Testing inflation with small-scale anisotropies

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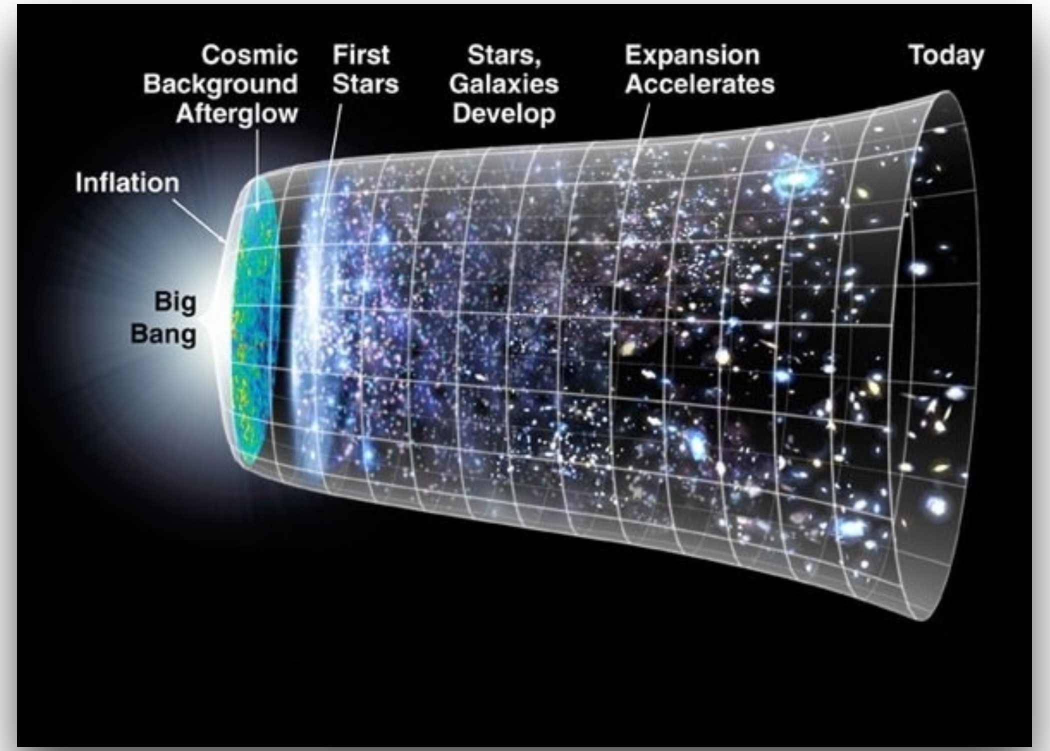
Talk based on papers in collaboration with:

Peter Adshead, Niayesh Afshordi, Matteo Fasiello, Tomohiro Fujita, Marc Kamionkowski, Eugene Lim, Ameek Malhotra, Daan Meerburg, Giorgio Orlando,Maresuke Shiraishi, Gianmassimo Tasinato, David Wands



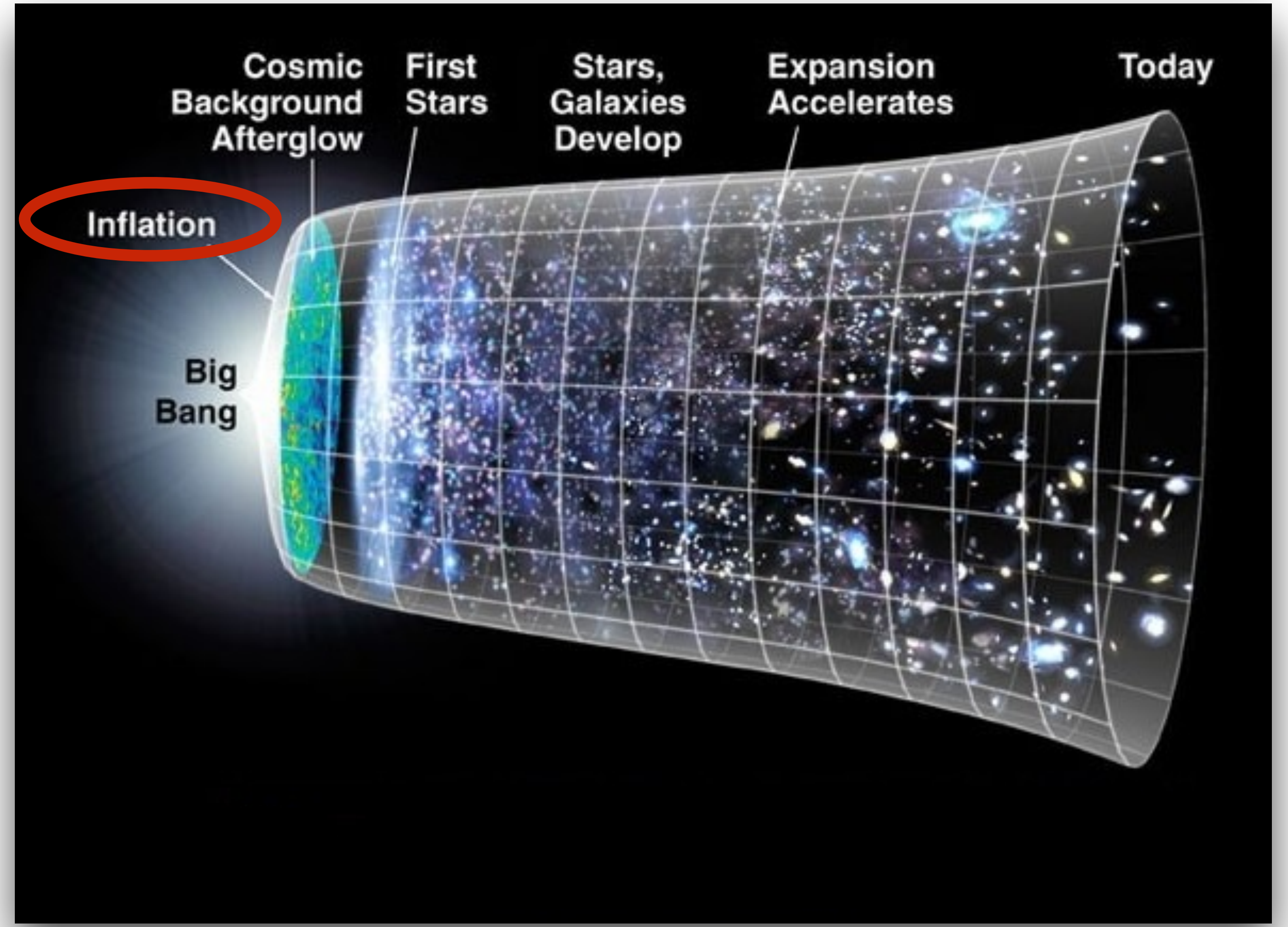
Inflation predicts a stochastic gravitational wave background

- How does it look like?
- What info does it provide on inflation?
- How do we characterise it?

— Frequency profile
— Chirality
— Non-Gaussianity
— Anisotropies

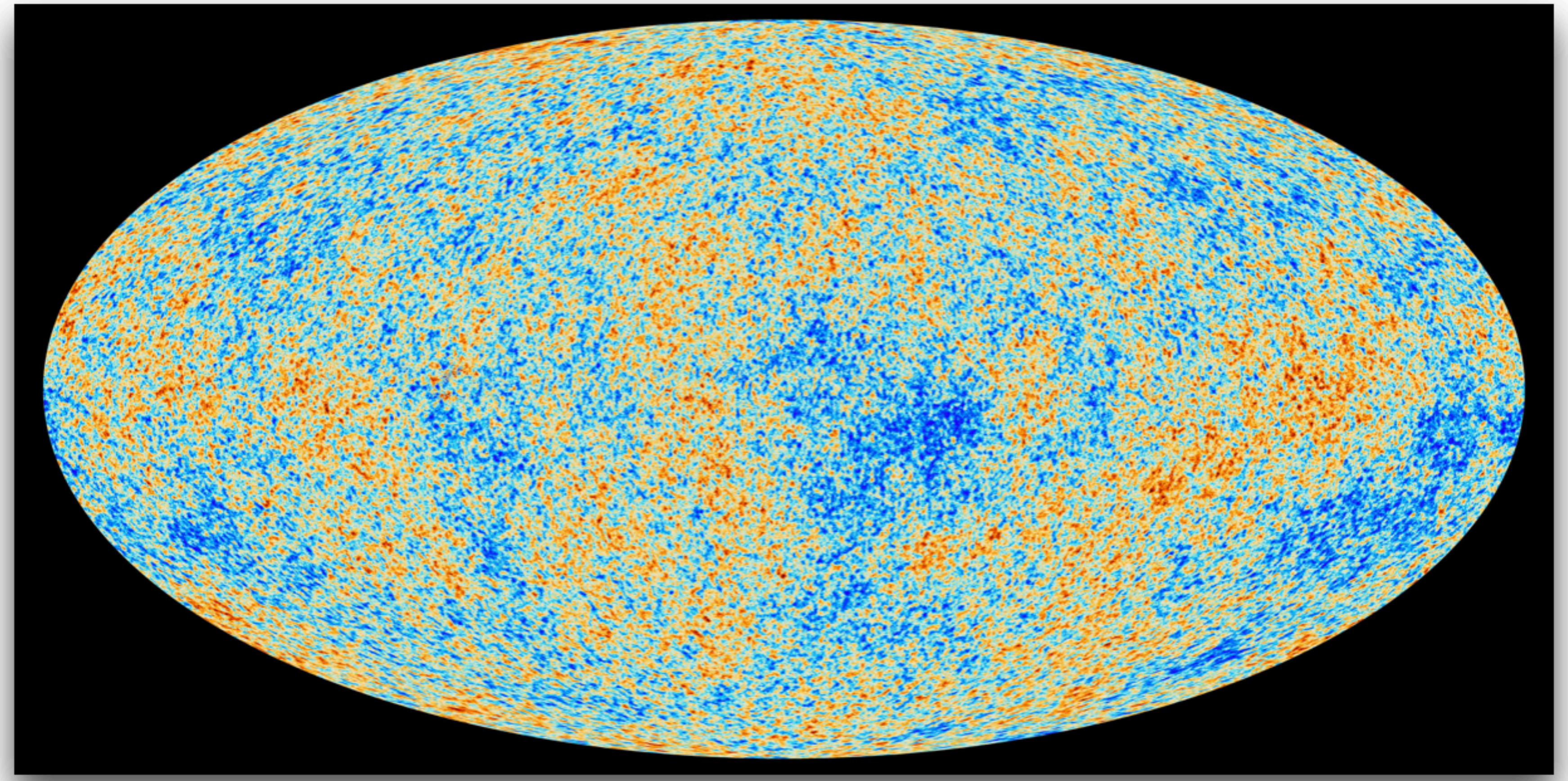
Inflation

- era of accelerated (exponential) expansion



Inflation

- era of accelerated (exponential) expansion
- explains why CMB is nearly uniform



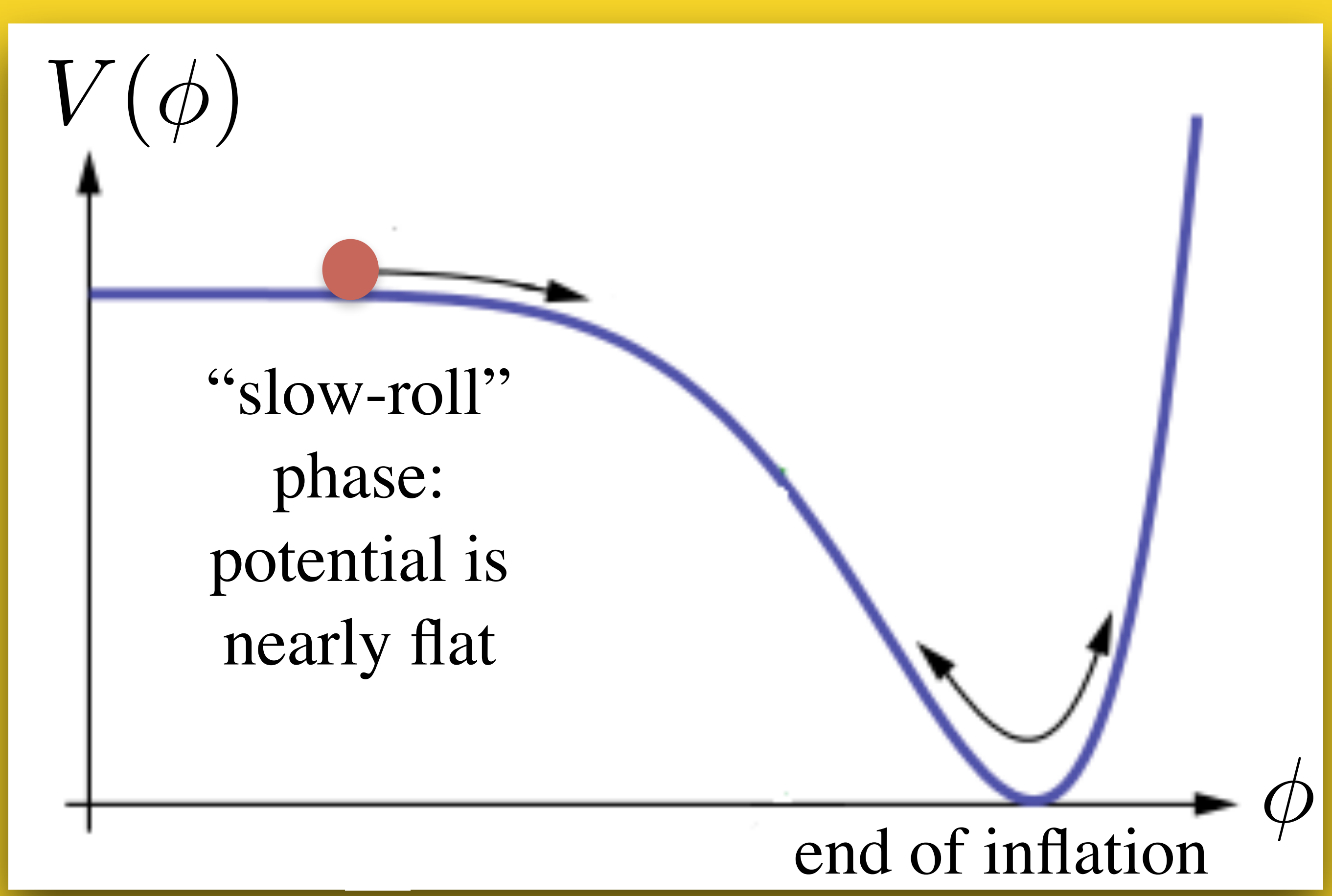
blackbody spectrum: $\bar{T}_{CMB} \sim 2.7\,K$

nearly isotropic: $\Delta T \sim 10^{-5}$

Inflation

- era of accelerated (exponential) expansion
- explains why CMB is nearly uniform
- explains how those fluctuations are generated

