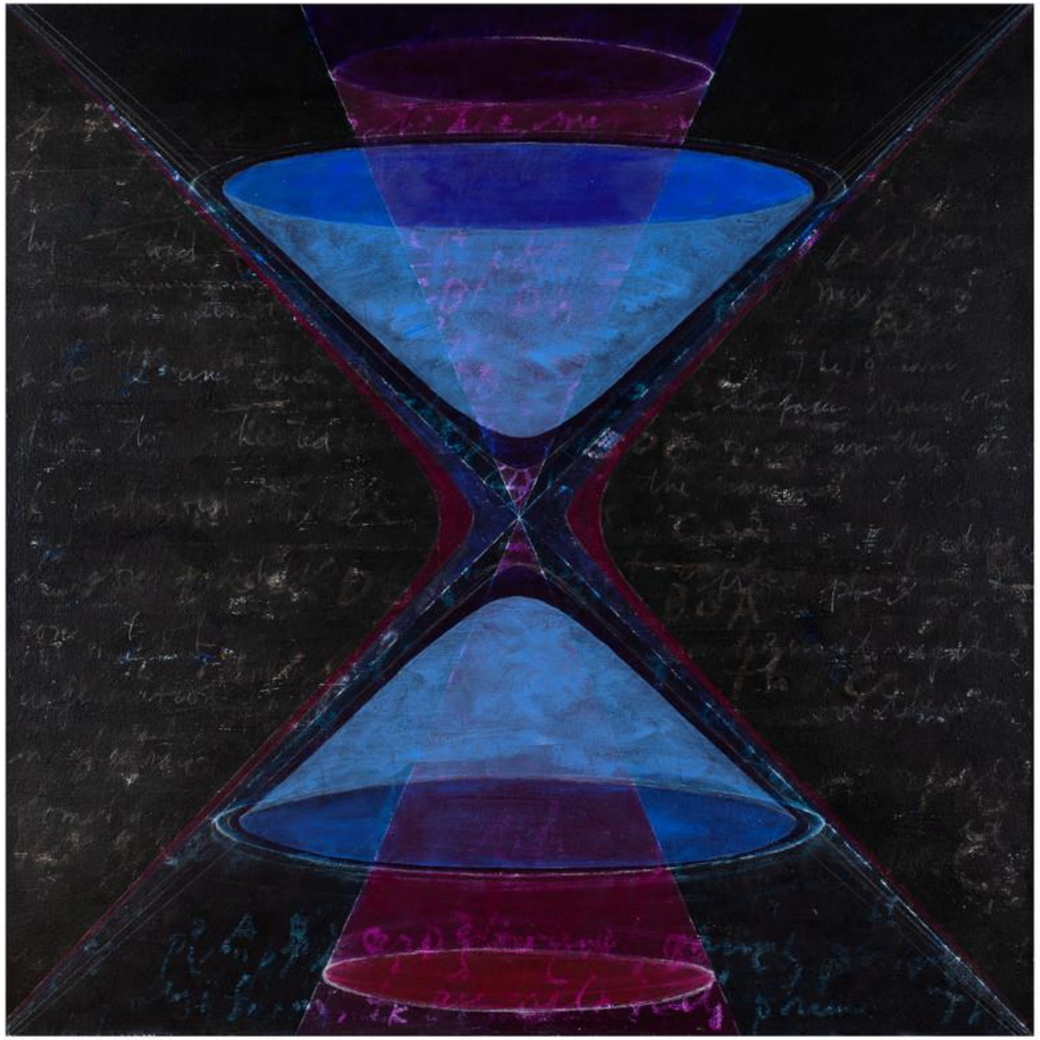
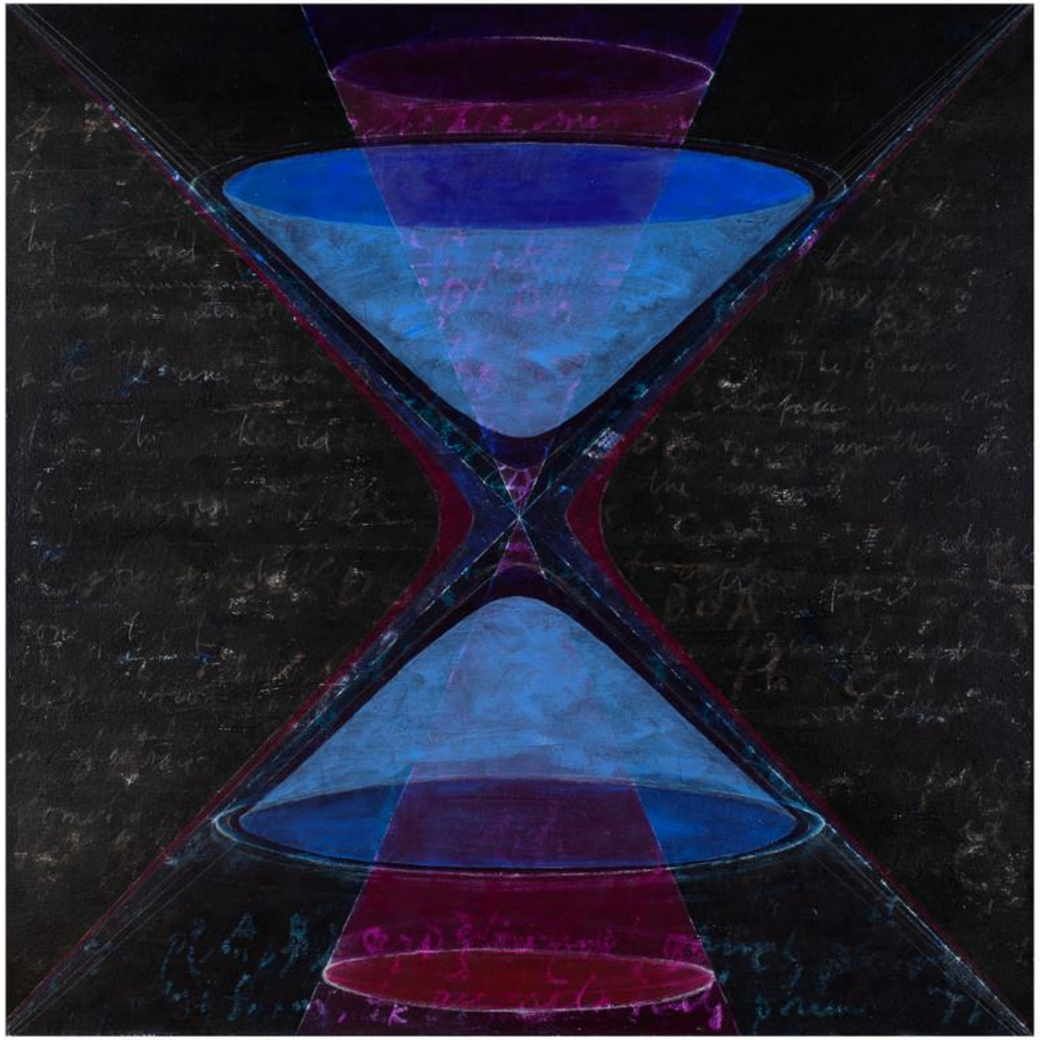
A cautionary case of casual causality



