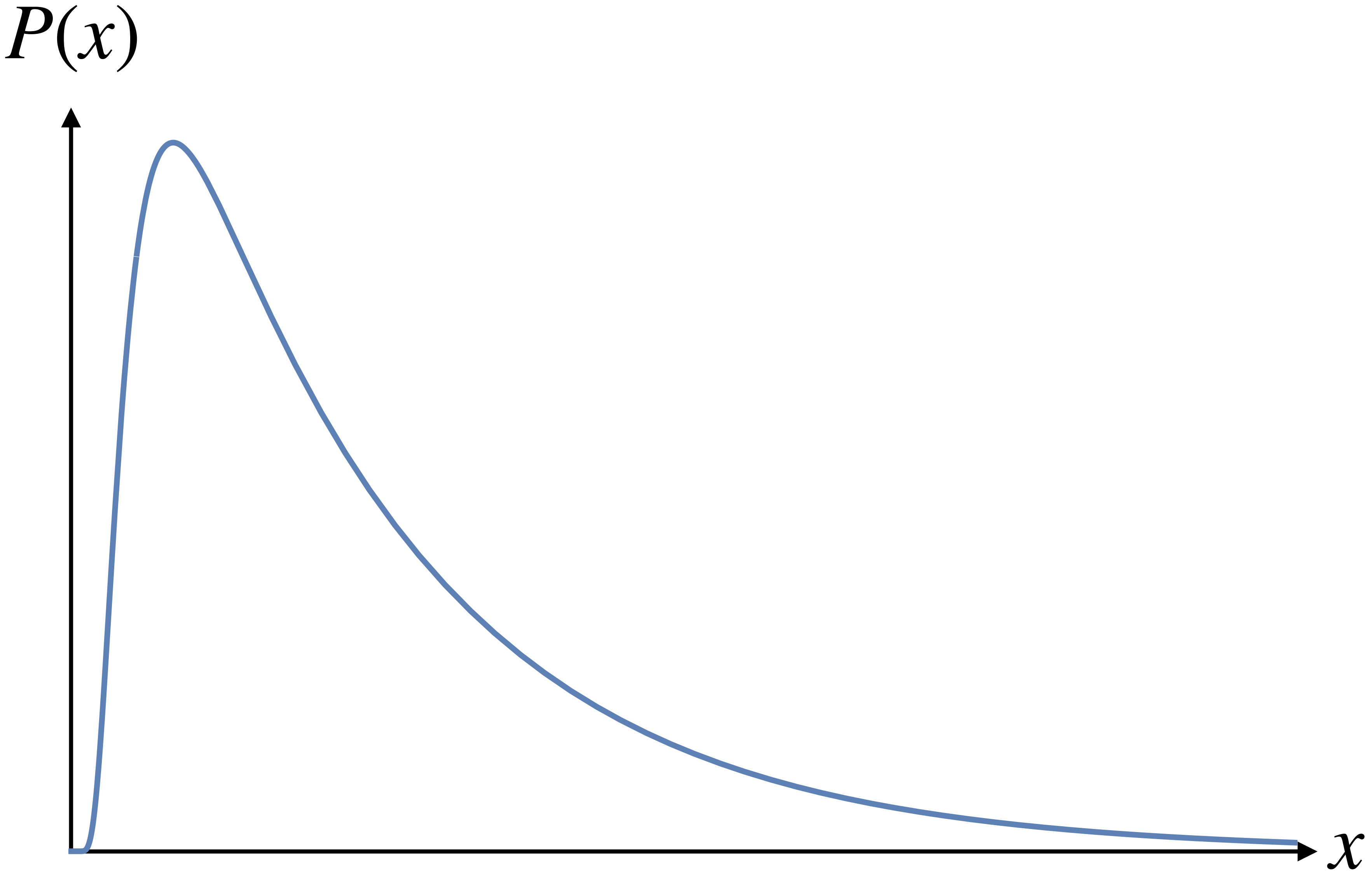
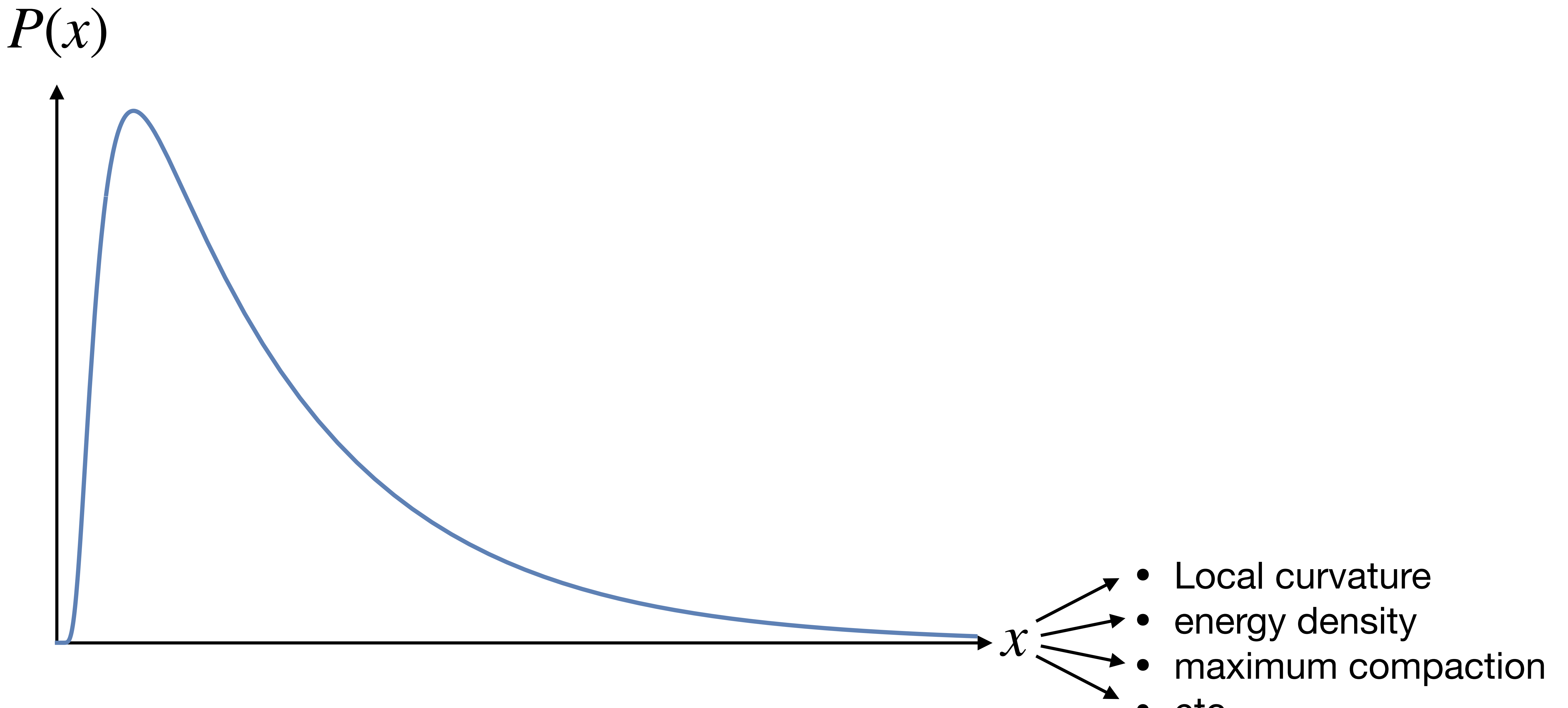
Non-Gaussianities from primordial quantum diffusion

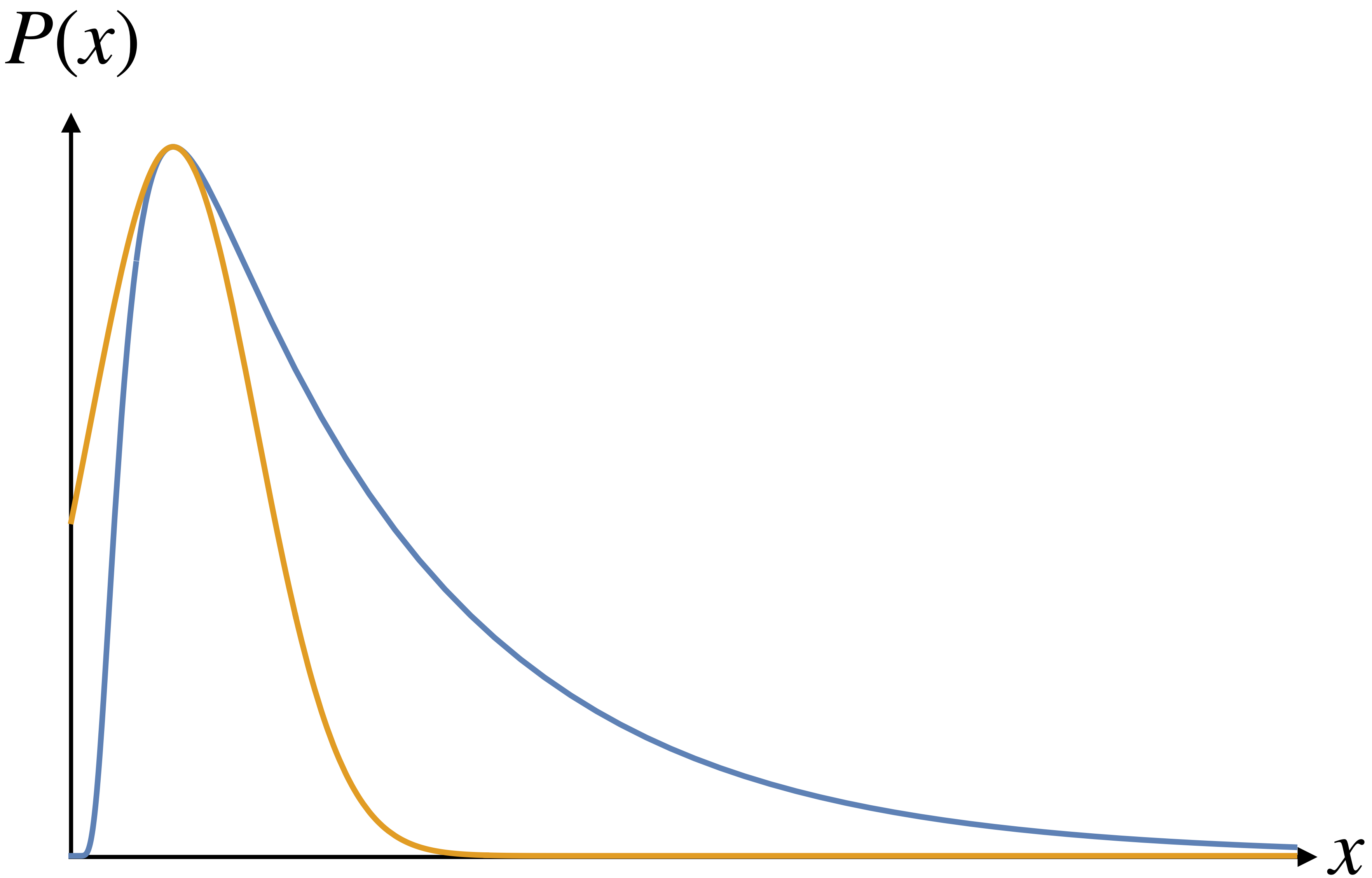
Vincent Vennin

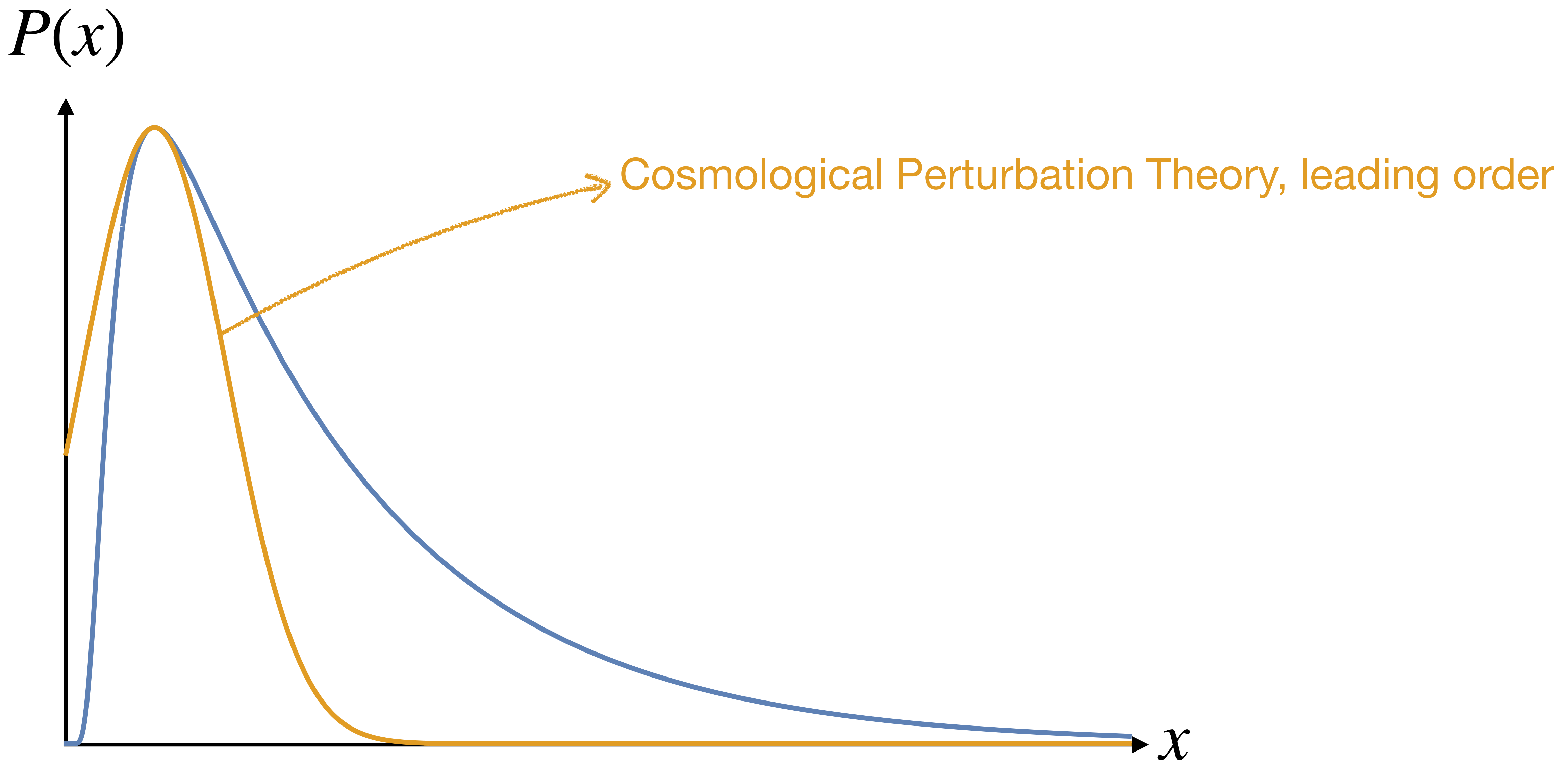
27 September 2022

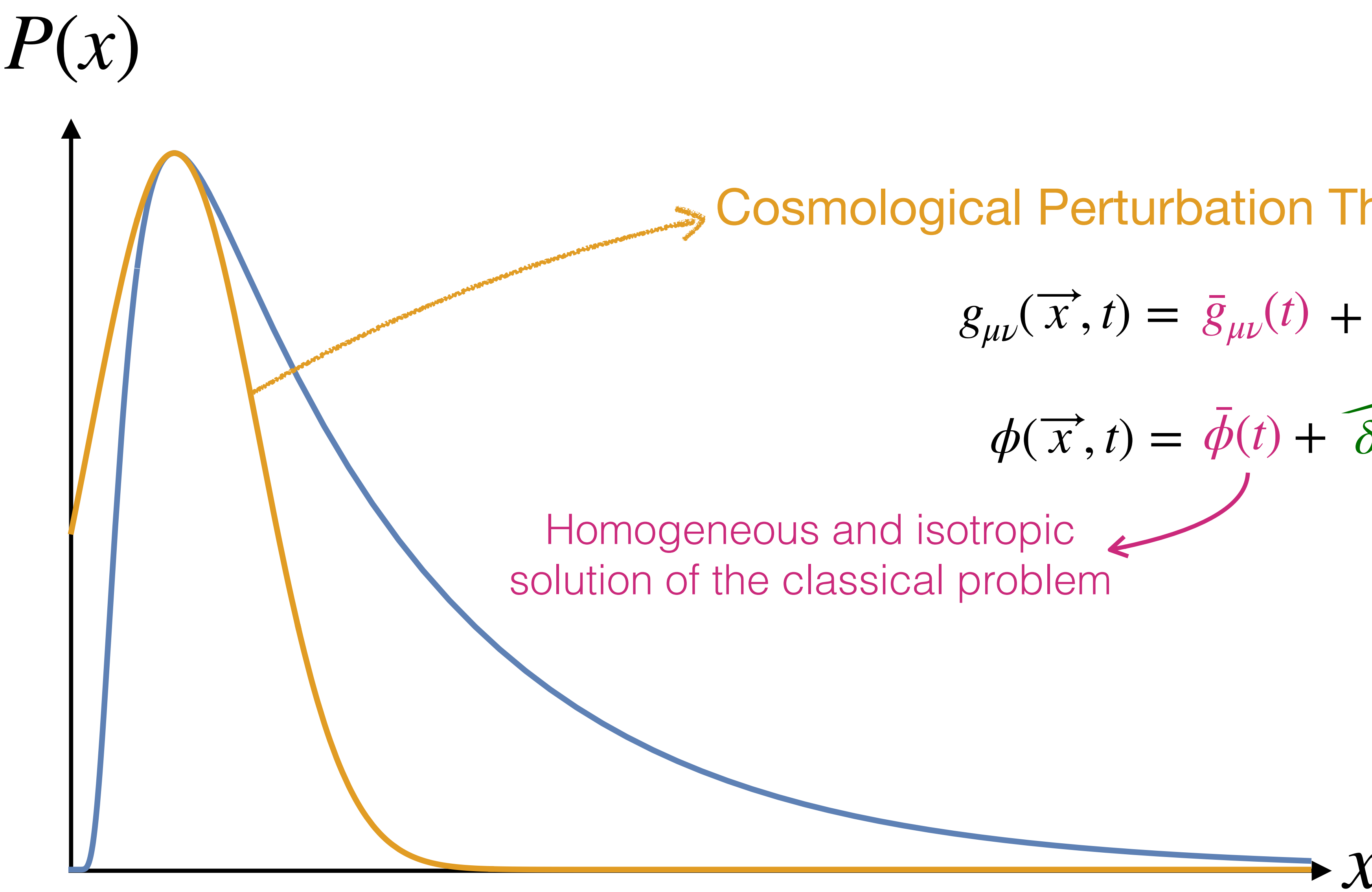
Copernicus Webinar


$P(x)$
x


$P(x)$
x
• Local curvature
• energy density
• maximum compaction
• etc


$P(x)$
x


$P(x)$
Cosmological Perturbation Theory, leading order
x



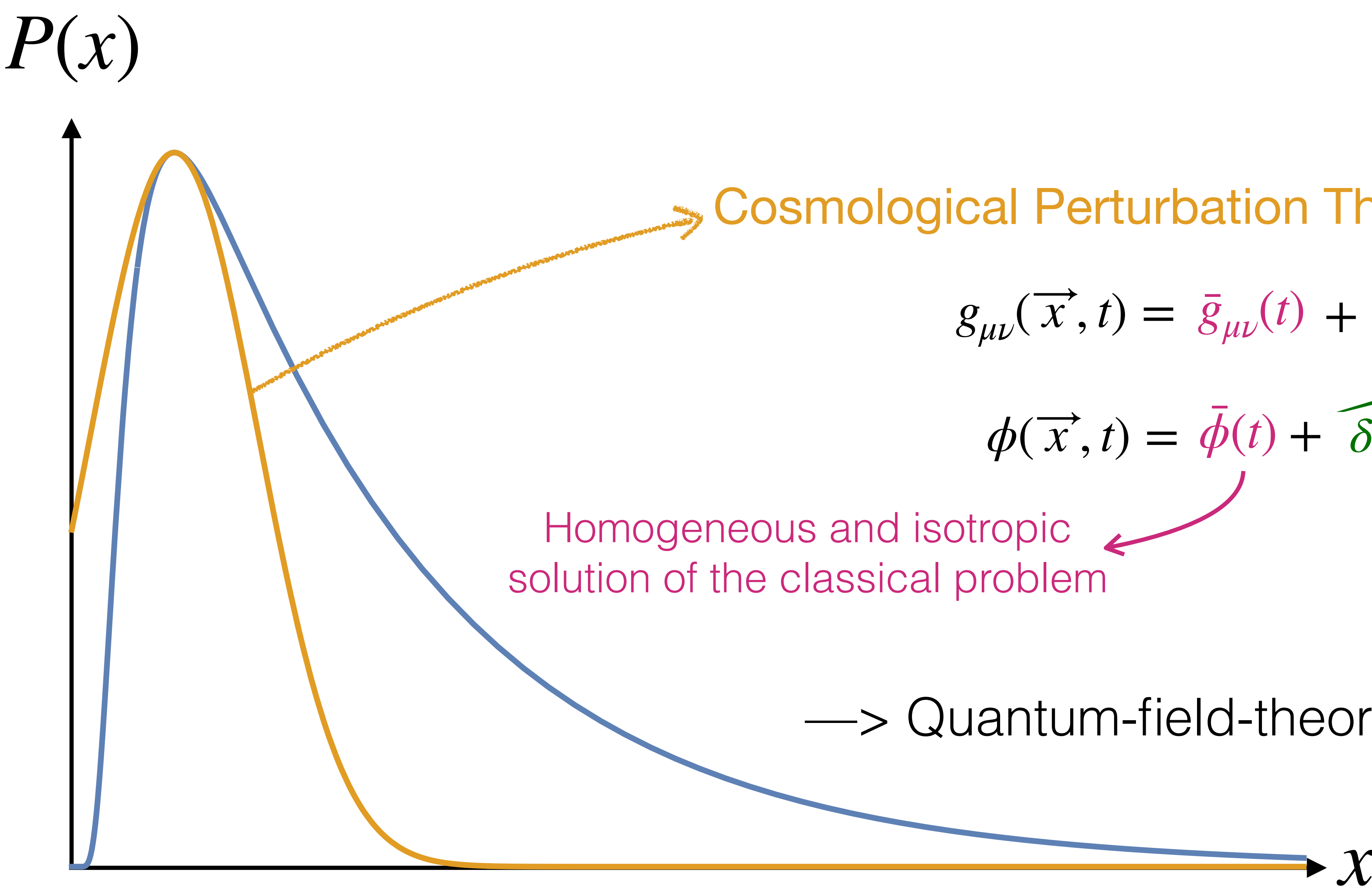
Cosmological Perturbation Theory, leading order

$$g_{\mu\nu}(\overrightarrow{x}, t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\overrightarrow{x}, t)$$

$$\phi(\overrightarrow{x}, t) = \bar{\phi}(t) + \widehat{\delta\phi}(\overrightarrow{x}, t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation



Cosmological Perturbation Theory, leading order

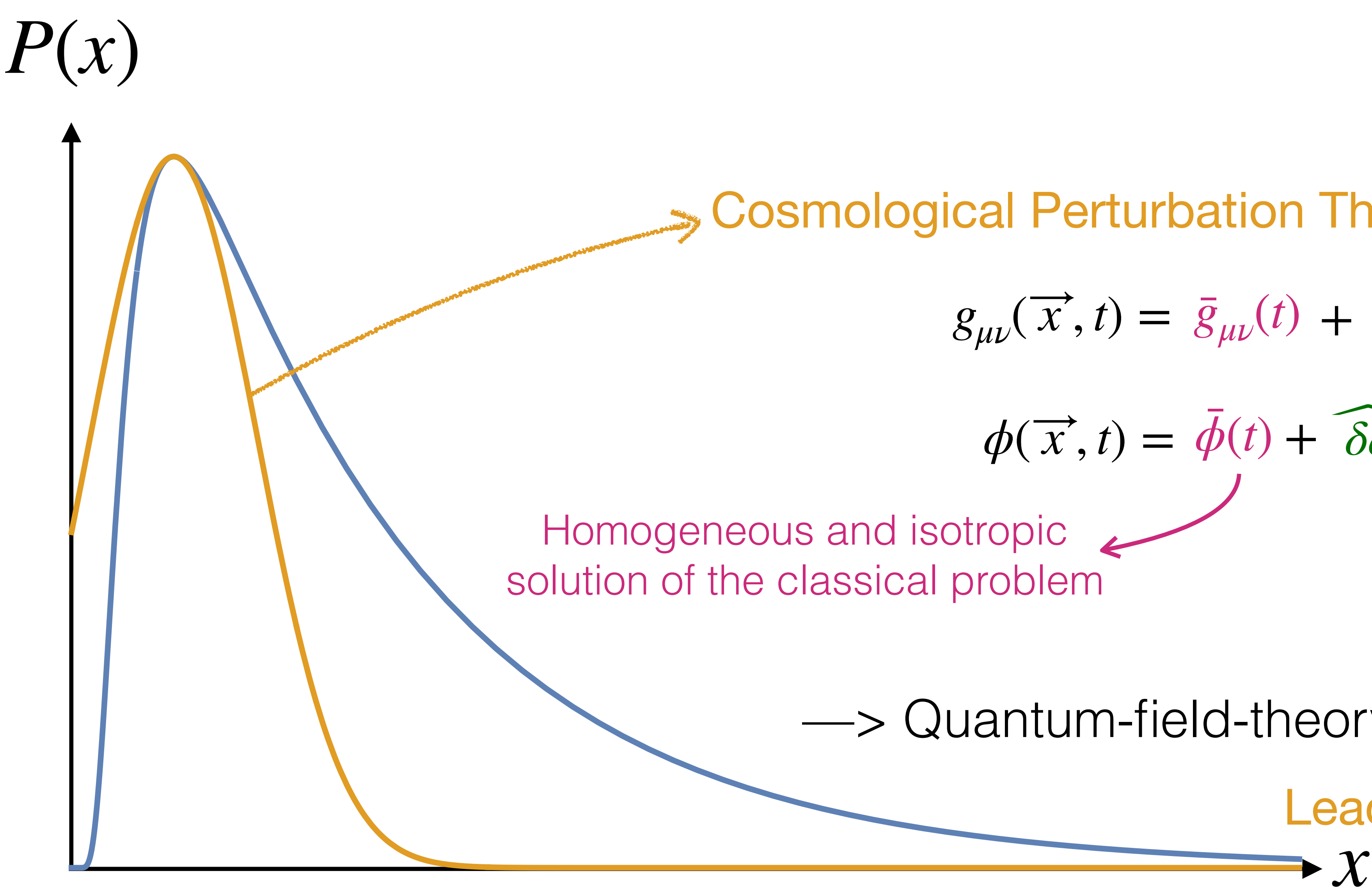
$$g_{\mu\nu}(\overrightarrow{x}, t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\overrightarrow{x}, t)$$

$$\phi(\overrightarrow{x}, t) = \bar{\phi}(t) + \widehat{\delta\phi}(\overrightarrow{x}, t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation

—> Quantum-field-theory on curved space-time



Cosmological Perturbation Theory, leading order

$$g_{\mu\nu}(\overrightarrow{x}, t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\overrightarrow{x}, t)$$

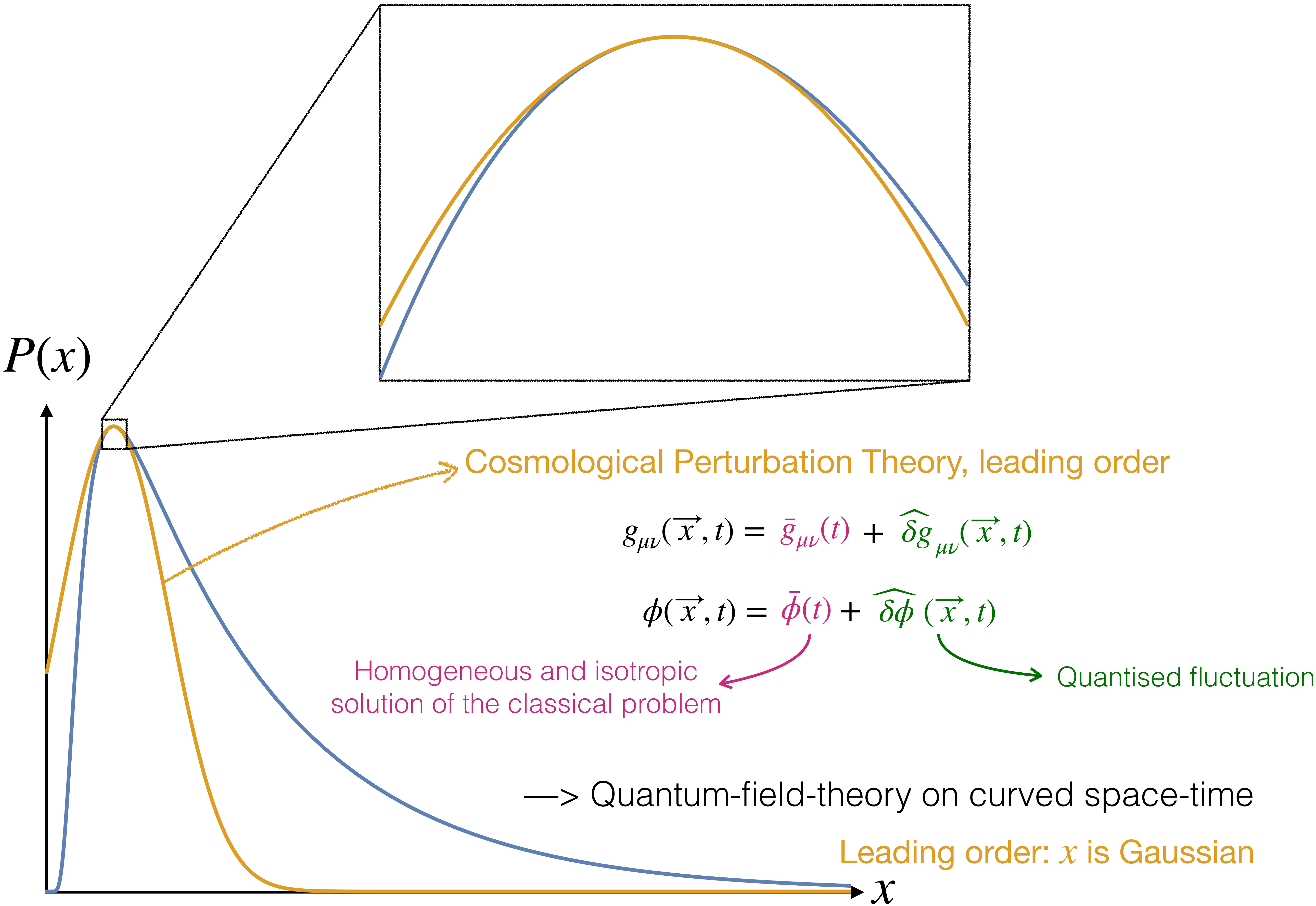
$$\phi(\overrightarrow{x}, t) = \bar{\phi}(t) + \widehat{\delta\phi}(\overrightarrow{x}, t)$$