Simplest realization: single-scalar field in slow-roll (SFSR)



Inflation

- era of accelerated exponential expansion
- explains why CMB is nearly uniform
- explains how those fluctuations are generated

$$\varphi(\vec{x}, t) = \phi(t) + \delta\phi(\vec{x}, t)$$

classical homogeneous background

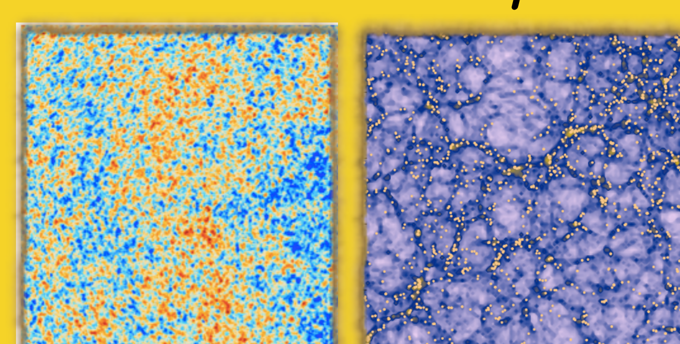
quantum fluctuations

perturbation modes are stretched by the expansion,
become super horizon and freeze out to their value at horizon exit

$$\lambda = a(t)\lambda_c$$

cosmological perturbations

ΔT $\delta\rho$



Inflation

- era of accelerated (exponential) expansion
- explains why CMB is nearly uniform
- explains how those fluctuations are generated
- stochastic gravitational wave background is generated (a key prediction!)

Gravitational waves

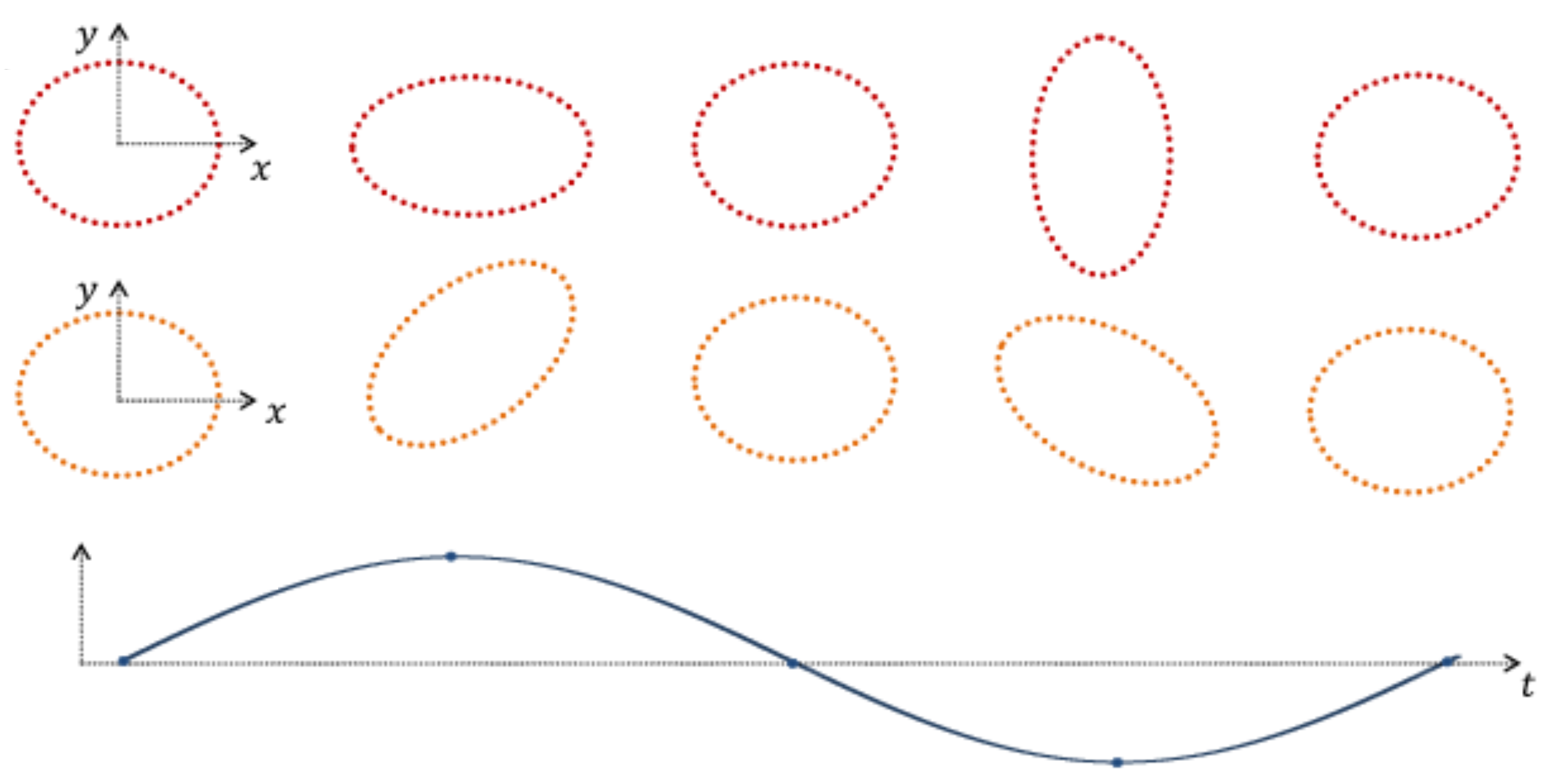
Einstein equations: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

geometry ($G_{\mu\nu}$) — matter/source ($T_{\mu\nu}$)

Perturbation around FRW (homogenous&isotropic) background

$$ds^2 = -dt^2 + a^2(t)\left(\delta_{ij} + \gamma_{ij}\right) dx^i dx^j$$

$$\gamma^i_i = \partial_i \gamma_{ij} = 0$$ → **two polarization states of the graviton: +,×**



Gravitational waves

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G\, \Pi_{ij}^{TT}$$

anisotropic stress-energy tensor
(source term from δT_{ij})

- **homogeneous** solution: GWs from **vacuum fluctuations**

Production of gravitons out of the vacuum
in an expanding universe!

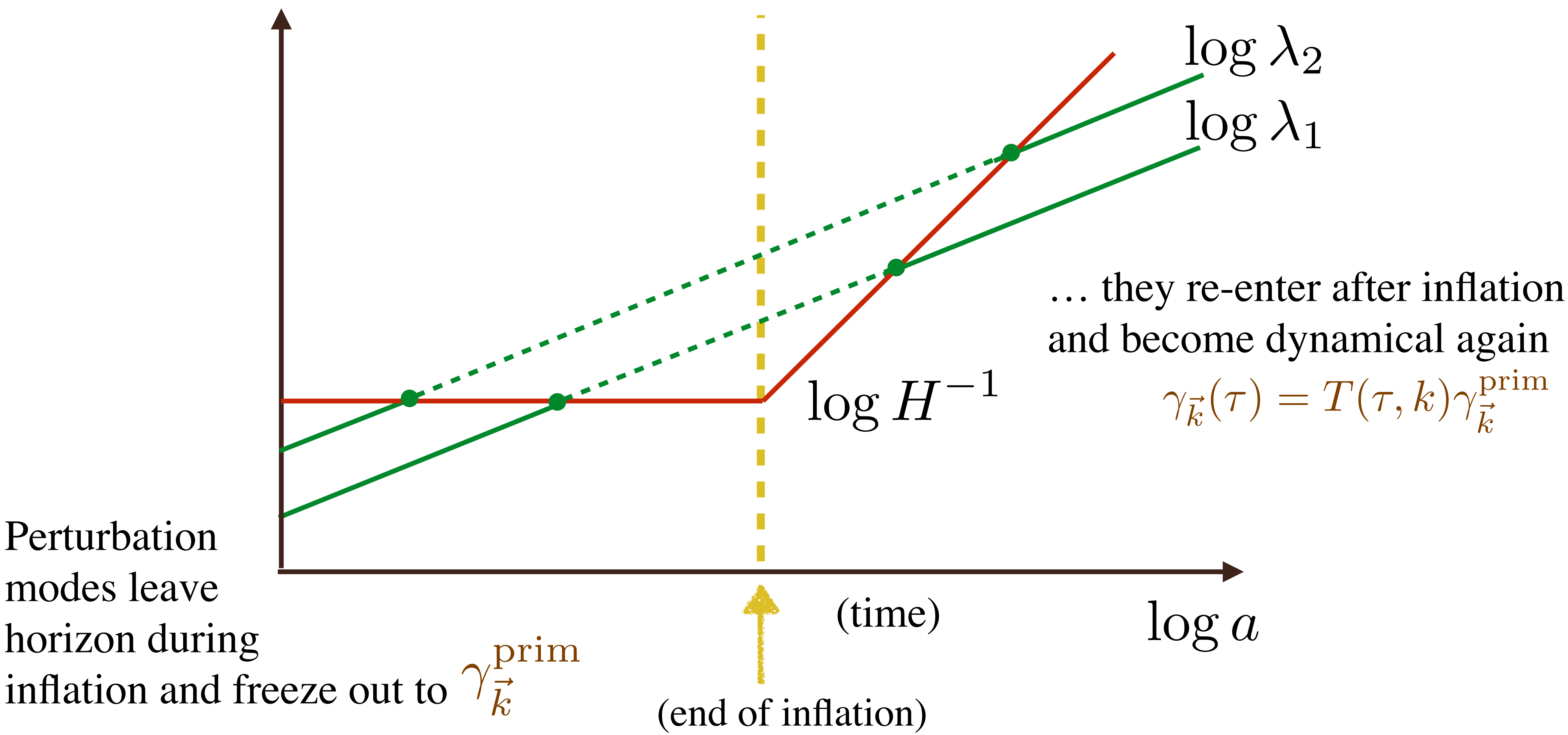
- **inhomogeneous** solution: GWs from **sources**

