ \mathrm{d}S + \sum_{i=a}^{N} \mu_a\ \mathrm{d}Q_a. \tag{12}$$

- The LHS is an exact one-form over the thermodynamic space.

- Chemical potentials and charges are related by the Gibbs-Duhem relation

$$S \, \mathrm{d}T + \sum_{i=a}^{N} Q_a \, \mathrm{d}\mu_a = 0. \tag{13}$$

- Together with the first law, this yields $E = TS + \sum_a \mu_a Q_a$.

- This equation relates E to the other charges and chemical potentials, e.g. $E = E(S, Q_a)$.

- $N + 1$ number of chemical potentials and/or charges may be taken to be 'independent' variables parameterizing the thermodynamical configuration space and the rest of $N + 1$ of them as functions of the former $N + 1$ variables.

Null Boundary Thermodynamical Phase Space

I. Null boundary thermodynamics consists of three parts:

I.1) $(D-1)$ dimensional 'thermodynamic sector' parametrized by $(\Gamma, \mathcal{G}^i)$ and conjugate charges $(\Omega, \mathcal{J}_i)$;

I.2) $\mathcal{P}$, only appears in the flux and not in the Noether charge and its conjugate chemical potential is $\Omega\Theta$;

I.3) bulk modes parameterized by determinant free part of Ω^{ij} and its 'conjugate charge' N_{ij} appear in the flux.

II. N_{ij} take the boundary system out-of-thermal-equilibrium (OTE)

whereas $\mathcal{P}$ parameterizes OTE within the boundary dynamics.

Put differently, OTE may come from inner boundary dynamics and/or from the gravity-waves passing through the null boundary.

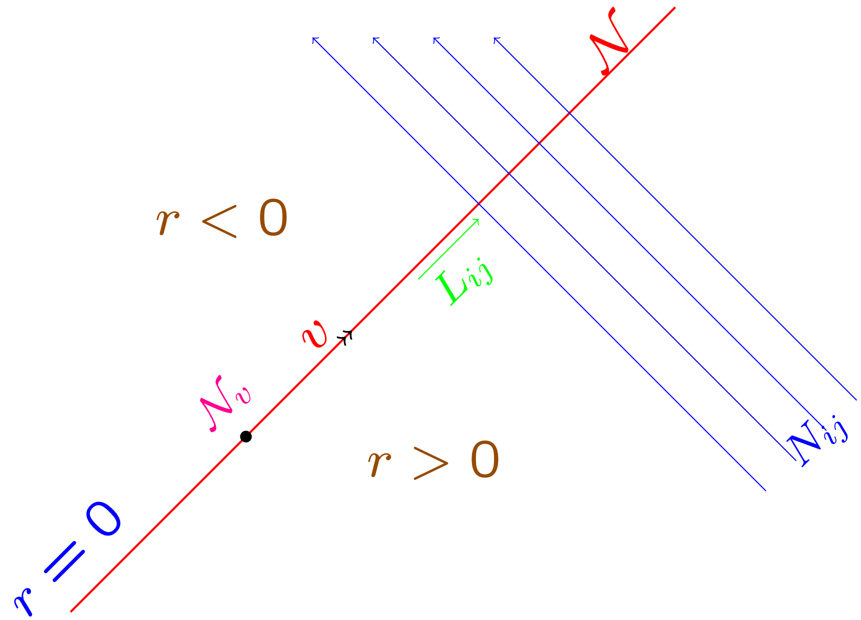
III. Expansion parameter Θ is a measure of OTE, from both bulk and boundary viewpoints. When $\Theta = 0$ the system is completely specified by the $D-1$ dimensional thermodynamic phase space.

IV. The rest of the in-falling graviton modes parameterized as L_{ij}, do not enter in the boundary/thermo dynamics, recalling usual causality and that the boundary is a null surface.

Below we give local first law, then local Gibbs-Duhem equation and come to local zeroth law, specifying the subsectors which can be brought to a (local) equilibrium.

Notation: $\boldsymbol{\mathcal{X}}$ we will denote the density of the quantity $\mathbf{X}$,

$$\mathbf{X} := \int_{\mathcal{N}_v} \mathrm{d}^{D-2}x \; \boldsymbol{\mathcal{X}}.$$



b.d.o.f. parametrized by $\Omega, \mathcal{P}, \mathcal{J}_i$ are residing on $\mathcal{N}$.

b.d.o.f. interact with themselves and with infalling flux N_{ij}.