Homogeneous and isotropic solution of the classical problem

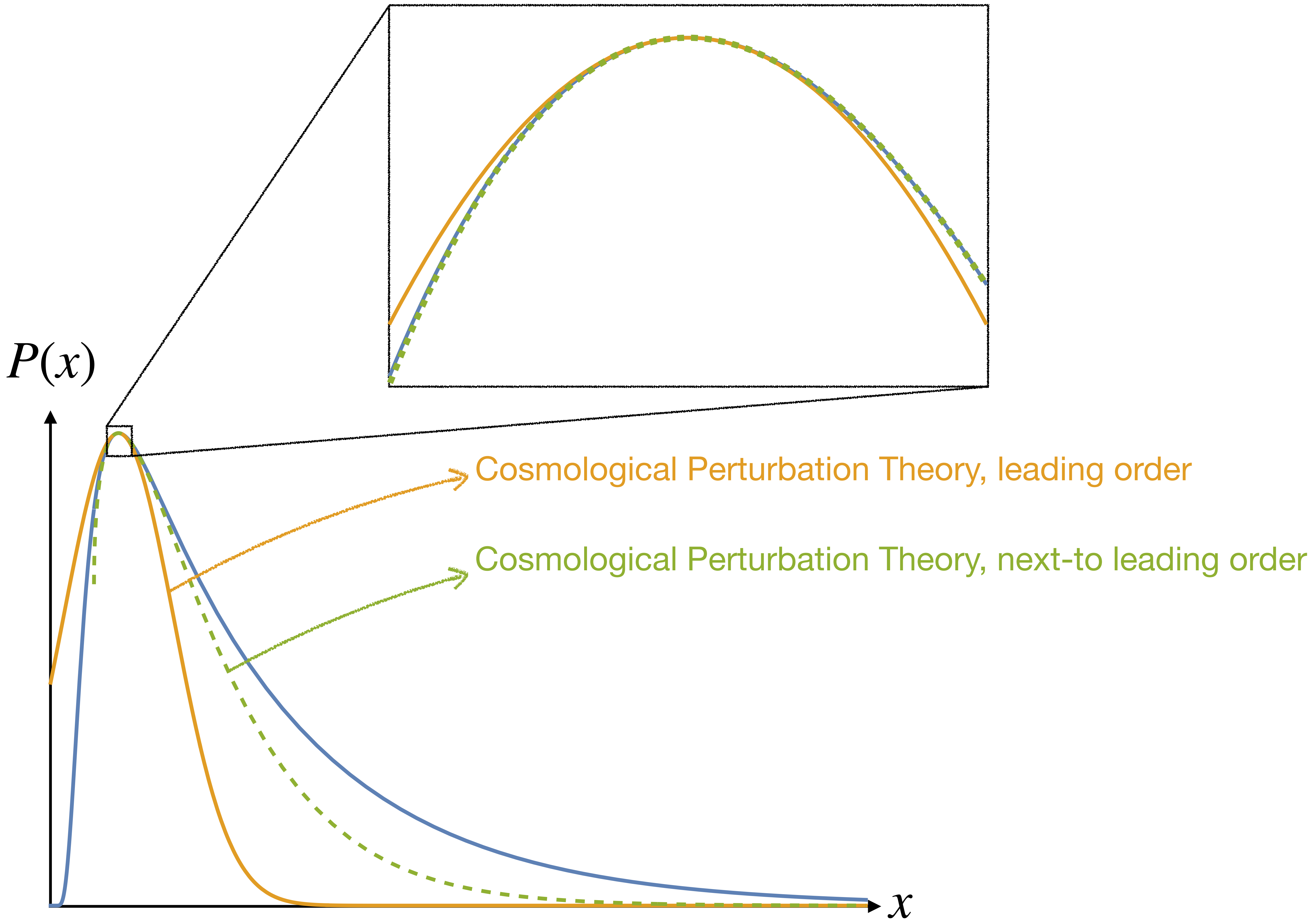
Quantised fluctuation

—> Quantum-field-theory on curved space-time

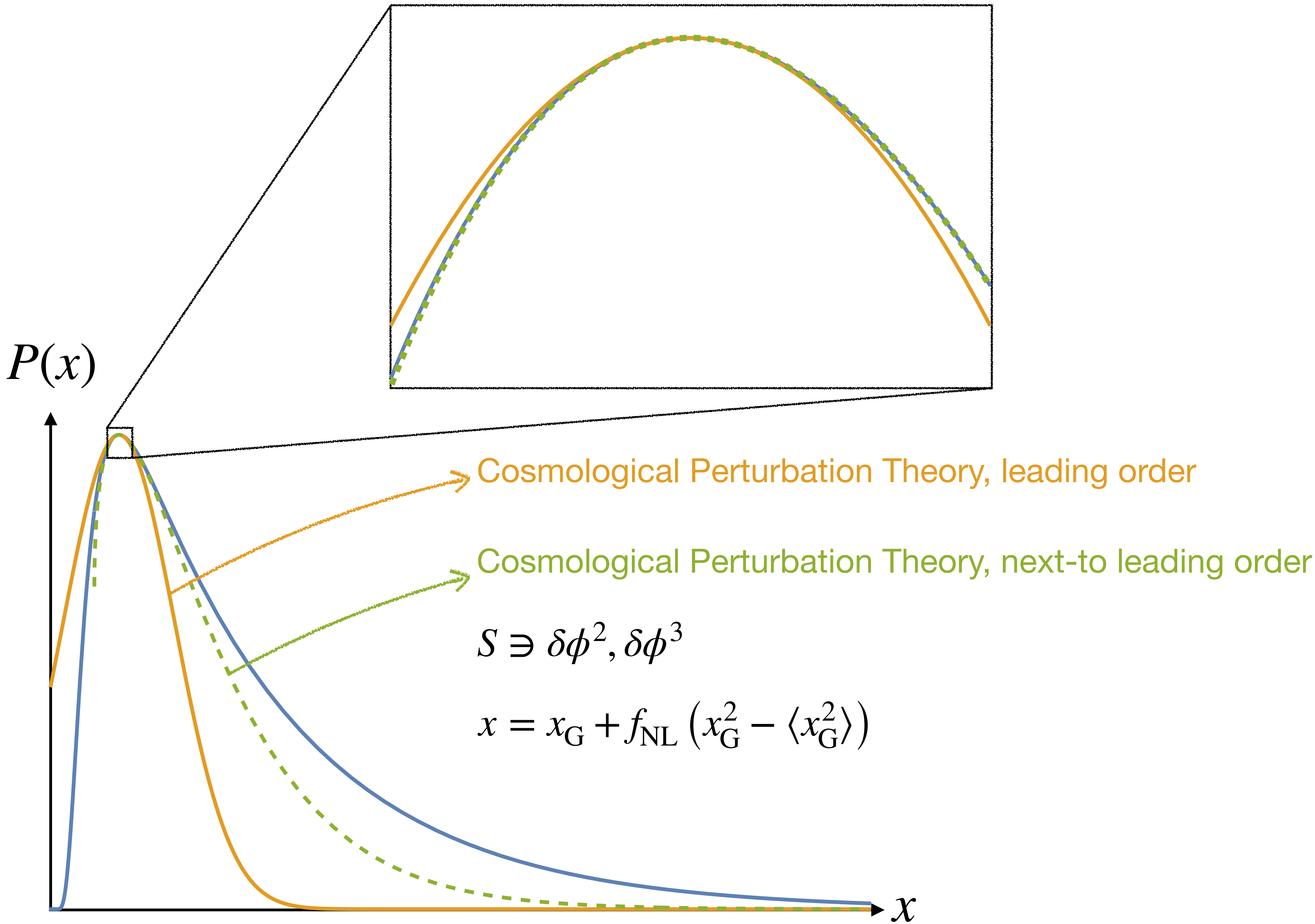
Leading order: x is Gaussian



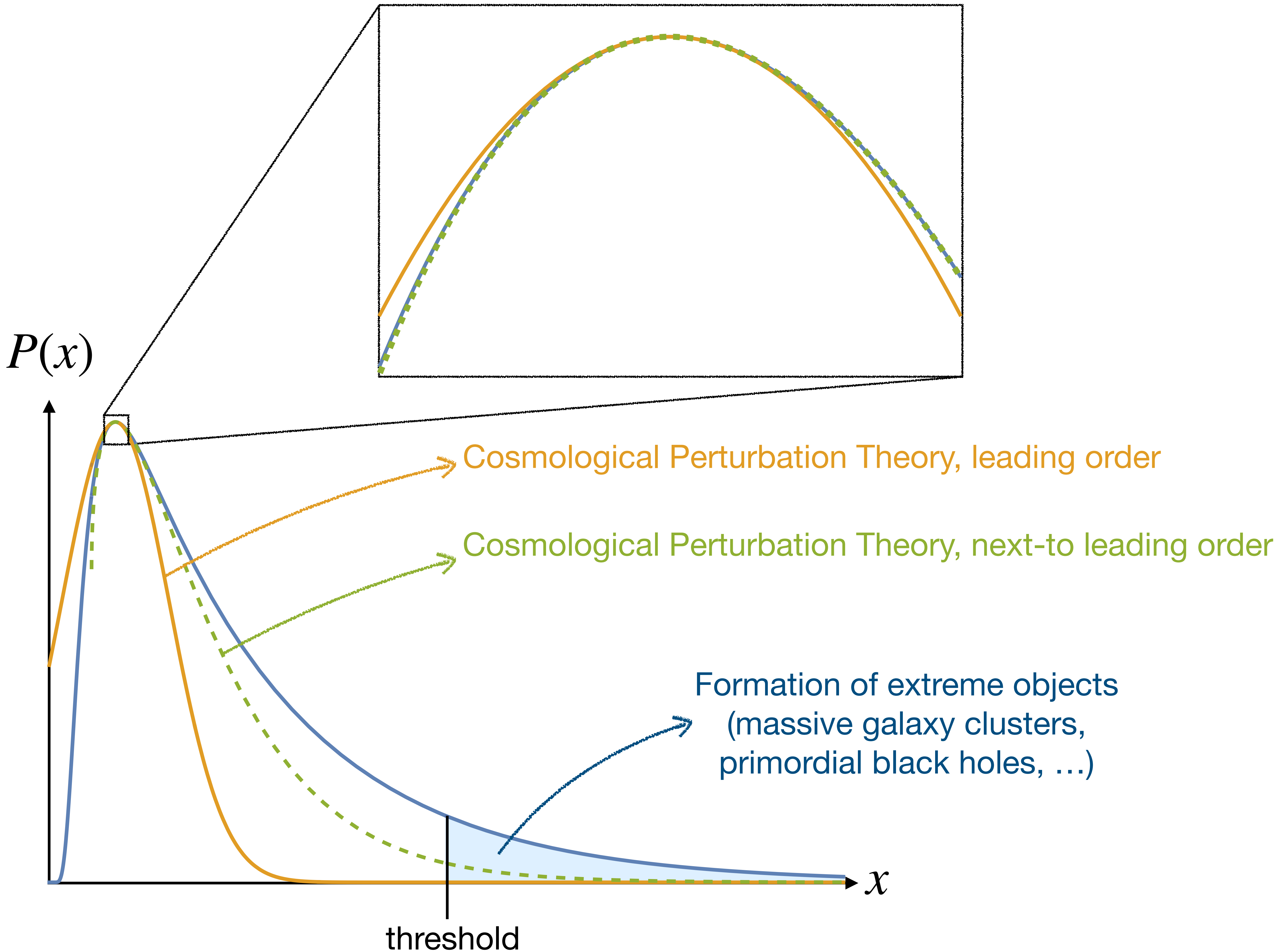
$P(x)$
Cosmological Perturbation Theory, leading order
$g_{\mu\nu}(\overrightarrow{x},t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\overrightarrow{x},t)$
$\phi(\overrightarrow{x},t) = \bar{\phi}(t) + \widehat{\delta\phi}(\overrightarrow{x},t)$
Homogeneous and isotropic solution of the classical problem
Quantised fluctuation
—> Quantum-field-theory on curved space-time
Leading order: x is Gaussian
x



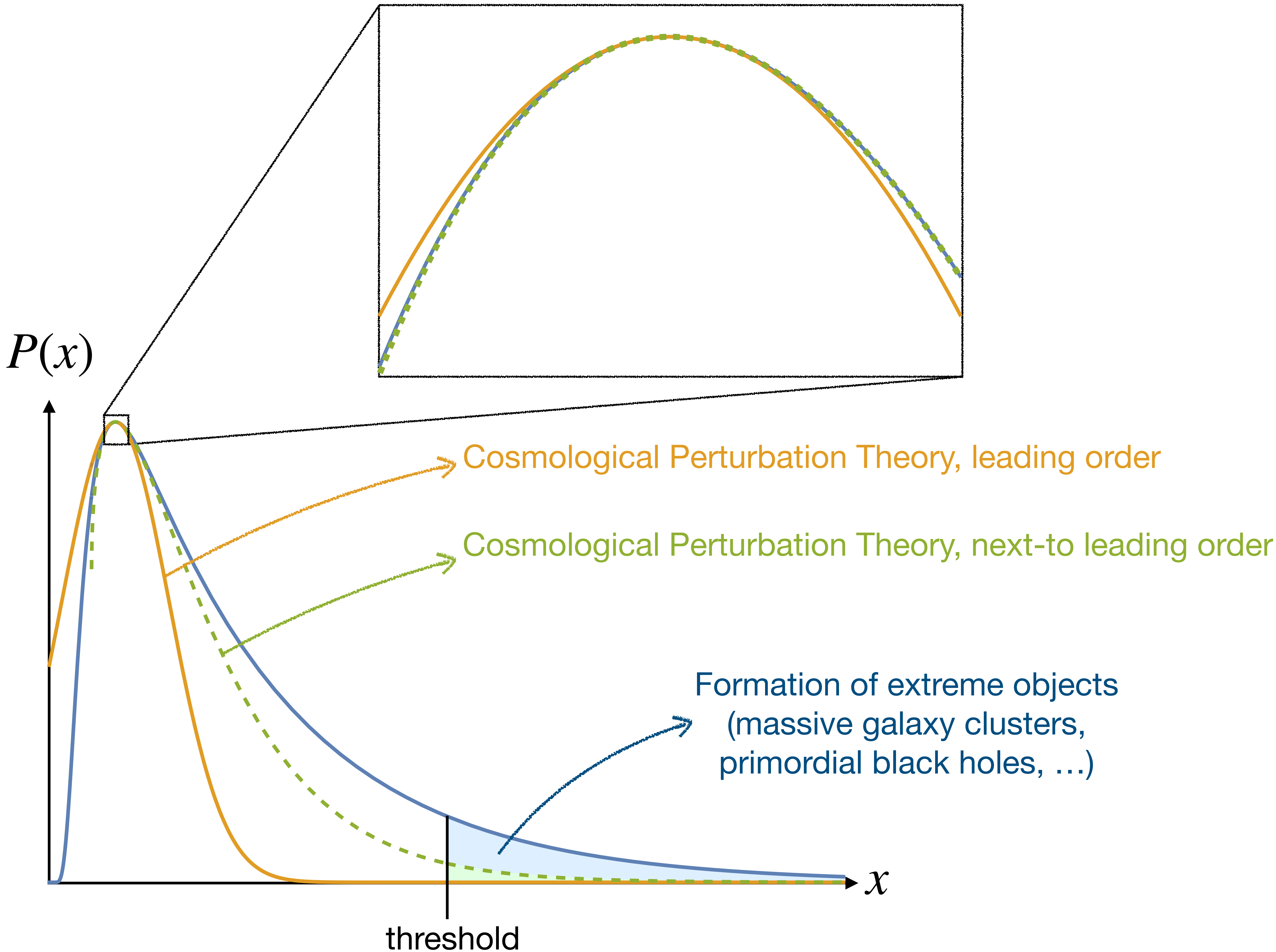
$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
x



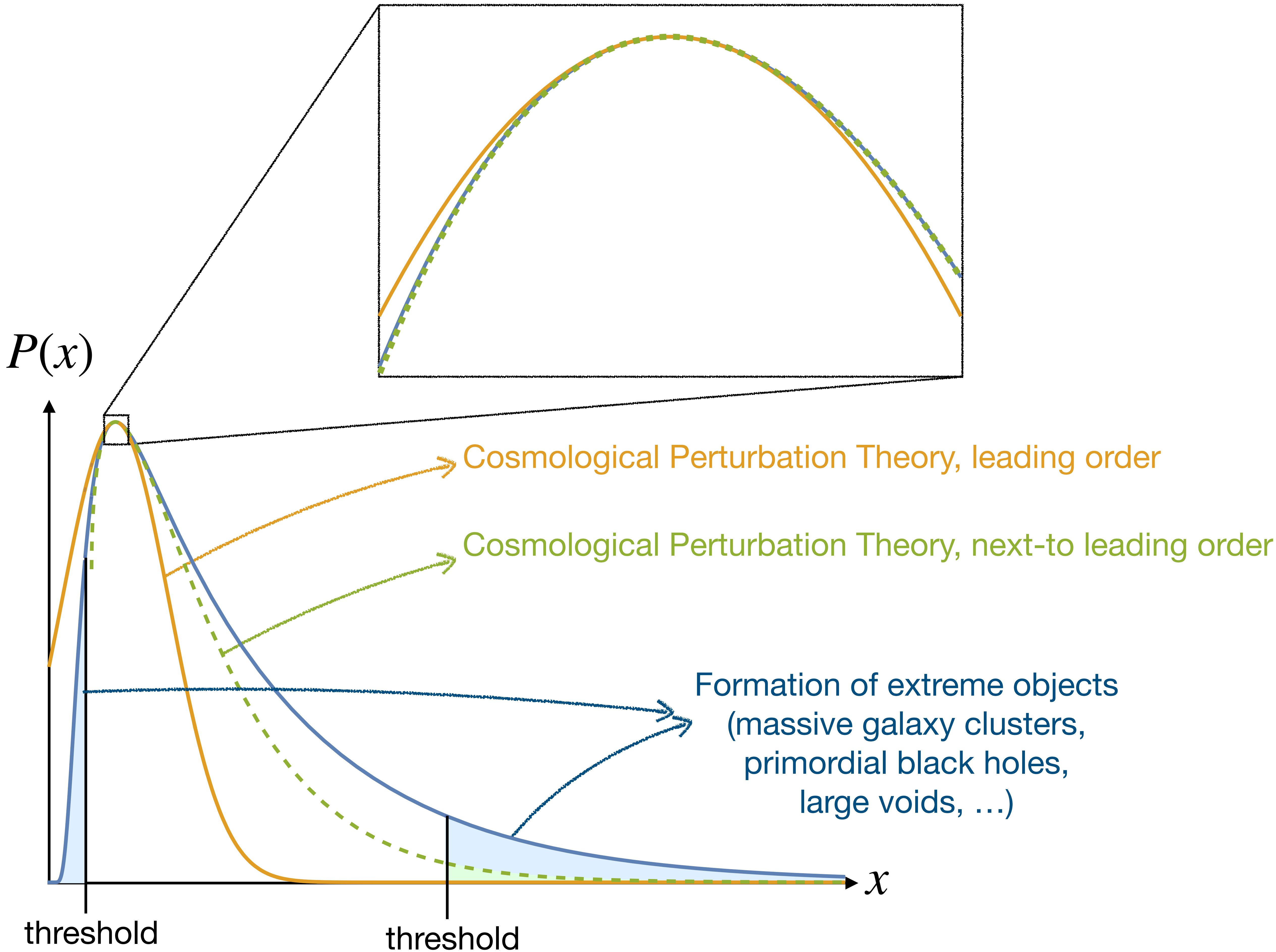
$P(x)$
x
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
$S \ni \delta\phi^2, \delta\phi^3$
$x = x_{\rm G} + f_{\rm NL}\left(x_{\rm G}^2 - \langle x_{\rm G}^2 \rangle\right)$



$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes, …)
x
threshold

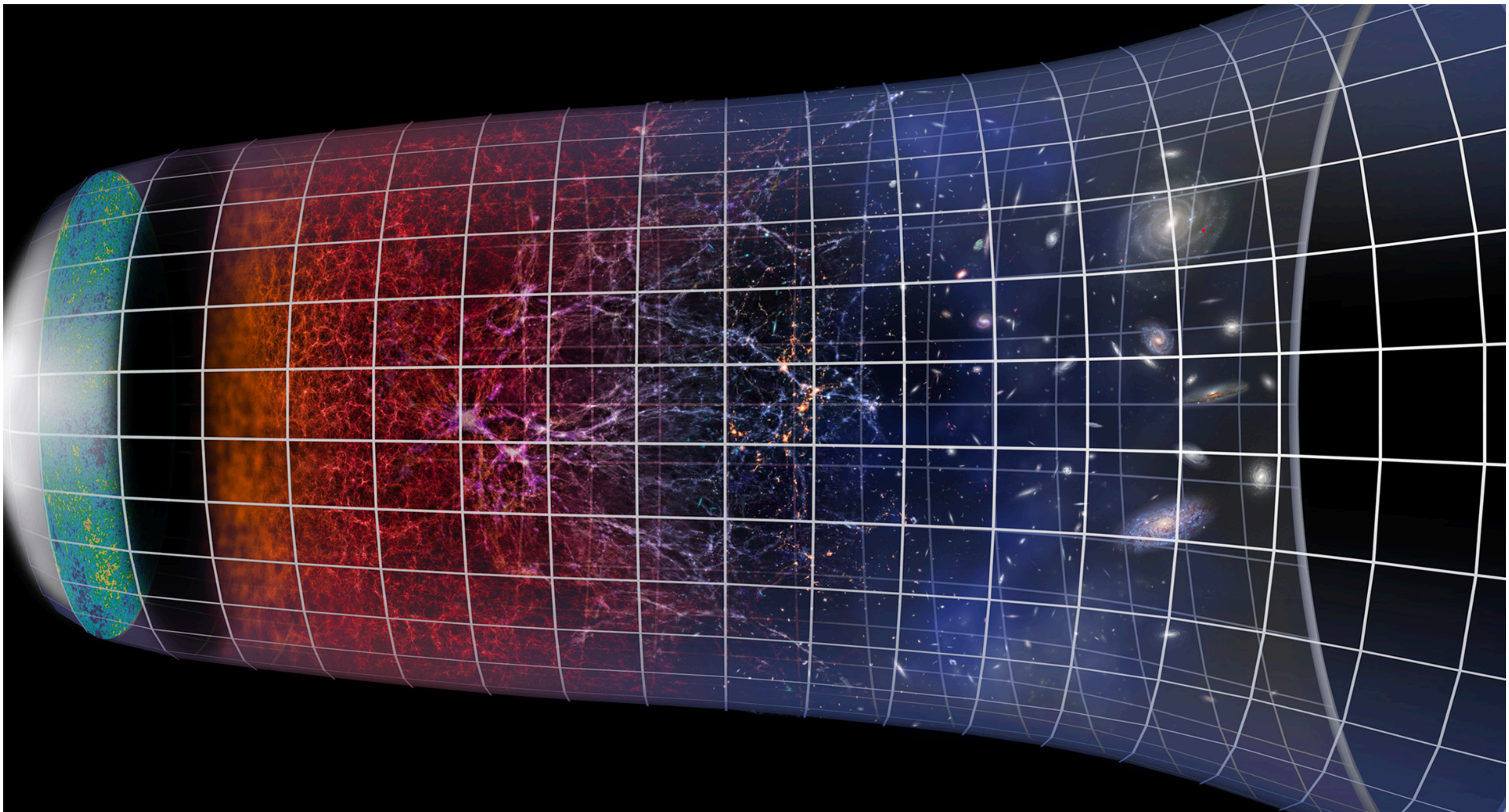


$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes, …)
x
threshold


$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes,
large voids, …)
x
threshold
threshold

Cosmic Inflation

$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$ with $\ddot{a} > 0$ and $(10\,\mathrm{MeV})^4 < \rho < (10^{16}\,\mathrm{GeV})^4$



Cosmic Inflation

$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2\,(t)\,\mathrm{d}\vec{x}^2$ with $\ddot{a} > 0$ and $(10\,\mathrm{MeV})^4 < \rho < \left(10^{16}\,\mathrm{GeV}\right)^4$

Hubble parameter $H = \dot{a}/a$

Cosmic Inflation

$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$ with $\ddot{a} > 0$ and $(10\,\mathrm{MeV})^4 < \rho < \left(10^{16}\,\mathrm{GeV}\right)^4$

Hubble parameter $H = \dot{a}/a$

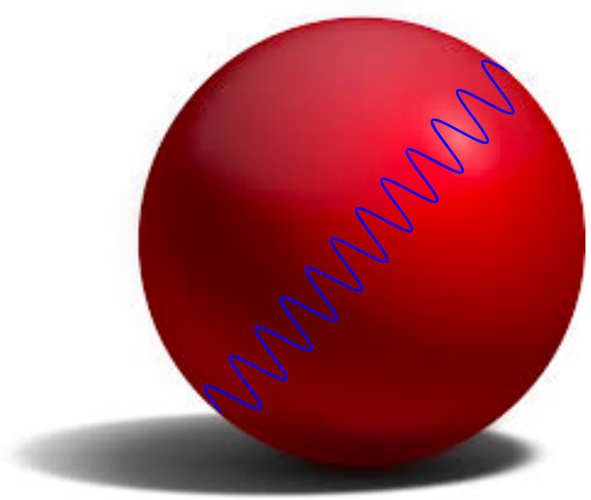
H^{-1} : characteristic time scale, or length scale ($c = 1$), of the expansion

Cosmic Inflation

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2\left(t\right)\mathrm{d}\vec{x}^2 \quad \text{with} \quad \ddot{a} > 0 \quad \text{and} \quad \left(10\,\mathrm{MeV}\right)^4 < \rho < \left(10^{16}\,\mathrm{GeV}\right)^4$$

