Cutting Rule for Cosmological Collider Signals: A Bulk Evolution Perspective

Yuhang Zhu (祝浴航)
Department of Physics, HKUST

With Xi Tong and Yi Wang
Based on arXiv: 2112.03448

Outline

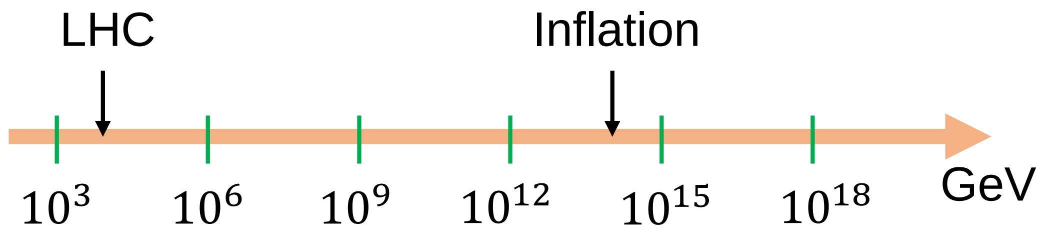
- Introduction
- Classification and interpretation of cosmological collider
- The procedure of cutting rule
- Application
- Conclusion & outlooks

The cosmological collider program

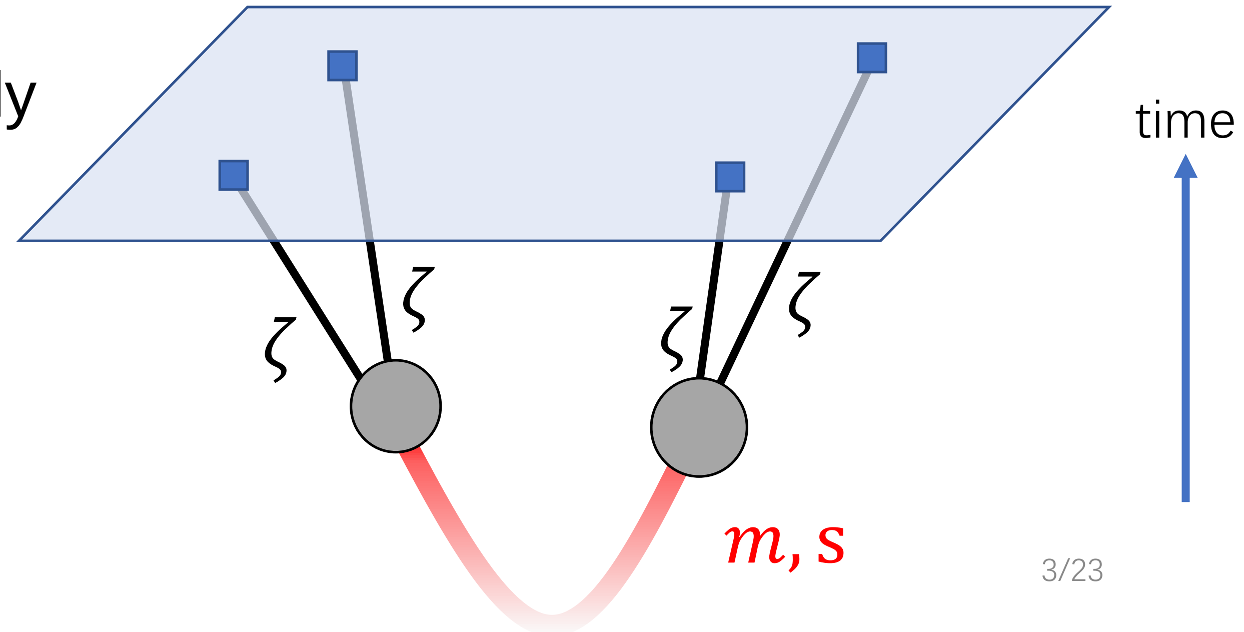
[Chen&Wang, 2009] [Baumann&Green, 2011] [Arkani-Hamed&Maldacena, 2015]

- Inflation of the very early universe

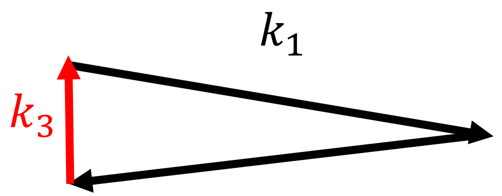
$$a(t) \sim e^{Ht}$$

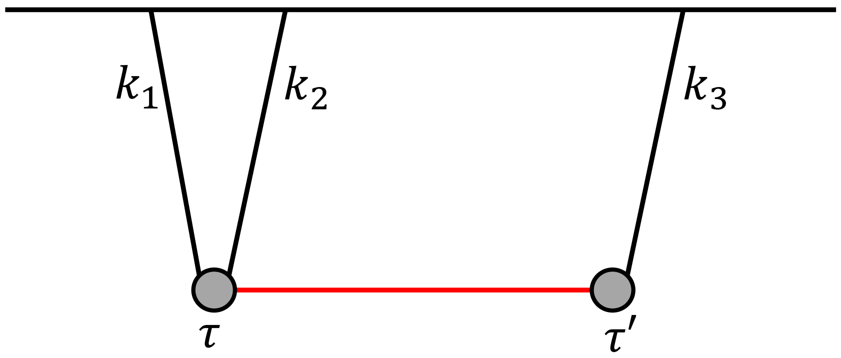


- Use this high energies period of the early universe to study fundamental physics



The cosmological collider program



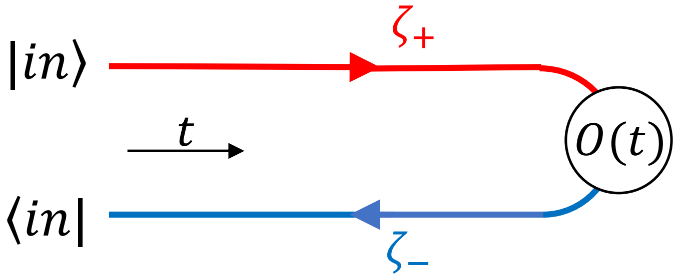


$$S \longrightarrow e^{-\pi\mu}\left(\frac{k_1}{k_3}\right)^{\frac{1}{2}\pm \overbrace{i\mu}^{\text{mass}}} \underbrace{P_s}_{\text{spin}}(\cos\theta)$$

$$\mu = \begin{cases} \sqrt{\dfrac{m^2}{H^2}-\dfrac{9}{4}}, & s=0 \\ \sqrt{\dfrac{m^2}{H^2}-\left(s-\dfrac{1}{2}\right)^2} & s\neq 0 \end{cases}$$

Schwinger-Keldysh formalism

[Chen, Wang & Xianyu, 2017]



$$Z_0[J_+, J_-] \equiv \int \mathcal{D}\zeta_+ \mathcal{D}\zeta_- \exp\left[i\int_{\tau_i}^{\tau_f} d\tau d^3x \left(\mathcal{L}_0[\zeta_+] - \mathcal{L}_0[\zeta_-] + J_+\zeta_+ - J_-\zeta_-\right)\right]$$