Interactions of b.d.o.f with infalling flux are fixed by diff invariance, governed by the balance equation.

Interactions among b.d.o.f themselves are still free to be chosen.

Local First Law at Null Boundary

- Defining $\boldsymbol{\mathcal{P}} := \mathcal{P}/(16\pi G)$ and $\boldsymbol{\mathcal{N}}_{ij} := (16\pi G)^{-1} N_{ij}$,

$$\not{\delta}\boldsymbol{\mathcal{H}} = T_{\mathcal{N}}\ \delta\boldsymbol{\mathcal{S}} + \mathcal{G}^i\ \delta\boldsymbol{\mathcal{J}}_i + \Omega\Theta\delta\boldsymbol{\mathcal{P}} - \Omega\Omega^{ij}\ \delta\boldsymbol{\mathcal{N}}_{ij}, \qquad T_{\mathcal{N}} := \frac{\Gamma}{4\pi}$$

- The above is true at each v, x^i over the null surface and represents the local null boundary first law, unlike its usual thermodynamic counterpart or as in black hole thermodynamics.

- LHS, unlike the usual first law, is not a complete variation; the system is describing an open thermodynamic system due to the existence of the expansion and the flux.

- The above reduces to a usual first law for closed systems when $N_{ij} = 0$ or in the non-expanding $\Theta = 0$ case.

Local Extended Gibbs-Duhem Equation at Null Boundary

- For the Noether charge densities in our notation, we have

$$\boxed{\mathcal{E} = T_{\mathcal{N}}\boldsymbol{\mathcal{S}} + \mathcal{G}^i\boldsymbol{\mathcal{J}}_i}$$

  an analogue of the Gibbs-Duhem equation if $\mathcal{E}$ is viewed as energy, $\boldsymbol{\mathcal{S}}$ as entropy and $\boldsymbol{\mathcal{J}}_i$ as other conserved charges and $\Gamma, \mathcal{G}^i$ as the respective chemical potentials.

- It is a local equation at the null boundary, unlike its usual thermodynamic counterpart.

- This equation also holds for non-stationary/non-adiabatic cases when the system is out-of-thermal-equilibrium (OTE) it is 'local extended Gibbs-Duhem' (LEGD) equation at the null boundary.

- LEGD equation, like the local first law, is a manifestation of diffeomorphism invariance of the theory.

- We expect them to be universally true for any diff-invariant theory of gravity in any dimension.

- This equation is on par with the first law of thermodynamics but extends it in two important ways:

  it is a local equation in v, x^i and holds also for OTE.

- The integrable parts of the charge are (by definition) independent of the bulk flux N_{ij} & $\mathcal{P}$, so the LEGD also do not involve $\mathcal{P}$ and N_{ij}.

- Chemical potentials Γ and $\mathcal{G}^i$ implicitly depend on N_{ij} and $\mathcal{P}$ through Raychaudhuri and Damour equations.

Local Zeroth Law

- Zeroth law is a statement of thermal equilibrium: as a consequence of the zeroth law, two (sub)systems with the same temperature and chemical potentials are in thermal equilibrium.

- Flow of charges is proportional to the gradient of associated chemical potentials and hence the absence of such fluxes can be taken as a statement of the zeroth law.

- Here the system is parameterized by chemical potentials $\Gamma, \mathcal{G}^i$ and γ^{ij} which are functions of charges $Q_A \in \{\Omega, \mathcal{P}, \mathcal{J}_i, N_{ij}\}$.

- This system is not in general in equilibrium but there could be special subsectors which are. The zeroth law is to specify such subsectors.

- Zeroth law requires existence of $\mathcal{G} = \mathcal{G}(\Omega, \mathcal{P}, \mathcal{J}_i, N_{ij})$ such that,

$$\delta\mathcal{G} = -\mathcal{S}\ (\delta T_{\mathcal{N}} - 4G\Theta\delta\mathcal{P}) - \mathcal{J}_i\ \delta\mathcal{G}^i + \Omega\mathcal{N}_{ij}\delta\Omega^{ij} \tag{14}$$

admits non-zero solutions.