$\Pi_{ij}^{TT} \propto$ {scalar fields, vector fields, fermions, tensors … }

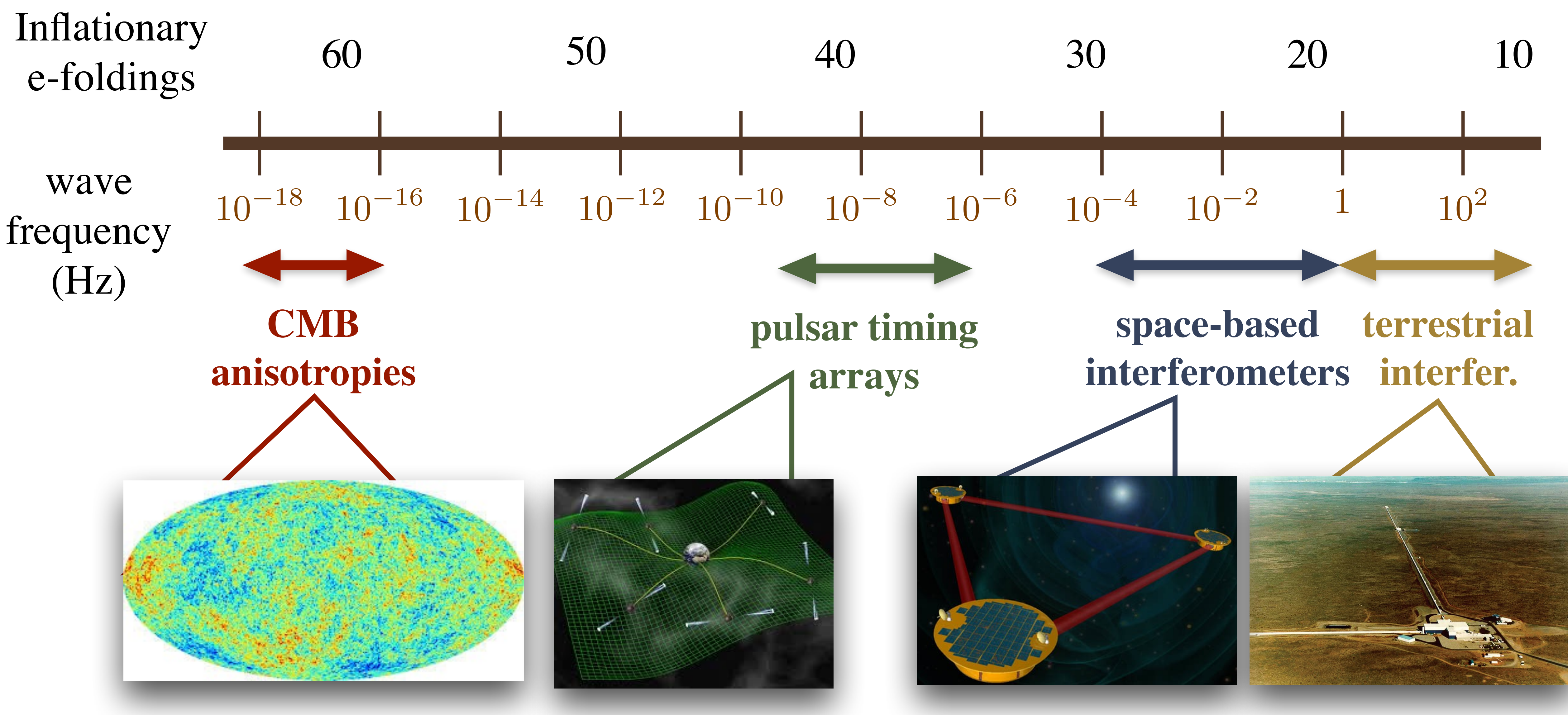
Scales

wavenumber → e-folding → time of re-entry

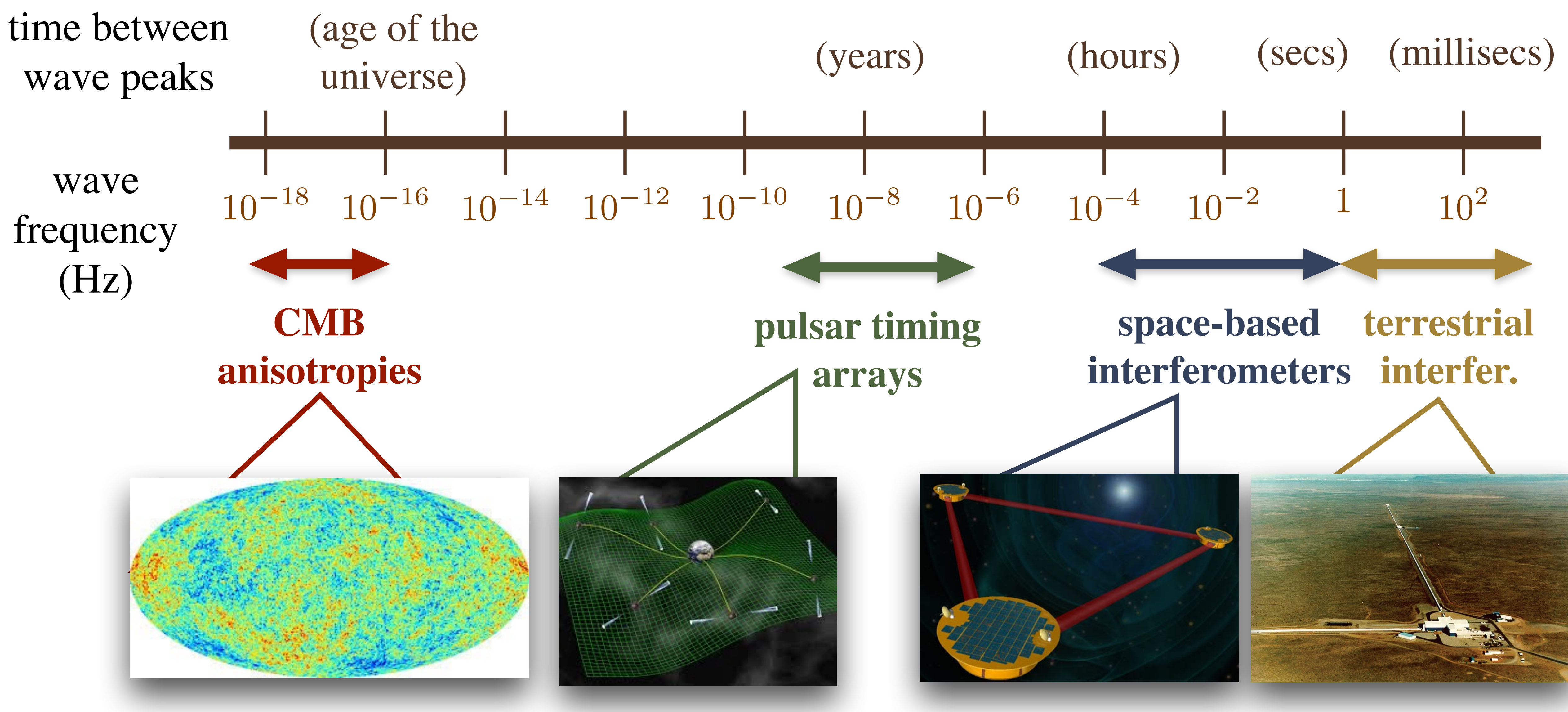
$k \sim f$ $\qquad$ N_k



Scales — Experiments



Scales — Experiments



Inflationary GW from vacuum fluctuations (SFSR)

- **Energy scale** of inflation: $$V_{\rm inf}^{1/4} \simeq 10^{16}{\rm GeV}(r/0.01)^{1/4}$$ $$H \simeq 2\times 10^{13}{\rm GeV}(r/0.01)^{1/2}$$
- Red **tilt**: $n_T \simeq -2\epsilon = -r/8$
- Non-**chiral**: $P_L = P_R$
- Nearly **Gaussian**: $f_{\rm NL} \ll 1$

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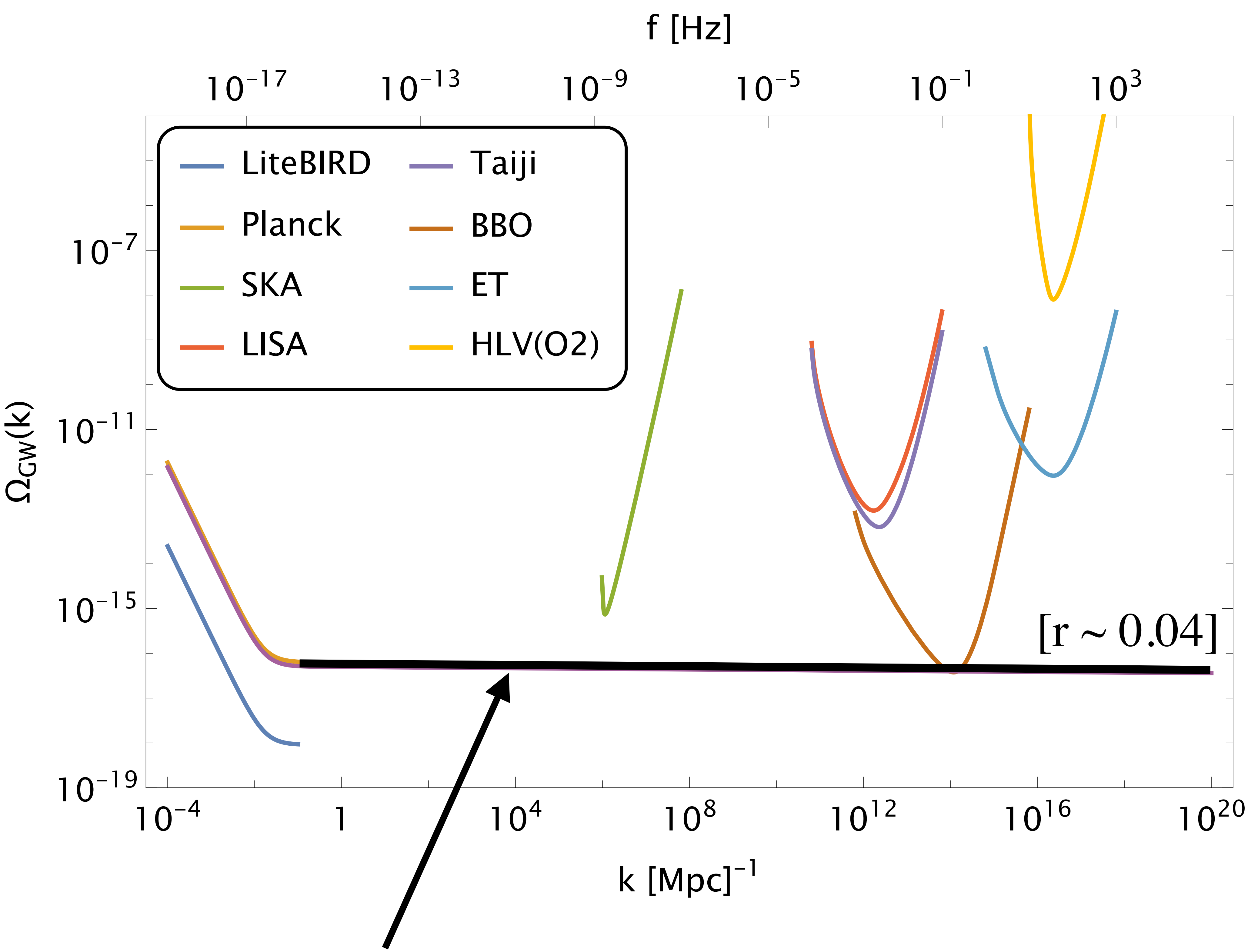
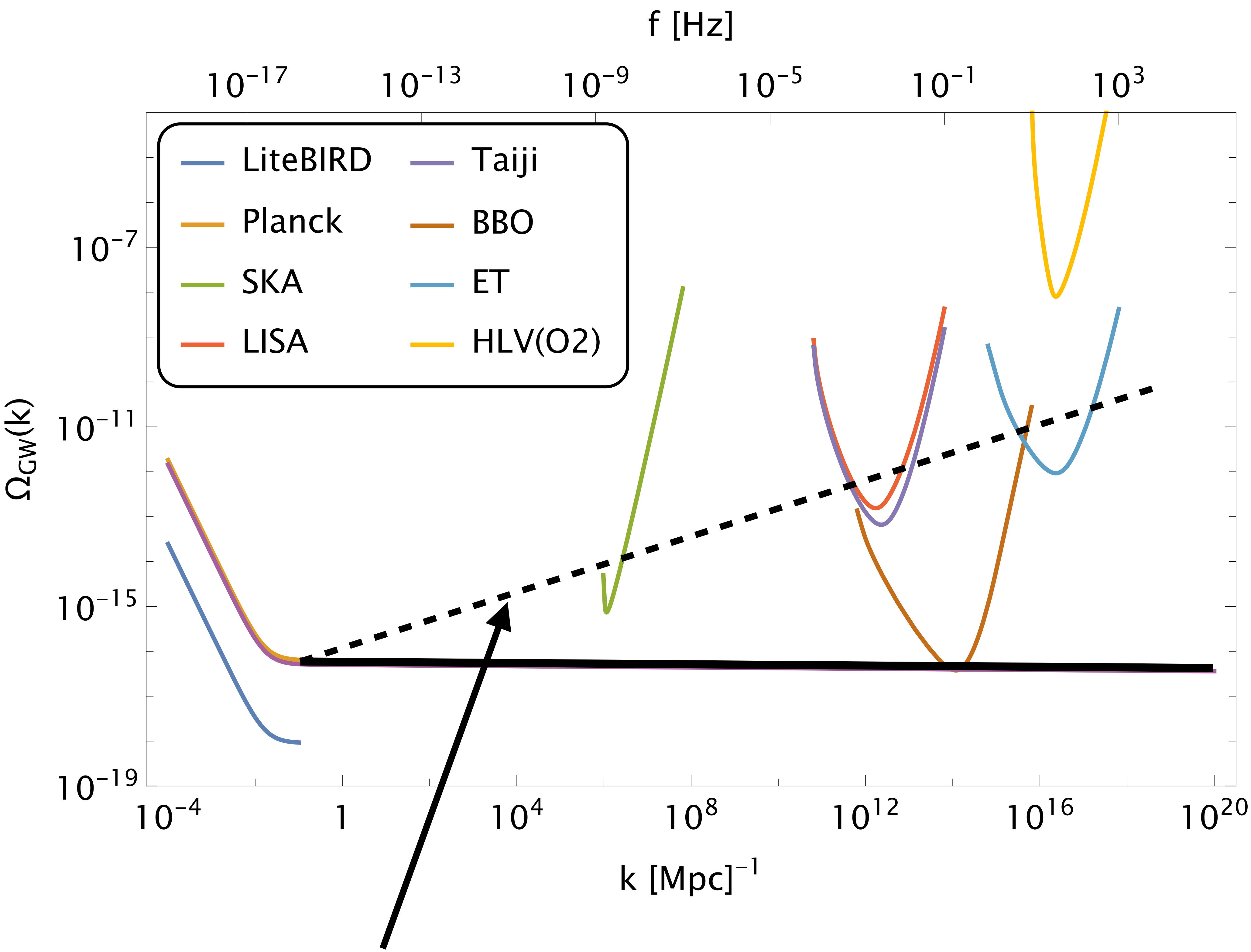
Prediction and sensitivity limits