Two copies of evolution histories

Time ordering (+): ● Anti-time ordering (-): ○

Classification and interpretation of CCS

Different events during inflation

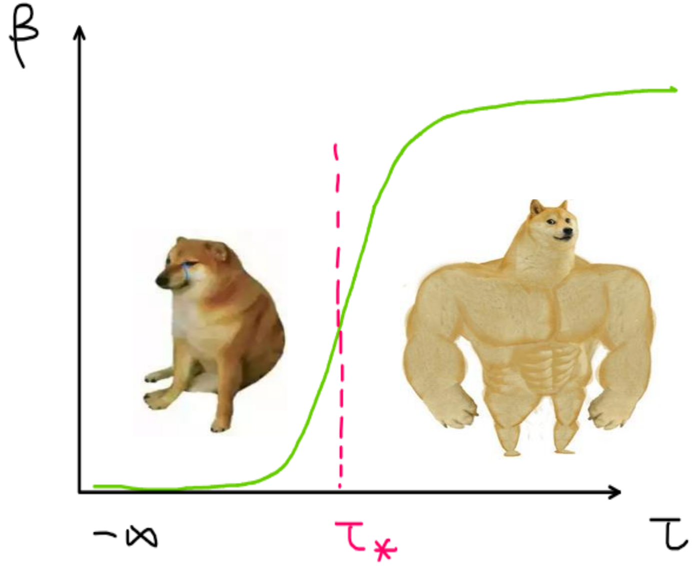
- **Gravitational particle production**

$$v_{k_I}(\tau) = \alpha\, f(\tau) + \beta\, f^*(\tau)$$

with

$$f(\tau) \equiv \frac{1}{\sqrt{2\omega(\tau)}} e^{-i\int \omega(\tau')d\tau'}$$

β is related to production amount $\sim e^{-\pi\mu}$

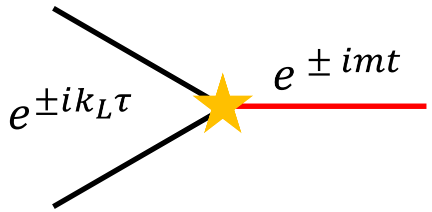


Production time

$$\boxed{|k_I\, \tau_*| \approx \mathcal{O}(1)\, \mu}$$

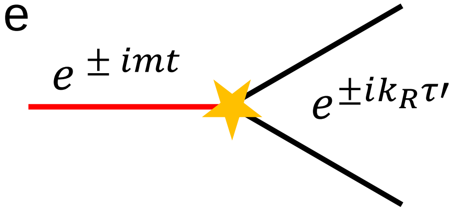
[Sou, Tong & Wang, 2021]

- **Resonant production/decay**



If these two coincide

$$\int e^{-ik\tau} e^{imt} d\tau$$



Resonance time

$$\boxed{\begin{aligned}|k_L\, \tau| &\approx \mu \\ |k_R\, \tau'| &\approx \mu\end{aligned}}$$

Classification and interpretation of CCS

- There are two different types of CCS

- Some hints can be found from the late-time limit of massive propagators

Non-analytic in k_I

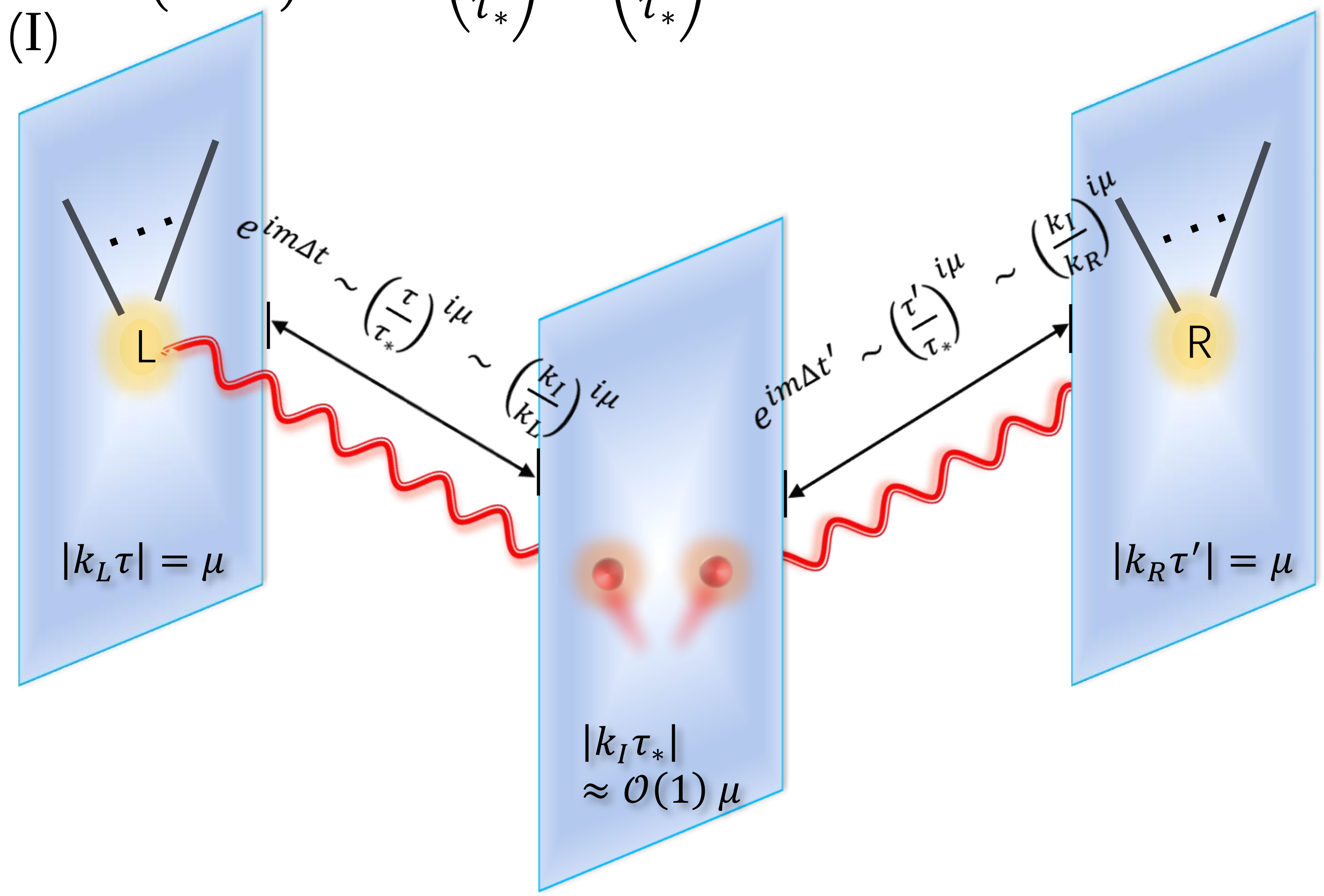
$$\lim_{\tau,\tau'\to 0} D_{-+}^{\text{non-local}}(\mathbf{k}_I,\tau,\tau') = \frac{H^2(\tau\tau')^{3/2}}{4\pi}\left[\Gamma(-i\mu)^2\left(\frac{\boxed{k_I{}^2}\tau\tau'}{4}\right)^{i\mu} + \Gamma(i\mu)^2\left(\frac{\boxed{k_I{}^2}\tau\tau'}{4}\right)^{-i\mu}\right]$$

$$\lim_{\tau,\tau'\to 0} D_{-+}^{\text{local}}(\mathbf{k}_I,\tau,\tau') = \frac{H^2(\tau\tau')^{3/2}}{4\pi}\Gamma(-i\mu)\Gamma(i\mu)\left[e^{\pi\mu}\left(\frac{\tau}{\tau'}\right)^{i\mu} + e^{-\pi\mu}\left(\frac{\tau}{\tau'}\right)^{-i\mu}\right].$$