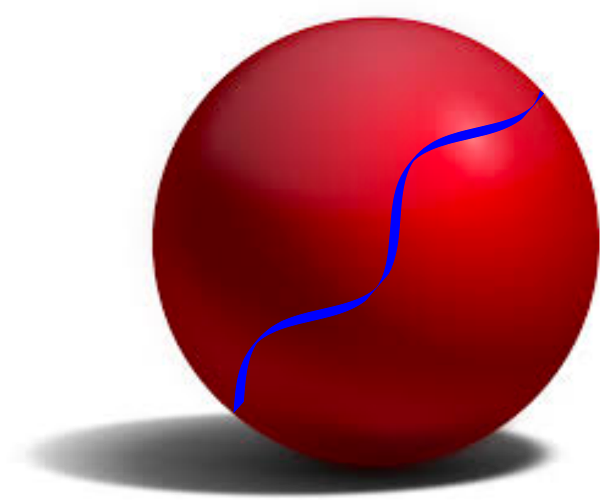
Hubble parameter $\quad H = \dot{a}/a$

H^{-1} : characteristic time scale, or length scale ($c = 1$), of the expansion



$\lambda \ll H^{-1}$

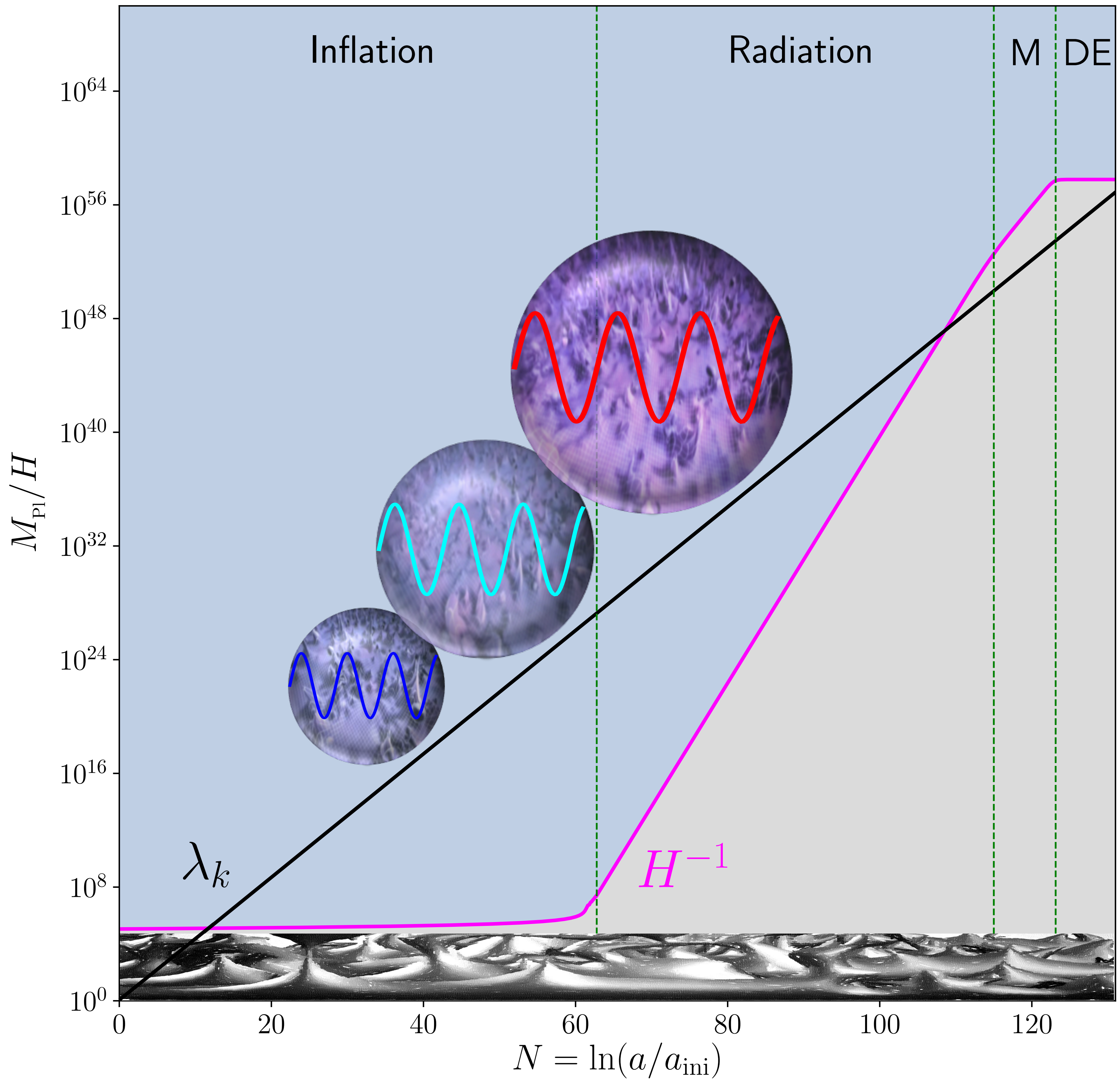
Insensitive to space-time curvature



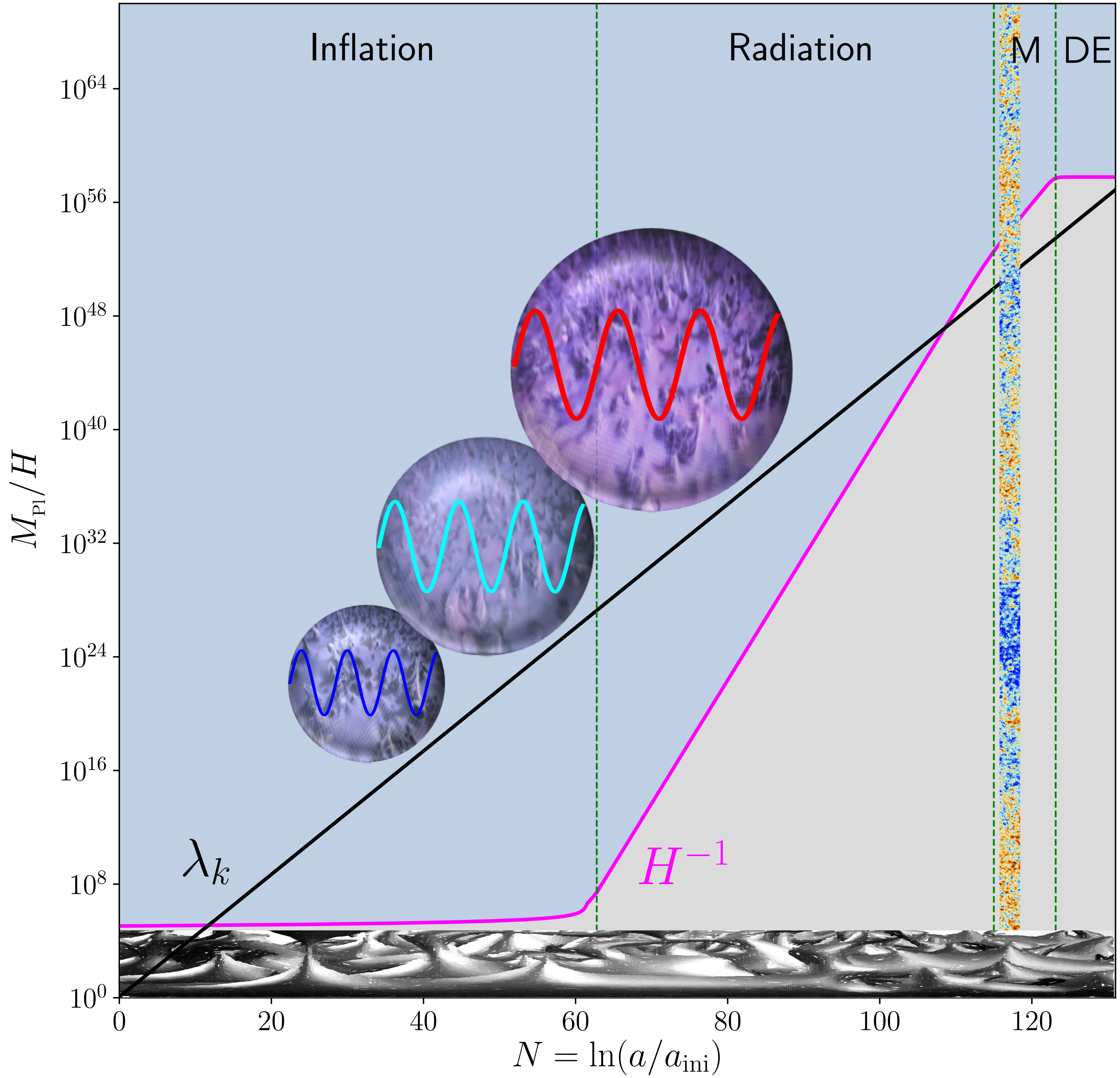
$\lambda \gtrsim H^{-1}$

Feels space-time curvature

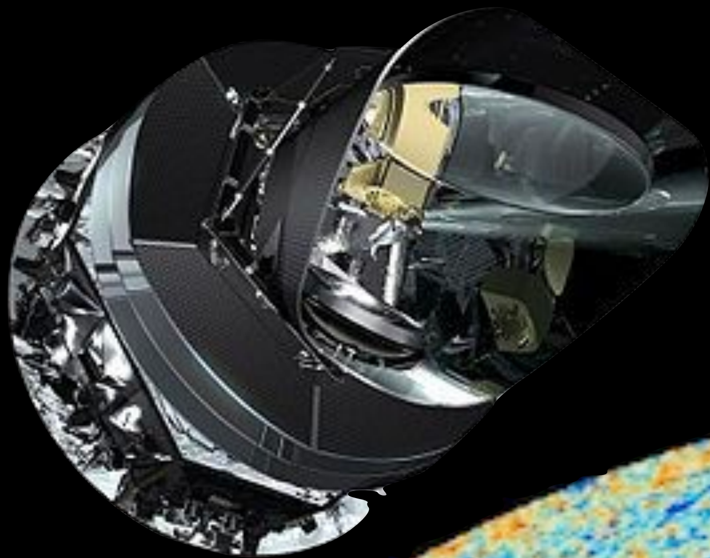
Cosmic Inflation



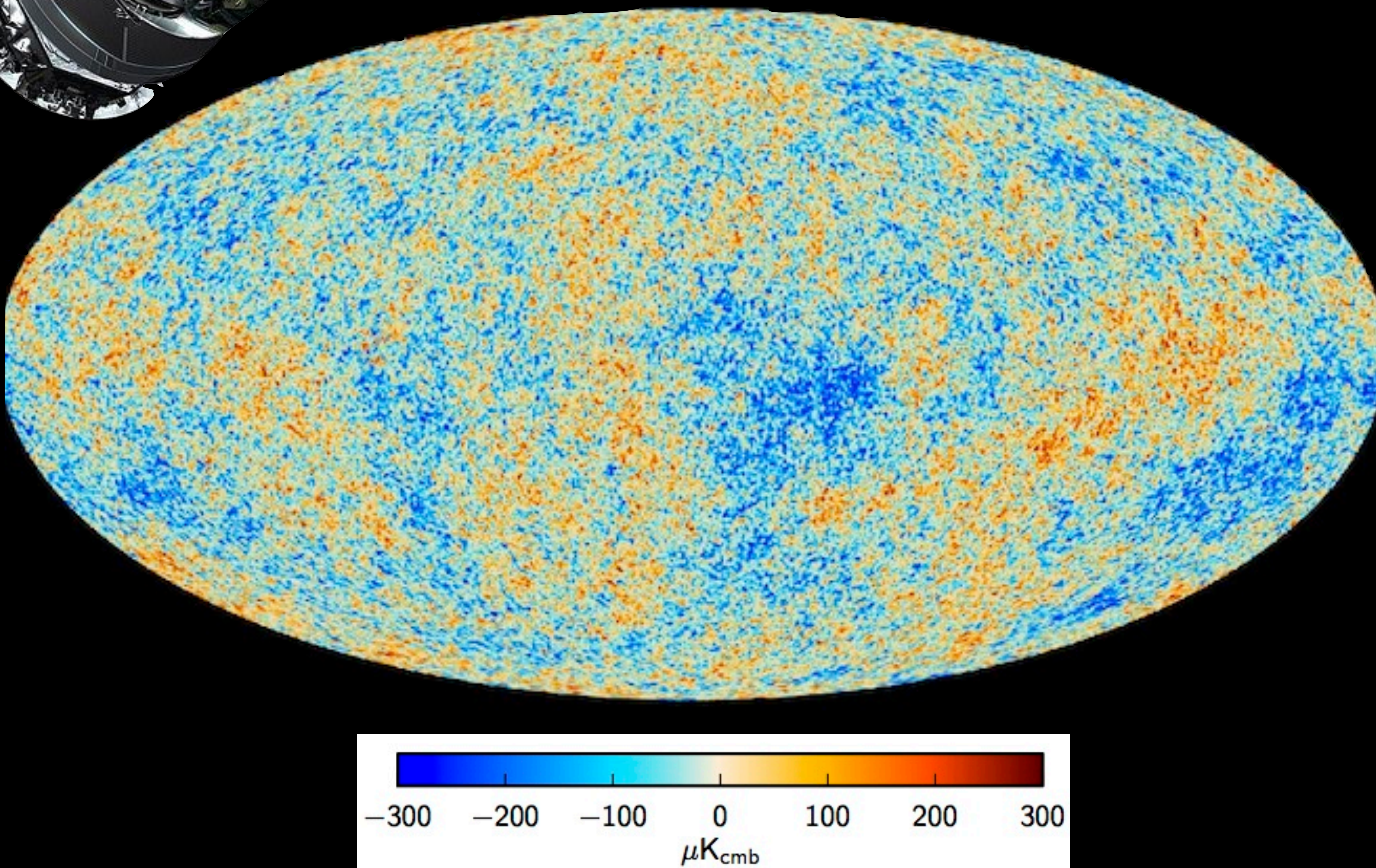
Cosmic Inflation



Planck satellite



$$\frac{\delta T}{T} \sim 10^{-5} \ll 1$$



Cosmological Perturbation Theory

Density fluctuations are small at CMB scales $\longrightarrow$ Perturbation Theory

Cosmological Perturbation Theory

Density fluctuations are small at CMB scales $\longrightarrow$ Perturbation Theory

$$g_{\mu\nu}(\overrightarrow{x}, t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\overrightarrow{x}, t)$$

$$\phi(\overrightarrow{x}, t) = \bar{\phi}(t) + \widehat{\delta\phi}(\overrightarrow{x}, t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation

Cosmological Perturbation Theory

Density fluctuations are small at CMB scales $\longrightarrow$ Perturbation Theory

$$g_{\mu\nu}(\overrightarrow{x}, t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\overrightarrow{x}, t)$$

$$\phi(\overrightarrow{x}, t) = \bar{\phi}(t) + \widehat{\delta\phi}(\overrightarrow{x}, t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation

—> Quantum-field-theory on curved space-time

Cosmological Perturbation Theory

Density fluctuations are small at CMB scales $\longrightarrow$ Perturbation Theory

$$g_{\mu\nu}(\overrightarrow{x},t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\overrightarrow{x},t)$$

$$\phi(\overrightarrow{x},t) = \bar{\phi}(t) + \widehat{\delta\phi}(\overrightarrow{x},t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation

—> Quantum-field-theory on curved space-time

Strong assumption: universe is quasi homogeneous and isotropic at <u>all scales</u>

Cosmological Perturbation Theory

Density fluctuations are small at CMB scales $\longrightarrow$ Perturbation Theory

$$g_{\mu\nu}(\overrightarrow{x},t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\overrightarrow{x},t)$$

$$\phi(\overrightarrow{x},t) = \bar{\phi}(t) + \widehat{\delta\phi}(\overrightarrow{x},t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation

—> Quantum-field-theory on curved space-time

Strong assumption: universe is quasi homogeneous and isotropic at <u>all scales</u>

Cosmological Perturbation Theory

Density fluctuations are small at CMB scales $\longrightarrow$ Perturbation Theory

$$g_{\mu\nu}(\vec{x},t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\vec{x},t)$$

$$\phi(\vec{x},t) = \bar{\phi}(t) + \widehat{\delta\phi}(\vec{x},t)$$

Homogeneous and isotropic solution of the classical problem

Quantised fluctuation

—> Quantum-field-theory on curved space-time

Strong assumption: universe is quasi homogeneous and isotropic at all scales

This may be broken at:

- Larger scales: space-time structure beyond the observable universe
- Smaller scales: formation of extreme objects such as primordial black holes, heavy clusters, large voids etc

Separate Universe

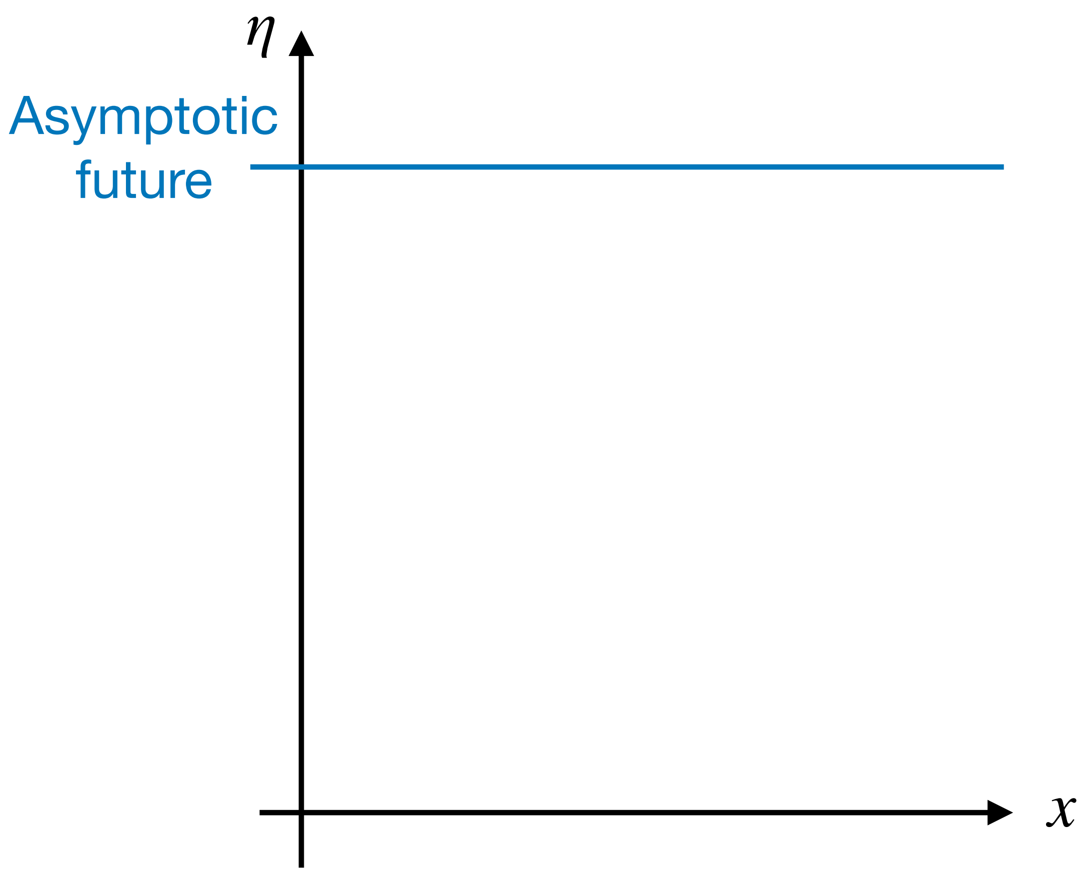
$$\mathrm{d}s^2 = a^2\left(-\mathrm{d}\eta^2 + \mathrm{d}\overrightarrow{x}^2\right)$$

de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$

Separate Universe

$$\mathrm{d}s^2 = a^2\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$$

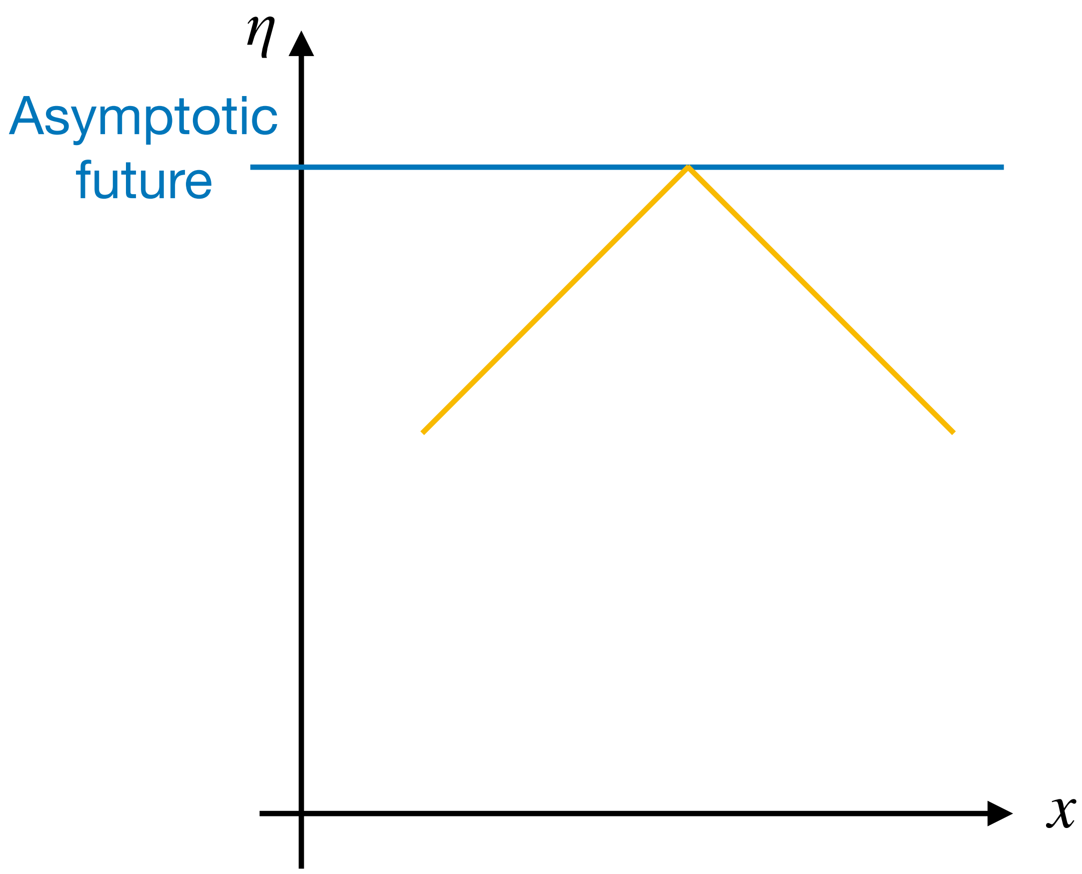
de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$



Separate Universe

$$\mathrm{d}s^2 = a^2\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$$

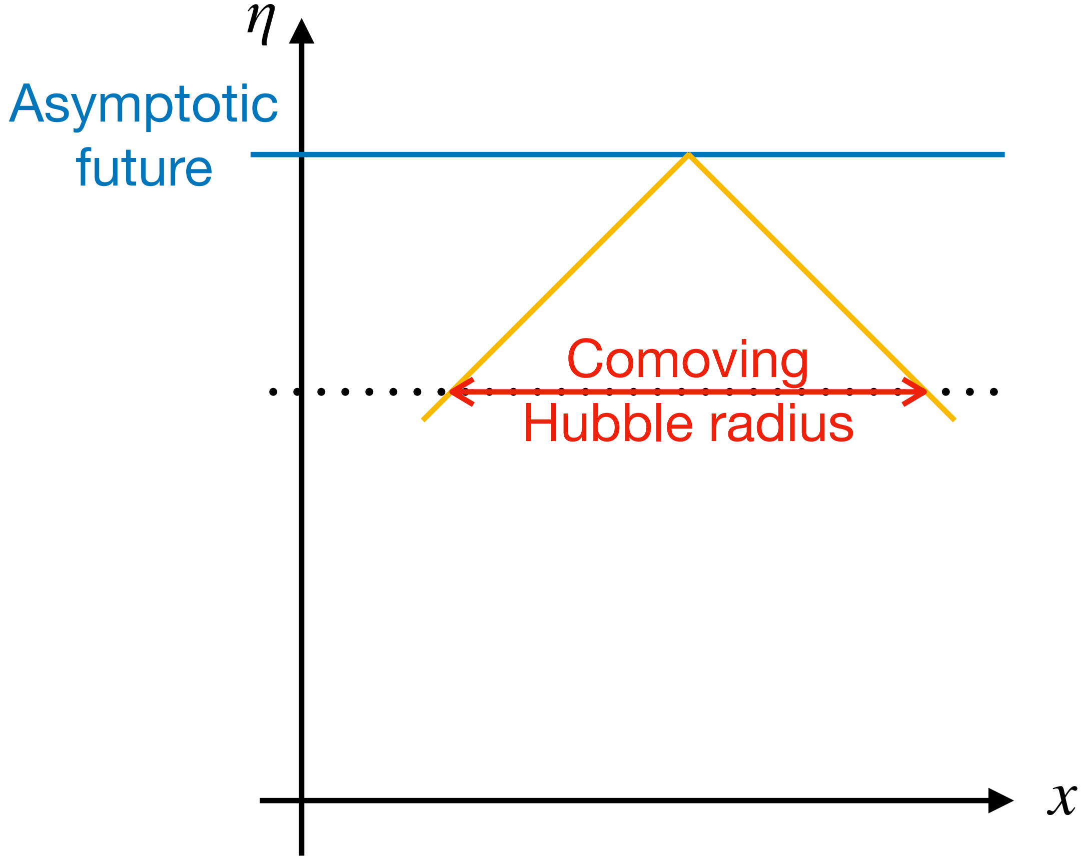
de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$



Separate Universe

$$\mathrm{d}s^2 = a^2\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$$

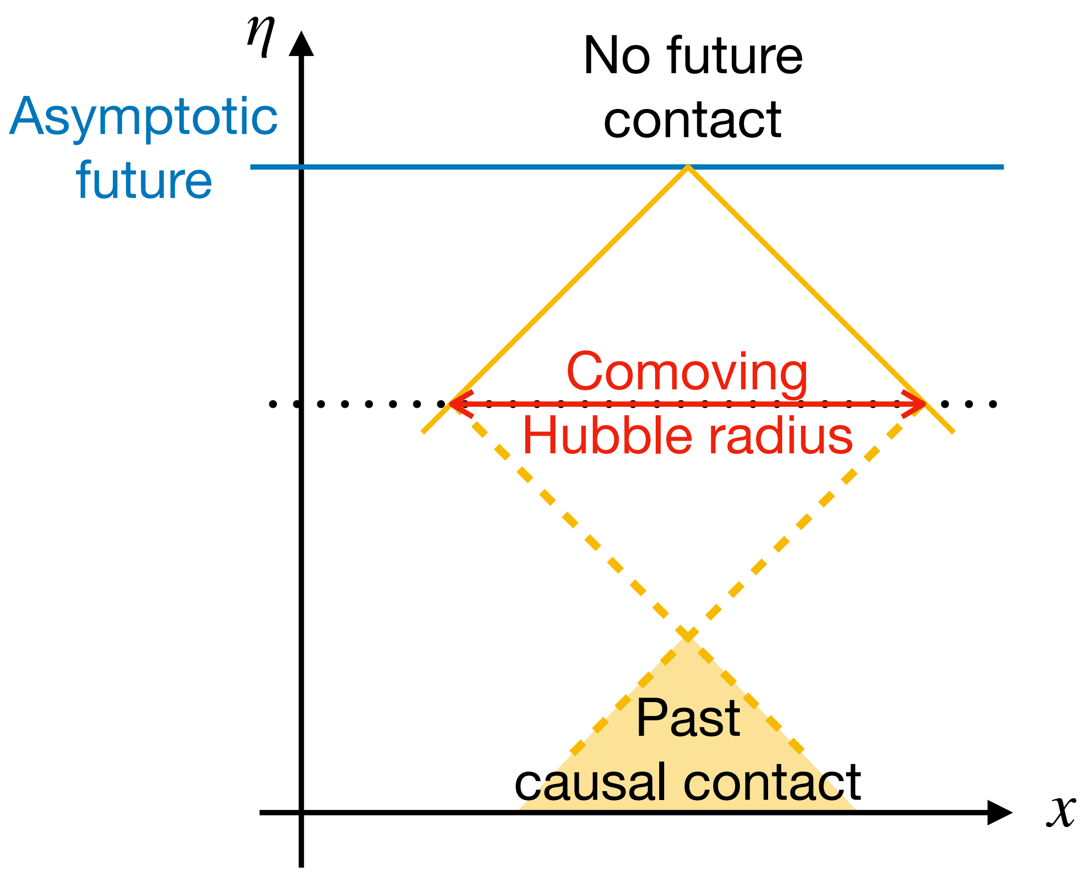
de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$



Separate Universe

$$\mathrm{d}s^2 = a^2\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$$

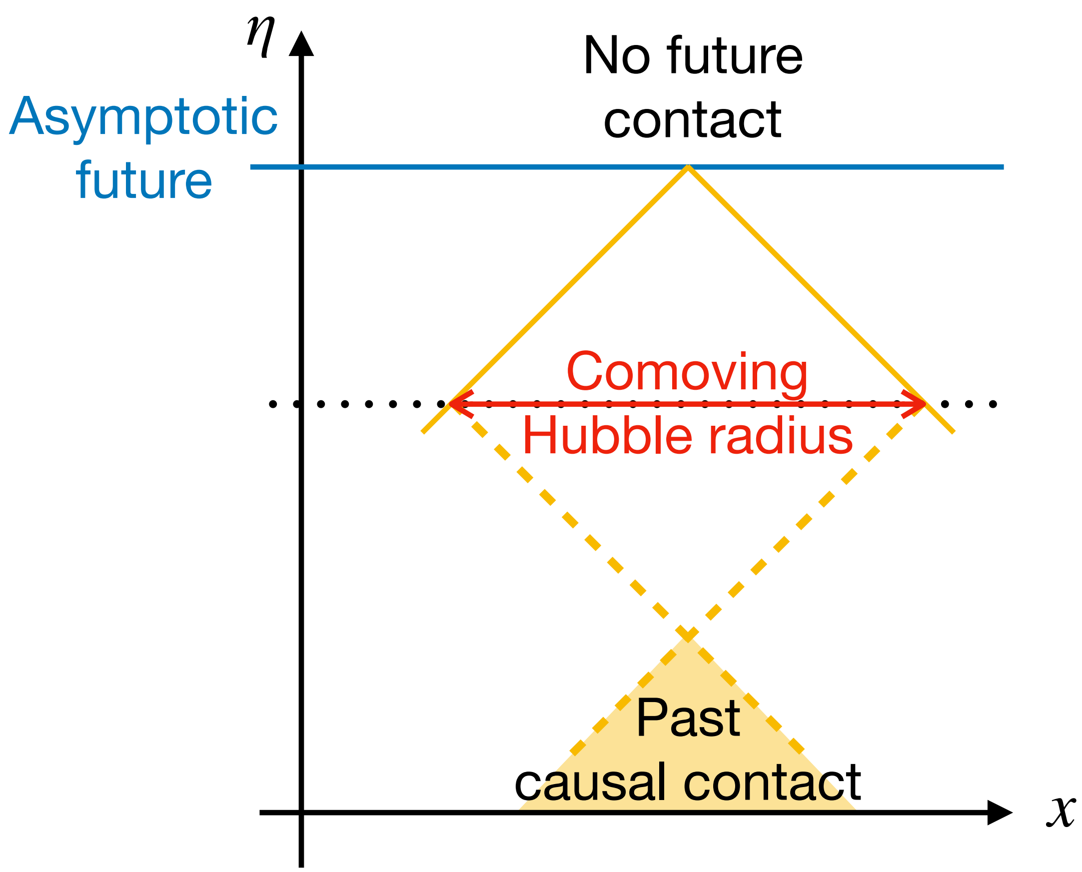
de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$

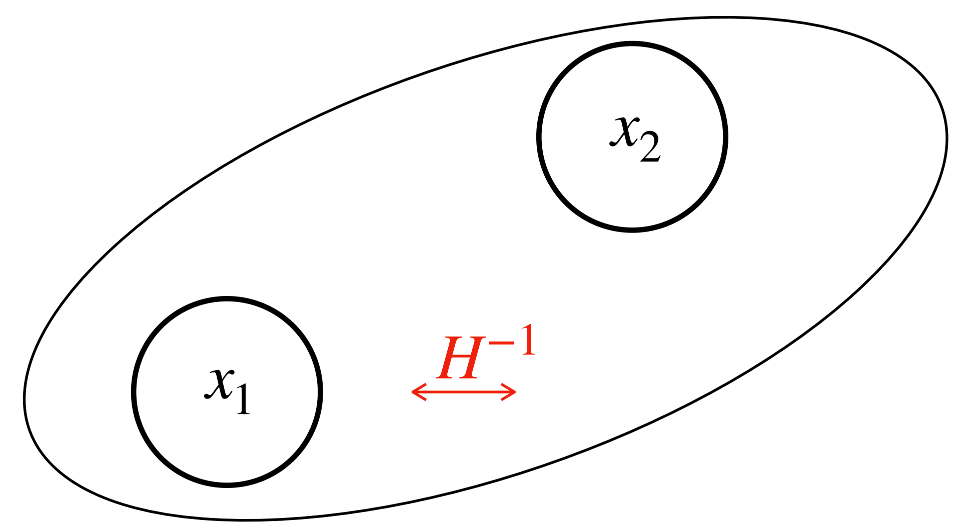


Separate Universe

$$\mathrm{d}s^2 = a^2\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$$

de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$



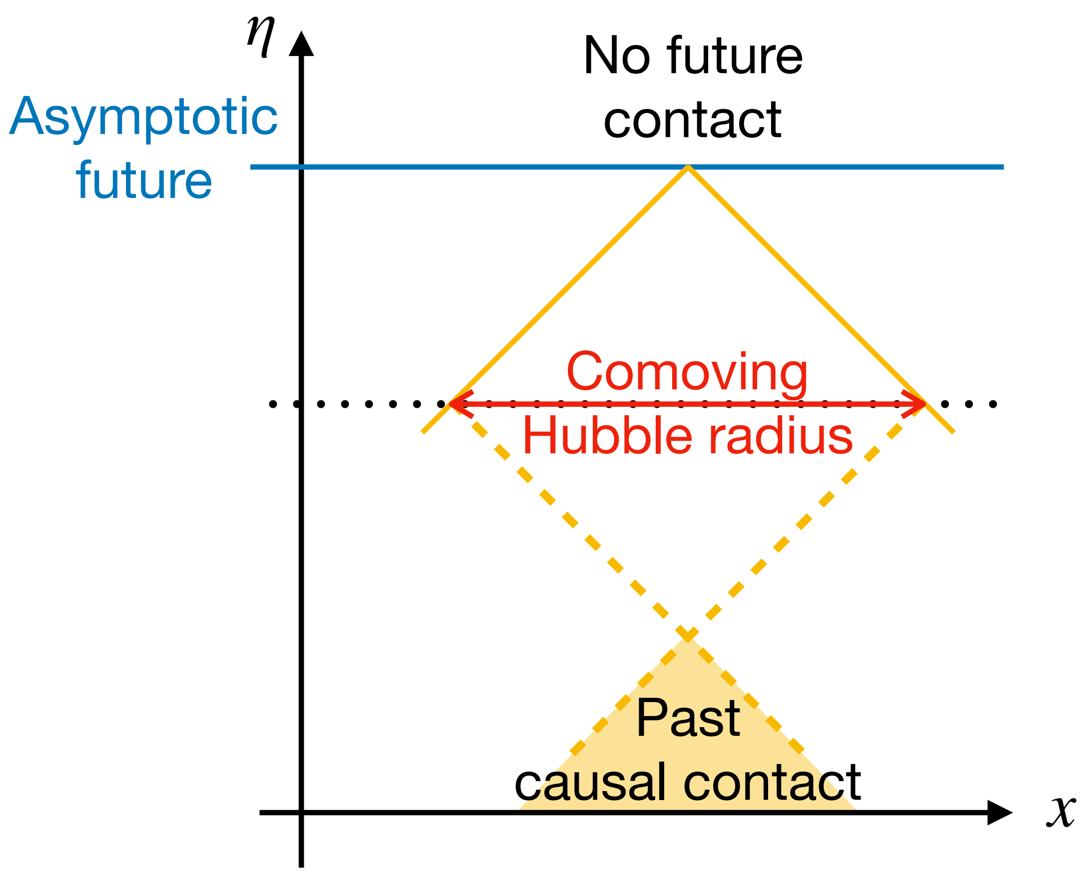


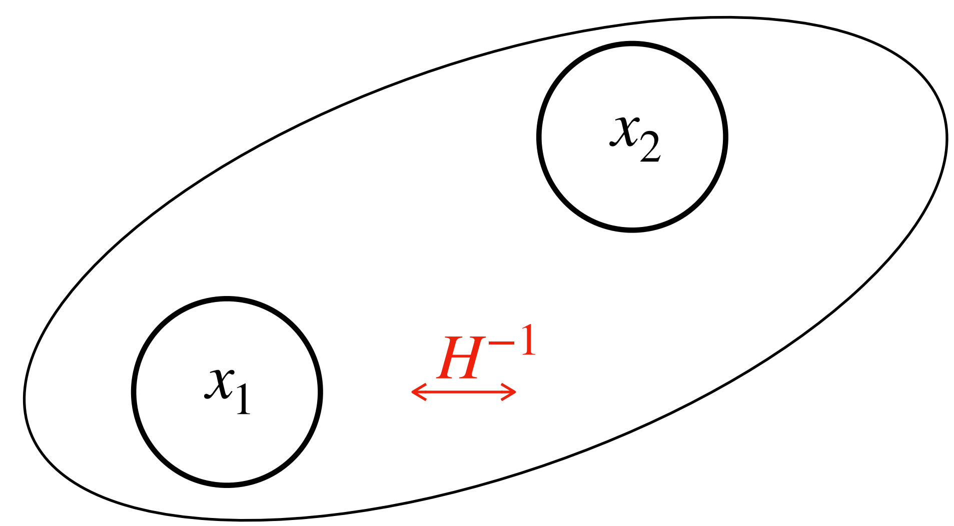
If a large fluctuation develops at $\boldsymbol{x}_1$, this cannot affect the local geometry at $\boldsymbol{x}_2$