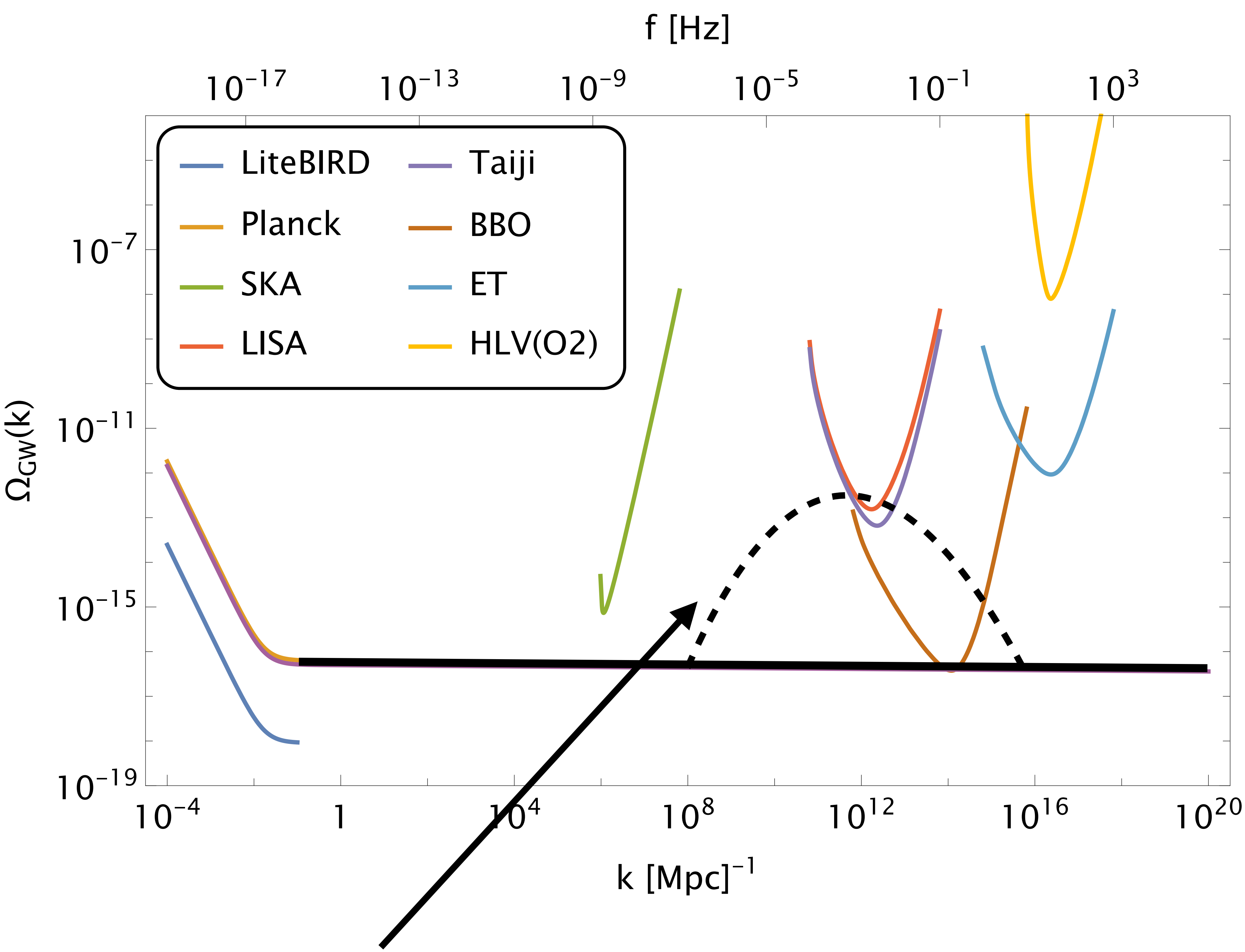
Standard SFSR would go undetected at small scales (**red tilt**)

Prediction and sensitivity limits



Power spectrum larger at small scales: e.g. **blue tilt**

Prediction and sensitivity limits



Power spectrum larger at small scales: e.g. **bump**

Inflationary GW from vacuum fluctuations

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- Red **tilt**: $n_T \simeq -2\epsilon = -r/8$

- Non-**chiral**: $\mathcal{P}_{\gamma,\,L}^{\rm prim} = \mathcal{P}_{\gamma,\,R}^{\rm prim}$

- Nearly **Gaussian**: $f_{\rm NL} \ll 1$

The astrophysical SGWB is expected to be non-chiral

A chiral signal would most certainly be of cosmological origin

Axion-Gauge fields models: Chern-Simons coupling

$$g(\phi)\, F\, \tilde{F}$$

gauge-field strength and its dual

axion-like field

- naturally light inflaton
- support reheating
- mechanism for baryogenesis
- primordial black holes formation
- **sourced chiral gravitational waves**

[Freese - Frieman - Olinto 1990, Anber - Sorbo 2009, Cook - Sorbo 2011, Barnaby - Peloso 2011, Adshead - Wyman 2011, Maleknejad - Sheikh-Jabbari, 2011, ED - Fasiello - Tolley 2012, ED - Peloso 2012, Namba - ED - Peloso 2013, Adshead - Martinec -Wyman 2013, ED - Fasiello - Fujita 2016, Garcia-Bellido - Peloso - Unal 2016, Agrawal - Fujita - Komatsu 2017, Fujita - Namba - Obata 2018, Domcke - Mukaida 2018, Iarygina - Sfakianakis 2021, …]

Axion-Gauge fields models: SU(2)

[Adshead - Wyman 2011]

$$\mathcal{L} = \mathcal{L}_{\rm inflaton} \overbrace{- \frac{1}{2}(\partial\chi)^2 - U(\chi) - \frac{1}{4}FF + \frac{\lambda\chi}{4f}F\tilde{F}}$$

$\mathcal{L}_{\rm inflaton} \rightarrow P_{\gamma,\rm vacuum}$

$\mathcal{L}_{\rm spectator} \rightarrow P_{\gamma,\rm sourced}$

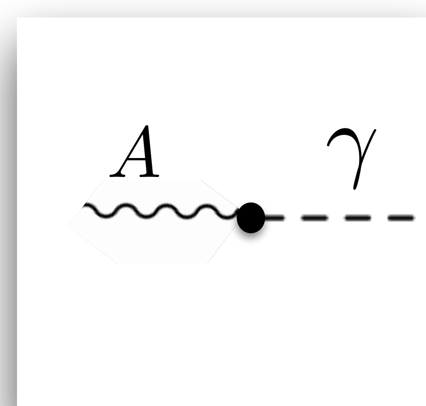
- Inflaton field dominates energy density of the universe
- Spectator sector contribution to curvature fluctuations negligible

$$A_0^a = 0$$
$$A_i^a = aQ\delta_i^a$$

slow-roll background attractor solution

$$\delta A_i^a = t_{ai} + \ldots$$

TT-component



[ED-Fasiello-Fujita 2016]

GW background from sources

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

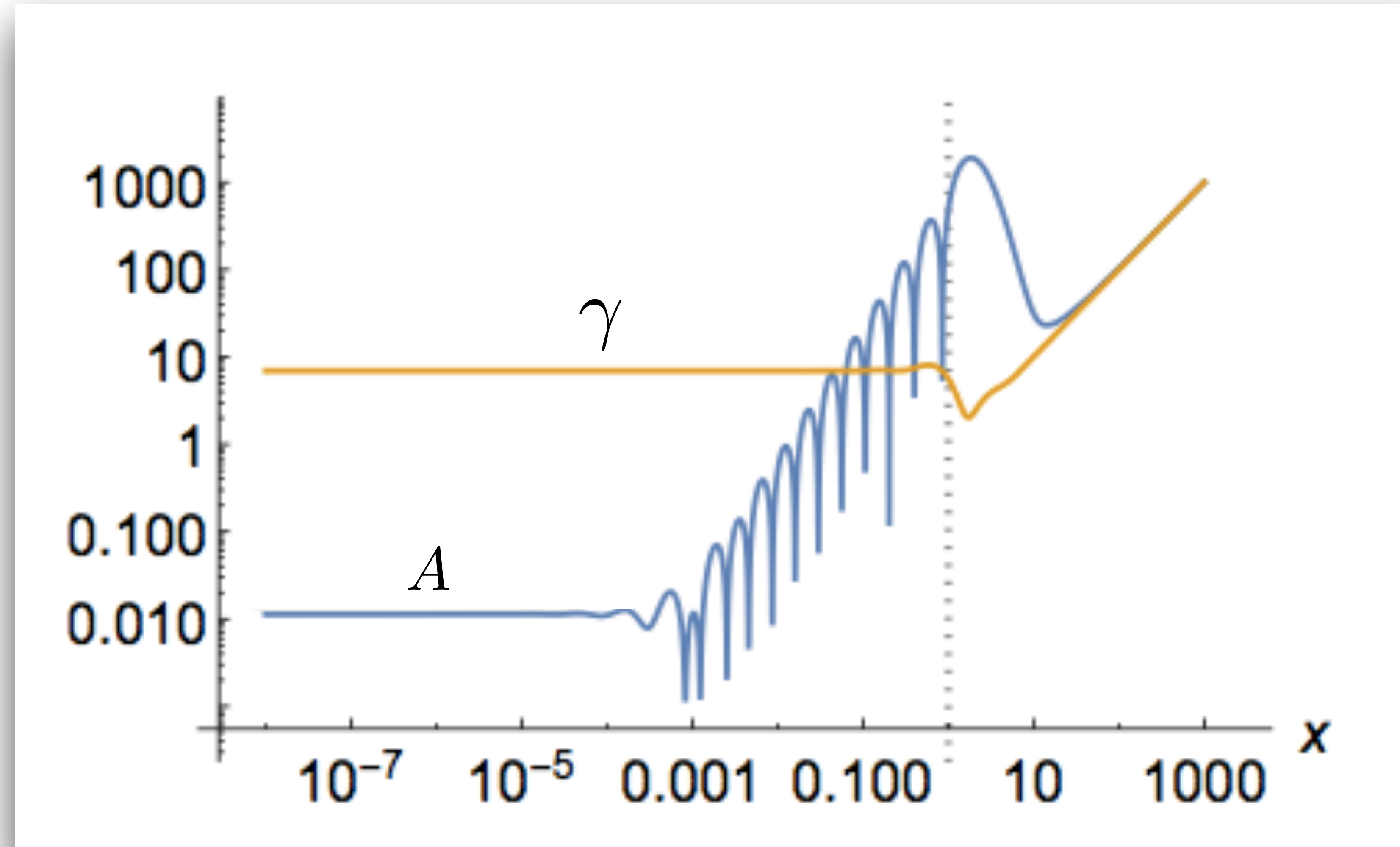
- **homogeneous** solution: GWs from **vacuum fluctuations**

- **inhomogeneous** solution: GWs from **sources**

$\Pi_{ij}^{TT} \supset$ {scalar fields, vector fields, fermions, tensors ... }

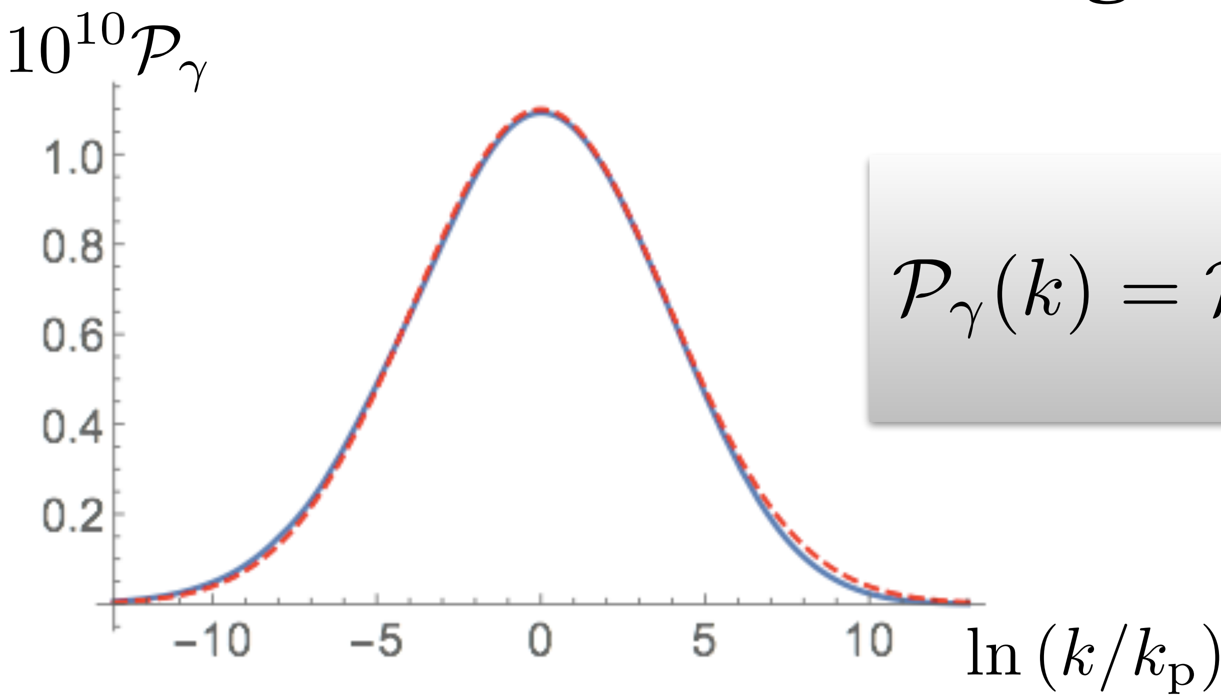
Axion-Gauge fields models: SU(2)

One helicity of the gauge field fluctuations is amplified from coupling with axion → the same helicity of the tensor mode is amplified