(I) Non-local Type CCS

- Massive particle's dynamical phases are accumulated from the gravitational production event at τ_* to the resonant decay event at τ, τ'

$$\left(k_I^2 \tau\tau'\right)^{\pm i\mu} \sim \left(\frac{\tau}{\tau_*}\right)^{\pm i\mu} \left(\frac{\tau'}{\tau_*}\right)^{\pm i\mu} \sim e^{\mp im(t-t_*)} e^{\mp im(t'-t_*)}$$



Production time

$$|k_I\, \tau_*| \approx \mathcal{O}(1)\, \mu$$

Resonance time

$$|k_L\, \tau| \approx \mu$$

$$|k_R\, \tau'| \approx \mu$$

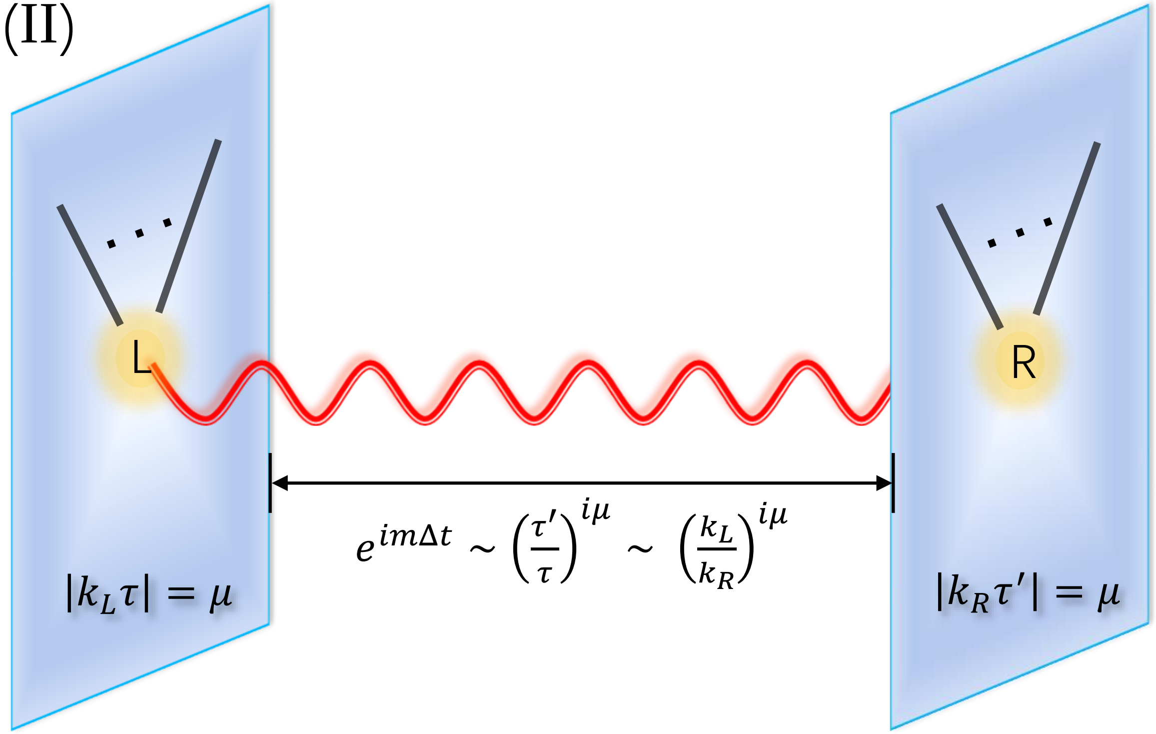
$$\boxed{\left(\frac{\tau}{\tau_*}\right)^{\pm i\mu} \left(\frac{\tau'}{\tau_*}\right)^{\pm i\mu} \Longrightarrow \left(\frac{k_L k_R}{k_I^2}\right)^{\pm i\mu}}$$

(II) local Type CCS

- Massive particle's dynamical phases are accumulated from the resonance production event at τ' to the resonant decay event at τ

$$\left(\frac{\tau}{\tau'}\right)^{\pm i\mu} \sim e^{\mp i m (t-t')}$$

(II)



Production time

$|k_I \, \tau_*| \approx \mathcal{O}(1)\, \mu$

Resonance time

$|k_L \, \tau| \approx \mu$

$|k_R \, \tau'| \approx \mu$

$$\boxed{\left(\frac{\tau}{\tau'}\right)^{\pm i\mu} \Longrightarrow \left(\frac{k_L}{k_R}\right)^{\mp i\mu}}$$

Challenge: The process of computing CCS can be laborious.

Analytical calculation?

- Integrands are some special functions (e.g. Hankel, Whittaker)
- Nested integrals are too complicated to solve analytically

It is not until the recent works, the single tree-level exchange diagram is fully evaluated in a closed form

[Chen, Namjoo & Wang, 2015]
[Arkani-Hamed, Baumann Lee, Pimentel 2018]
……

Numerical calculation?

- It can be extremely inefficient for large masses
- Slow integral convergence

- Memory consumption quickly becomes unmanageable when mass become larger

Q: With our physical picture, can we find an efficient way to extract these CCS?