- The zeroth law as mentioned above is closely related to the notion of charge integrability & variational principle.

- Integrability condition for the zeroth law is $\delta(\delta\mathcal{G}) = 0$, yielding an equation like

$$\sum_{AB} C_{AB}\delta Q_A \wedge \delta Q_B = 0,$$

where Q_A are generic charges and C_{AB} is skew-symmetric. This equation is satisfied only for $C_{AB} = 0$.

- One can immediately see $N_{ij} = 0 = \delta N_{ij}$ is a necessary (but not sufficient) condition for the zeroth law to have non-trivial solutions.

- When zeroth law is fulfilled the charge $\boldsymbol{\mathcal{H}}$, which appears in the LHS of the local first law, becomes integrable and we obtain

$$\boxed{\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{G}} + T_{\mathcal{N}}\boldsymbol{\mathcal{S}} + \mathcal{G}^i \boldsymbol{\mathcal{J}}_i}$$

- Besides $N_{ij} = 0$, in terms of $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}(\boldsymbol{\mathcal{S}}, \boldsymbol{\mathcal{J}}_i, \boldsymbol{\mathcal{P}})$ local zeroth law implies,

$$\boxed{T_{\mathcal{N}} = \frac{\delta \boldsymbol{\mathcal{H}}}{\delta \boldsymbol{\mathcal{S}}}, \qquad \mathcal{G}^i = \frac{\delta \boldsymbol{\mathcal{H}}}{\delta \boldsymbol{\mathcal{J}}_i}, \qquad \mathcal{D}_v \boldsymbol{\mathcal{S}} = \boldsymbol{\mathcal{S}} \Theta = \frac{1}{4G} \frac{\delta \boldsymbol{\mathcal{H}}}{\delta \boldsymbol{\mathcal{P}}}}$$

- For $\Theta = 0$ case, one simply deduces that $\boldsymbol{\mathcal{H}}$ does not depend on $\boldsymbol{\mathcal{P}}$.

Generic $\Theta \neq 0$ case.

Zeroth law requires $N_{ij} = 0$ and we have Einstein boundary field equations

$$T_{\mathcal{N}} = -4G\mathcal{D}_v\boldsymbol{\mathcal{P}}, \qquad \mathcal{D}_v\left[\boldsymbol{\mathcal{J}}_i + 4G\bar{\nabla}_i(\boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{P}})\right] = 0.$$

Zeroth law is satisfied for any $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}(\boldsymbol{\mathcal{S}}, \boldsymbol{\mathcal{P}}, \boldsymbol{\mathcal{J}}_i)$, when $\boldsymbol{\mathcal{S}}, \boldsymbol{\mathcal{P}}$ and $\boldsymbol{\mathcal{J}}_i$ have the following basic Poisson brackets:

$$\begin{aligned}
&\{\boldsymbol{\mathcal{S}}(x,v), \boldsymbol{\mathcal{P}}(y,v)\} = \frac{1}{4G}\delta^{D-2}(x-y),\\
&\{\boldsymbol{\mathcal{S}}(x,v), \boldsymbol{\mathcal{S}}(y,v)\} = \{\boldsymbol{\mathcal{P}}(x,v), \boldsymbol{\mathcal{P}}(y,v)\} = 0,\\
&\{\boldsymbol{\mathcal{S}}(x,v), \boldsymbol{\mathcal{J}}_i(y,v)\} = \boldsymbol{\mathcal{S}}(y,v)\frac{\partial}{\partial x^i}\delta^{D-2}(x-y),\\
&\{\boldsymbol{\mathcal{P}}(x,v), \boldsymbol{\mathcal{J}}_i(y,v)\} = \left(\boldsymbol{\mathcal{P}}(y,v)\frac{\partial}{\partial x^i} + \boldsymbol{\mathcal{P}}(x,v)\frac{\partial}{\partial y^i}\right)\delta^{D-2}(x-y),\\
&\{\boldsymbol{\mathcal{J}}_i(x,v), \boldsymbol{\mathcal{J}}_j(y,v)\} = \frac{1}{16\pi G}\left(\boldsymbol{\mathcal{J}}_i(y,v)\frac{\partial}{\partial x^j} - \boldsymbol{\mathcal{J}}_j(x,v)\frac{\partial}{\partial y^i}\right)\delta^{D-2}(x-y)
\end{aligned}$$

- The above Poisson brackets imply

$$\partial_v \boldsymbol{\mathcal{X}} = \{\boldsymbol{\mathcal{H}}, \boldsymbol{\mathcal{X}}\}.$$

- Therefore, $\boldsymbol{\mathcal{H}}$ is the Hamiltonian over the thermodynamic phase space.