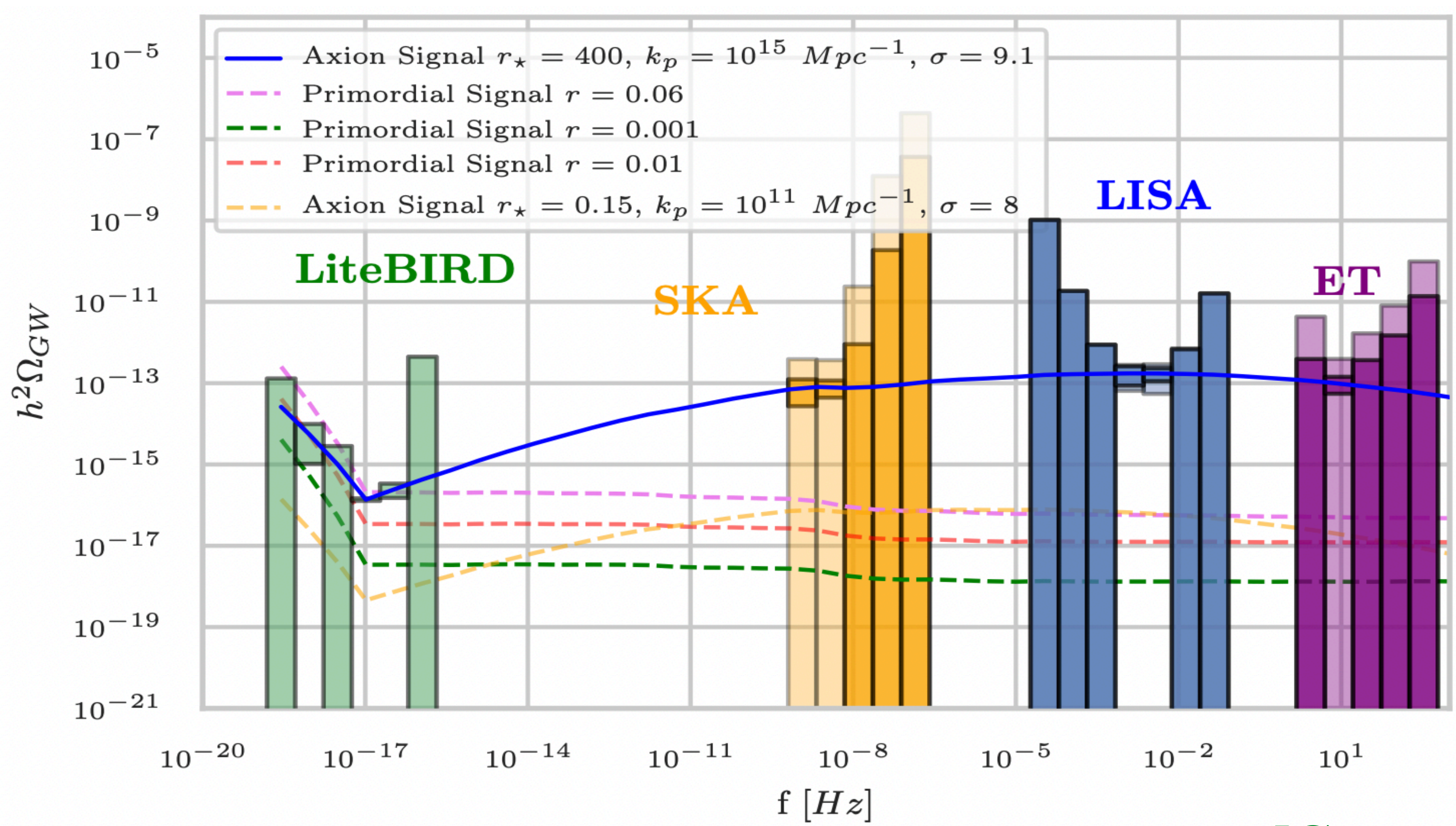
[ED-Fasiello-Fujita 2016]

Axion-Gauge fields models: SU(2)



$$\mathcal{P}_\gamma(k) = \mathcal{P}^{(\mathrm{sourced})}_{\gamma,\,L}(k) = r_*\,\mathcal{P}_\zeta(k)e^{-\frac{1}{2\sigma^2}\ln^2(k/k_\mathrm{p})}$$

[ED-Fasiello-Fujita, 2016 — Thorne et al, 2017]



[Campeti et al, 2020]

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- Non-**chiral**: $P_L = P_R$
- Nearly **Gaussian**: $f_{\rm NL} \ll 1$

How do we detect chirality?

CMB angular power spectra & chirality

$$\mathcal{C}_\ell^{XY} = \int dk\, \Delta_\ell^X(k,\eta_0)\Delta_\ell^Y(k,\eta_0)\left[\mathcal{P}_\gamma^R(k) + \epsilon\cdot\mathcal{P}_\gamma^L(k)\right]$$

$$X,\, Y = T,\ E,\ B$$

$$\epsilon = \begin{cases} 1 \text{ for TT, EE, BB, TE} \\ \text{-1 for TB, EB} \end{cases}$$

For parity-conserving theories $\langle TB\rangle,\ \langle EB\rangle = 0$

For parity-violating theories $\langle TB\rangle,\ \langle EB\rangle \neq 0$

Axion-Gauge fields models: SU(2)

axion-gauge field model

$\gamma_L \neq \gamma_R$

chiral

$\langle TB \rangle, \langle EB \rangle \neq 0$

Detectable at 2σ by LiteBIRD for r > 0.03

[Thorne et al, 2017]

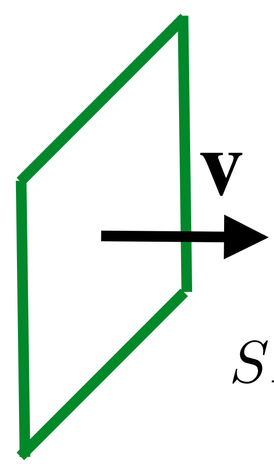
Constraining chirality at high frequencies (interferometers)

A **planar** detector cannot distinguish L from R
for an isotropic SGWB

An ‘effective’ **non-planar** geometry can be realised by:

— using different (non co-planar) detectors at once → monopole

— exploiting the motion of a detector → higher multiples



use of kinematically induced dipole:

$$SNR \simeq \frac{v}{10^{-3}} \frac{\Omega_{\rm GW,\it R} - \Omega_{\rm GW,\it L}}{1.4 \cdot 10^{-11}} \sqrt{\frac{T}{3\,\rm years}}$$

(LISA, ET)

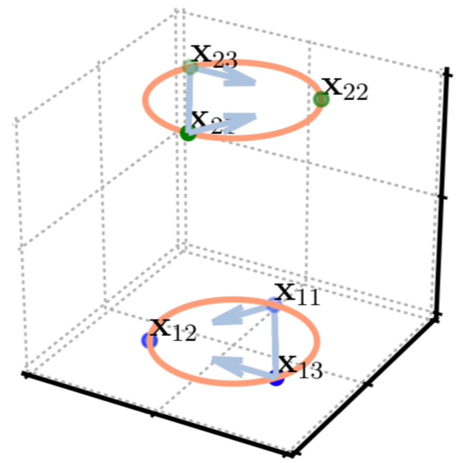
* For (networks of) space-based interferometers see: Domcke et al, 2020 - Orlando et al., 2021

* For ground-based networks see, e.g. : Seto-Taruya, 2007— Smith-Caldwell, 2017

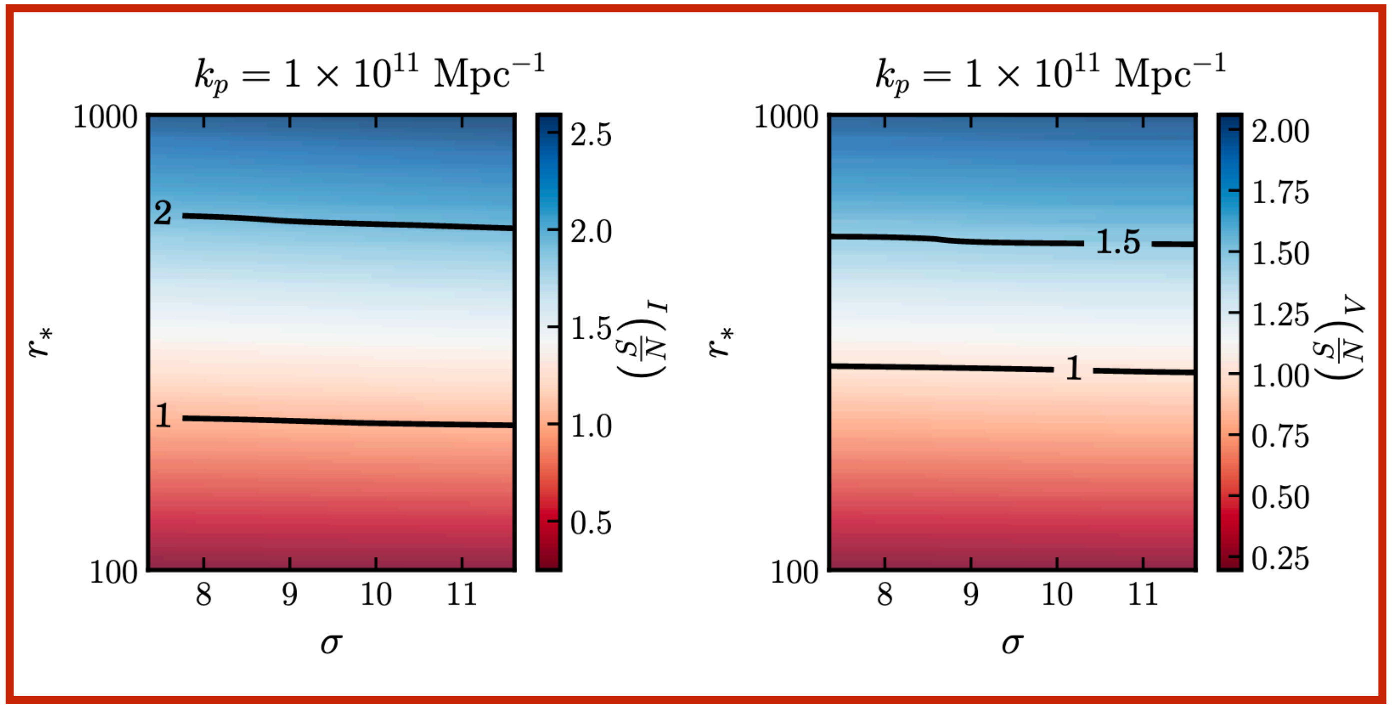
* For PTA: Belgacem-Kamionkowski, 2020

[See also Seto 2006-2007]

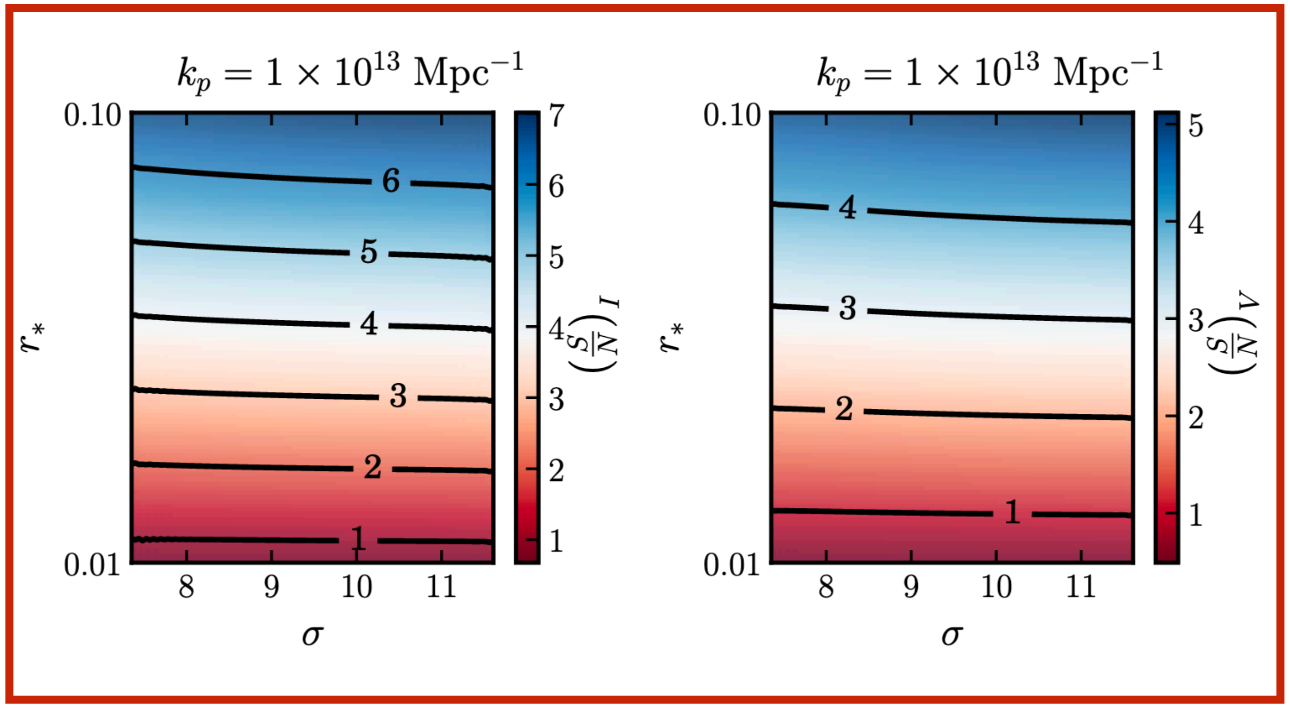
Cross-correlating signal from different detectors



Forecasts for axion-gauge field model [Komatsu et al 2017]