A cutting rule for CCS

- ➢ There are other recently proposed cosmological cutting rules

[Goodhew, Jazayeri & Pajer, 2020]
[Melville &Pajer,2021]
[Goodhew, Jazayeri, Lee &Pajer,2021]
[Baumann, Chen, Pueyo, Joyce, Lee & Pimentel,2021]
……

- Fundamental principles
- Wavefunction coefficients

- ➢ We propose a practical cutting rule to analytically extract the leading order CCS

- Practicality
- Correlation functions

The general algorithm

Step.1

Consider only all-black, all-white

Step.2

Focus on one massive propagator each time

Step.3

Compute the left/right blobs

Step.4

Flip & factorize

Step.5

Account $k_L > k_R$

Step.6

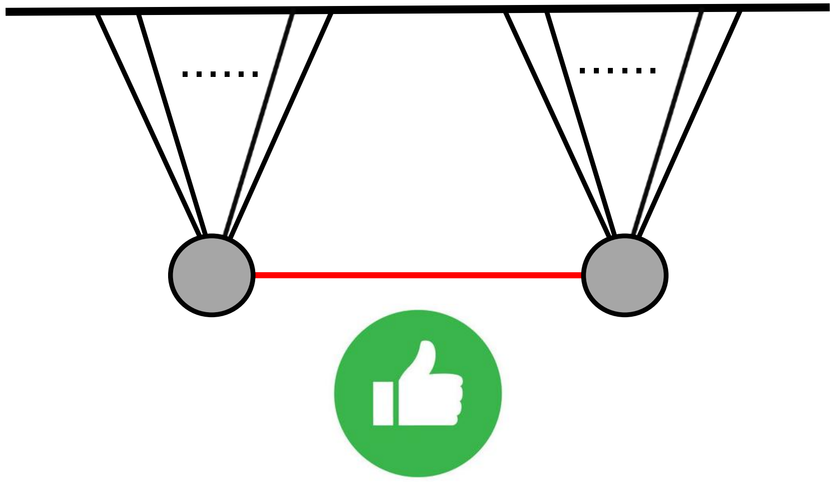
Repeat Step.2-5 for each MP

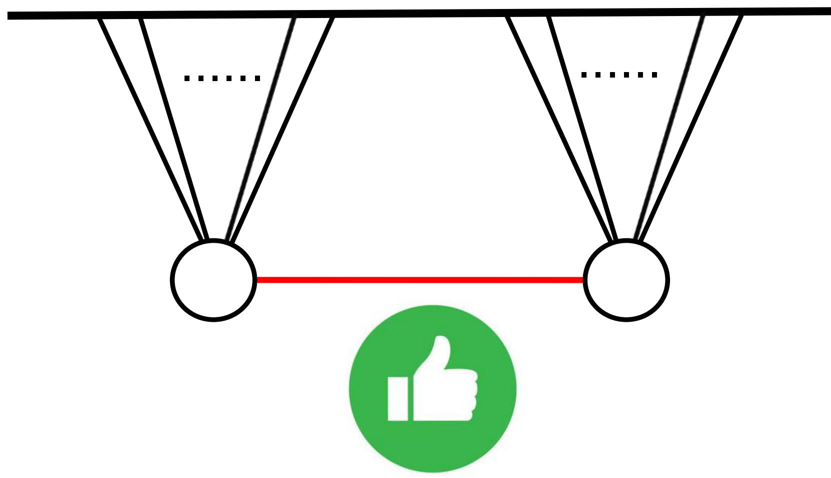
Step.7

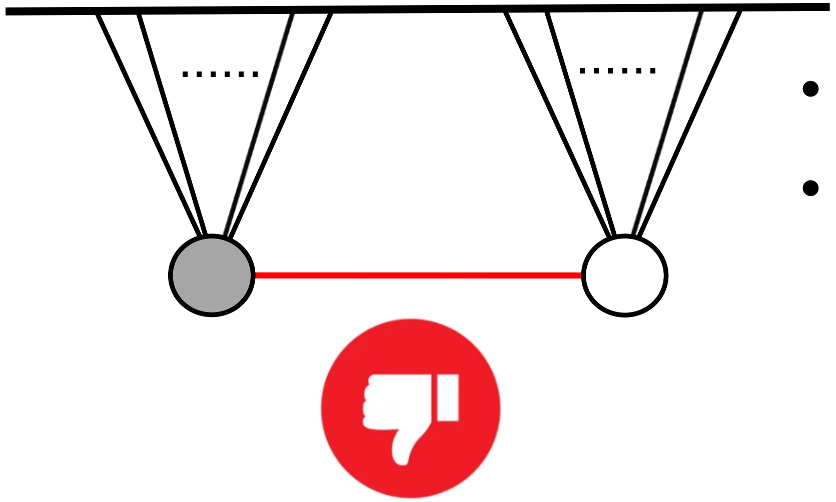
Add EFT Background

Step.1

Consider only all-black, all-white



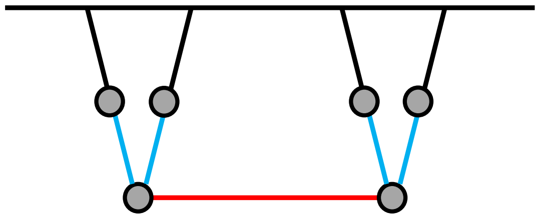


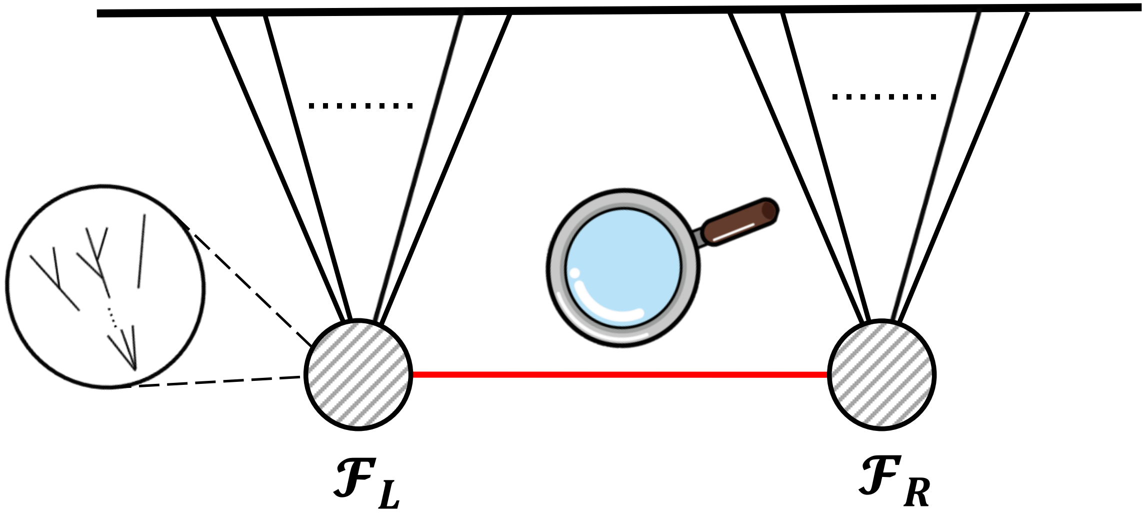


- Heavier suppression $\mathcal{O}(e^{-2\pi\mu})$
- Already factorized

Step.2

Focus on one massive propagator each time





$$\langle \zeta^n \rangle_{\rm TO} \sim \int_{-\infty}^{0} d\tau d\tau' \mathcal{F}_L(\tau)\mathcal{F}_R(\tau') \times \left[\theta(\tau-\tau')v_{k_I}(\tau)v^*_{k_I}(\tau') + \theta(\tau'-\tau)v^*_{k_I}(\tau)v_{k_I}(\tau')\right]$$