Separate Universe

$$\mathrm{d}s^2 = a^2\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$$

de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$





If a large fluctuation develops at $\boldsymbol{x}_1$, this cannot affect the local geometry at $\boldsymbol{x}_2$

Separate universe: On large scales, the universe can be described by an ensemble of independent, locally homogeneous and isotropic patches

Salopek & Bond; Sasaki & Stewart; Wands, Malik, Lyth & Liddle

The quantum state of cosmological perturbations

The quantum state of cosmological perturbations

- $|\Psi\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle$ with $|\Psi_{\boldsymbol{k}}\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{2in\varphi_k}(-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$

Two-mode squeezed state (Gaussian state)

The quantum state of cosmological perturbations

- $|\Psi\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle$ with $|\Psi_{\boldsymbol{k}}\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{2in\varphi_k}(-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$

Two-mode squeezed state (Gaussian state)

- Evolution equation $\frac{\partial}{\partial t}W(v,p,t) = -\{W(v,p,t)\ ,\ H(v,p,t)\}_{\text{Poisson Bracket}}$

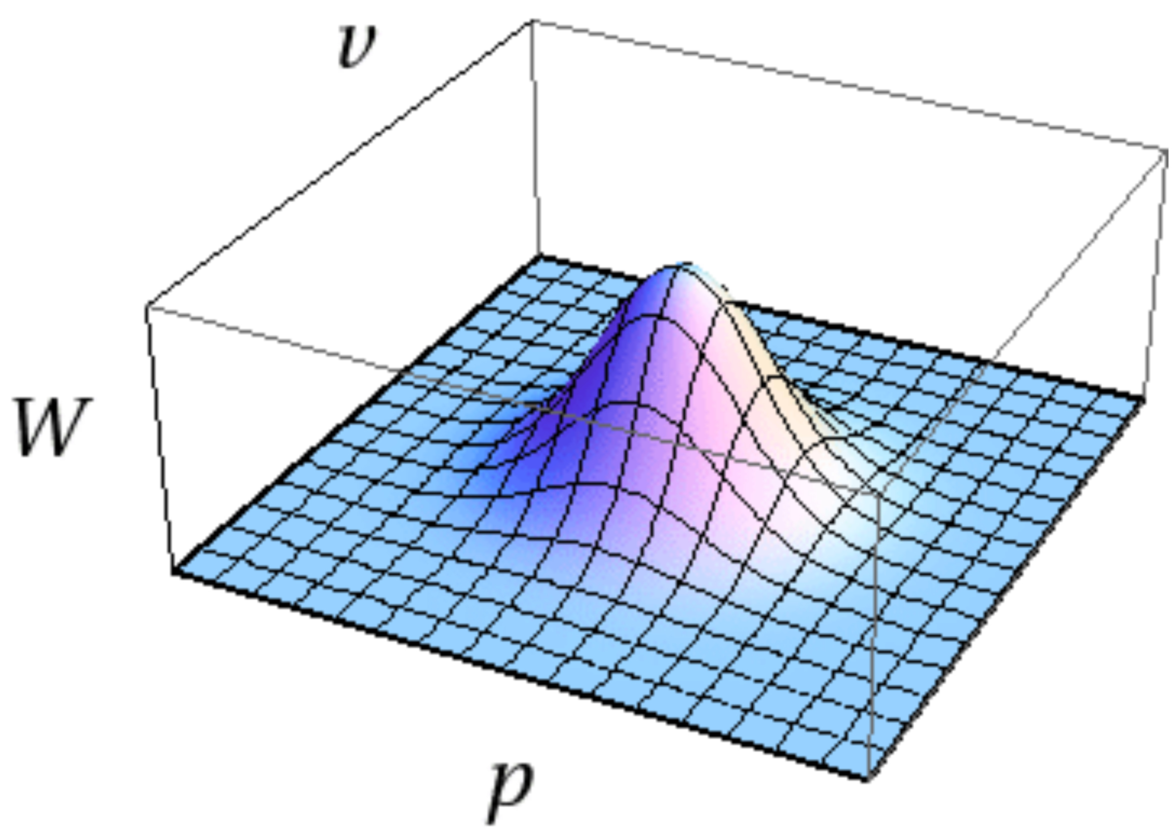
For quadratic Hamiltonians

Classicality in the Wigner approach

Wigner function $$W\left(q,p\right)=\int\Psi^{*}\left(q-\frac{u}{2}\right)e^{-ipu}\Psi\left(q+\frac{u}{2}\right)\frac{\mathrm{d}u}{2\pi}$$

Evolution equation: $$\frac{\partial}{\partial t}W\left(q,p,t\right)=-\left\{W(q,p,t),H(q,p,t)\right\}_{\mathrm{Poisson\ \ bracket}}$$

for quadratic Hamiltonians

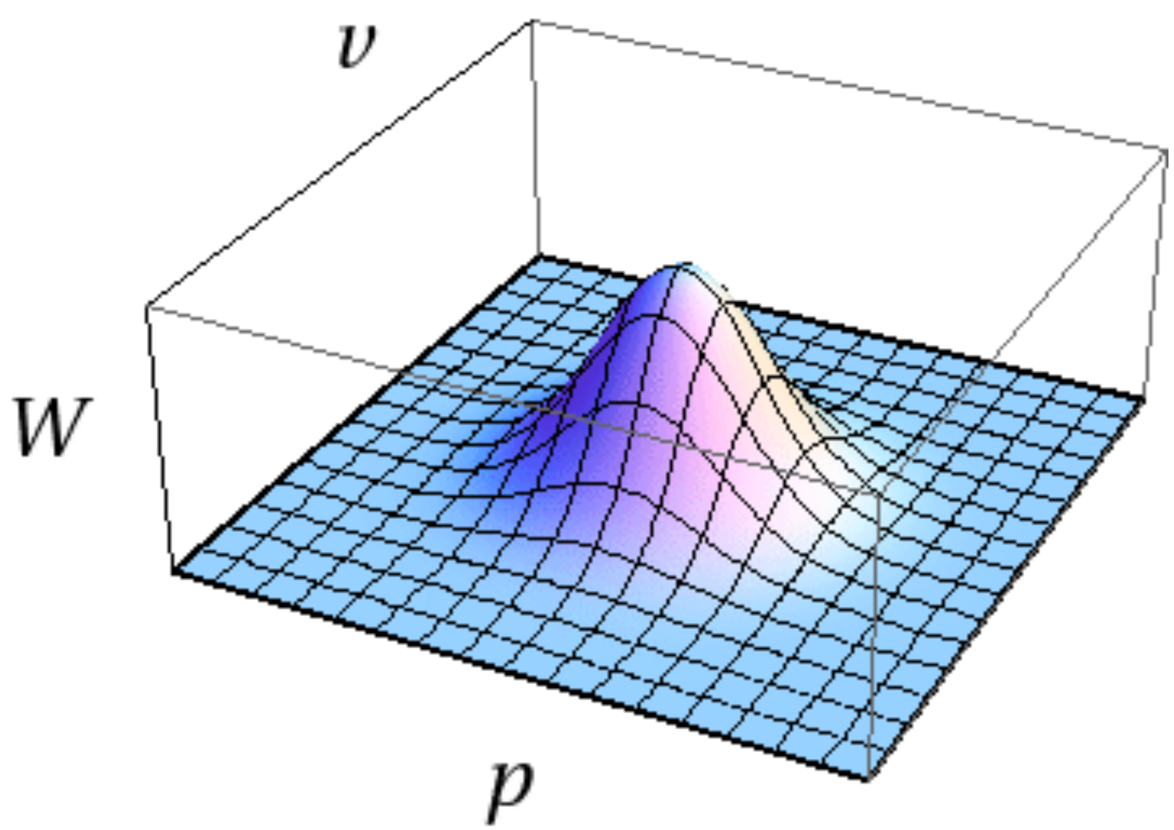


Classicality in the Wigner approach

Wigner function $$W\left(q,p\right)=\int\Psi^{*}\left(q-\frac{u}{2}\right)e^{-ipu}\Psi\left(q+\frac{u}{2}\right)\frac{\mathrm{d}u}{2\pi}$$

Evolution equation: $$\frac{\partial}{\partial t}W\left(q,p,t\right)=-\left\{W(q,p,t),H(q,p,t)\right\}_{\mathrm{Poisson\ \ bracket}}$$

for quadratic Hamiltonians

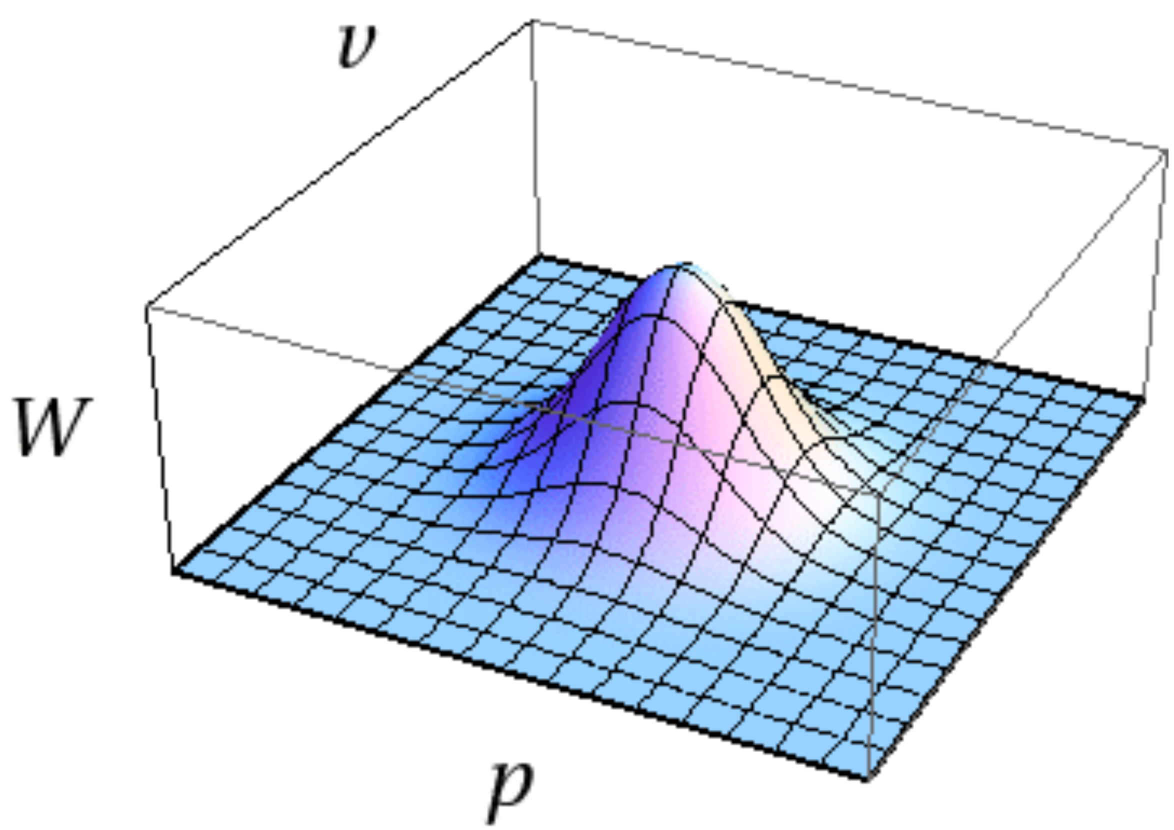


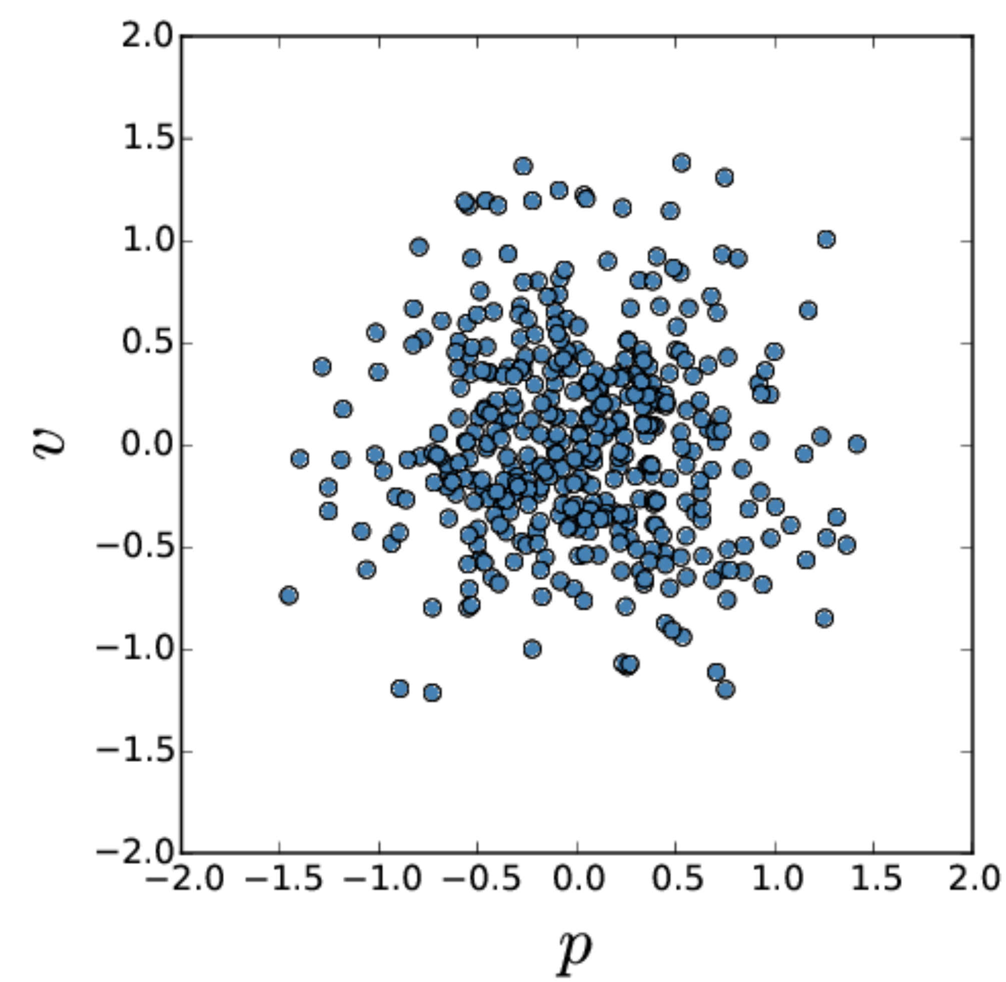
Classicality in the Wigner approach

Wigner function $$W\left(q,p\right)=\int\Psi^{*}\left(q-\frac{u}{2}\right)e^{-ipu}\Psi\left(q+\frac{u}{2}\right)\frac{\mathrm{d}u}{2\pi}$$

Evolution equation: $$\frac{\partial}{\partial t}W\left(q,p,t\right)=-\left\{W(q,p,t),H(q,p,t)\right\}_{\mathrm{Poisson\ bracket}}$$

for quadratic Hamiltonians



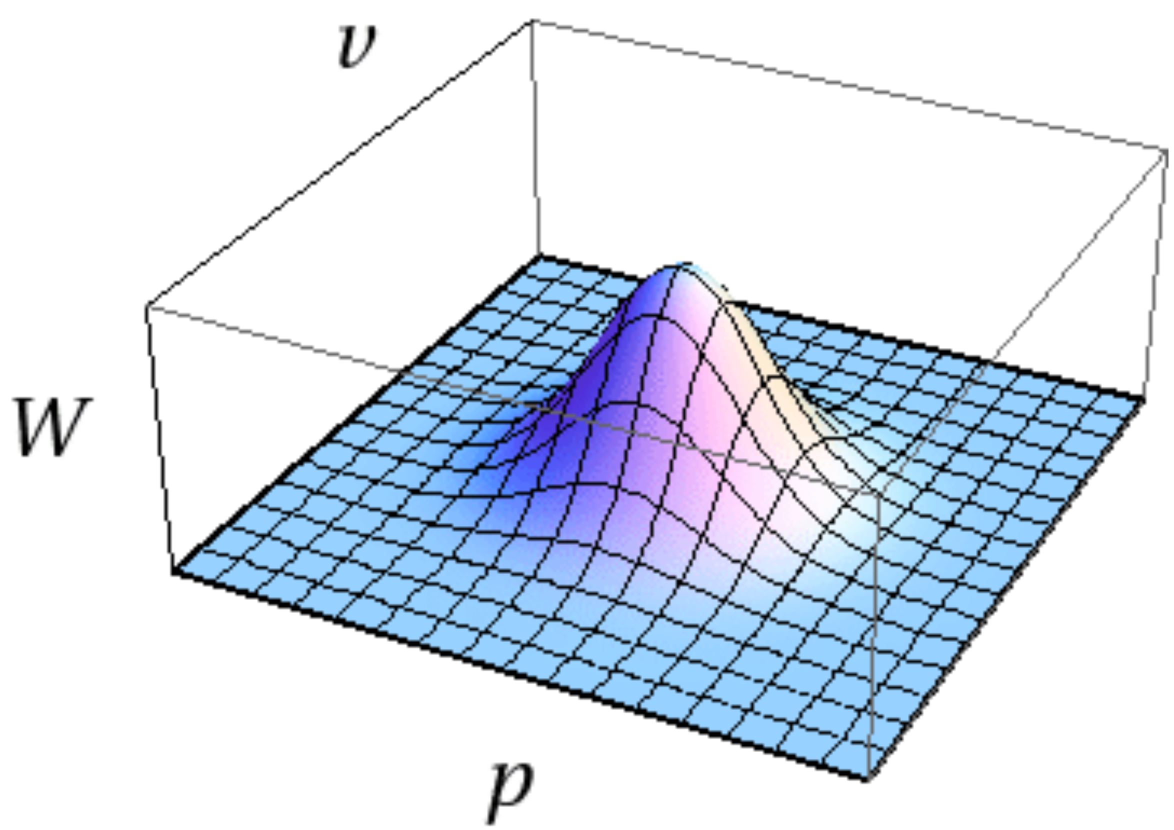


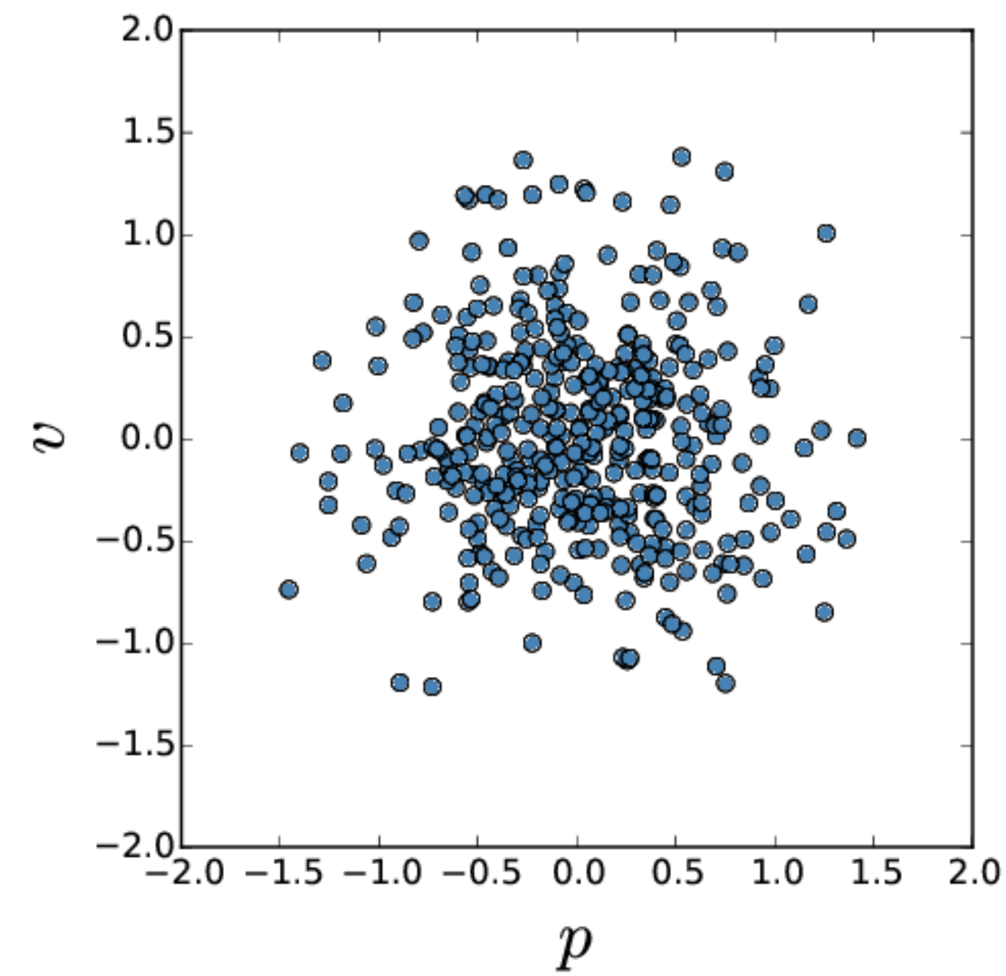
Classicality in the Wigner approach

Wigner function $$W(q,p) = \int \Psi^*\left(q - \frac{u}{2}\right) e^{-ipu} \Psi\left(q + \frac{u}{2}\right) \frac{\mathrm{d}u}{2\pi}$$

Evolution equation: $$\frac{\partial}{\partial t} W(q,p,t) = -\left\{W(q,p,t), H(q,p,t)\right\}_{\mathrm{Poisson\ bracket}}$$

for quadratic Hamiltonians





The quantum state of cosmological perturbations

- $|\Psi\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle$ with $|\Psi_{\boldsymbol{k}}\rangle = \dfrac{1}{\cosh r_k} \sum_{n=0}^{\infty} e^{2in\varphi_k} (-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$

  Two-mode squeezed state (Gaussian state)

- Evolution equation $\dfrac{\partial}{\partial t} W(v,p,t) = -\left\{W(v,p,t)\ ,\ H(v,p,t)\right\}_{\text{Poisson Bracket}}$

  For quadratic Hamiltonians

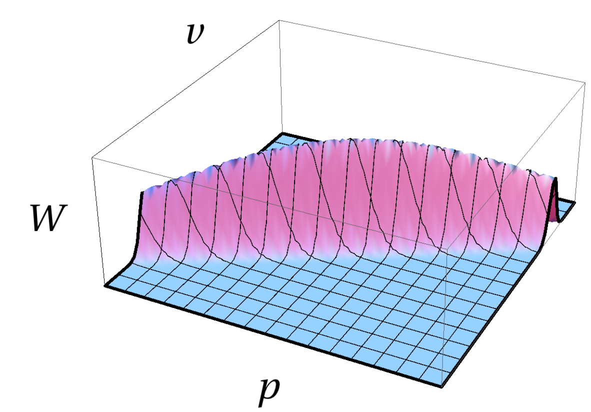
The quantum state of cosmological perturbations

- $|\Psi\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle$ with $|\Psi_{\boldsymbol{k}}\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{2in\varphi_k}(-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$

Two-mode squeezed state (Gaussian state)

- Evolution equation $\frac{\partial}{\partial t} W(v,p,t) = -\left\{W(v,p,t)\ ,\ H(v,p,t)\right\}_{\text{Poisson Bracket}}$

For quadratic Hamiltonians



- Quantum mean value and stochastic average

$$\left\langle \hat{\mathcal{O}}\left(\hat{v},\hat{p}\right)\right\rangle_{\text{quant}} = \int W(v,p)\,\tilde{\mathcal{O}}(v,p)\,\mathrm{d}v\,\mathrm{d}p$$

Large squeezing: $\tilde{O}(v,p) \longrightarrow O(v,p)$

Lesgourgues, Polarski, Starobinsky (1997)
Martin, VV (2016)

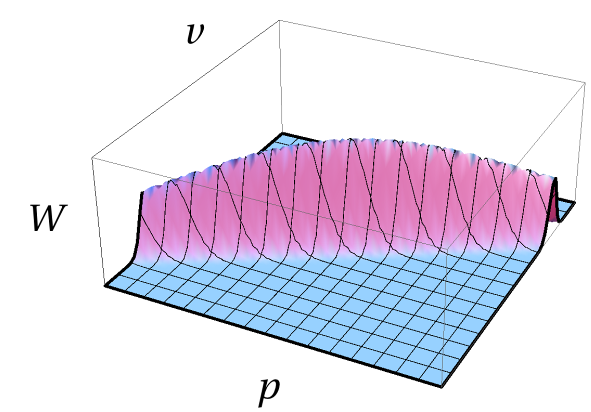
The quantum state of cosmological perturbations

- $|\Psi\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle$ with $|\Psi_{\boldsymbol{k}}\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{2in\varphi_k}(-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$

Two-mode squeezed state (Gaussian state)

- Evolution equation $\frac{\partial}{\partial t}W(v,p,t) = -\left\{W(v,p,t)\ ,\ H(v,p,t)\right\}_{\text{Poisson Bracket}}$

For quadratic Hamiltonians



- Quantum mean value and stochastic average

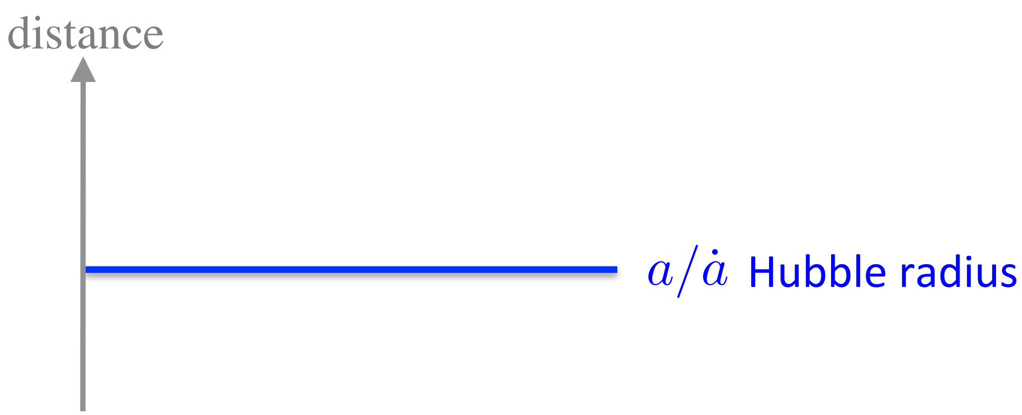
$$\left\langle \hat{\mathcal{O}}(\hat{v},\hat{p})\right\rangle_{\text{quant}} = \int W(v,p)\,\tilde{\mathcal{O}}(v,p)\,\mathrm{d}v\,\mathrm{d}p$$

Large squeezing: $\tilde{O}(v,p) \longrightarrow O(v,p)$

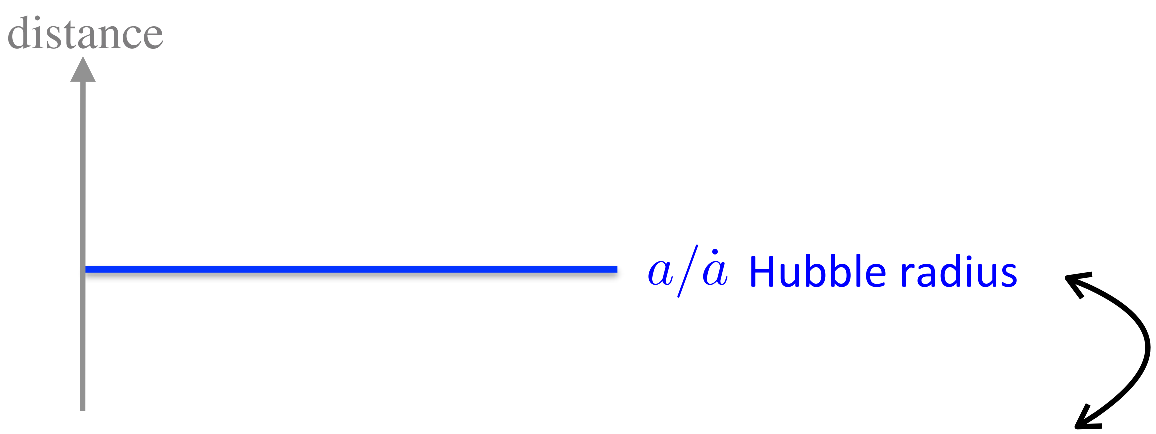
Lesgourgues, Polarski, Starobinsky (1997)
Martin, VV (2016)

(at least for proper operators…) Revzen (2006); Martin, VV (2017)