LISA-like



BBO-like

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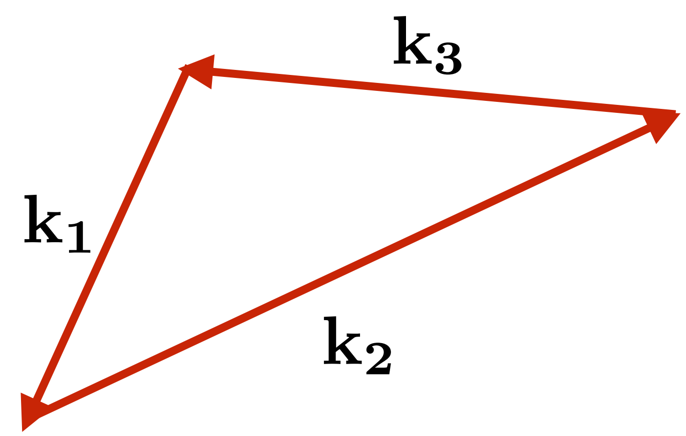
Non-Gaussianity: beyond the power spectrum

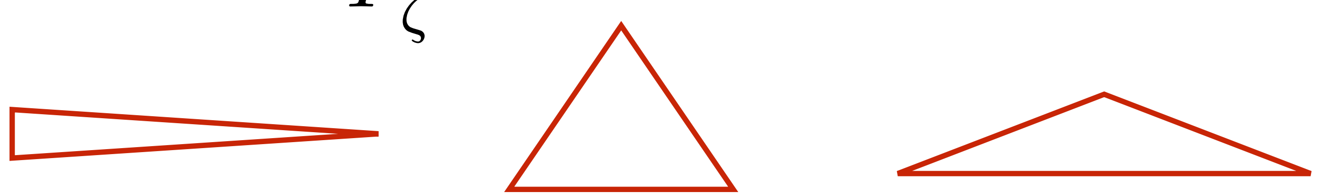


$$\langle \gamma^{\lambda_1}_{\mathbf{k_1}} \gamma^{\lambda_2}_{\mathbf{k_2}} \gamma^{\lambda_3}_{\mathbf{k_3}} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}) B_\gamma^{\lambda_1\lambda_2\lambda_3}(k_1,k_2,k_3)$$

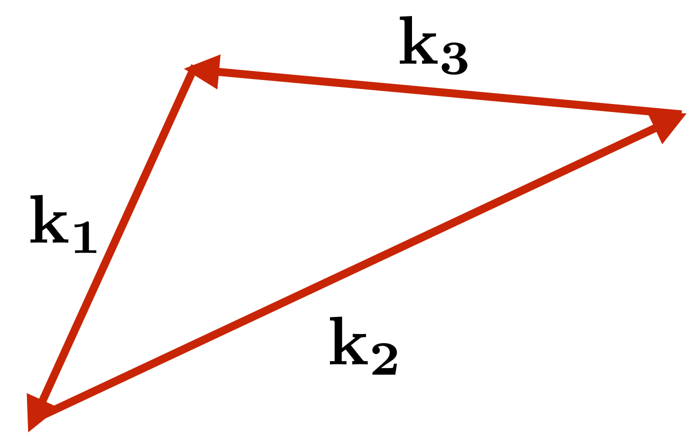
tensor bispectrum

amplitude: $f_{NL} = \dfrac{B}{P_\zeta^2}$

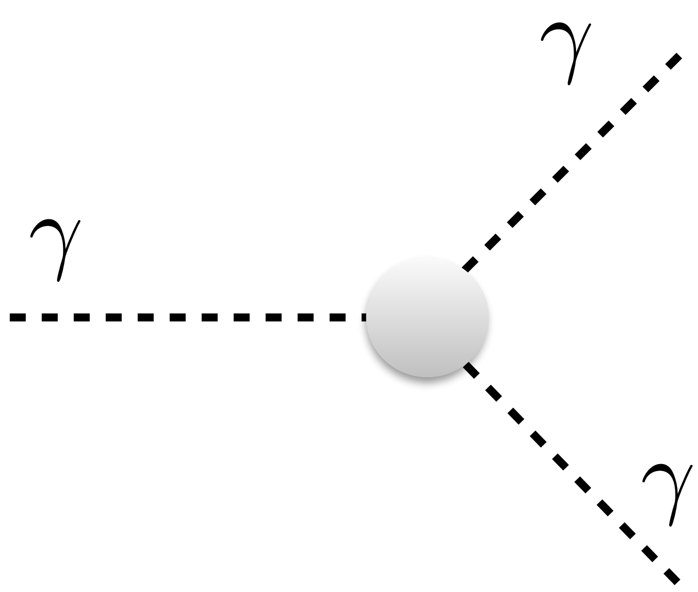
shape:



Tensor non-Gaussianity

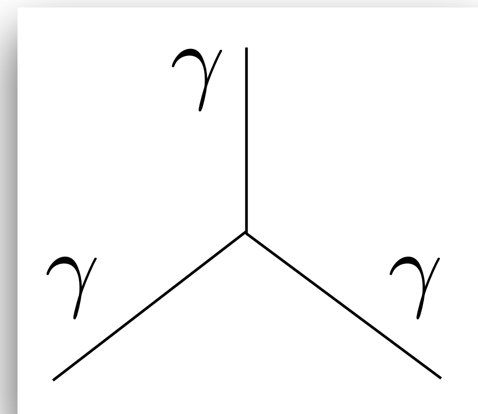


from interactions of the tensors with other fields or from self-interactions



Tensor non-Gaussianity

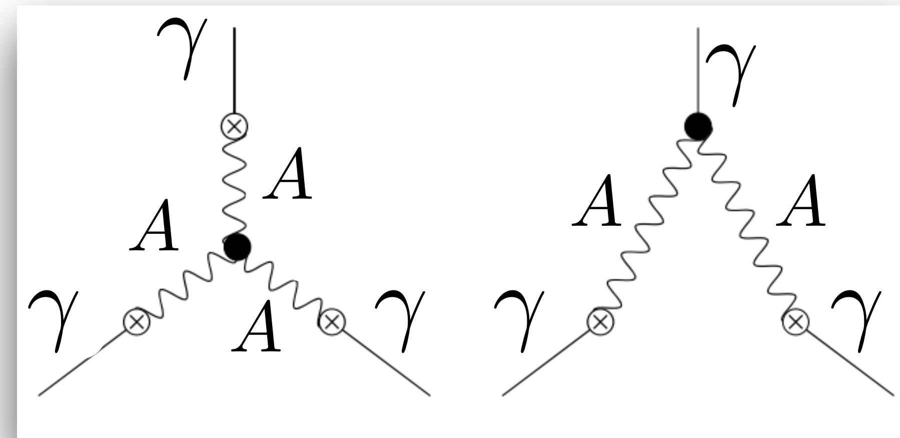
basic single-field inflation



$$f_{NL} = \mathcal{O}(r^2)$$

too small for detection

axion-gauge fields models



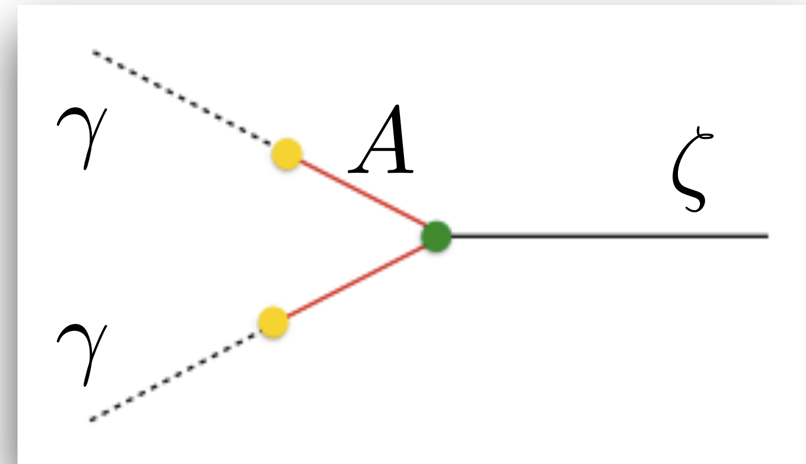
$$f_{NL} = r^2 \cdot \frac{25}{\Omega_A} \gtrsim \mathcal{O}(r^2 \cdot 10^6)$$

- detectable by upcoming CMB space missions

[Agrawal - Fujita - Komatsu 2017]

Mixed (scalar-tensor) non-Gaussianity

testing interactions of tensors and matter fields



[ED - Fasiello - Hardwick - Koyama - Wands 2018]

Mixed (scalar-tensor) non-Gaussianity

basic single-field inflation
(tensor-scalar-scalar)

$$f_{NL} = \mathcal{O}(r)$$

too small for detection

axion-gauge fields models
(tensor-scalar-scalar)

$$f_{NL} \gtrsim 10^5 \cdot r$$

[ED - Fasiello - Hardwick - Koyama - Wands 2018]

potentially observable with CMBS4

[CMBS4 science book, 2016]

Non-Gaussianity at interferometers

Shapiro time delay:

$$\gamma^{''} + 2\mathcal{H}\gamma^{'} - [1 + (12/5)\zeta]\,\gamma_{,kk} = 0$$

GW propagating in FRW background
+ long-wavelength perturbations

$$\gamma_{ij} = A_{ij}\, e^{ik\tau + ik\cdot 2\int^{\tau} d\tau'\,\zeta[\tau',(\tau'-\tau_0)\hat{k}]}$$

GW from different directions
undergo different phase shift
due to intervening structure

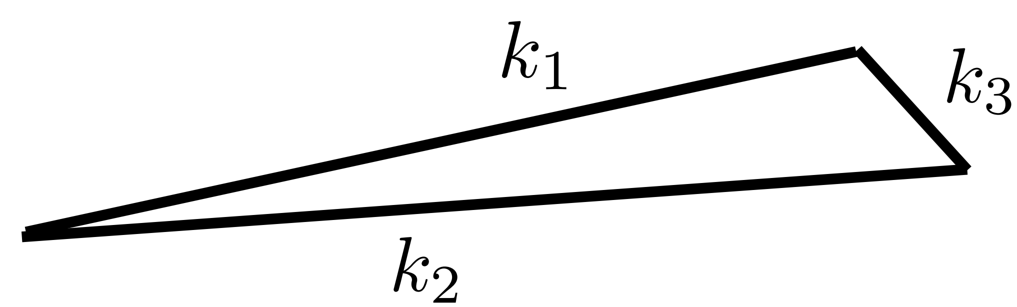
→ decorrelation → cannot measure bispectrum directly with interferometers

[Bartolo, De Luca, Franciolini, Lewis, Peloso, Riotto 2018]

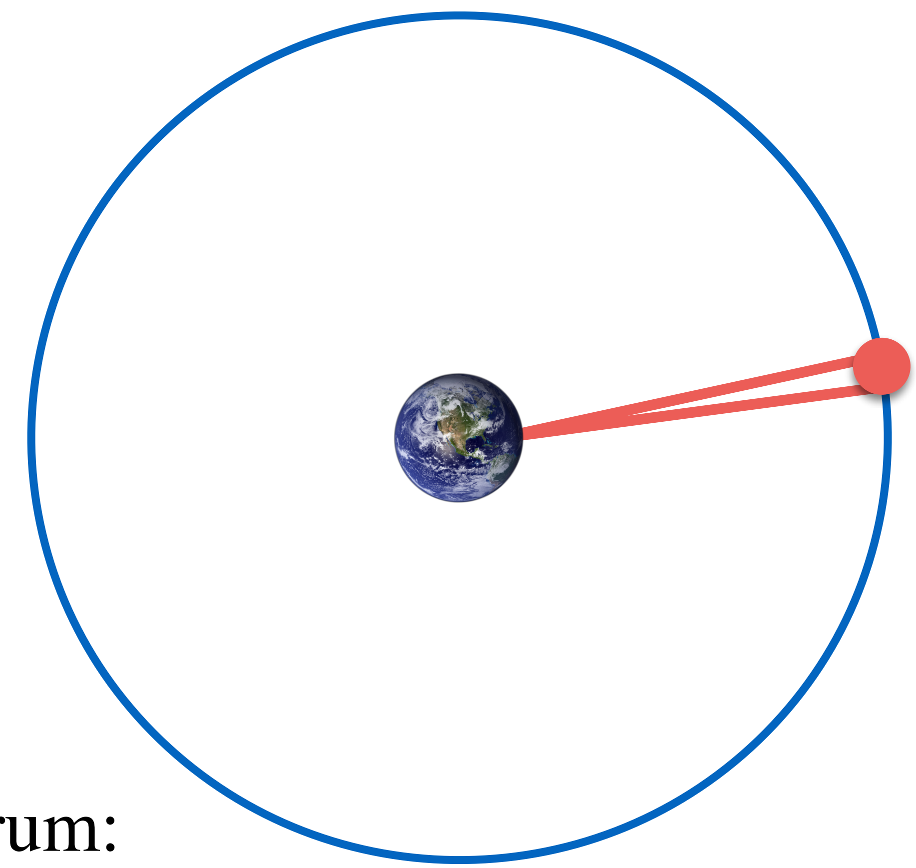
Note: signal measured by an interferometer arises from the superposition
of signals from a large number of Hubble patches (CLT)

[Adshead, Lim 2009 — Caprini, Figueroa 2018 — Bartolo, De Luca, Franciolini, Lewis, Peloso, Riotto 2018]

Ultra squeezed non-Gaussianity



Correlation among two short-wavelength modes (e.g. interferometer scale) and 1 very long-wavelength mode: the latter has not undergone propagation!



Signals originate from the same patch!