Step.3

Compute the left/right blobs

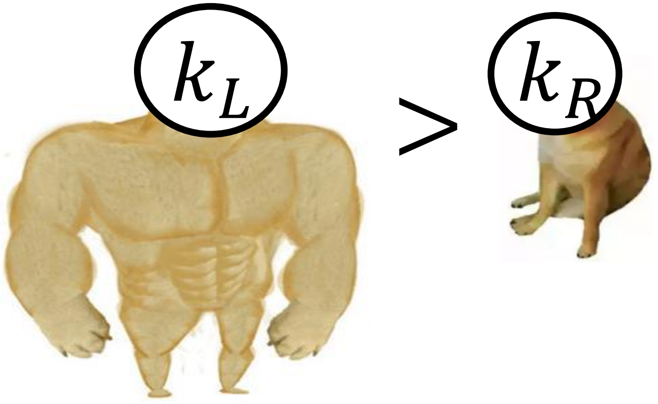
$$\mathcal{F}_L(\tau) = \sum_{k_L} \mathcal{P}_L(\tau) e^{ik_L\tau}$$

$$\mathcal{F}_R(\tau') = \sum_{k_R} \mathcal{P}_R(\tau') e^{ik_R\tau'}$$

Step.4

Flip & factorize

When $k_L > k_R$



$$\langle\zeta^n\rangle_{\rm TO} \sim \int_{-\infty}^{0} d\tau d\tau' \mathcal{F}_L(\tau)\mathcal{F}_R(\tau') \times \left[\boxed{\theta(\tau-\tau')} v_{k_I}(\tau) v^*_{k_I}(\tau') + \theta(\tau'-\tau) v^*_{k_I}(\tau) v_{k_I}(\tau')\right]$$

$$\boxed{1-\theta(\tau'-\tau)}$$

$$\langle\zeta^n\rangle_{\rm TO} \sim \int_{-\infty}^{0} d\tau \mathcal{F}_L(\tau) v_{k_I}(\tau) \int_{-\infty}^{0} d\tau' \mathcal{F}_R(\tau') v^*_{k_I}(\tau') \quad \text{Term 1}$$
$$+ \int_{-\infty}^{0} d\tau \int_{\tau}^{0} d\tau' \mathcal{F}_L(\tau)\mathcal{F}_R(\tau') \left[v^*_{k_I}(\tau) v_{k_I}(\tau') - v_{k_I}(\tau) v^*_{k_I}(\tau')\right] \quad \text{Term 2}$$

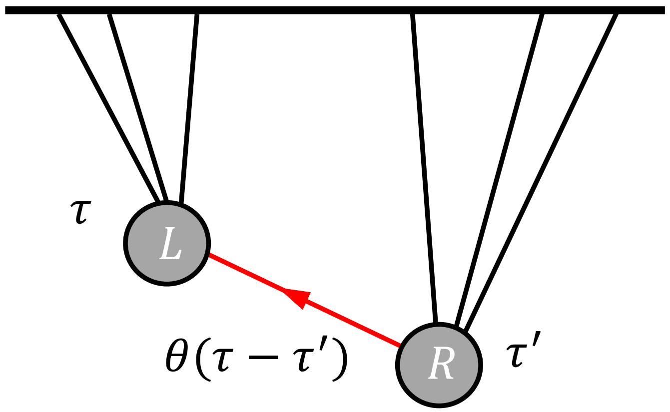
- Term 1 $\equiv S_I^{>}(k_L, k_R) \sim \mathcal{O}(e^{-\pi\mu})|\alpha|^2 \left(\frac{k_R}{k_L}\right)^{i\mu} + \mathcal{O}(1)\alpha\beta^* \left(\frac{k_L k_R}{k_I}\right)^{-i\mu}$
- Term 2 It cannot contribute CCS

- No non-local type CCS $D^{\text{non-local}}_{-+}(\mathbf{k}_I, \tau, \tau') = D^{\text{non-local}}_{-+}(\mathbf{k}_I, \tau, \tau')^*$
- No local type CCS

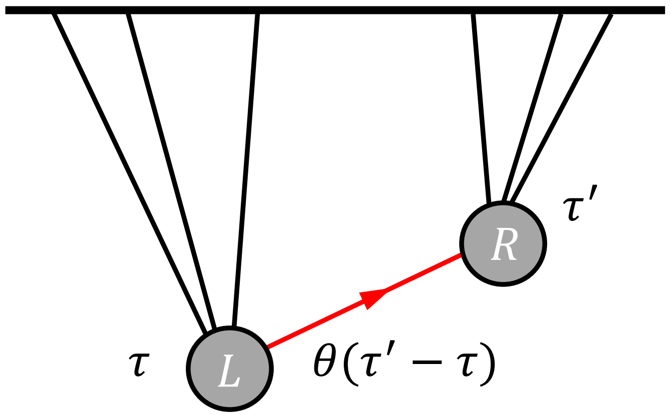
$$\begin{matrix} k_L > k_R \\ |\tau| > |\tau'| \end{matrix} \;\not\longrightarrow\; \begin{matrix} |k_L\,\tau| \approx \mu \\ |k_R\,\tau'| \approx \mu \end{matrix}$$

Step.4

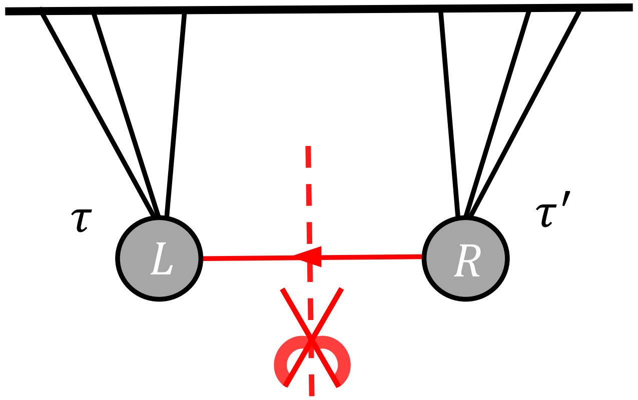
Flip & factorize



$+$



$=$



$+$ (Terms without CC signals)

Step.5

Account

$k_L < k_R$

$$S_I = \sum_{k_L, k_R} \left[\theta(k_L - k_R) S_I^{>}(k_L, k_R) + \theta(k_R - k_L) S_I^{>}(k_R, k_L) \right]$$

Step.6

Repeat Step.2-5 for each MP

$$S = \sum_{I \in \Sigma} S_I$$

Step.7

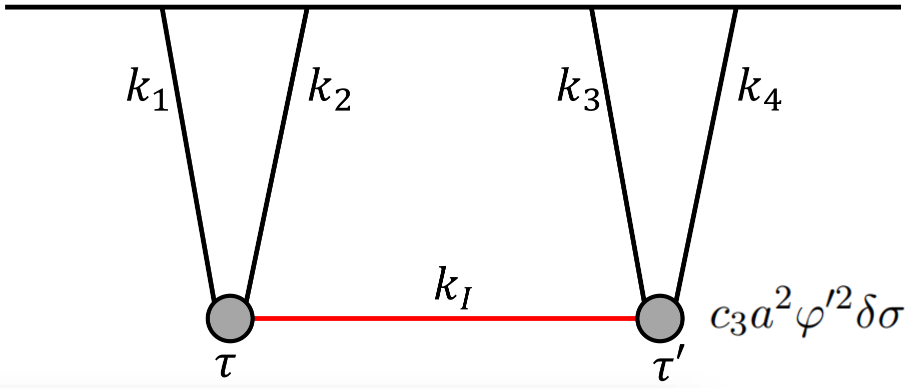
Add EFT Background

$$\langle \zeta^n \rangle \simeq (S+B) \sim \left[\mathcal{O}(|\beta|) \times (\text{non-local}) + \mathcal{O}(e^{-\pi\mu}) \times (\text{local})\right] + \mathcal{O}\left(\frac{\text{EFT}}{\mu^{\#}}\right)$$