Stochastic Inflation

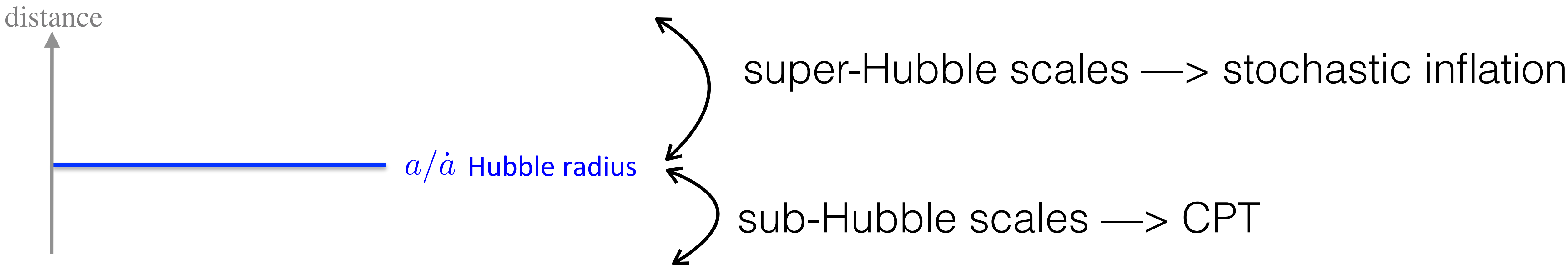


Stochastic Inflation

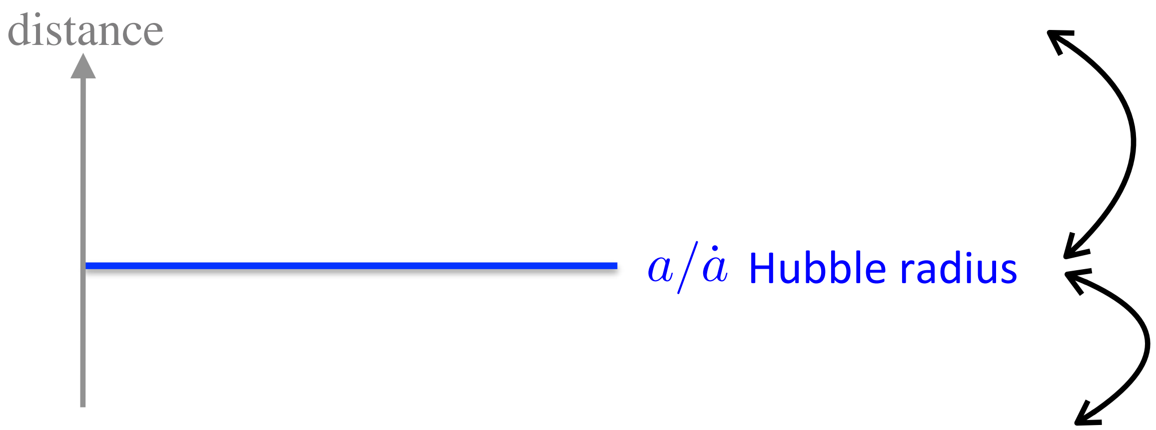


sub-Hubble scales —> CPT

Stochastic Inflation



Stochastic Inflation



super-Hubble scales —> stochastic inflation

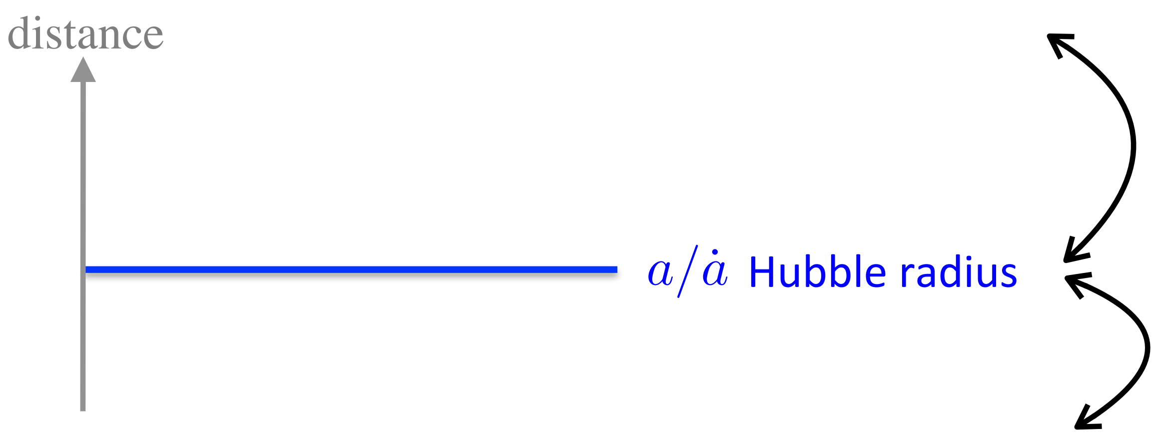
sub-Hubble scales —> CPT

Coarse-grained field

$N = \ln(a)$

$$\hat{\mathbf{\Phi}}_{\rm cg}(N, \vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k} \left[\mathbf{\Phi}_{\vec{k}}(N) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \mathbf{\Phi}^{\star}_{\vec{k}}(N) e^{i\vec{k}\cdot\vec{x}} \hat{a}^{\dagger}_{\vec{k}}\right]$$

Stochastic Inflation



super-Hubble scales —> stochastic inflation

sub-Hubble scales —> CPT

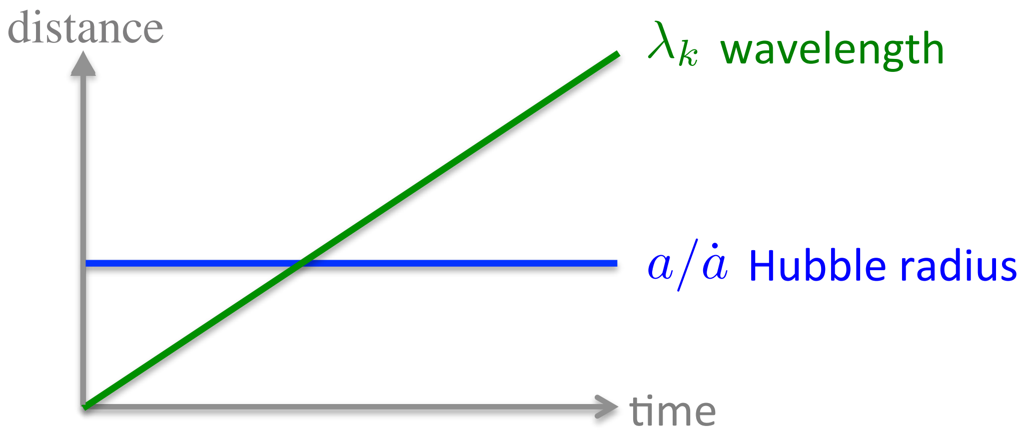
Coarse-grained field
$N = \ln(a)$

$$\hat{\boldsymbol{\Phi}}_{\mathrm{cg}}(N, \vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k} \left[\boldsymbol{\Phi}_{\vec{k}}(N) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \boldsymbol{\Phi}^{\star}_{\vec{k}}(N) e^{i\vec{k}\cdot\vec{x}} \hat{a}^{\dagger}_{\vec{k}} \right]$$

Equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}N}\boldsymbol{\Phi}_{\mathrm{cg}} = \mathscr{D}_{\mathrm{background}}(\boldsymbol{\Phi}_{\mathrm{cg}}) + \xi$$

Stochastic Inflation



super-Hubble scales —> stochastic inflation

sub-Hubble scales —> CPT

Coarse-grained field

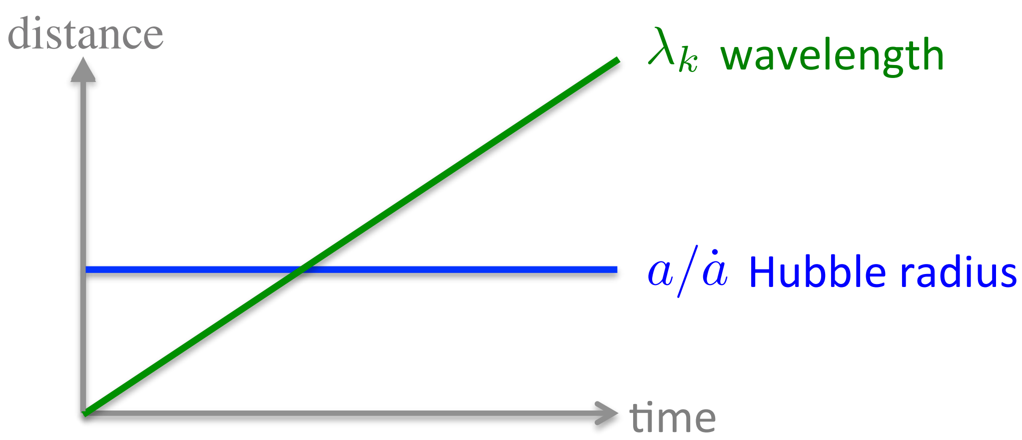
$N = \ln(a)$

$$\hat{\mathbf{\Phi}}_{\rm cg}(N, \vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k} \left[\mathbf{\Phi}_{\vec{k}}(N) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \mathbf{\Phi}^{\star}_{\vec{k}}(N) e^{i\vec{k}\cdot\vec{x}} \hat{a}^{\dagger}_{\vec{k}} \right]$$

Equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}N}\mathbf{\Phi}_{\rm cg} = \mathscr{D}_{\rm background}(\mathbf{\Phi}_{\rm cg}) + \xi$$

Stochastic Inflation



super-Hubble scales —> stochastic inflation

sub-Hubble scales —> CPT

Coarse-grained field
$N = \ln(a)$

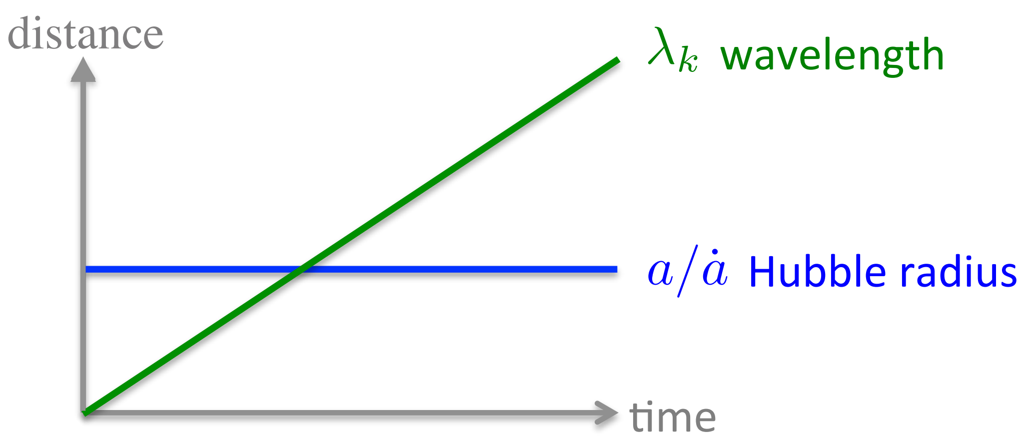
$$\hat{\mathbf{\Phi}}_{\mathrm{cg}}(N, \vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k} \left[\mathbf{\Phi}_{\vec{k}}(N) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \mathbf{\Phi}^{\star}_{\vec{k}}(N) e^{i\vec{k}\cdot\vec{x}} \hat{a}^{\dagger}_{\vec{k}} \right]$$

Quantum fluctuations source the **background**

Equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}N}\mathbf{\Phi}_{\mathrm{cg}} = \mathscr{D}_{\mathrm{background}}(\mathbf{\Phi}_{\mathrm{cg}}) + \xi$$

Stochastic Inflation



super-Hubble scales —> stochastic inflation

sub-Hubble scales —> CPT

Coarse-grained field $\hat{\mathbf{\Phi}}_{\rm cg}(N, \vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k} \left[\mathbf{\Phi}_{\vec{k}}(N) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \mathbf{\Phi}^{\star}_{\vec{k}}(N) e^{i\vec{k}\cdot\vec{x}} \hat{a}^{\dagger}_{\vec{k}} \right]$

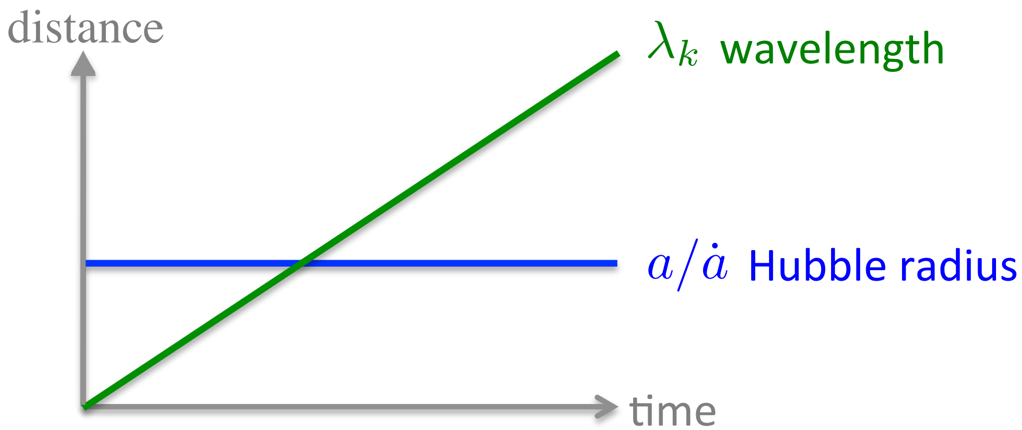
$N = \ln(a)$

Quantum fluctuations source the **background**

Equation of motion $$\frac{\mathrm{d}}{\mathrm{d}N}\mathbf{\Phi}_{\rm cg} = \mathscr{D}_{\rm background}(\mathbf{\Phi}_{\rm cg}) + \xi$$

Starobinsky, (1982) 1986

Stochastic Inflation



super-Hubble scales —> stochastic inflation

sub-Hubble scales —> CPT

Coarse-grained field $N = \ln(a)$

$$\hat{\mathbf{\Phi}}_{\mathrm{cg}}(N, \vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k} \left[\mathbf{\Phi}_{\vec{k}}(N) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \mathbf{\Phi}^{\star}_{\vec{k}}(N) e^{i\vec{k}\cdot\vec{x}} \hat{a}^{\dagger}_{\vec{k}} \right]$$

Quantum fluctuations
source the **background**

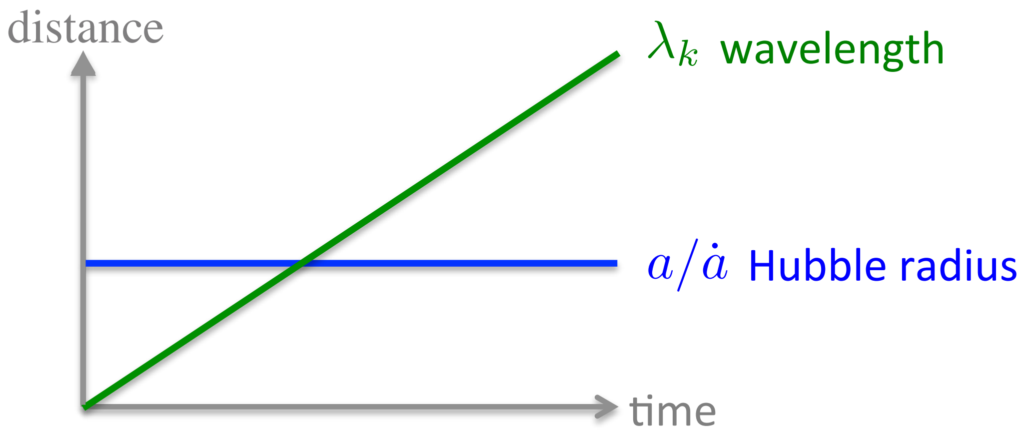
Equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}N}\mathbf{\Phi}_{\mathrm{cg}} = \mathscr{D}_{\mathrm{background}}(\mathbf{\Phi}_{\mathrm{cg}}) + \xi$$

Starobinsky, (1982) 1986

Over one e-fold: $\dfrac{\Delta\phi_{\mathrm{quant}}}{\Delta\phi_{\mathrm{classical}}} \sim \zeta_{\mathrm{classical}}$

Stochastic Inflation



super-Hubble scales —> stochastic inflation

sub-Hubble scales —> CPT

Coarse-grained field $\hat{\boldsymbol{\Phi}}_{\mathrm{cg}}(N, \vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k} \left[\boldsymbol{\Phi}_{\vec{k}}(N) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \boldsymbol{\Phi}^{\star}_{\vec{k}}(N) e^{i\vec{k}\cdot\vec{x}} \hat{a}^{\dagger}_{\vec{k}} \right]$

$N = \ln(a)$

Quantum fluctuations source the **background**

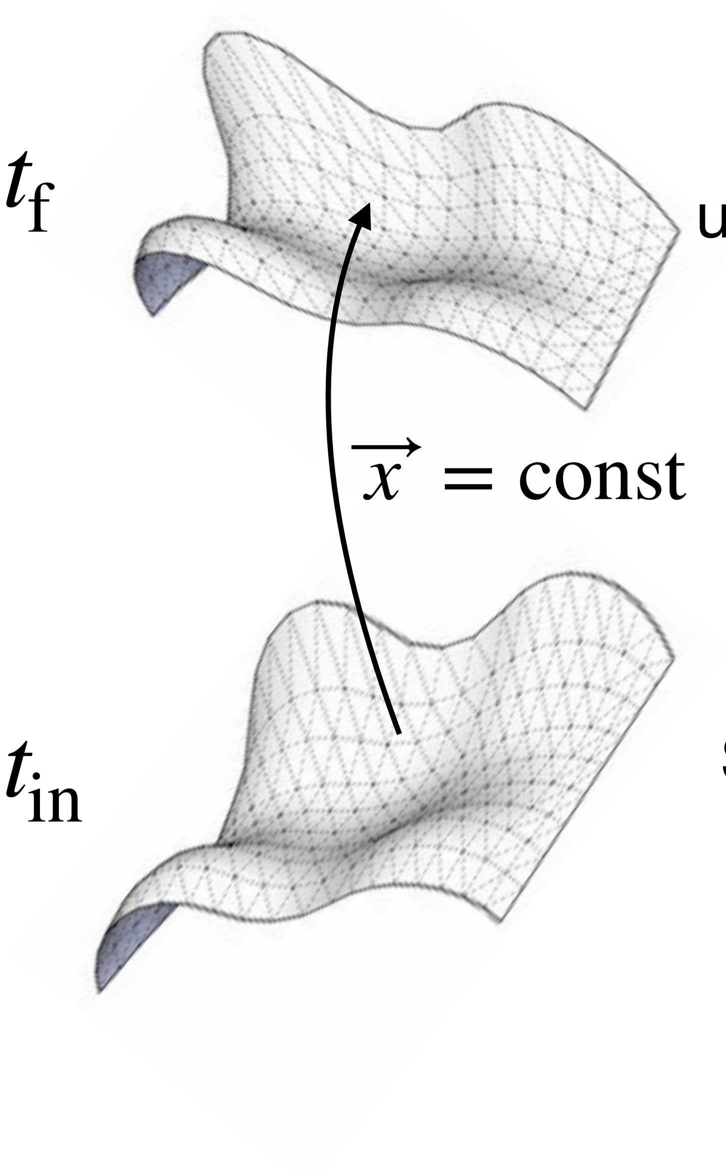
Equation of motion $\frac{\mathrm{d}}{\mathrm{d}N}\boldsymbol{\Phi}_{\mathrm{cg}} = \mathscr{D}_{\mathrm{background}}(\boldsymbol{\Phi}_{\mathrm{cg}}) + \xi$ Starobinsky, (1982) 1986

Over one e-fold: $\frac{\Delta\phi_{\mathrm{quant}}}{\Delta\phi_{\mathrm{classical}}} \sim \zeta_{\mathrm{classical}}$

What about far from the classical regime?

What about tail effects?

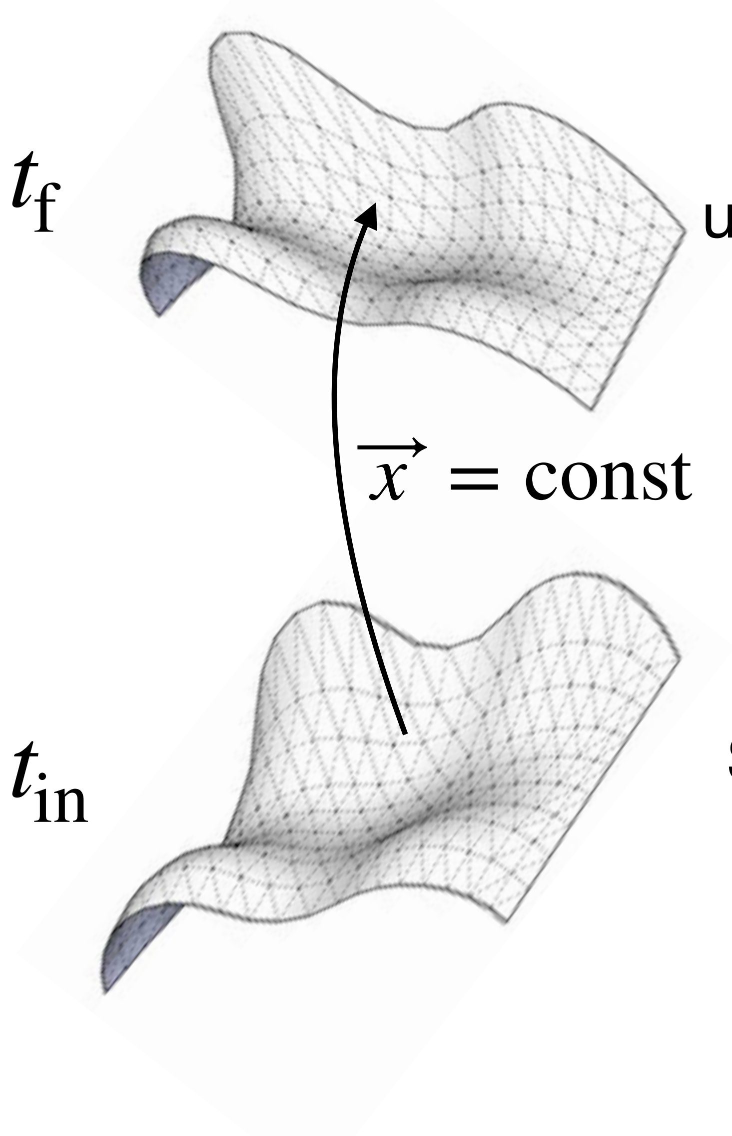
Stochastic-δN formalism



$$\zeta(t,x) = N(t,x) - N_0(t) \equiv \delta N$$

Lifshitz, Khalatnikov (1960)
Starobinsky (1983)
Wands, Malik, Lyth, Liddle (2000)

Stochastic-δN formalism



uniform density slice
$\delta\rho = 0$

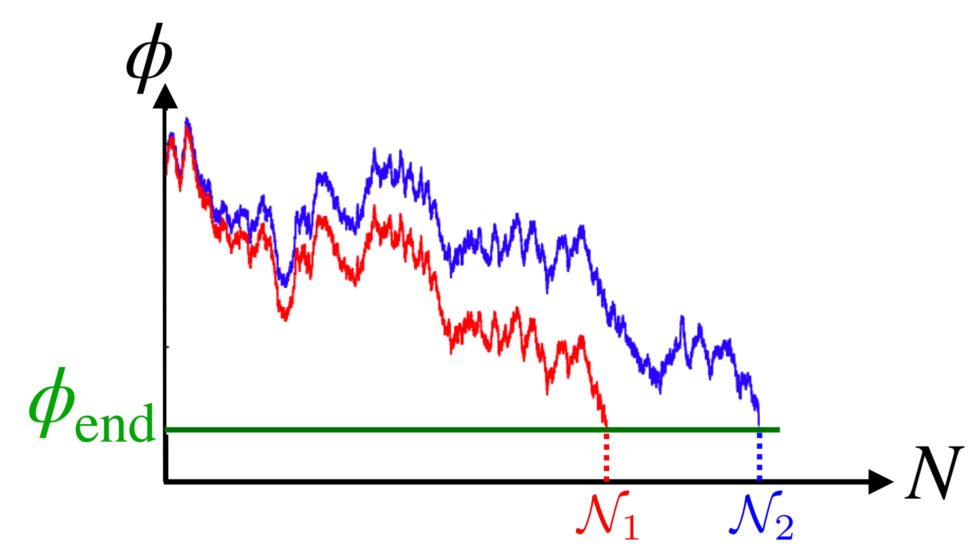
Spatially flat slice
$\psi = 0$

$$\zeta(t,x) = N(t,x) - N_0(t) \equiv \delta N$$

Lifshitz, Khalatnikov (1960)
Starobinsky (1983)
Wands, Malik, Lyth, Liddle (2000)

The realised number of e-folds
is a stochastic quantity:

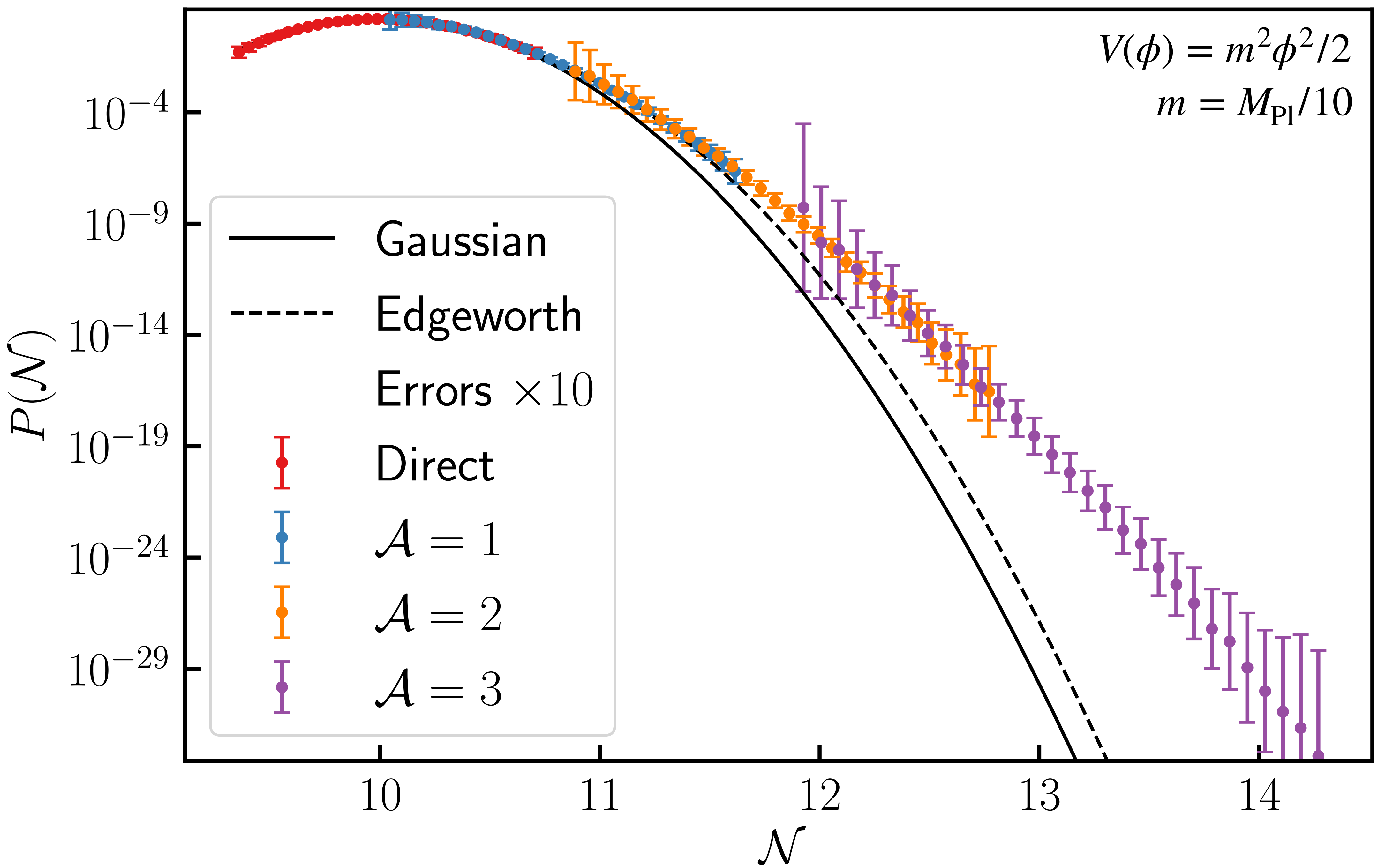
$$\zeta_{\mathrm{coarse\ grained}} = \mathcal{N} - \langle \mathcal{N} \rangle$$



Enqvist, Nurmi, Podolsky, Rigopoulos (2008) ; Fujita, Kawasaki, Tada, Takesako (2014); VV, Starobinsky (2015)

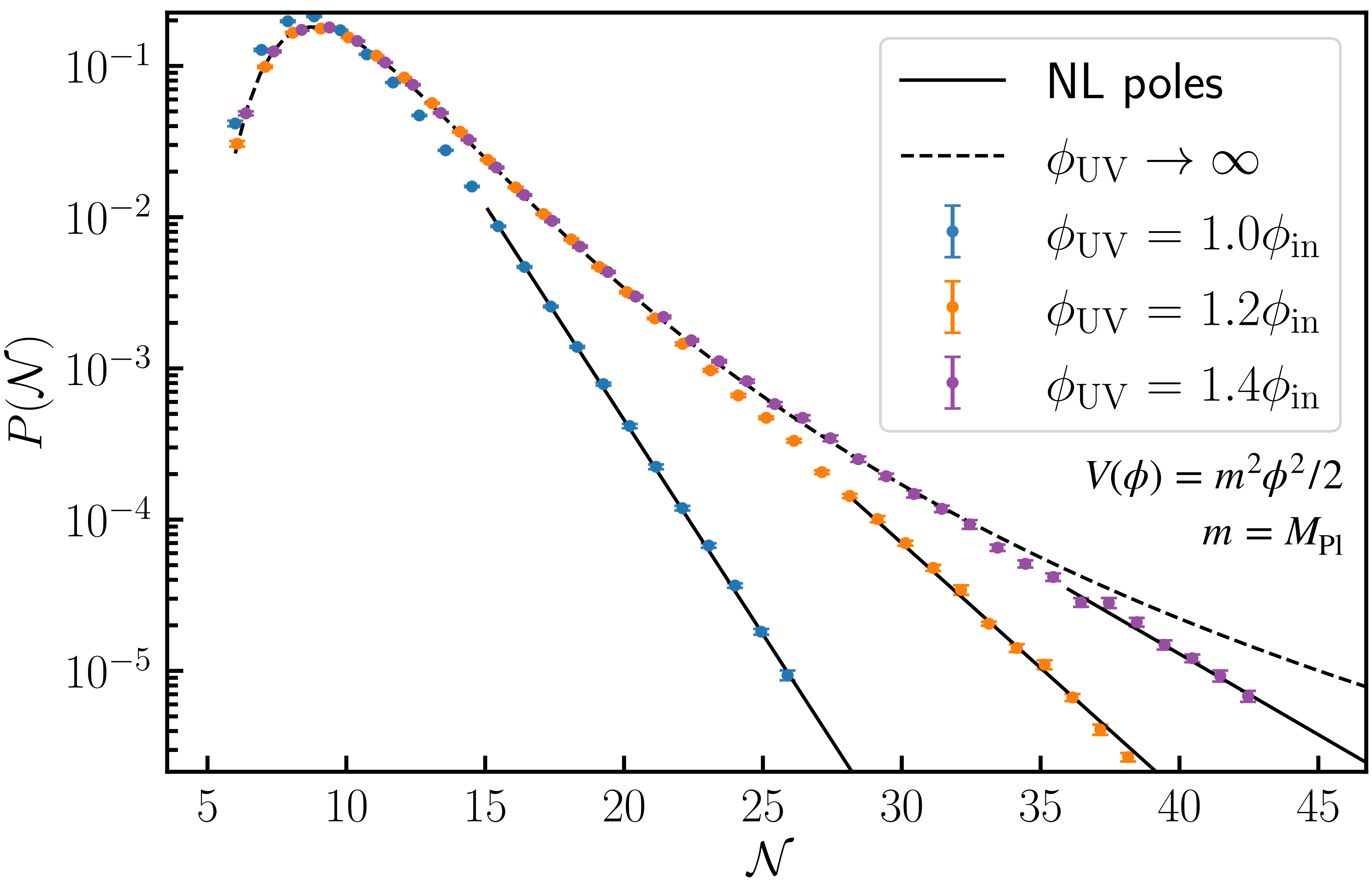
How to solve for the first-passage time problem?