How do we constrain this ultra-squeezed bispectrum:

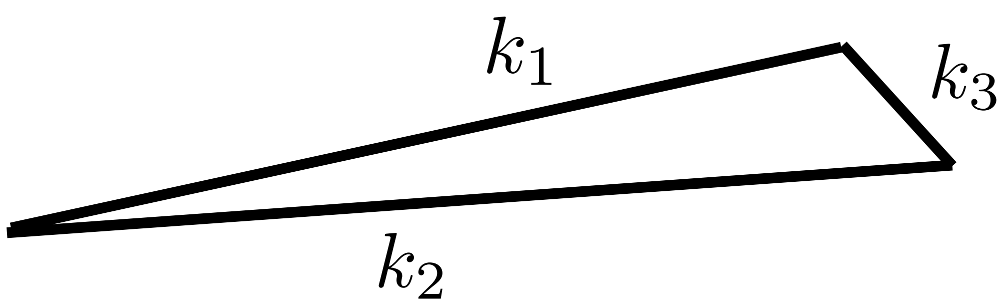
Look for anisotropies in the SGWB!

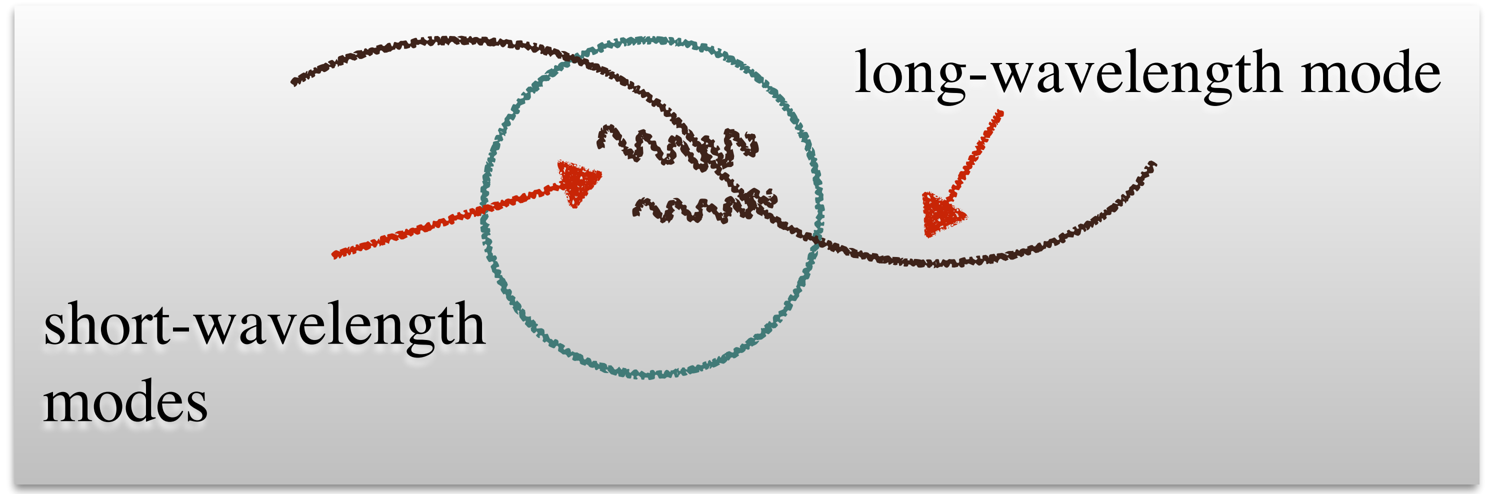
$$\Omega_{\rm GW}(k) = \bar{\Omega}_{\rm GW}(k)\left[1 + \frac{1}{4\pi}\int d^2\hat{n}\,\delta_{\rm GW}(k,\hat{n})\right]$$

[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

Soft limits and ‘fossils’

$k_1 \simeq k_2 \gg k_3$





long wavelength modes introduces a modulation
in the primordial power spectrum of the short wavelength modes

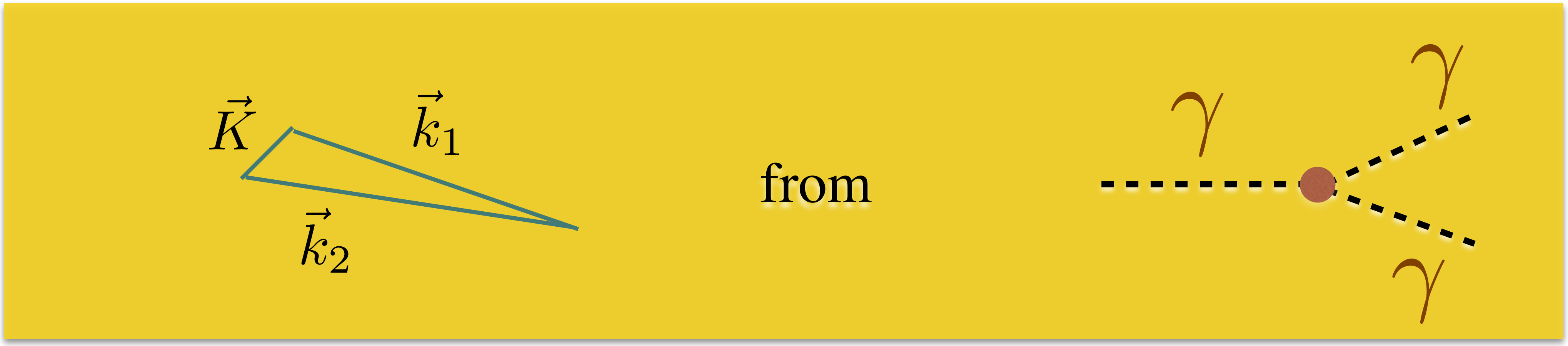
$$B^{F\gamma\gamma} \equiv \langle F_L \gamma_S \gamma_S \rangle' \sim F_L \cdot \langle \gamma_S \gamma_S \rangle'_{F_L}$$

$$\delta\langle \gamma_S \gamma_S \rangle \equiv \langle \gamma_S \gamma_S \rangle_{F_L} \sim \frac{B^{F\gamma\gamma}}{P_F(k_3)} \cdot F_L^* = P_\gamma(k_1) \cdot \underbrace{\frac{B^{F\gamma\gamma}}{P_F(k_3) P_\gamma(k_1)}}_{f_{\rm NL}^{F\gamma\gamma}} \cdot F_L^*$$

$$\langle \gamma_S \gamma_S \rangle'_{\rm total} = P_\gamma(k_1) \left(1 + f_{\rm NL}^{F\gamma\gamma} \cdot F_L^*\right)$$

[ED, Fasiello, Jeong, Kamionkowski - 2014, ED, Fasiello, Kamionkowski - 2015, …]

Soft limits and fossils



$$P_\gamma^{\rm mod}(\mathbf{k},\mathbf{x}) = P_\gamma(k)\left[1+\mathcal{Q}_{\ell m}(\mathbf{k},\mathbf{x})\,\hat{n}_\ell \hat{n}_m\right]$$

$$\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}} \sum_{\lambda_3} h_{\ell m}^{\lambda_3}(\mathbf{q}) F_{\rm NL}^{\rm ttt}(\mathbf{k},\mathbf{q})$$

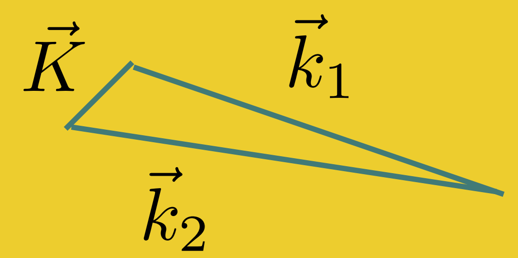
$$\delta_{\rm GW}(k,\hat{n}) = \mathcal{Q}_{\ell m}(\mathbf{k},\mathbf{d})\,\hat{n}_\ell \hat{n}_m$$

$$\mathbf{d} = -(\eta_0-\eta_{\rm in})\hat{n}$$

$$\Omega_{\rm GW}(k) = \bar{\Omega}_{\rm GW}(k)\left[1+\frac{1}{4\pi}\int d^2\hat{n}\,\delta_{\rm GW}(k,\hat{n})\right]$$

[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

Soft limits and fossils



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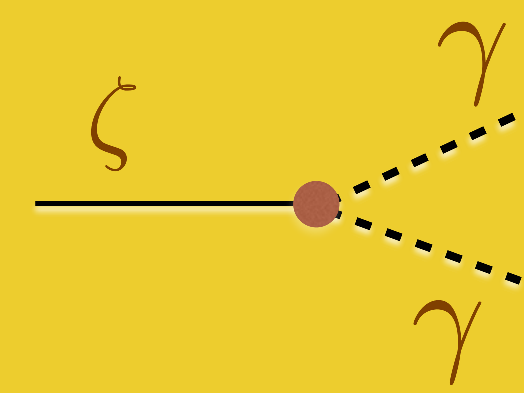


$$\delta_{\rm GW}(k,\hat{n}) = \int \frac{d^3q}{(2\pi)^3} e^{-id\hat{n}\cdot\mathbf{q}} \zeta(\mathbf{q}) F_{\rm NL}^{\rm stt}(\mathbf{k},\mathbf{q})$$

$$\mathbf{d} = -(\eta_0 - \eta_{\rm in})\hat{n}$$

$$\Omega_{\rm GW}(k) = \bar{\Omega}_{\rm GW}(k)\left[1 + \frac{1}{4\pi}\int d^2\hat{n}\,\delta_{\rm GW}(k,\hat{n})\right]$$

[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

Soft limits in inflation

- ***Extra*** *fields* / superhorizon ***evolution***

  [Chen - Wang 2009, Baumann - Green 2011, Chen et al 2013,
  ED - Fasiello - Kamionkowski 2015, …]

- ***Non-Bunch Davies*** initial states

  [Holman - Tolley 2007, Ganc - Komatsu 2012, Brahma - Nelson - Shandera 2013, …]

- ***Broken space diffs***
  (e.g. space-dependent background)

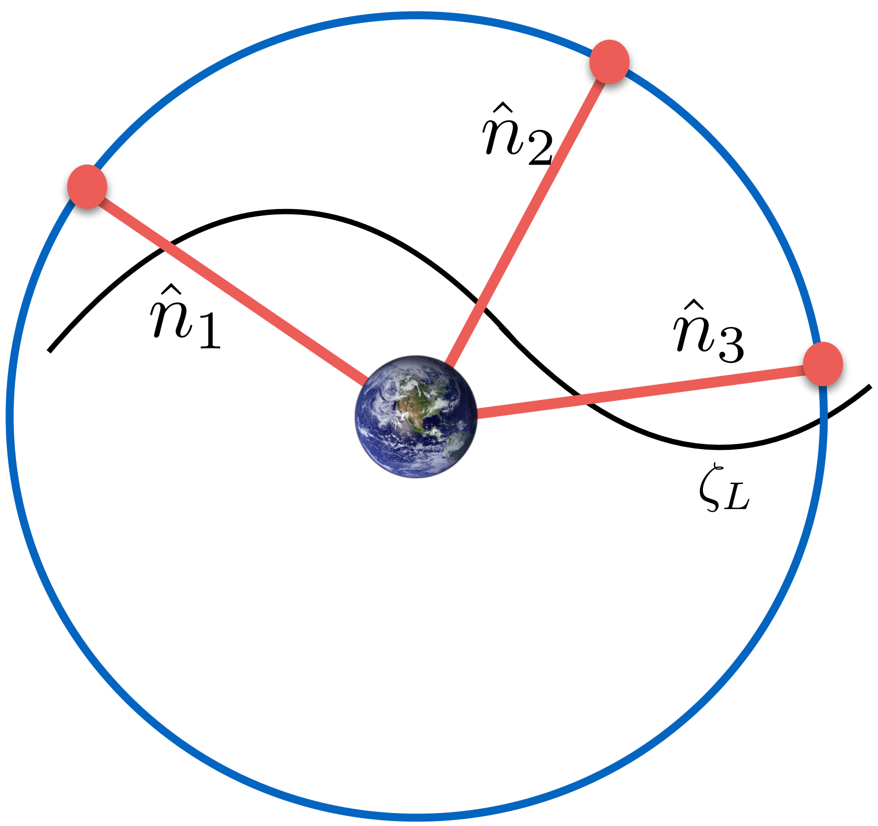
  [Endlich et al. 2013, ED - Fasiello - Jeong - Kamionkowski 2014, …]

Ideal probe for (extra) fields, pre-inflationary dynamics, (non-standard) symmetry patterns

Any other kinds of anisotropies expected in the SGWB?

GW propagate through the perturbed universe, so they are subject to Sachs-Wolfe / integrated Sachs-Wolfe, … , just like CMB photons

Simplified treatment in **[Alba, Maldacena 2015]**:
large-scale gravitational potential ⟶ SW dominates



$$\frac{\delta f(\hat{n})}{f} = \frac{1}{5}\left[\zeta_{L(\text{today})} - \zeta_L(\hat{n}\cdot\eta_0)\right]$$

Gravitational redshift/blueshift of gravitons

$$\zeta_L(\hat{n}_1\cdot\eta_0) \neq \zeta_L(\hat{n}_2\cdot\eta_0)$$

↓

Direction-dependent frequency shift

↓

Anisotropy in the GW energy density $\delta_{\rm GW}(f,\,\hat{n}) \sim \frac{\alpha}{5}\cdot\zeta_{\rm L}(\hat{n}\cdot\eta_0)$

$\bar{\Omega}_{\rm GW}(f) \sim \left(\frac{f}{f_0}\right)^{\alpha}$

[See: Contaldi, 2017 — Bartolo, Bertacca, Matarrese, Peloso, Ricciardone, Riotto, Tasinato, 2019 — for full Boltzmann treatment of GW anisotropies]

Any other kinds of anisotropies expected in the SGWB?