- $\Theta = 0$ **case.** may be worked out similarly
  - in this case $\mathcal{P} = 0 = N_{ij}$ and the thermodynamic phase space is described by $\boldsymbol{\mathcal{S}}, \mathcal{J}_i$ and their chemical potentials.
  - Local zeroth law is satisfied by any scalar Hamiltonian $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}(\boldsymbol{\mathcal{S}}, \boldsymbol{\mathcal{J}}_i)$, together with basic Poisson brackets given above, but with $\boldsymbol{\mathcal{P}}$ dropped and again with $\partial_v \boldsymbol{\mathcal{X}} = \{\boldsymbol{\mathcal{H}}, \boldsymbol{\mathcal{X}}\}$.

- Zeroth law is just defining the Poisson bracket structure over the thermodynamic phase space and existence of Hamiltonian dynamics, but it does not specify the boundary Hamiltonian $\mathcal{H}$.

- Choice of Hamiltonian $\mathcal{H}$ fixes a boundary Lagrangian and boundary dynamical equations and hence local dynamics of charges on the null boundary $\mathcal{N}$.

- In analogy with isolated horizon of black holes, if the zeroth law holds the null surface may be called an 'isolated null surface'.

- Our zeroth law is a weaker condition than stationarity as ∂_v of the chemical potentials need not vanish.

Discussion, Concluding Remarks and Outlook

⊛ Presence of boundaries brings in new ‘boundary d.o.f.’.

- b.d.o.f. may be classified and labelled by surface charges associated with nontrivial diffeos.

- CPSF can be used to construct the boundary phase space which govern b.d.o.f.

- Motivated by identification and formulation of BH microstates we studied spacetimes with a null boundary $\mathcal{N}$.

- $\mathcal{N} \sim R_v \ltimes \mathcal{N}_v$, where $\mathcal{N}_v$ is a codim. two compact surface.

- $\mathcal{N}$ may be viewed as the null limit of the stretched horizon.

- Physics in the outside horizon region is then described by
  boundary d.o.f $\oplus$ bulk d.o.f.

- Hilbert space of b.d.o.f, $\mathcal{H}_{bdof}$ may be labeled by surface charges associated with nontrivial diffeos.

- Poisson bracket of charges is v independent; $\mathcal{H}_{bdof}$ is defined at $\mathcal{N}_v$.

- Boundary d.o.f interact with bulk d.o.f through the *Bondi news*, the energy and angular momentum flux through the horizon.

- Balance equation equates time derivative of boundary charges to the flux through the boundary. It tells us how b.d.o.f should rearrange themselves as a consequence of passage of the flux.

- We identified null surface thermodynamic phase space, which in general describes an open system.

- Thermodynamics phase space is described by $D-1$ charges and associated chemical potentials as well as the flux.

- Local laws of thermodynamics govern thermodynamic phase space.

- Local zeroth law ensures we have a phase space by specifying the Poisson bracket structure, which is v independent.

- Our local laws of thermodynamics

  – manifest diffeomorphism invariance of the theory at the boundary.

  – account for the dynamics of the part of spacetime 'behind the boundary' which is excised from our spacetime.

- Einstein field equations appear as boundary Hamilton equations, but boundary Hamiltonian is still free to be chosen.

- Second law of thermodynamics and how it can be realized in our setting is an important problem that should be tackled. Focusing theorem may be of use.

- Our analysis provides a new framework to formulate a general memory effect, especially a horizon memory effect.

- The analysis so far is classical and we should quantize the system.

- It should be possible to perform a semiclassical analysis in which the boundary d.o.f are quantized while the bulk is classical.

Focusing on the boundary instead of the bulk and formulating quantum dynamics of the boundary thermodynamic phase space will hopefully shed light on BH micorstate & information puzzle and more generally on the very nature of gravity itself.

Thank You For Your Attention