EFT Tower	CC Signals
$\frac{1}{\mu^2}$ (Leading Order EFT)	$e^{-\pi\mu}\left\{\left(\frac{k_L}{k_R}\right)^{i\mu}, \left(\frac{k_L k_R}{k_I^2}\right)^{i\mu}\right\}$
$\frac{1}{\mu^4}$ (NL Order EFT)	$e^{-\#\pi\mu}$ (CC signals $\cdots$)
$\vdots$	$\vdots$

Example I

Scalar exchange

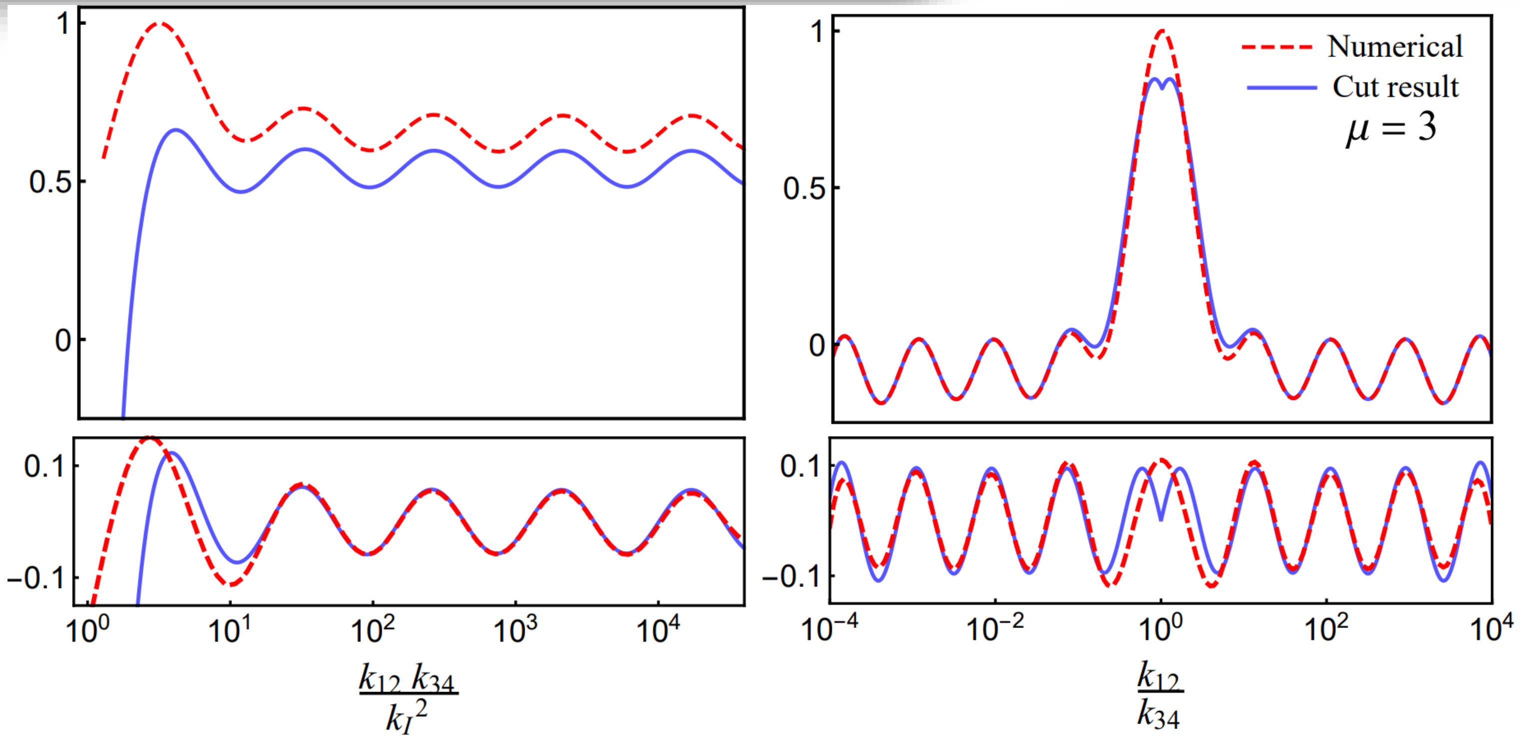


$$\mathcal{F}_L(\tau) = 2i\tilde{c}_3 u_{k_1} u_{k_2}(0) a^2 u'^*_{k_1} u'^*_{k_2}(\tau)$$

$$\mathcal{F}_R(\tau') = 2i\tilde{c}_3 u_{k_3} u_{k_4}(0) a^2 u'^*_{k_3} u'^*_{k_4}(\tau')$$

$$S_I^{>}(k_{12}, k_{34}) = -\left(\frac{H}{\dot{\phi}_0}\right)^4 \frac{\pi^2 {c_3}^2 H^6 (16\mu^4 + 40\mu^2 + 9)^2 \operatorname{sech}^2(\pi\mu)}{2^{17} k_1 k_2 k_3 k_4 k_I^5} \times {}_2F_1\left[\begin{matrix} \frac{5}{2} - i\mu, \frac{5}{2} + i\mu \\ 3 \end{matrix} \middle| \frac{k_I - k_{34}}{2k_I}\right] \times {}_2F_1\left[\begin{matrix} \frac{5}{2} - i\mu, \frac{5}{2} + i\mu \\ 3 \end{matrix} \middle| \frac{k_I + k_{12}}{2k_I}\right]$$

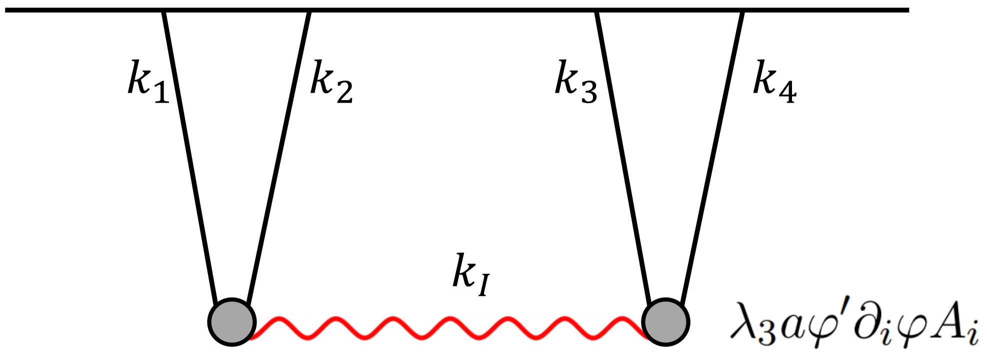
$$B_I = \left(\frac{H}{\dot{\phi}_0}\right)^4 \frac{12 H^6 c_3^2}{\mu^2 k_1 k_2 k_3 k_4 k_{1234}^5}$$