Using importance sampling: J Jackson, H Assadullahi, K. Koyama, VV, D. Wands (2022)



How to solve for the first-passage time problem?

Using importance sampling: J Jackson, H Assadullahi, K. Koyama, VV, D. Wands (2022)



How to solve for the first-passage time problem?

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N)$$

Langevin equation

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right)$$

Langevin equation

Fokker-Planck equation

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Langevin equation

Fokker-Planck equation

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N)$$

Langevin equation

$$\longrightarrow$$

$$\frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Fokker-Planck equation

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

Equation for the PDF of the first passage time

$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\mathrm{FPT}}$$

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Langevin equation

Fokker-Planck equation

Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\mathrm{FPT}}$$

Computational program:

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Langevin equation

Fokker-Planck equation

Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\mathrm{FPT}}$$

Computational program:

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Langevin equation

Fokker-Planck equation

Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\mathrm{FPT}}$$

Computational program:

- Solve the first-passage-time problem

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Langevin equation

Fokker-Planck equation

Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}^\dagger_\phi \cdot P_{\mathrm{FPT}}$$

Computational program:

- Solve the first-passage-time problem

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Langevin equation — Fokker-Planck equation

Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\mathrm{FPT}}$$

Computational program:

- Solve the first-passage-time problem
- This gives the one-point PDF of curvature perturbation coarse-grained at H_{end}

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Langevin equation

Fokker-Planck equation

Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\mathrm{FPT}}$$

Computational program:

- Solve the first-passage-time problem
- This gives the one-point PDF of curvature perturbation coarse-grained at H_{end}

How to solve for the first-passage time problem?

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Langevin equation

Fokker-Planck equation

Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

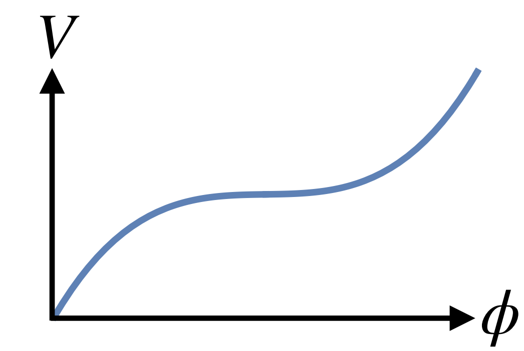
$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\mathrm{FPT}}$$

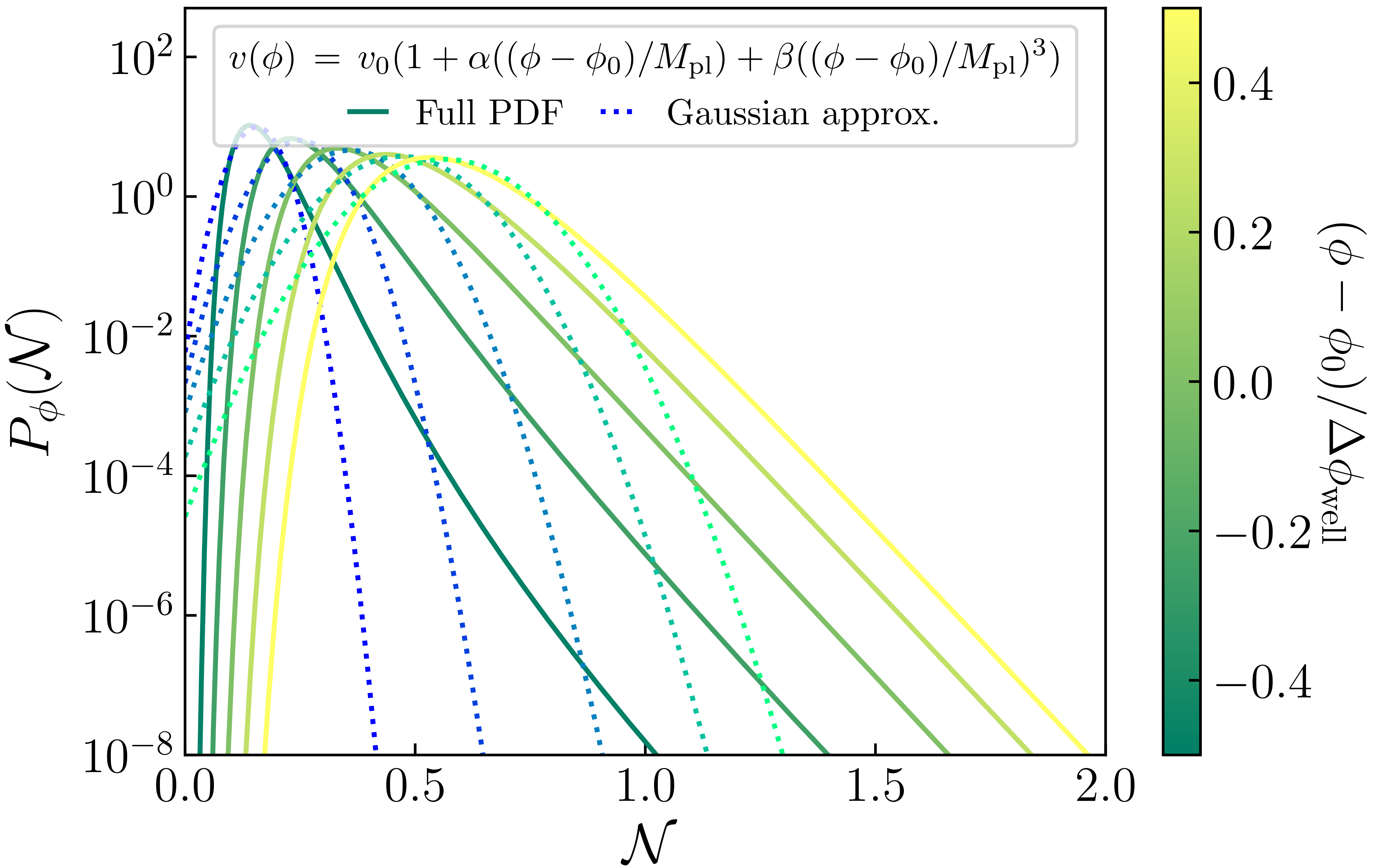
Computational program:

- Solve the first-passage-time problem
- This gives the one-point PDF of curvature perturbation coarse-grained at H_{end}
- Extract cosmologically relevant quantities (power spectrum, mass functions, compaction function, etc)

Exponential tails

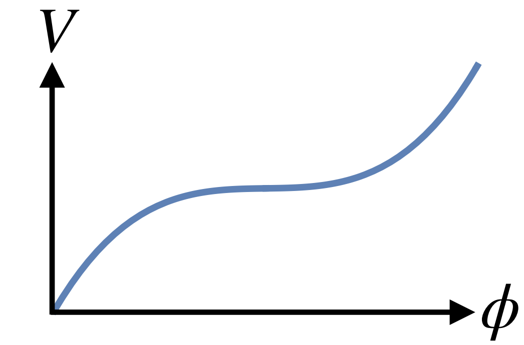
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)

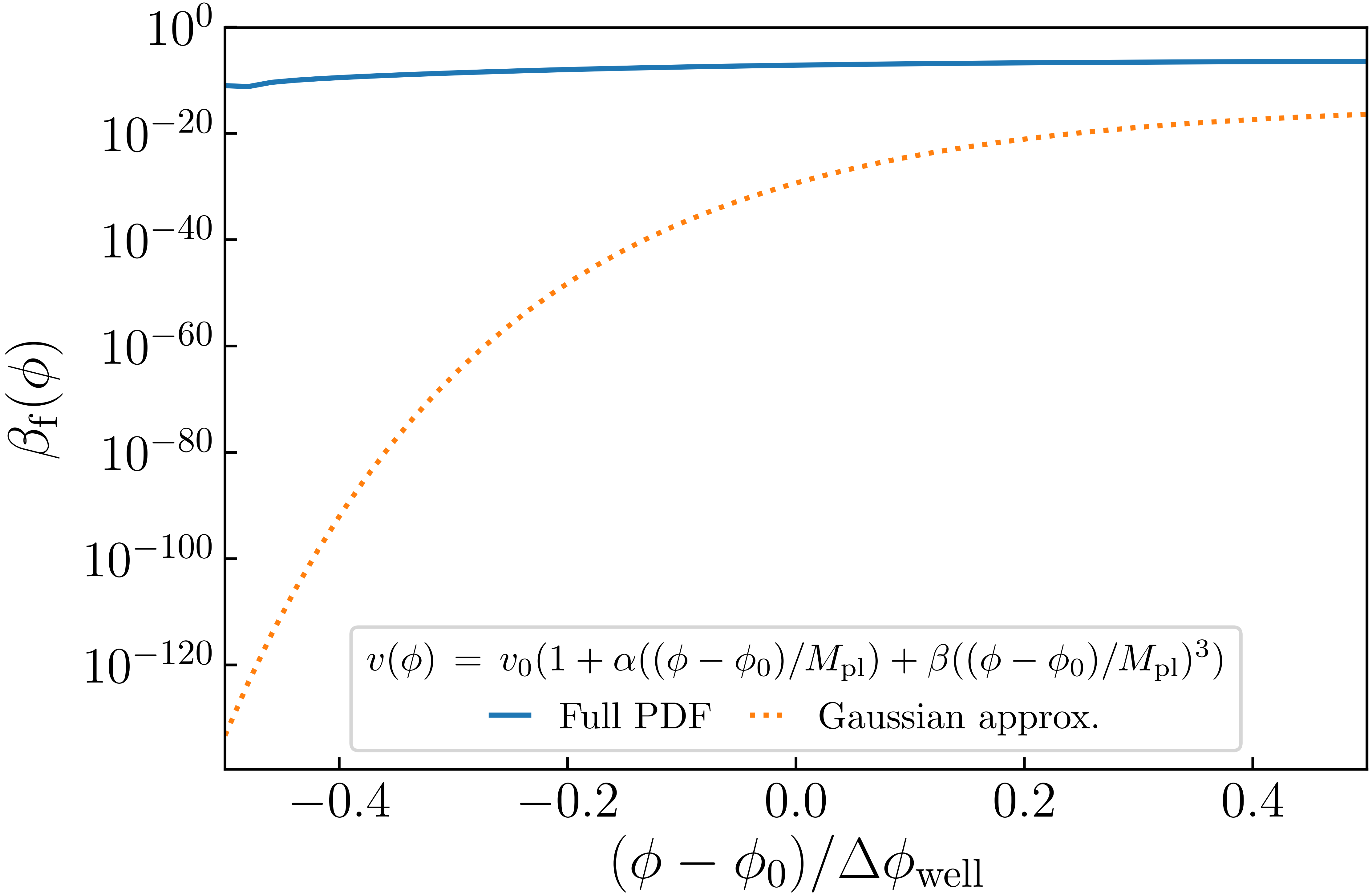




Impact on PBHs

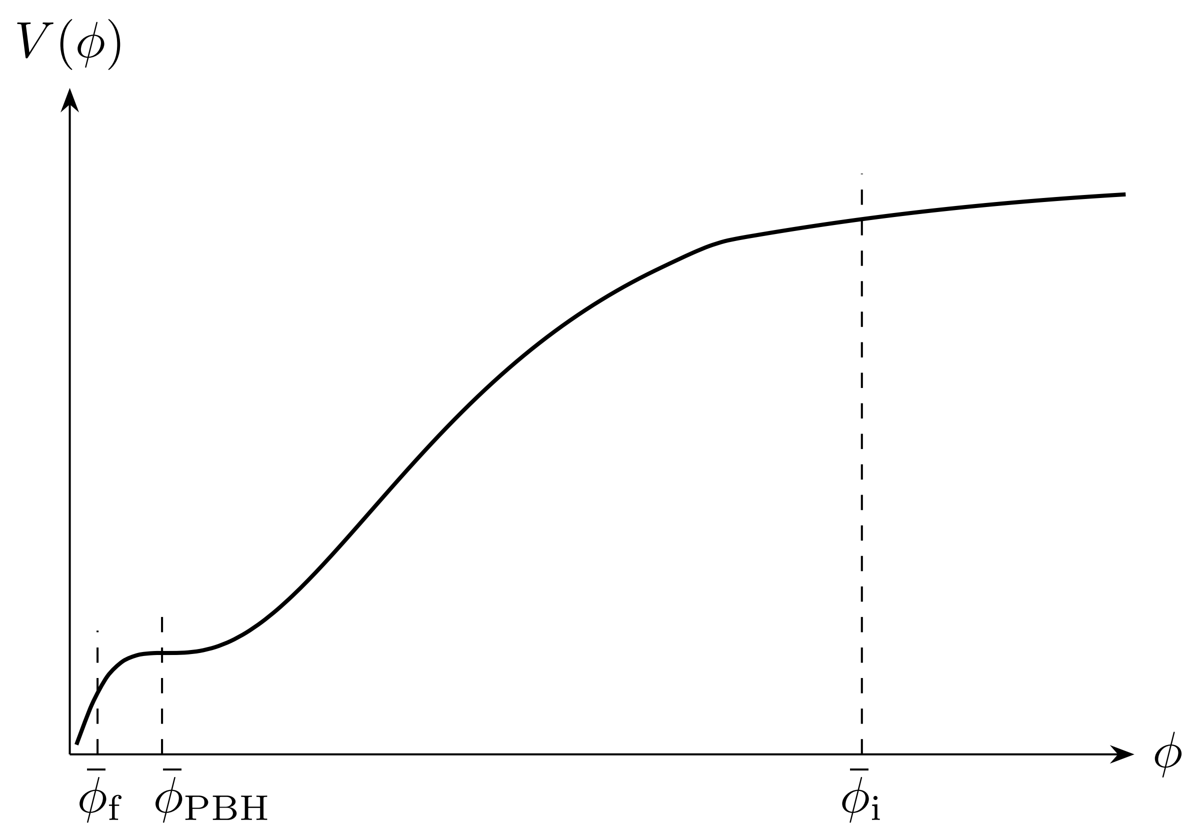
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)

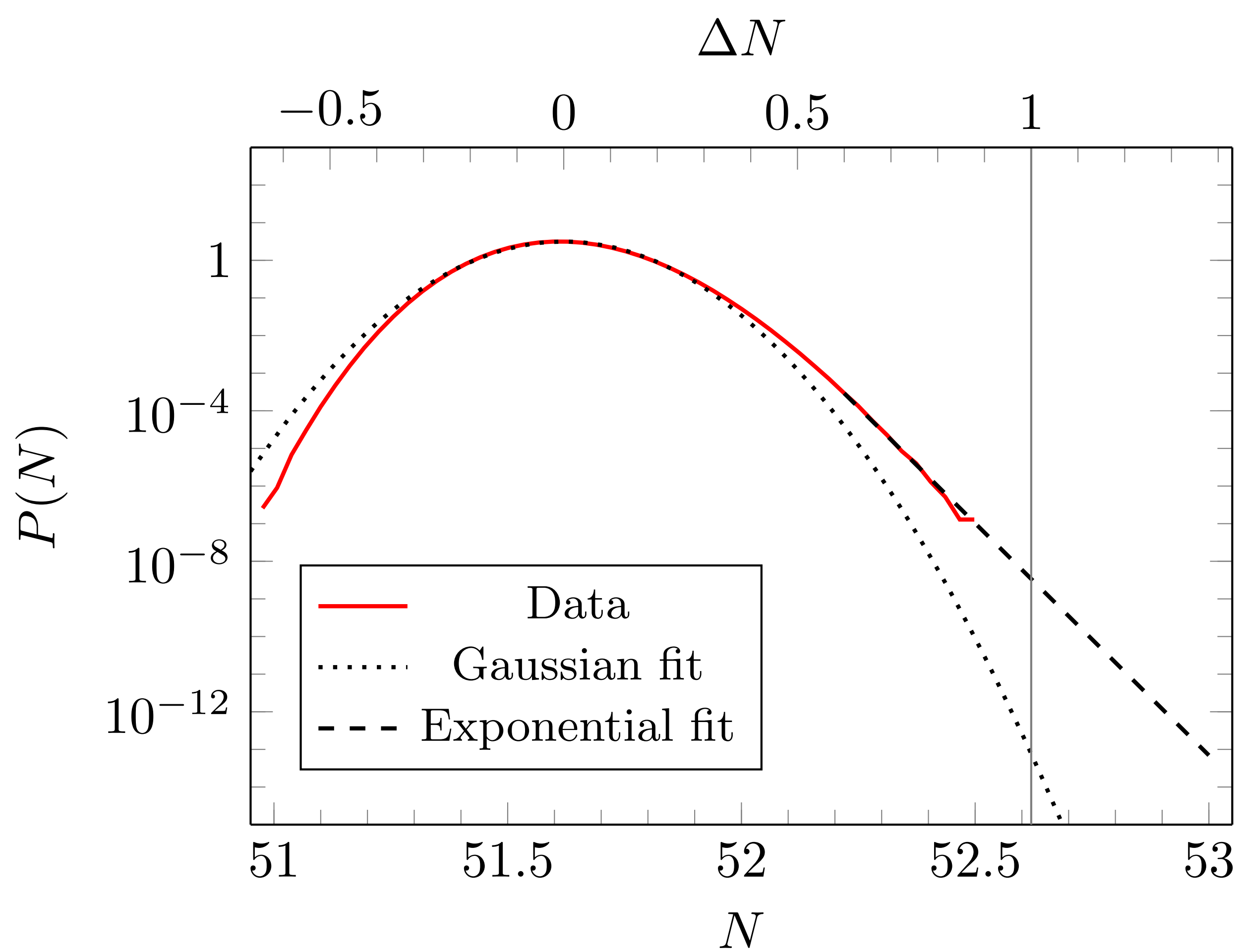




Exponential tails
in ultra slow roll models

D. Figueroa, S. Raatikainen, S. Räsänen, E. Tomberg (2020)

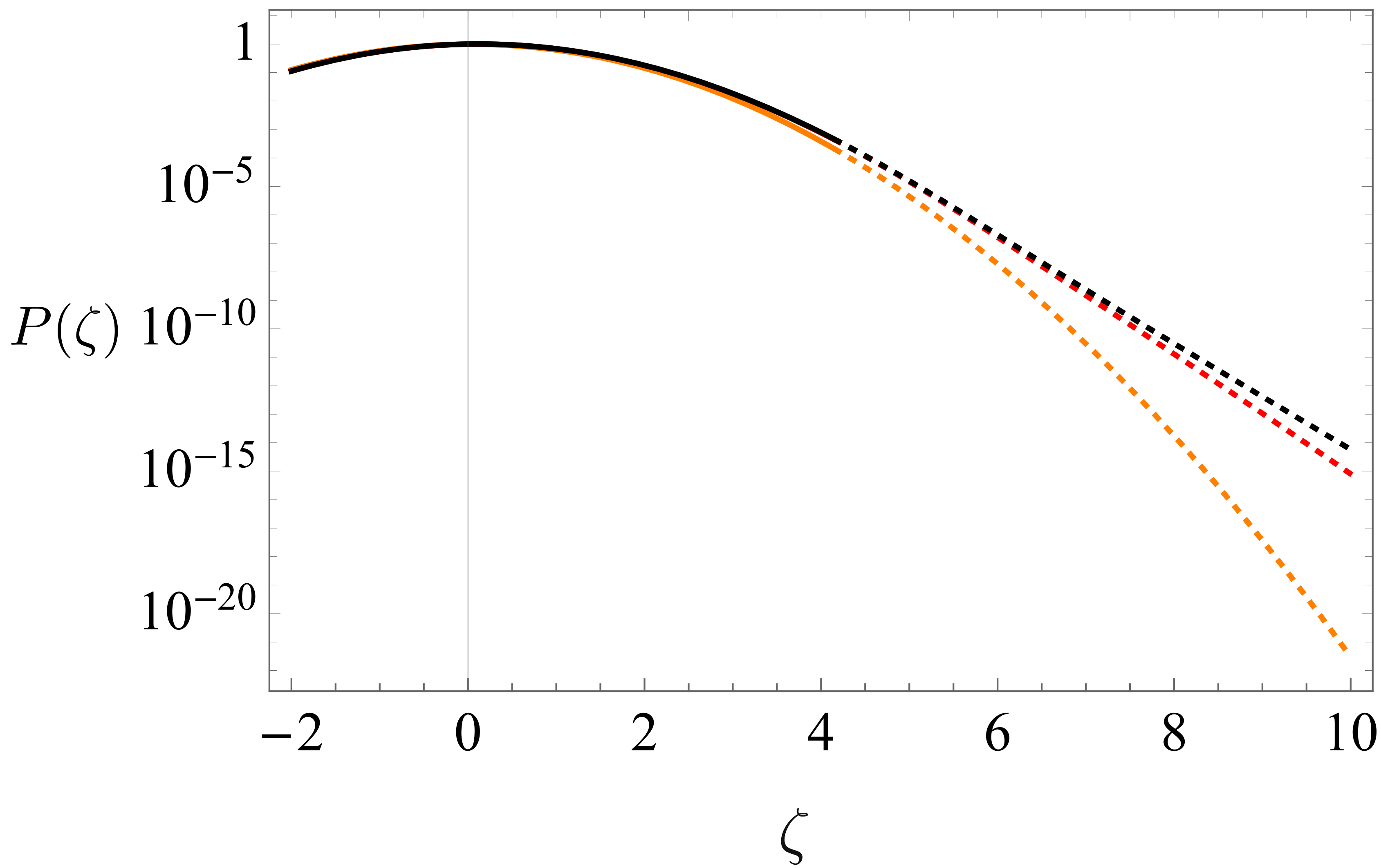




See also Pattison, Vennin, Wands, Assadullahi (2021)

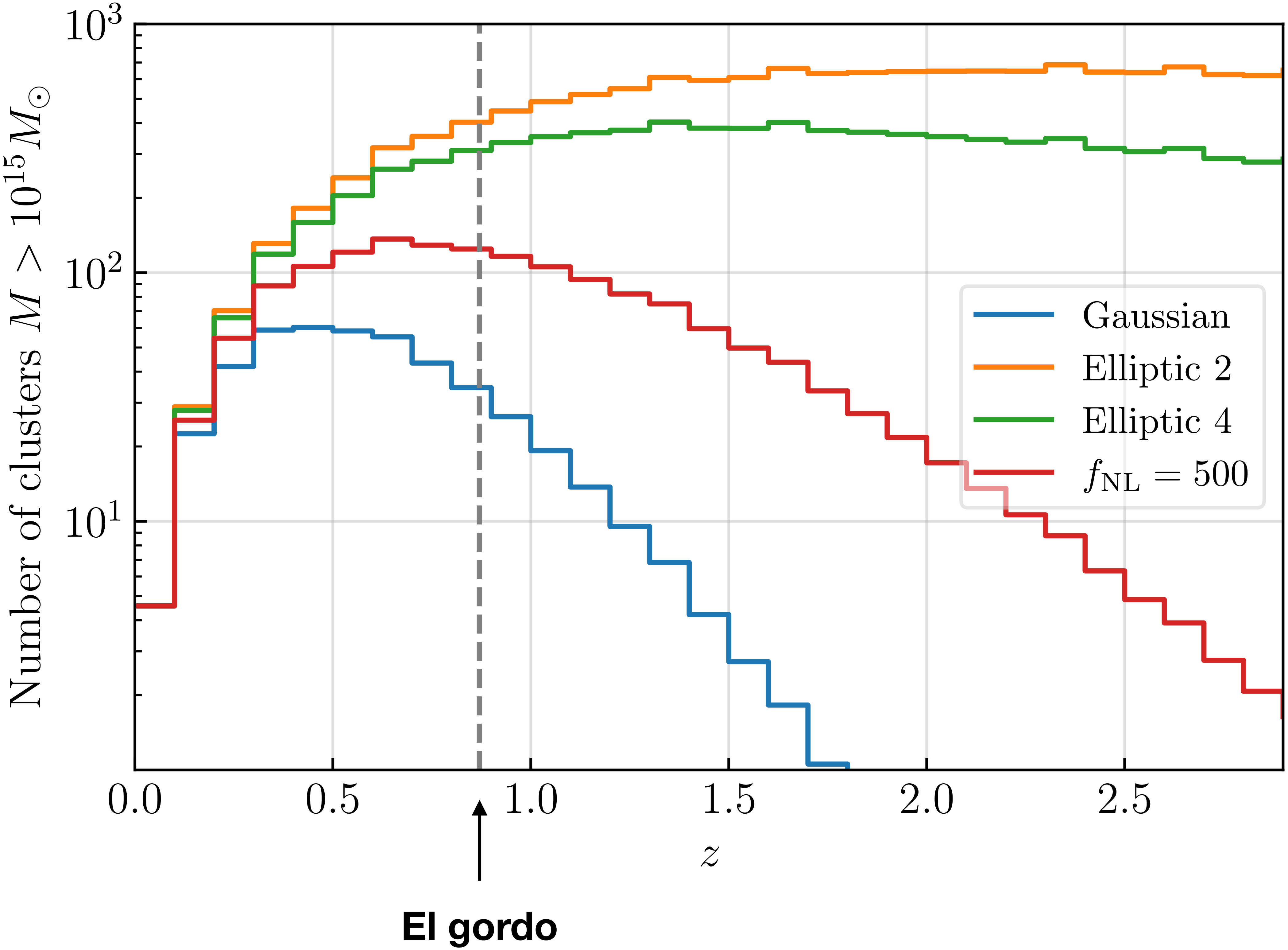
Exponential tails
in multi-field models

Achucarro, Cespedes, Davies, Palma (2021)



Impact on LSS

Ezquiaga, Garcia-Bellido, VV (2022)



Extracting cosmological observables

Scale k $\longrightarrow$ Hubble-crossing time $\longrightarrow$ Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

$$N_{\rm bw}(k) = \ln(a_{\rm end} H/k)$$

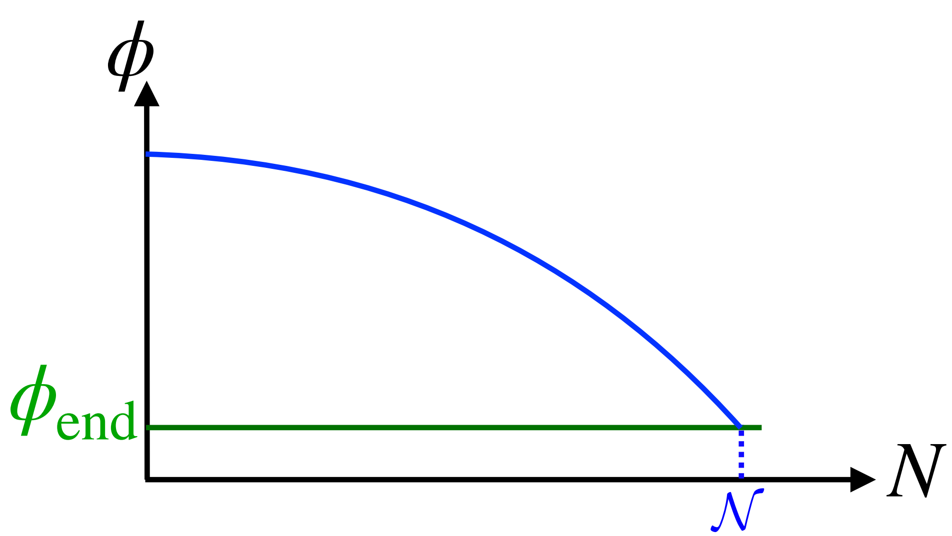
Extracting cosmological observables

Scale k $\longrightarrow$ Hubble-crossing time $\longrightarrow$ Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

$$N_{\rm bw}(k) = \ln(a_{\rm end} H/k)$$

<u>Classical picture</u>



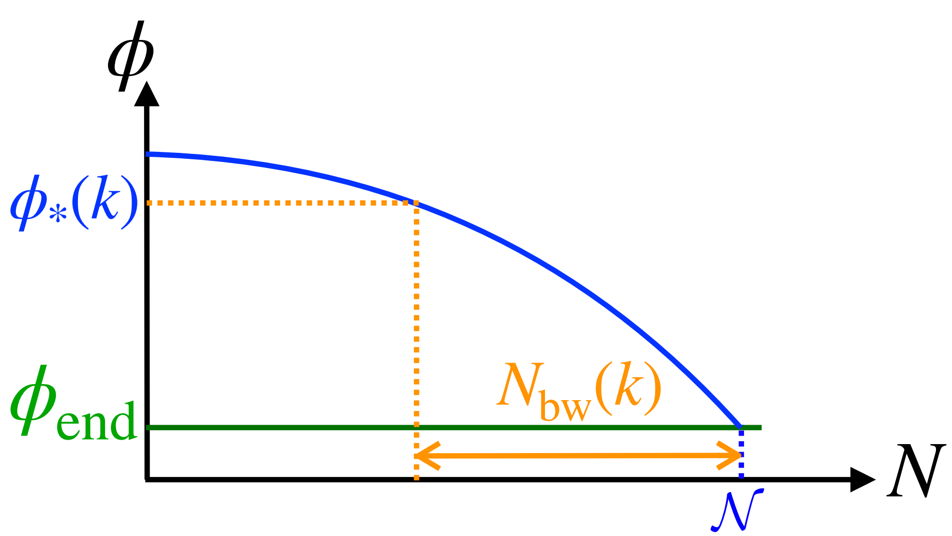
Extracting cosmological observables

Scale k $\longrightarrow$ Hubble-crossing time $\longrightarrow$ Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

$$N_{\rm bw}(k) = \ln(a_{\rm end}H/k)$$

Classical picture



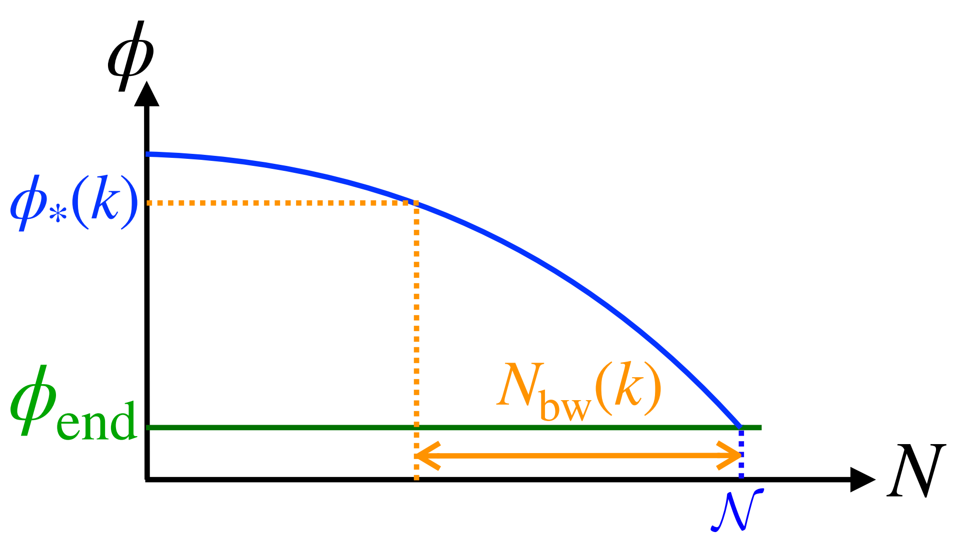
Extracting cosmological observables

Scale k $\longrightarrow$ Hubble-crossing time $\longrightarrow$ Hubble-crossing field ϕ_*

$$N_* = N_{\text{end}} - N_{\text{bw}}(k)$$

$$N_{\text{bw}}(k) = \ln(a_{\text{end}} H/k)$$

<u>Classical picture</u>



Observables (power spectrum etc) at scale k depend on <u>**local properties**</u> of the potential at location $\phi_*(k)$