GW propagate through the perturbed universe, so they are subject to Sachs-Wolfe / integrated Sachs-Wolfe, … , just like CMB photons

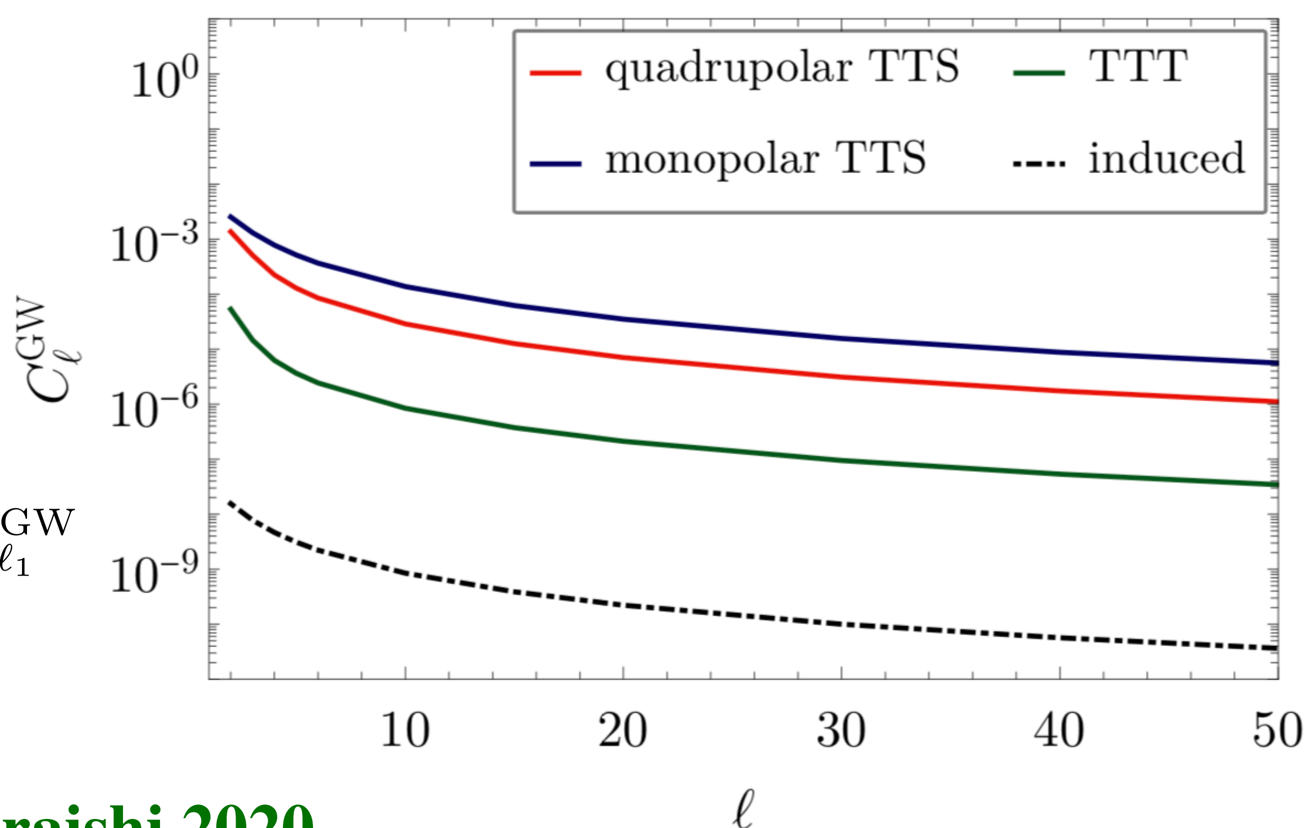
large-scale gravitational potential ⟶ SW dominates

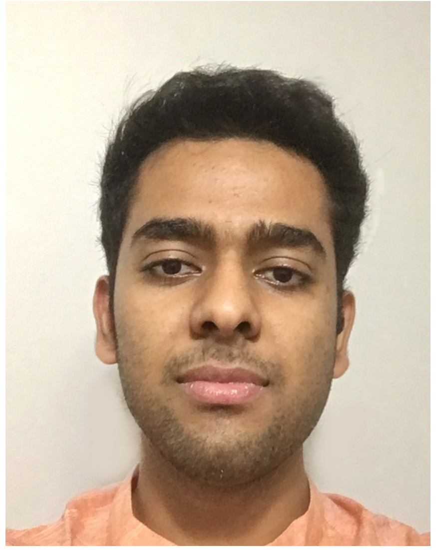
$$\frac{\delta f}{f} \sim -\frac{\zeta_L}{5} \longrightarrow \delta_{\rm GW} \sim \zeta_L \sim 10^{-5}$$

Note: to be compared with

$$\delta^{\rm stt}_{\rm GW} \sim F^{\rm stt}_{\rm NL} \cdot \zeta_L$$

$$\langle \delta_{{\rm GW},\ell_1 m_1} \delta_{{\rm GW},\ell_2 m_2} \rangle = \delta_{\ell_1 \ell_2} \delta_{m_1 m_2} C^{\rm GW}_{\ell_1}$$





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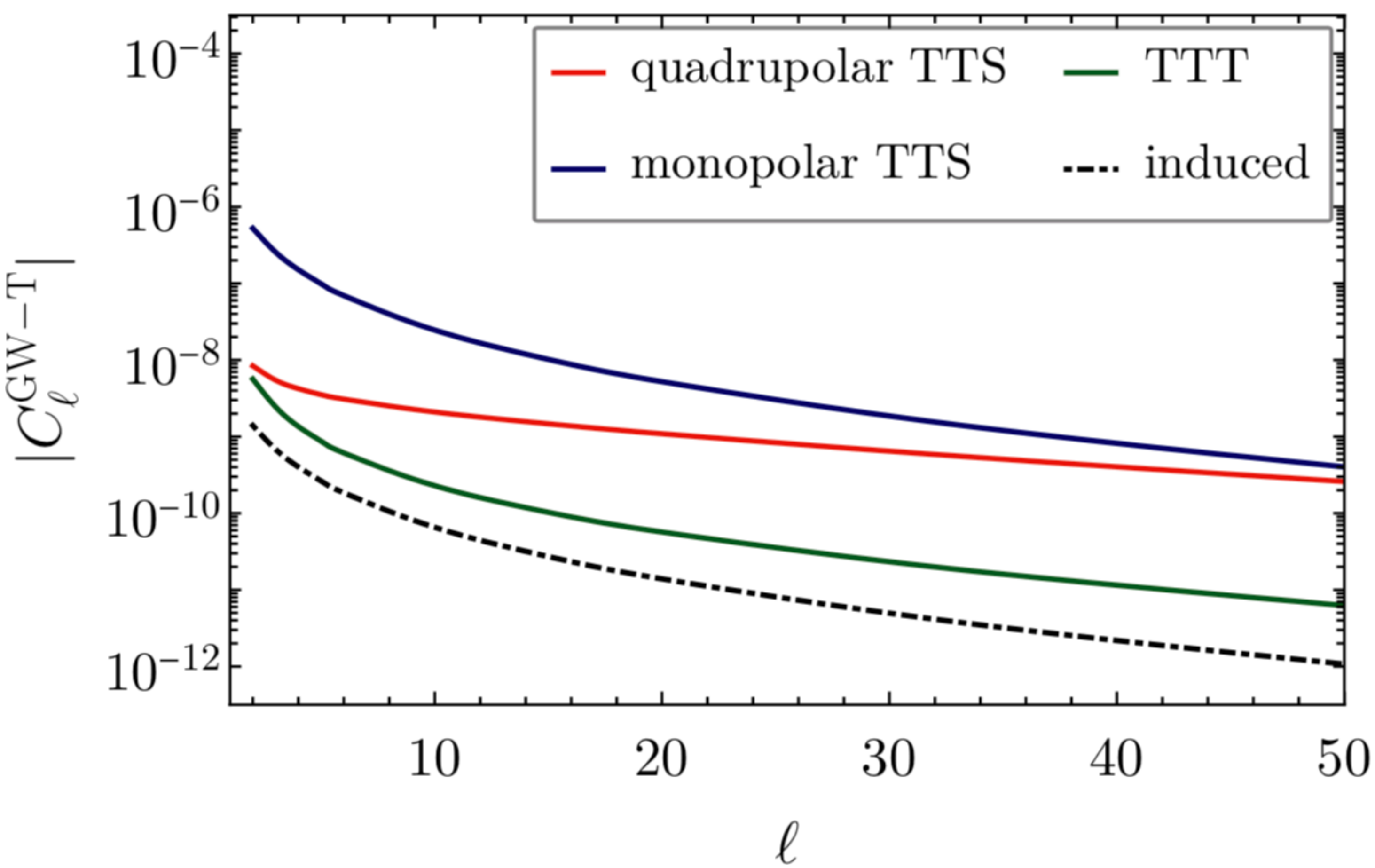
[Malhotra, ED, Fasiello, Shiraishi 2020 -
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

Cross-correlations of GW and CMB anisotropies

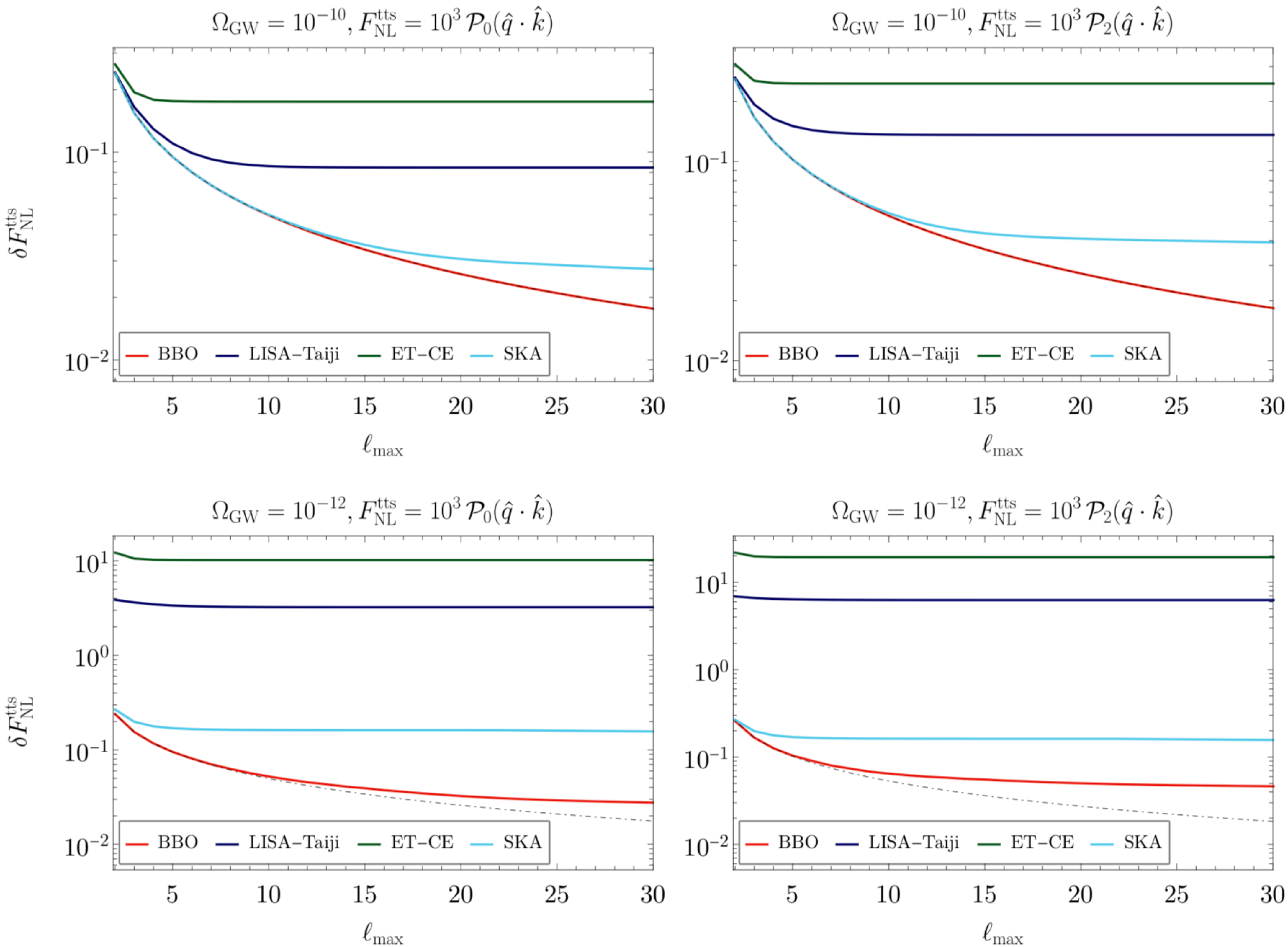
$$\delta_{\rm GW}^{\rm propagation} \sim \zeta_L$$

$$\delta_{\rm GW}^{\rm stt} \sim F_{\rm NL}^{\rm stt} \cdot \zeta_L$$

$$\left.\begin{aligned} \delta_{\rm GW}^{\rm stt} &\sim F_{\rm NL}^{\rm stt} \cdot \zeta_L \\ \frac{\Delta T}{T} &\sim \zeta_L \end{aligned}\right\} \; C_\ell^{\text{GW-T}} \sim F_{\rm NL}^{\rm stt} \cdot C_\ell^{TT}$$



[Adshead, Afshordi, ED, Fasiello, Lim, Tasinato 2020
Malhotra, ED, Fasiello, Shiraishi 2020
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]



(See [ED, Fasiello, Malhotra, Meerburg, Orlando 2021] also for applications to concrete models)

Gravitational waves

- Extremely useful in testing inflation, also at interferometer scales

- A variety of classes of models (beyond the vanilla scenario) generate interesting gravitational waves signatures

- Crucial for disentangling inflationary SGWB from the one due to other cosmological sources and from the astrophysical SGWB:
    - spectral shape
    - chirality
    - non-Gaussianity and anisotropies