Example II

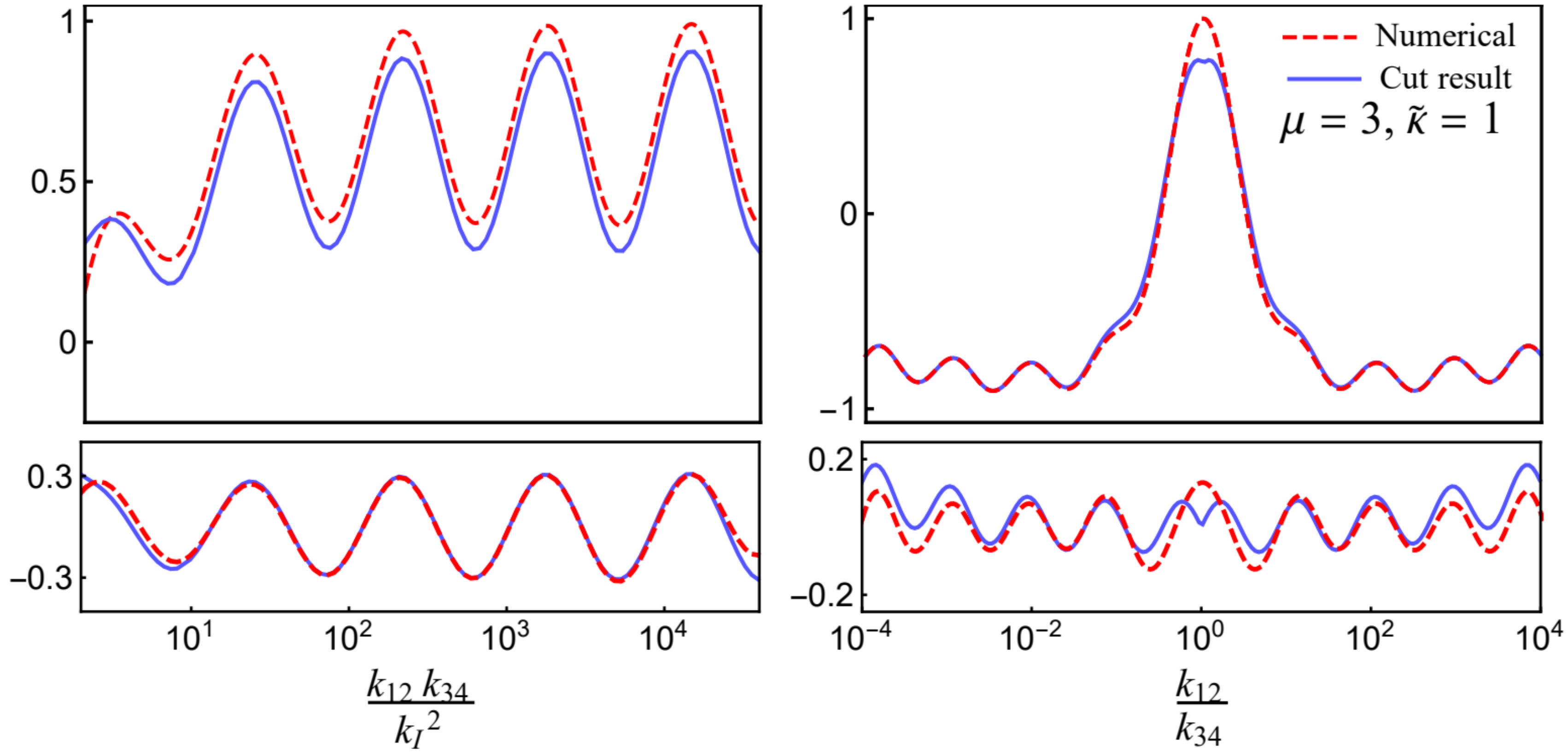
Vector exchange with chemical potential

$\phi F\tilde{F}$



$$\mathcal{F}_L(\tau) = i\lambda_3 u_{k_1}u_{k_2}(0)au^*_{k_1}u'^*_{k_2}(\tau)$$

$$\mathcal{F}_R(\tau') = i\lambda_3 u_{k_3}u_{k_4}(0)au^*_{k_3}u'^*_{k_4}(\tau)$$



$$\begin{aligned}\mathcal{I}_L^\lambda &\equiv \int_0^\infty \left(1+i\frac{k_1}{k_I}x\right)W_{i\lambda\tilde{\kappa},i\mu}e^{-i\frac{k_{12}}{k_I}x}(-2ix)dx \\ &= i\pi(4\mu^2+1)\,\mathrm{sech}(\pi\mu) \\ &\times\frac{1}{8}\left\{{}_2\mathbf{F}_1\left[\left.\begin{matrix}\frac{3}{2}-i\mu,\frac{3}{2}+i\mu\\ 2-i\kappa\lambda\end{matrix}\right|\frac{k_I+k_{12}}{2k_I}\right]-\frac{2\left(\mu^2+\frac{9}{4}\right)^2k_1}{k_I}{}_2\mathbf{F}_1\left[\left.\begin{matrix}\frac{5}{2}-i\mu,\frac{5}{2}+i\mu\\ 3-i\kappa\lambda\end{matrix}\right|\frac{k_I+k_{12}}{2k_I}\right]\right\}\end{aligned}$$

$$\begin{aligned}\mathcal{I}_R^\lambda &\equiv \int_0^\infty \left(1+i\frac{k_3}{k_I}x\right)W_{-i\lambda\tilde{\kappa},-i\mu}e^{-i\frac{k_{34}}{k_I}x}(2ix)dx \\ &= -i\pi(4\mu^2+1)\,\mathrm{sech}(\pi\mu) \\ &\times\frac{1}{8}\left\{{}_2\mathbf{F}_1\left[\left.\begin{matrix}\frac{3}{2}-i\mu,\frac{3}{2}+i\mu\\ 2+i\kappa\lambda\end{matrix}\right|\frac{k_I-k_{34}}{2k_I}\right]+\frac{2\left(\mu^2+\frac{9}{4}\right)^2k_3}{k_I}{}_2\mathbf{F}_1\left[\left.\begin{matrix}\frac{5}{2}-i\mu,\frac{5}{2}+i\mu\\ 3+i\kappa\lambda\end{matrix}\right|\frac{k_I-k_{12}}{2k_I}\right]\right\}\end{aligned}$$