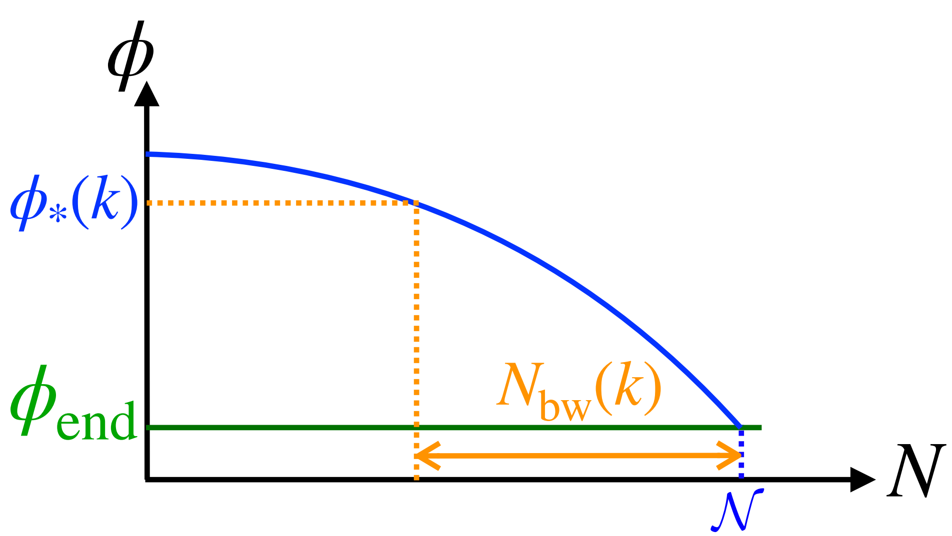
Extracting cosmological observables

Scale k $\longrightarrow$ Hubble-crossing time $\longrightarrow$ Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

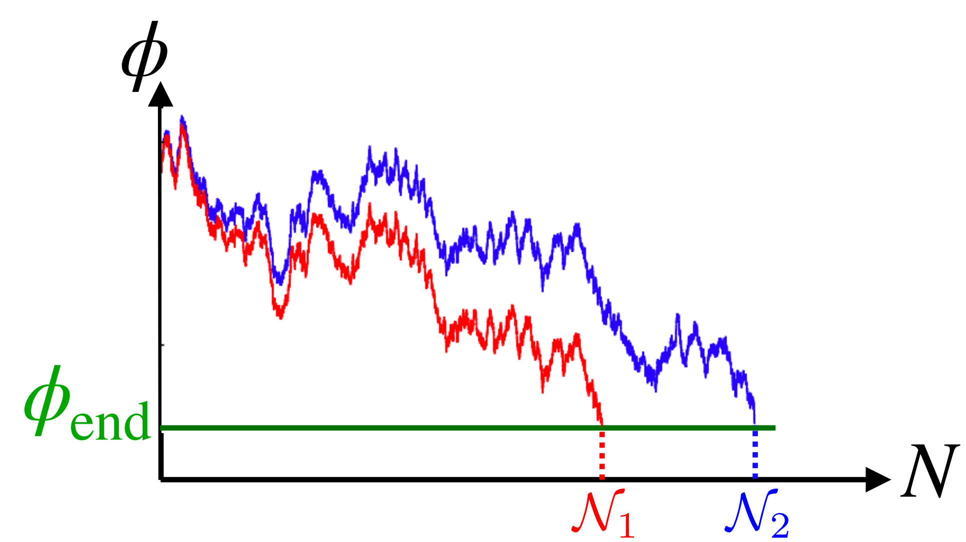
$$N_{\rm bw}(k) = \ln(a_{\rm end} H/k)$$

Classical picture



Observables (power spectrum etc) at scale k depend on **local properties** of the potential at location $\phi_*(k)$

Stochastic picture



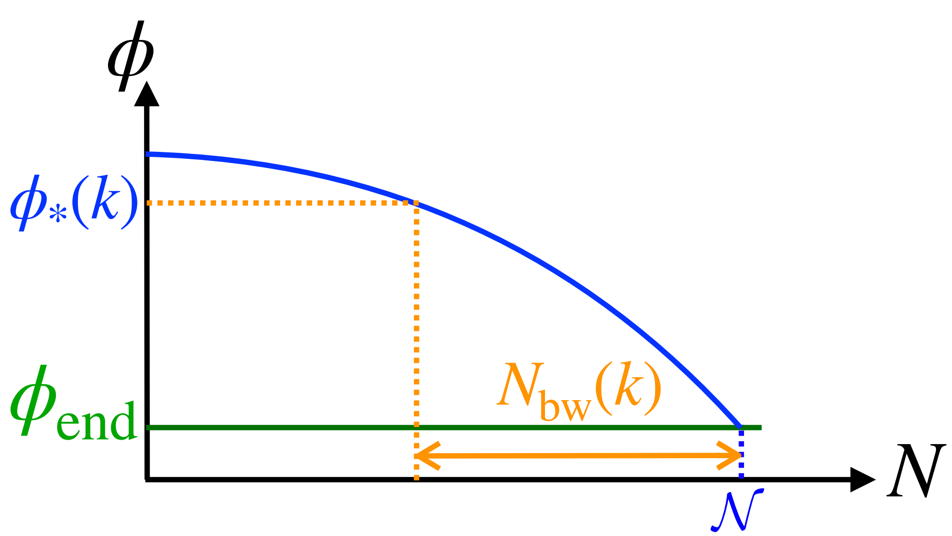
Extracting cosmological observables

Scale k $\longrightarrow$ Hubble-crossing time $\longrightarrow$ Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

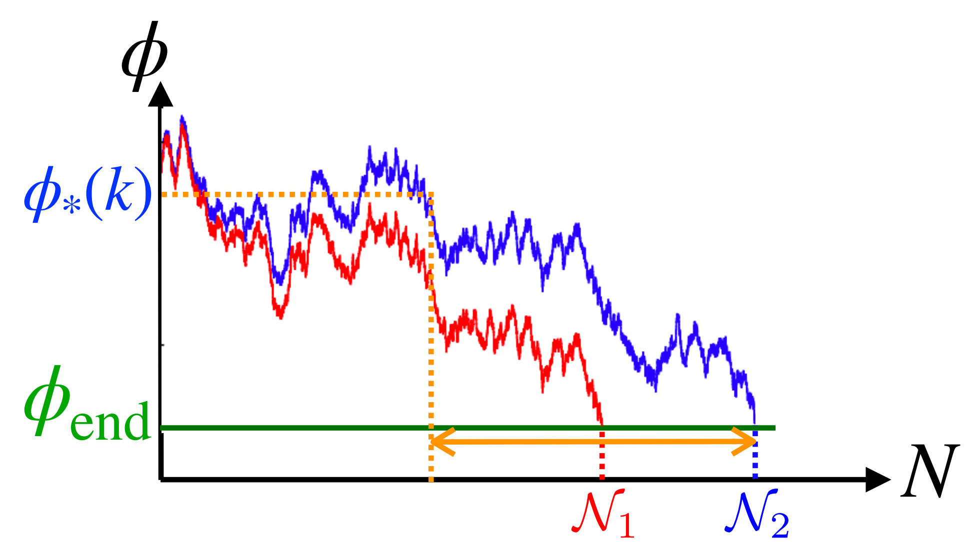
$$N_{\rm bw}(k) = \ln(a_{\rm end}H/k)$$

Classical picture



Observables (power spectrum etc) at scale k depend on **local properties** of the potential at location $\phi_*(k)$

Stochastic picture



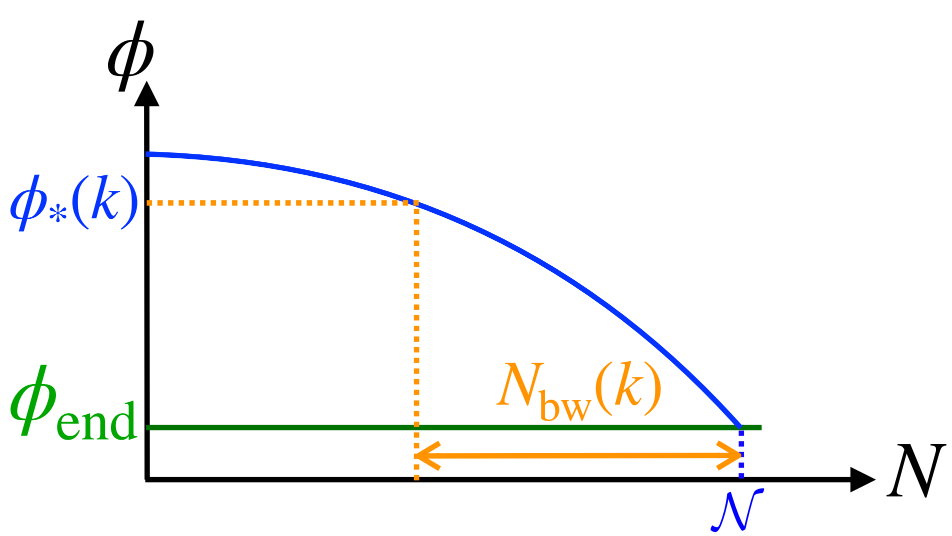
Extracting cosmological observables

Scale k $\longrightarrow$ Hubble-crossing time $\longrightarrow$ Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

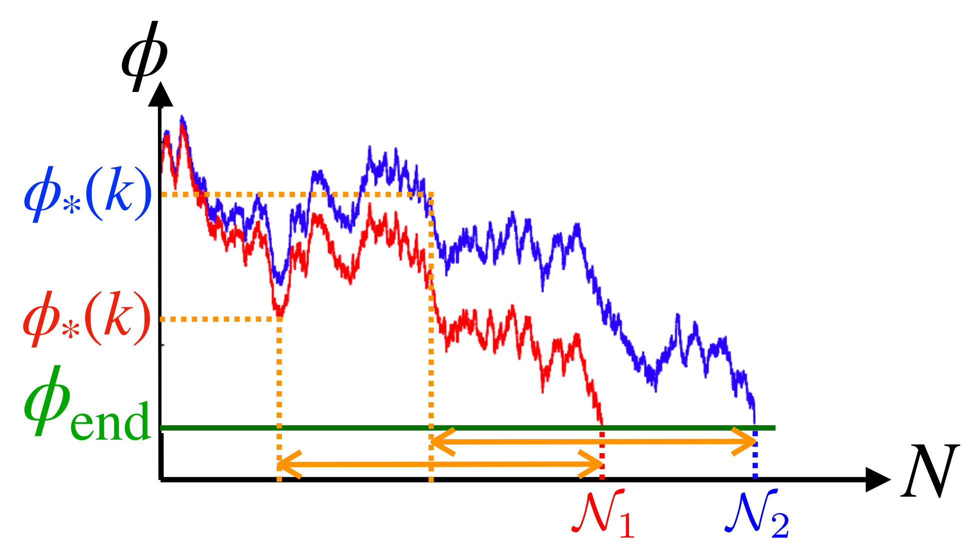
$$N_{\rm bw}(k) = \ln(a_{\rm end} H/k)$$

<u>Classical picture</u>



Observables (power spectrum etc) at scale *k* depend on **<u>local properties</u>** of the potential at location $\phi_*(k)$

<u>Stochastic picture</u>



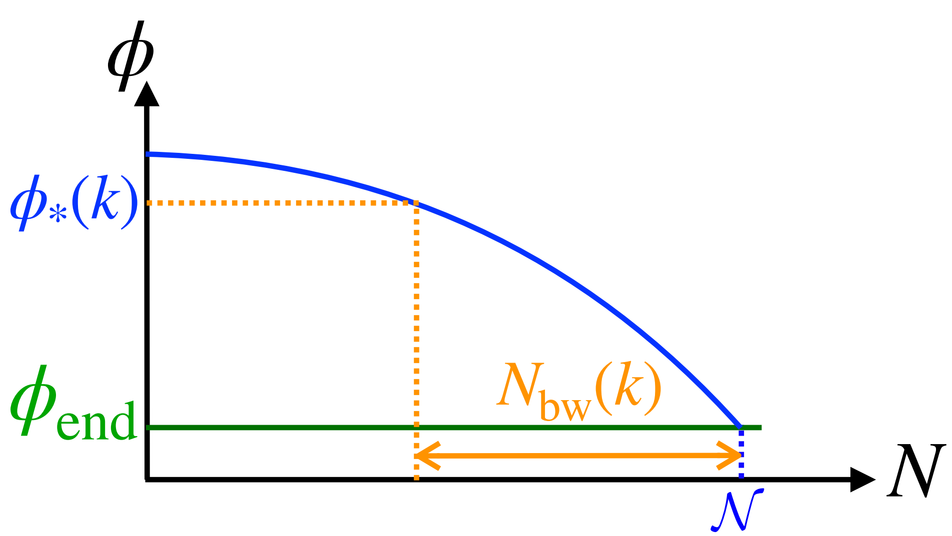
Extracting cosmological observables

Scale k $\longrightarrow$ Hubble-crossing time $\longrightarrow$ Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

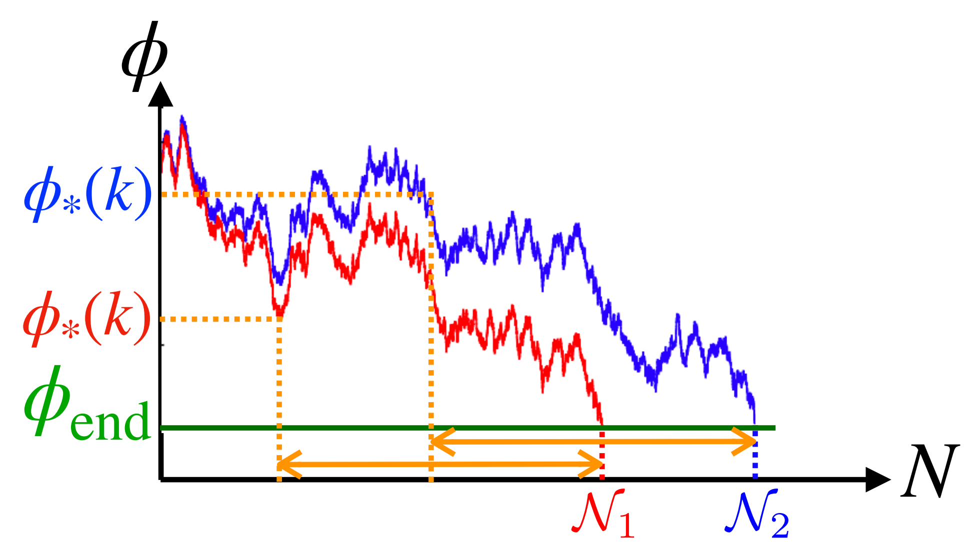
$$N_{\rm bw}(k) = \ln(a_{\rm end} H/k)$$

<u>Classical picture</u>



Observables (power spectrum etc) at scale *k* depend on **<u>local properties</u>** of the potential at location $\phi_*(k)$

<u>Stochastic picture</u>



$$P_{\rm bw}(\phi_*;k) = P_{\rm FPT}\left[N_{\rm bw}(k);\phi_*\right] \frac{\int_0^\infty P(\phi_*;N)\,\mathrm{d}N}{\int_{N_{\rm bw}(k)}^\infty P_{\rm FPT}(\mathcal{N};\phi_{\rm in})\,\mathrm{d}\mathcal{N}}$$

Kenta Ando, VV (2020)

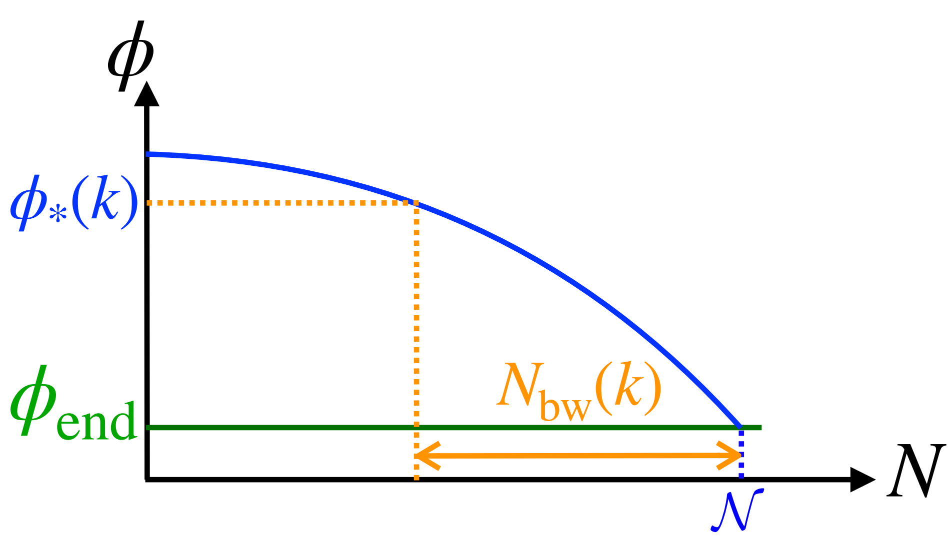
Extracting cosmological observables

Scale k ⟶ Hubble-crossing time ⟶ Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

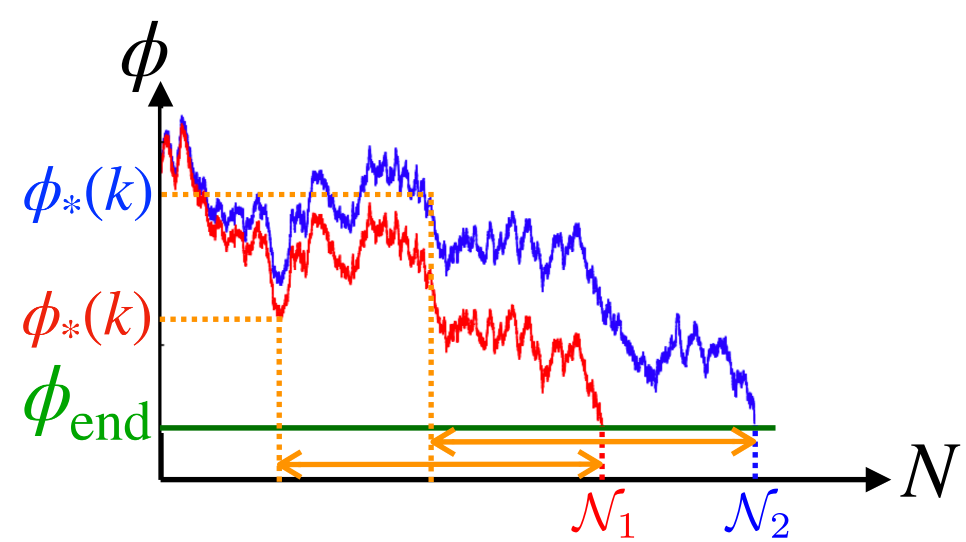
$$N_{\rm bw}(k) = \ln(a_{\rm end}H/k)$$

<u>Classical picture</u>



Observables (power spectrum etc) at scale k depend on <u>**local properties**</u> of the potential at location $\phi_*(k)$

<u>Stochastic picture</u>



$$P_{\rm bw}(\phi_*;k) = P_{\rm FPT}\left[N_{\rm bw}(k);\phi_*\right]\frac{\int_0^\infty P(\phi_*;N)\,\mathrm{d}N}{\int_{N_{\rm bw}(k)}^\infty P_{\rm FPT}(\mathcal{N};\phi_{\rm in})\,\mathrm{d}\mathcal{N}}$$

Kenta Ando, VV (2020)

Observables at scale k depend on the <u>**whole potential**</u> and on the <u>**initial condition**</u>

Extracting cosmological observables

Power Spectrum

Kenta Ando, VV (2020)

$$\mathscr{P}_\zeta(k) = - \int_\Omega \mathrm{d}\boldsymbol{\Phi}_* \left. \frac{\partial P_{\mathrm{bw}}(\boldsymbol{\Phi}_*; N_{\mathrm{bw}})}{\partial N_{\mathrm{bw}}} \right|_{N_{\mathrm{bw}}(k)} \left\langle \delta \mathcal{N}^2(\boldsymbol{\Phi}_0 \to \boldsymbol{\Phi}_*) \right\rangle$$

Extracting cosmological observables

Power Spectrum

Kenta Ando, VV (2020)

$$\mathscr{P}_\zeta(k) = -\boxed{\int_\Omega \mathrm{d}\boldsymbol{\Phi}_*} \frac{\partial P_{\mathrm{bw}}(\boldsymbol{\Phi}_*; N_{\mathrm{bw}})}{\partial N_{\mathrm{bw}}}\bigg|_{N_{\mathrm{bw}}(k)} \left\langle \delta\mathcal{N}^2(\boldsymbol{\Phi}_0 \to \boldsymbol{\Phi}_*)\right\rangle$$

Integration over the full inflating domain

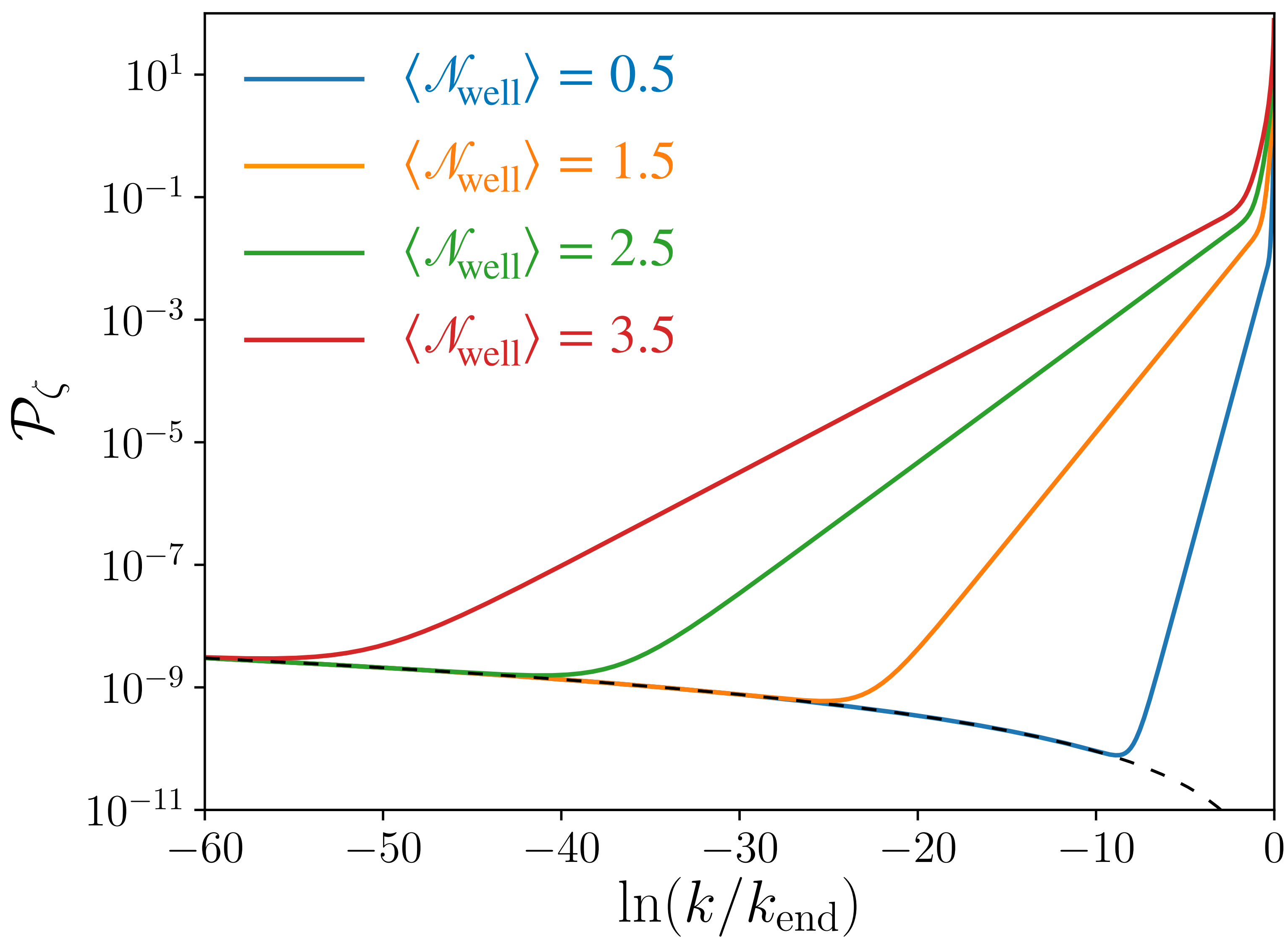
Extracting cosmological observables

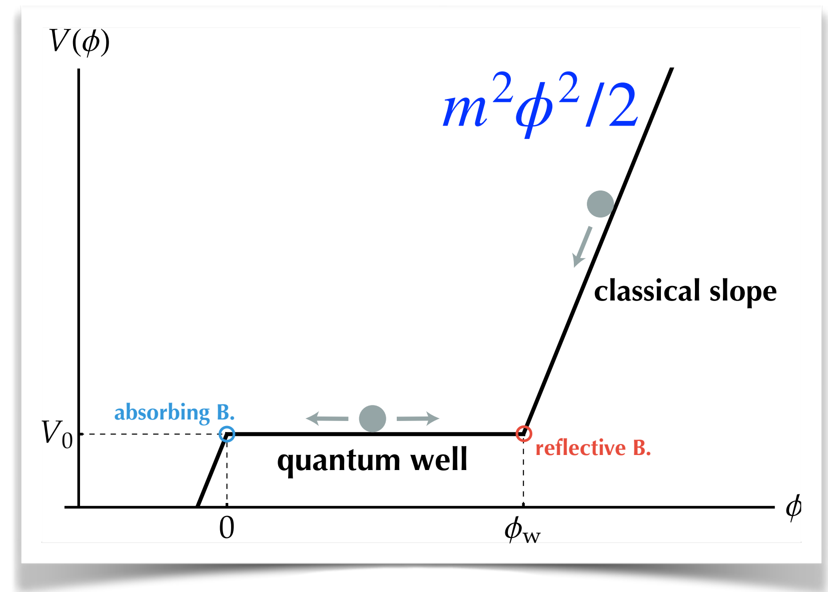
Power Spectrum

Kenta Ando, VV (2020)

$$\mathscr{P}_\zeta(k) = -\boxed{\int_\Omega \mathrm{d}\mathbf{\Phi}_*} \frac{\partial P_{\mathrm{bw}}(\mathbf{\Phi}_*; N_{\mathrm{bw}})}{\partial N_{\mathrm{bw}}}\bigg|_{N_{\mathrm{bw}}(k)} \left\langle \delta\mathcal{N}^2(\mathbf{\Phi}_0 \to \mathbf{\Phi}_*) \right\rangle$$

Integration over the full inflating domain





Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

$$P\left(\zeta_R\right) = \int_\Omega \mathrm{d}\boldsymbol{\Phi}_* P_{\mathrm{bw}}\left[\boldsymbol{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT},\boldsymbol{\Phi}_0\to\boldsymbol{\Phi}_*}\left[\zeta_R - \langle\mathcal{N}(\boldsymbol{\Phi}_*)\rangle + \langle\mathcal{N}(\boldsymbol{\Phi}_0)\rangle\right]$$

R

Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

$$P\left(\zeta_R\right) = \boxed{\int_\Omega \mathrm{d}\mathbf{\Phi}_*} P_{\mathrm{bw}}\left[\mathbf{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT},\mathbf{\Phi}_0\to\mathbf{\Phi}_*}\left[\zeta_R - \langle\mathcal{N}(\mathbf{\Phi}_*)\rangle + \langle\mathcal{N}(\mathbf{\Phi}_0)\rangle\right]$$

R

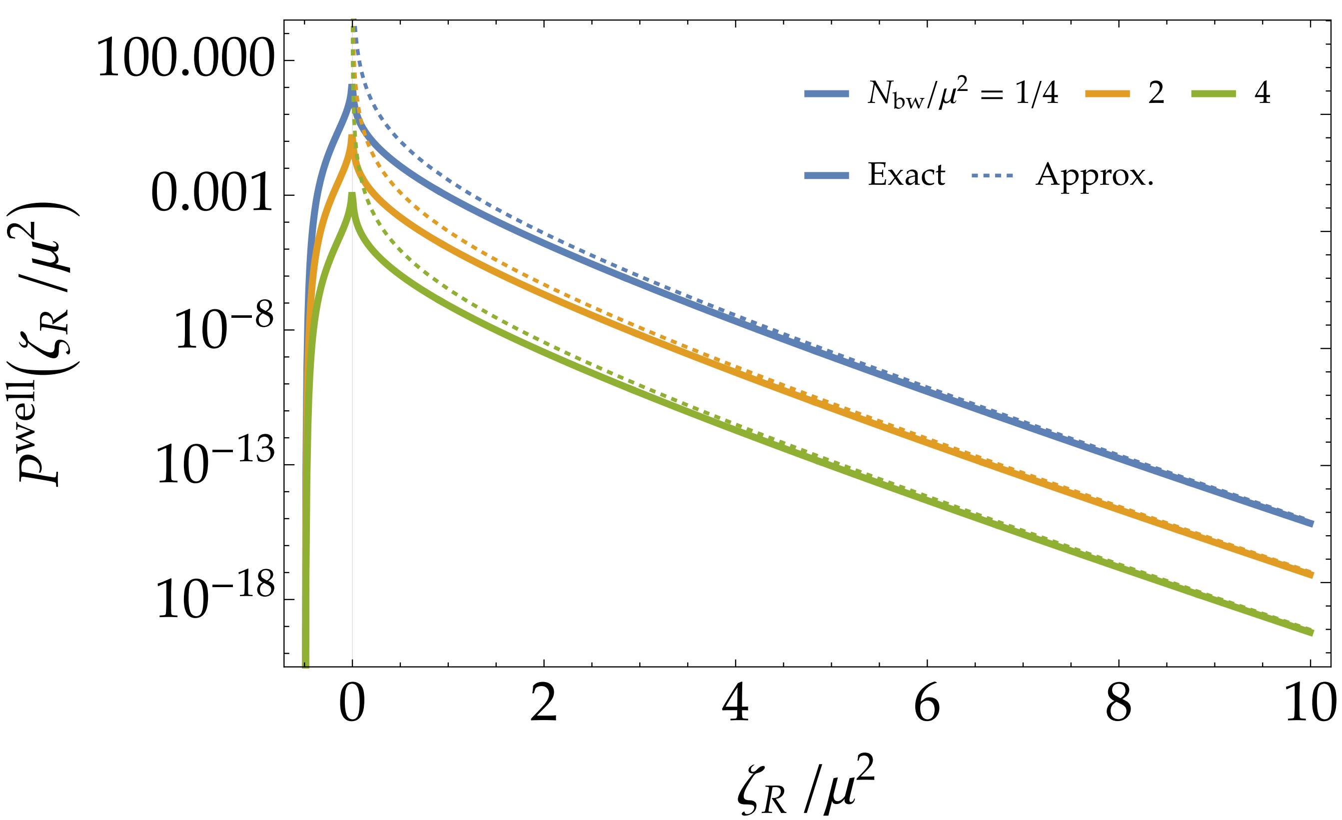
Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

$$P\left(\zeta_R\right)=\boxed{\int_\Omega \mathrm{d}\mathbf{\Phi}_*} P_{\mathrm{bw}}\left[\mathbf{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT}, \mathbf{\Phi}_0 \rightarrow \mathbf{\Phi}_*}\left[\zeta_R-\langle\mathcal{N}(\mathbf{\Phi}_*)\rangle+\langle\mathcal{N}(\mathbf{\Phi}_0)\rangle\right]$$

R



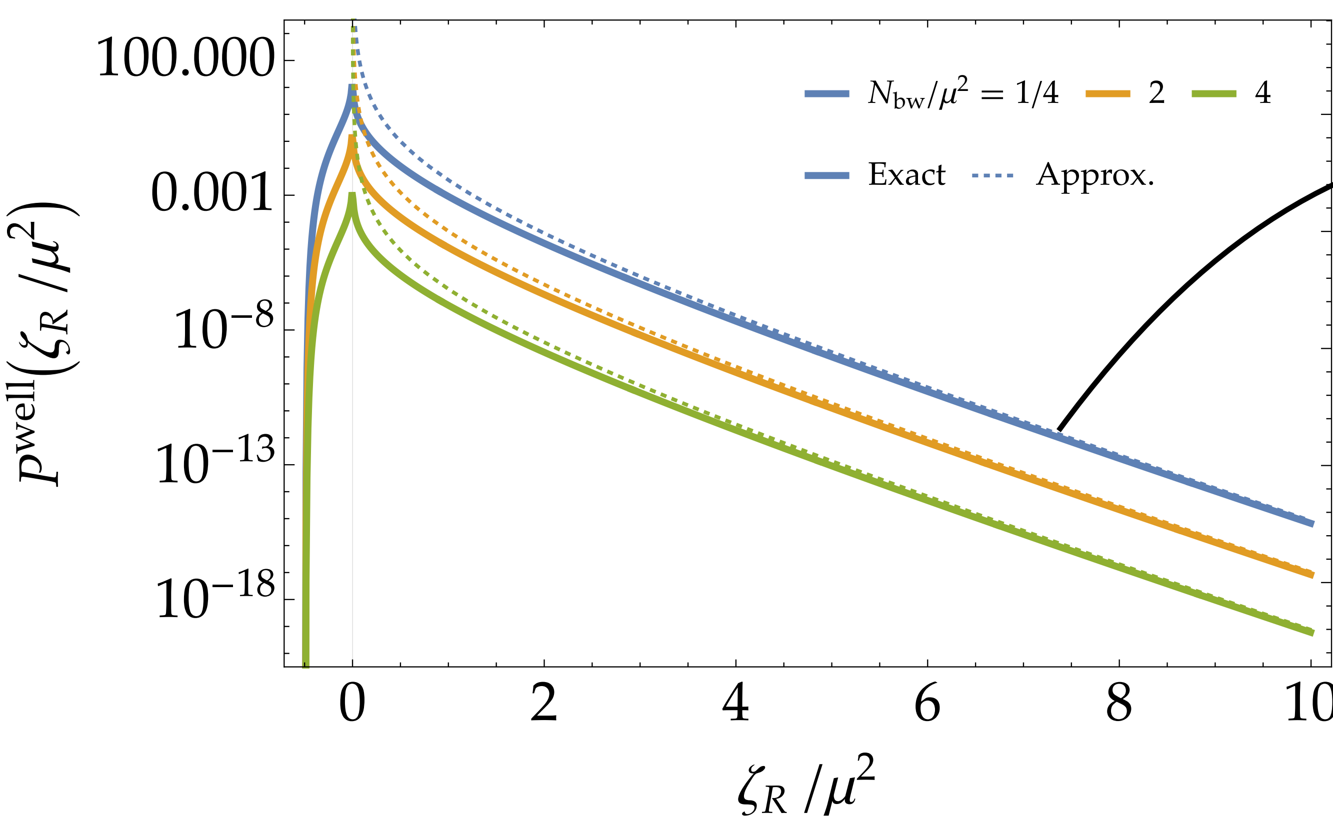
Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

$$P\left(\zeta_R\right) = \boxed{\int_\Omega \mathrm{d}\boldsymbol{\Phi}_*} P_{\mathrm{bw}}\left[\boldsymbol{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT},\boldsymbol{\Phi}_0\to\boldsymbol{\Phi}_*}\left[\zeta_R - \langle\mathcal{N}(\boldsymbol{\Phi}_*)\rangle + \langle\mathcal{N}(\boldsymbol{\Phi}_0)\rangle\right]$$

R



$$P\left(\zeta_R\right) \propto \frac{e^{-\frac{\pi^2}{4}\frac{\zeta_R}{\mu^2}}}{\left(\zeta_R/\mu^2\right)^3}$$

Quasi-exponential tail

Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

$$P\left(\zeta_R\right)=\boxed{\int_\Omega \mathrm{d}\boldsymbol{\Phi}_*} P_{\mathrm{bw}}\left[\boldsymbol{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT}, \boldsymbol{\Phi}_0 \rightarrow \boldsymbol{\Phi}_*}\left[\zeta_R-\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*\right)\right\rangle+\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_0\right)\right\rangle\right]$$

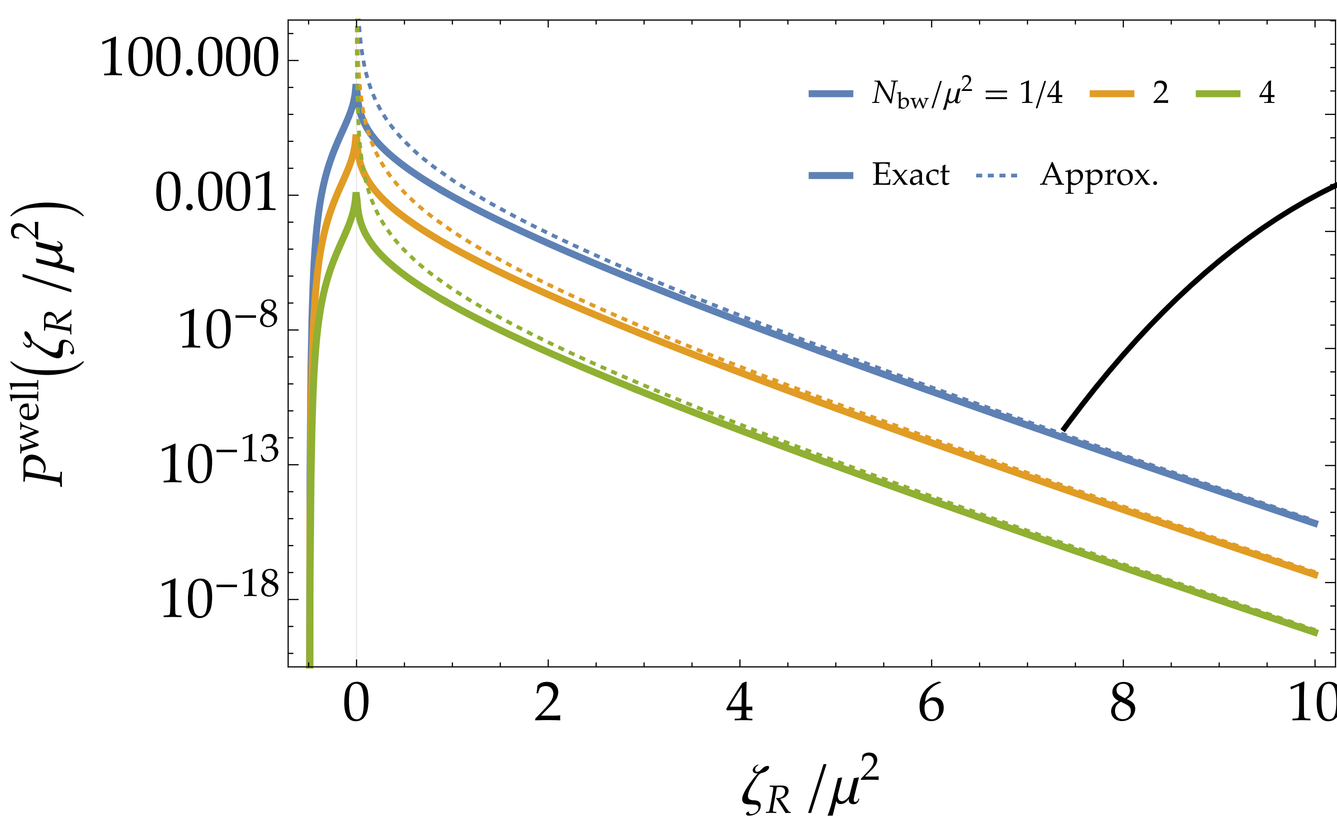
R

$$P(\Delta\zeta)=\int_\Omega \mathrm{d}\boldsymbol{\Phi}_*^{(1)} \mathrm{d}\boldsymbol{\Phi}_*^{(2)} P_{\mathrm{bw}}\left(\boldsymbol{\Phi}_*^{(1)}, \boldsymbol{\Phi}_*^{(2)} \middle| N_{\mathrm{bw}}^{(1)}, N_{\mathrm{bw}}^{(2)}\right) \delta\left[\Delta\zeta+\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*^{(1)}\right)\right\rangle-\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*^{(2)}\right)\right\rangle-\ln(1+\beta)\right]$$

$R^{(1)}$ $R^{(2)}$

$\longrightarrow$ Comoving density contrast

$\longrightarrow$ Compaction function



$$P\left(\zeta_R\right) \propto \frac{e^{-\frac{\pi^2}{4} \frac{\zeta_R}{\mu^2}}}{\left(\zeta_R/\mu^2\right)^3}$$

Quasi-exponential tail

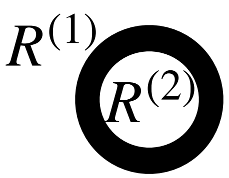
Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

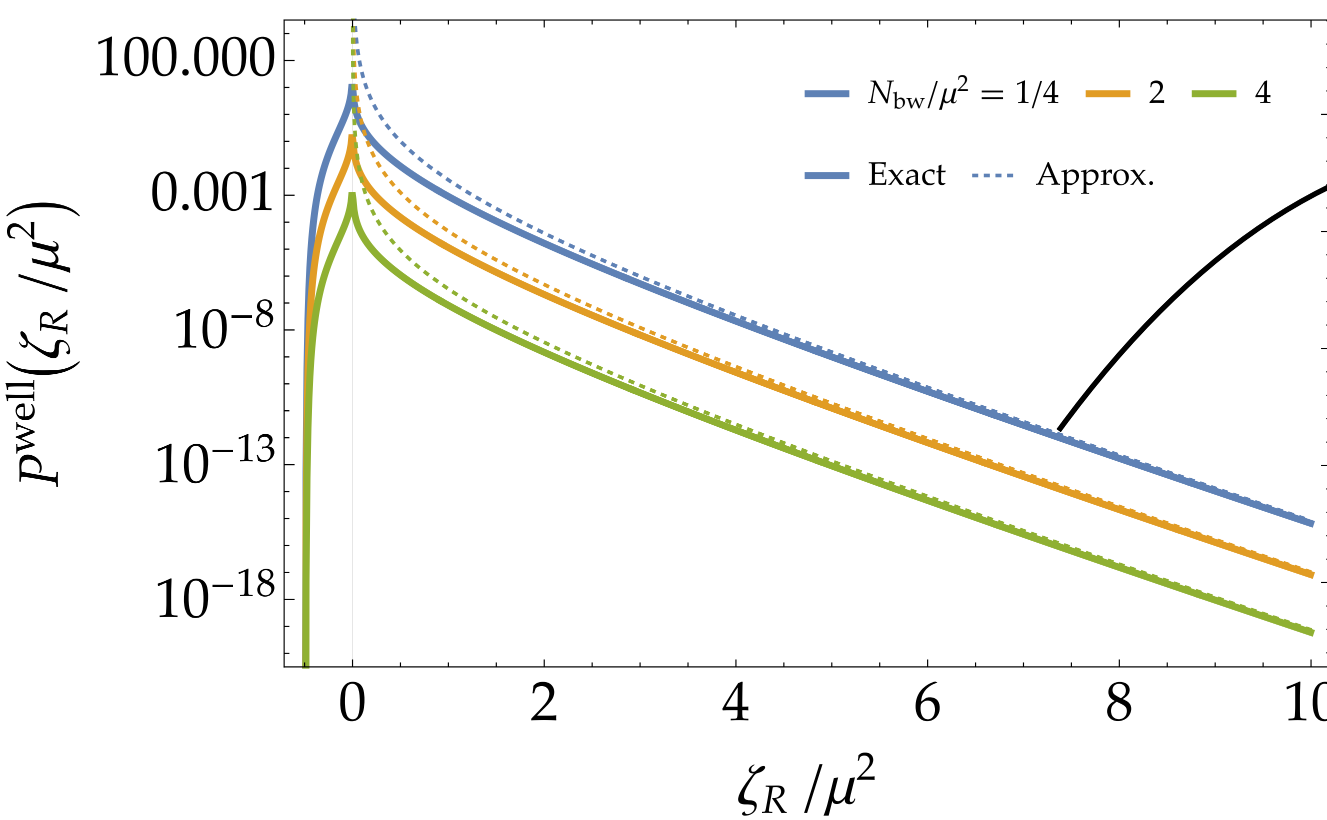
$$P\left(\zeta_R\right) = \boxed{\int_\Omega \mathrm{d}\mathbf{\Phi}_*} P_{\mathrm{bw}}\left[\mathbf{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT},\mathbf{\Phi}_0\to\mathbf{\Phi}_*}\left[\zeta_R - \langle\mathcal{N}(\mathbf{\Phi}_*)\rangle + \langle\mathcal{N}(\mathbf{\Phi}_0)\rangle\right]$$

$$P(\Delta\zeta) = \boxed{\int_\Omega \mathrm{d}\mathbf{\Phi}_*^{(1)}\mathrm{d}\mathbf{\Phi}_*^{(2)}} P_{\mathrm{bw}}\left(\mathbf{\Phi}_*^{(1)}, \mathbf{\Phi}_*^{(2)} \middle| N_{\mathrm{bw}}^{(1)}, N_{\mathrm{bw}}^{(2)}\right) \delta\left[\Delta\zeta + \left\langle\mathcal{N}\left(\mathbf{\Phi}_*^{(1)}\right)\right\rangle - \left\langle\mathcal{N}\left(\mathbf{\Phi}_*^{(2)}\right)\right\rangle - \ln(1+\beta)\right]$$



→ Comoving density contrast

→ Compaction function



$$P\left(\zeta_R\right) \propto \frac{e^{-\frac{\pi^2}{4}\frac{\zeta_R}{\mu^2}}}{\left(\zeta_R/\mu^2\right)^3}$$

Quasi-exponential tail

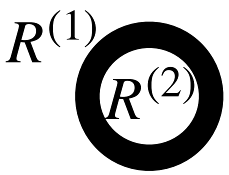
Extracting cosmological observables

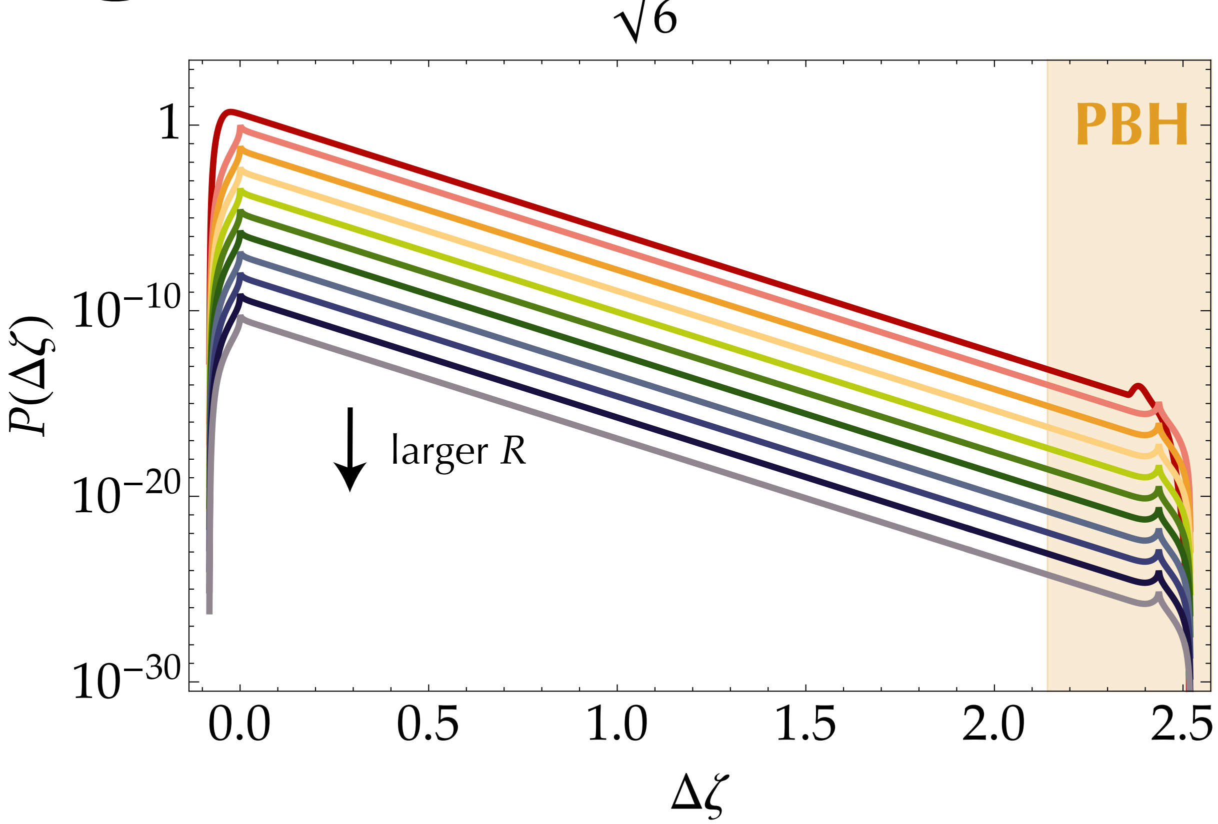
One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

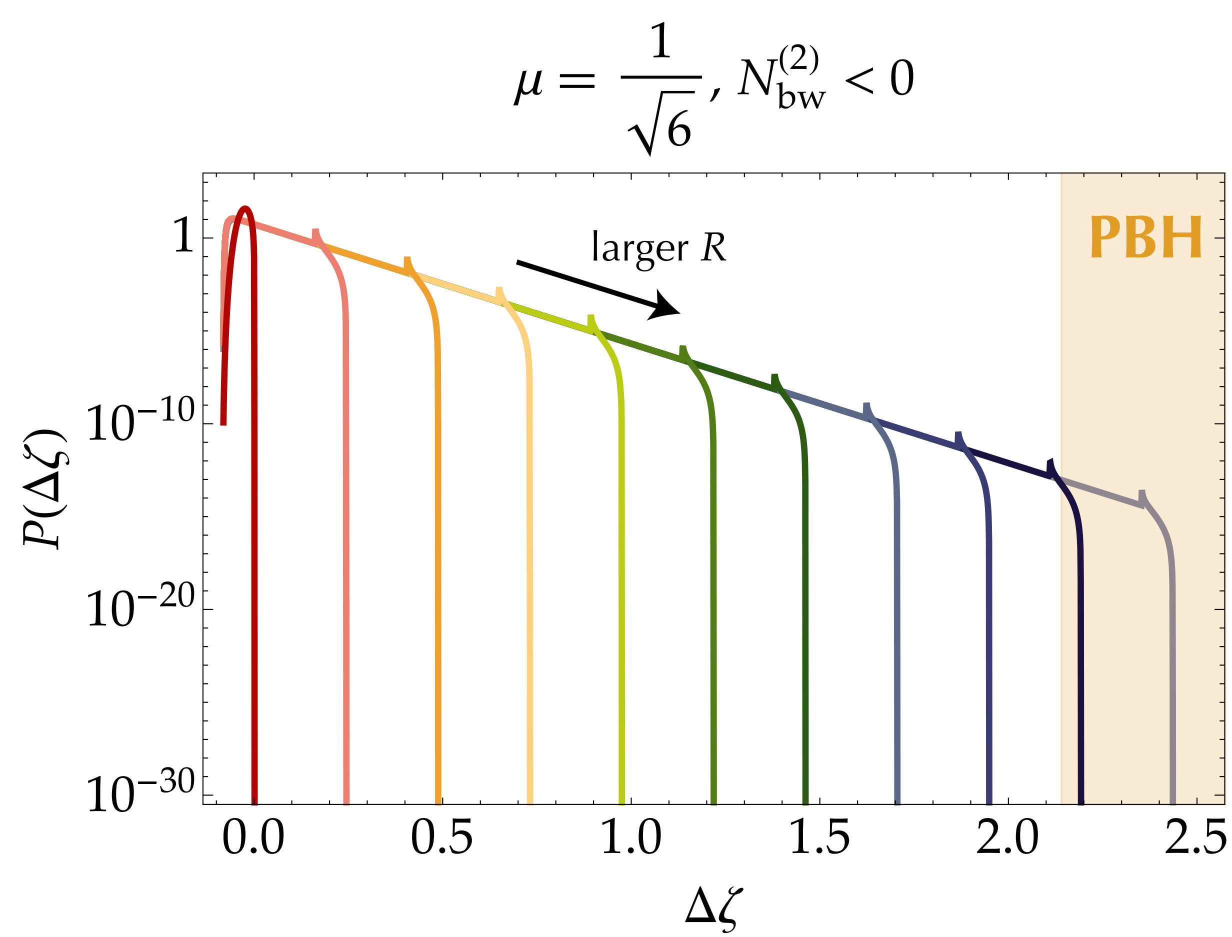
$$P\left(\zeta_R\right)=\boxed{\int_\Omega \mathrm{d}\mathbf{\Phi}_*} P_{\mathrm{bw}}\left[\mathbf{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT}, \mathbf{\Phi}_0 \rightarrow \mathbf{\Phi}_*}\left[\zeta_R-\left\langle\mathcal{N}\left(\mathbf{\Phi}_*\right)\right\rangle+\left\langle\mathcal{N}\left(\mathbf{\Phi}_0\right)\right\rangle\right]$$

$$P(\Delta\zeta)=\boxed{\int_\Omega \mathrm{d}\mathbf{\Phi}_*^{(1)} \mathrm{d}\mathbf{\Phi}_*^{(2)}} P_{\mathrm{bw}}\left(\mathbf{\Phi}_*^{(1)}, \mathbf{\Phi}_*^{(2)} \middle| N_{\mathrm{bw}}^{(1)}, N_{\mathrm{bw}}^{(2)}\right) \delta\left[\Delta\zeta+\left\langle\mathcal{N}\left(\mathbf{\Phi}_*^{(1)}\right)\right\rangle-\left\langle\mathcal{N}\left(\mathbf{\Phi}_*^{(2)}\right)\right\rangle-\ln(1+\beta)\right]$$





R_2 exits within the quantum well



R_2 exits below the quantum well

Extracting cosmological observables

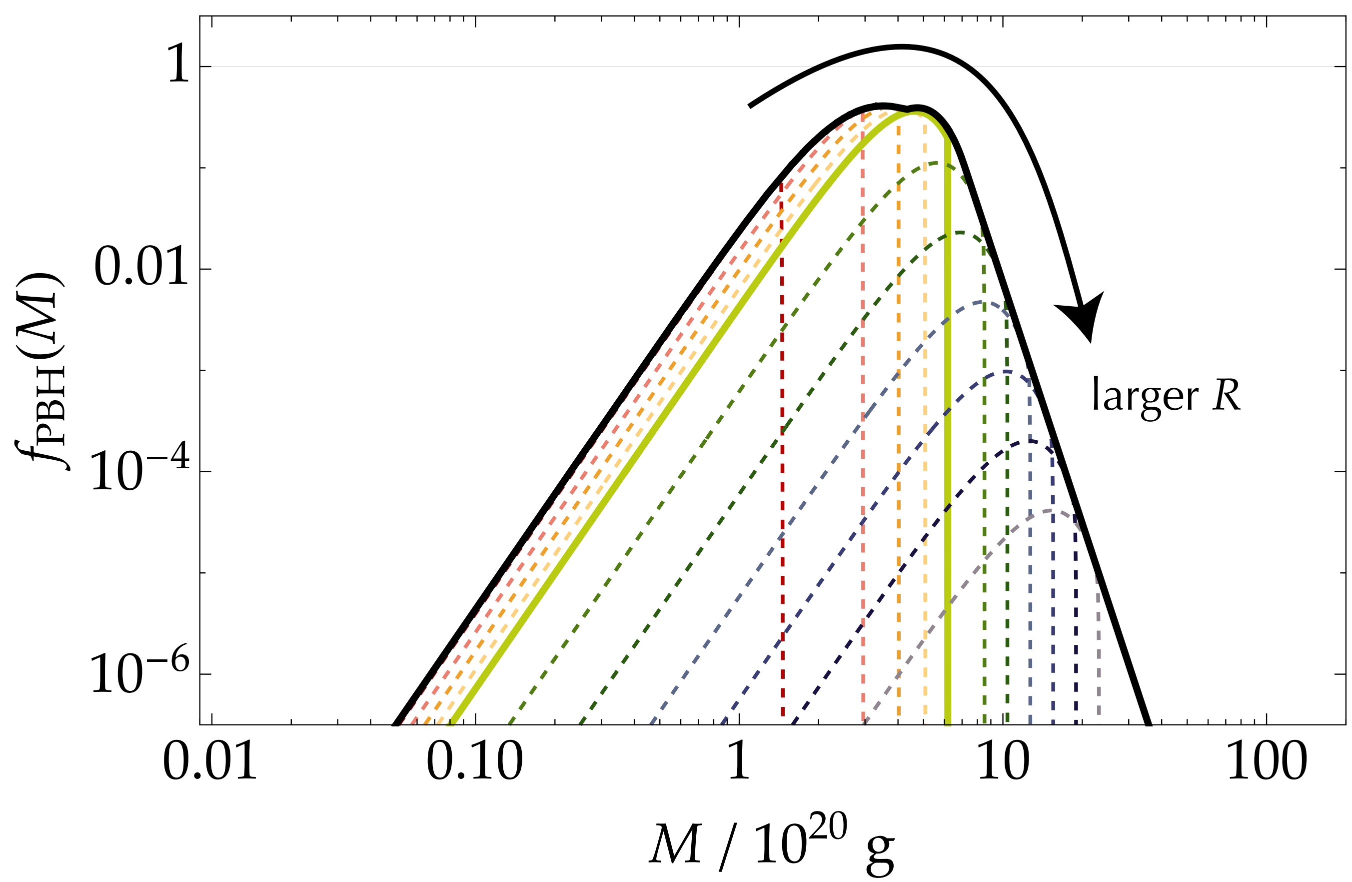
One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

$$P\left(\zeta_R\right)=\boxed{\int_\Omega \mathrm{d}\boldsymbol{\Phi}_*} P_{\mathrm{bw}}\left[\boldsymbol{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT}, \boldsymbol{\Phi}_0 \rightarrow \boldsymbol{\Phi}_*}\left[\zeta_R-\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*\right)\right\rangle+\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_0\right)\right\rangle\right]$$

$$P(\Delta\zeta)=\boxed{\int_\Omega \mathrm{d}\boldsymbol{\Phi}_*^{(1)} \mathrm{d}\boldsymbol{\Phi}_*^{(2)}} P_{\mathrm{bw}}\left(\boldsymbol{\Phi}_*^{(1)}, \boldsymbol{\Phi}_*^{(2)} \middle| N_{\mathrm{bw}}^{(1)}, N_{\mathrm{bw}}^{(2)}\right) \delta\left[\Delta\zeta+\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*^{(1)}\right)\right\rangle-\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*^{(2)}\right)\right\rangle-\ln(1+\beta)\right]$$

$$\mu=\frac{1}{\sqrt{6}}$$



Conclusions

- The back-reaction of vacuum quantum fluctuations on the background dynamics can be incorporated within the formalism of stochastic inflation
- This is necessary to describe regimes leading to large fluctuations, such as those yielding primordial black holes
- Quantum diffusion leads to exponential tails: non-perturbative breakdown of Gaussian statistics
- Most cosmological observables can be reconstructed from first-passage time analysis (power spectrum, mass functions, n-point functions?)
- Quantum diffusion makes the CMB probe the whole potential: models leading to PBHs are constrained by the CMB, even if those two sets of scales are well separated
- What is the best strategy to look for exponential tails in the data?

Advertisement

Two postdoctoral-position openings at ENS Paris:

https://academicjobsonline.org/ajo/jobs/22611

and

https://academicjobsonline.org/ajo/jobs/22047