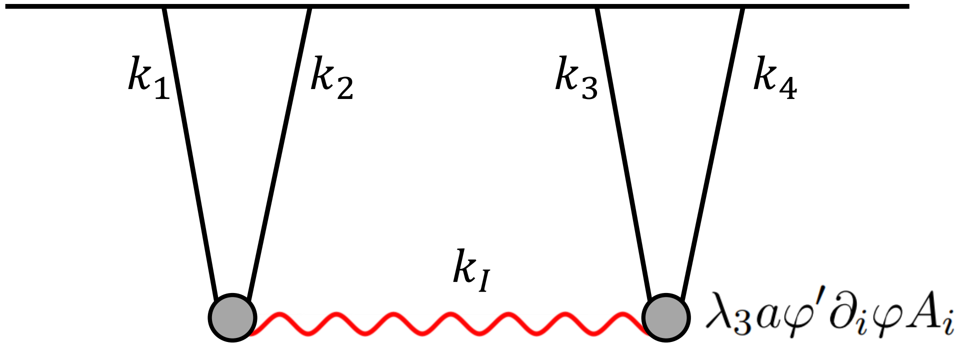
$$S_I^>(k_{12},k_{34}) = \sum_\lambda \Pi^\lambda(\mathbf{k}_1,\mathbf{k}_3,\mathbf{k}_I)\left(\frac{H}{\dot{\phi}_0}\right)^4\frac{\lambda_3^2H^6e^{-\pi\lambda\tilde{\kappa}}}{2^5k_1^3k_2k_3^3k_4k_I^3}\mathcal{I}_L^\lambda\mathcal{I}_R^\lambda$$

$$B_I = \left(\frac{H}{\dot{\phi}_0}\right)^4\frac{\lambda_3^2H^6\mathbf{k_1}\cdot\mathbf{k_3}}{\mu^2}\left[\frac{4k_1^2+(4k_1+k_{1234})(k_{234}+3k_3)}{4k_1^3k_2k_3^3k_4k_{1234}^5}\right]$$

Example II

Vector exchange with chemical potential

$\phi F \tilde{F}$



$$\mathcal{F}_L(\tau) = i\lambda_3 u_{k_1} u_{k_2}(0) a u^*_{k_1} u'^*_{k_2}(\tau)$$

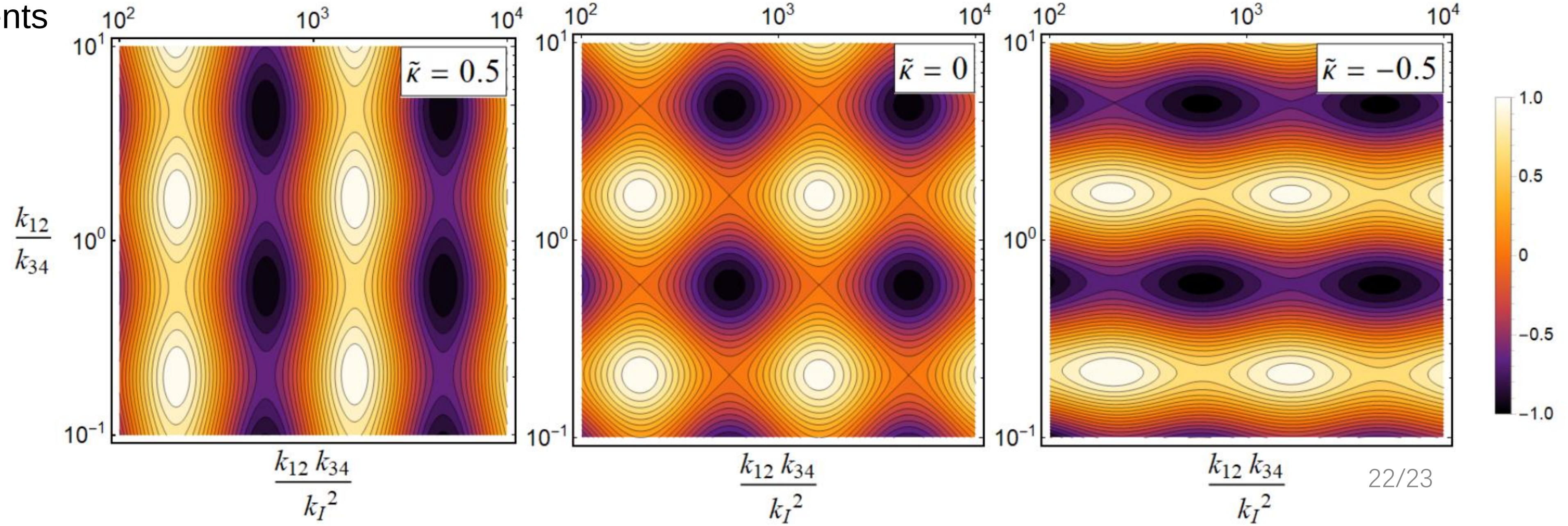
$$\mathcal{F}_R(\tau') = i\lambda_3 u_{k_3} u_{k_4}(0) a u^*_{k_3} u'^*_{k_4}(\tau)$$

$$S_I^\lambda \supset \boxed{e^{-\pi(\mu+\lambda\tilde{\kappa})}} \left(\frac{k_{12}k_{34}}{k_I^2}\right)^{i\mu} + \boxed{e^{-\pi\mu}} \left(\frac{k_{12}}{k_{34}}\right)^{i\mu}$$

Non-Local

Local

- Non-local type based on gravitational production events
- Chemical potential assists particle production



Conclusion & outlooks

- ✓ Different events during inflation
- ✓ Two different types of CCS
- ✓ Cutting rule for extracting CCS
- ✓ Application

- ☐ EFT truncation error?
- ☐ Loop level?
- ☐ Cutting algorithm as a computer program

Thank you for listening!

Schwinger-Keldysh formalism

[Chen, Wang & Xianyu, 2017]

$$Z_0[J_+, J_-] \equiv \int \mathcal{D}\zeta_+ \mathcal{D}\zeta_- \exp\left[i\int_{\tau_i}^{\tau_f} d\tau d^3x \left(\mathcal{L}_0[\zeta_+] - \mathcal{L}_0[\zeta_-] + J_+\zeta_+ - J_-\zeta_-\right)\right]$$

Two copies of evolution histories

Time ordering (+): ●

Anti-time ordering (-): ○

$$\begin{aligned}
G_{++}(\mathbf{k},\tau,\tau') &= \theta(\tau-\tau')u_k(\tau)u_k^*(\tau') + \theta(\tau'-\tau)u_k^*(\tau)u_k(\tau') \\
G_{+-}(\mathbf{k},\tau,\tau') &= u_k^*(\tau)u_k(\tau') \;, \\
G_{-+}(\mathbf{k},\tau,\tau') &= u_k(\tau)u_k^*(\tau') \;, \\
G_{--}(\mathbf{k},\tau,\tau') &= \theta(\tau-\tau')u_k^*(\tau)u_k(\tau') + \theta(\tau'-\tau)u_k(\tau)u_k^*(\tau')
\end{aligned}$$