Calvin Y.-R. Chen
Imperial College London

11th October 2022
Copernicus Webinar

A cautionary case of casual causality



based on arxiv:2112.05031 in collaboration with C. de Rham, A. Margalit, A. J. Tolley

Outline

III. Causality and the EFT of gravity

II. Causality in curved spacetime

I. EFTs and causality

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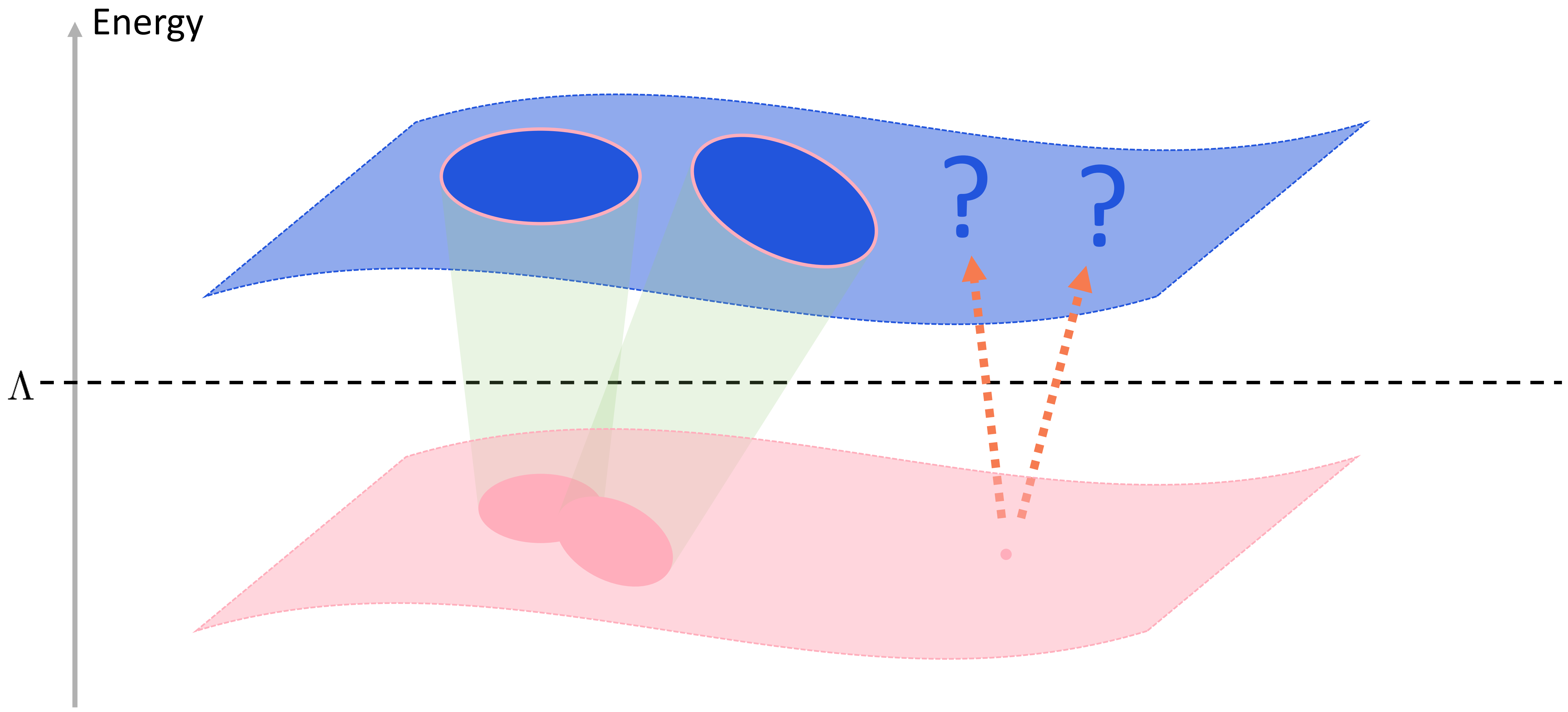
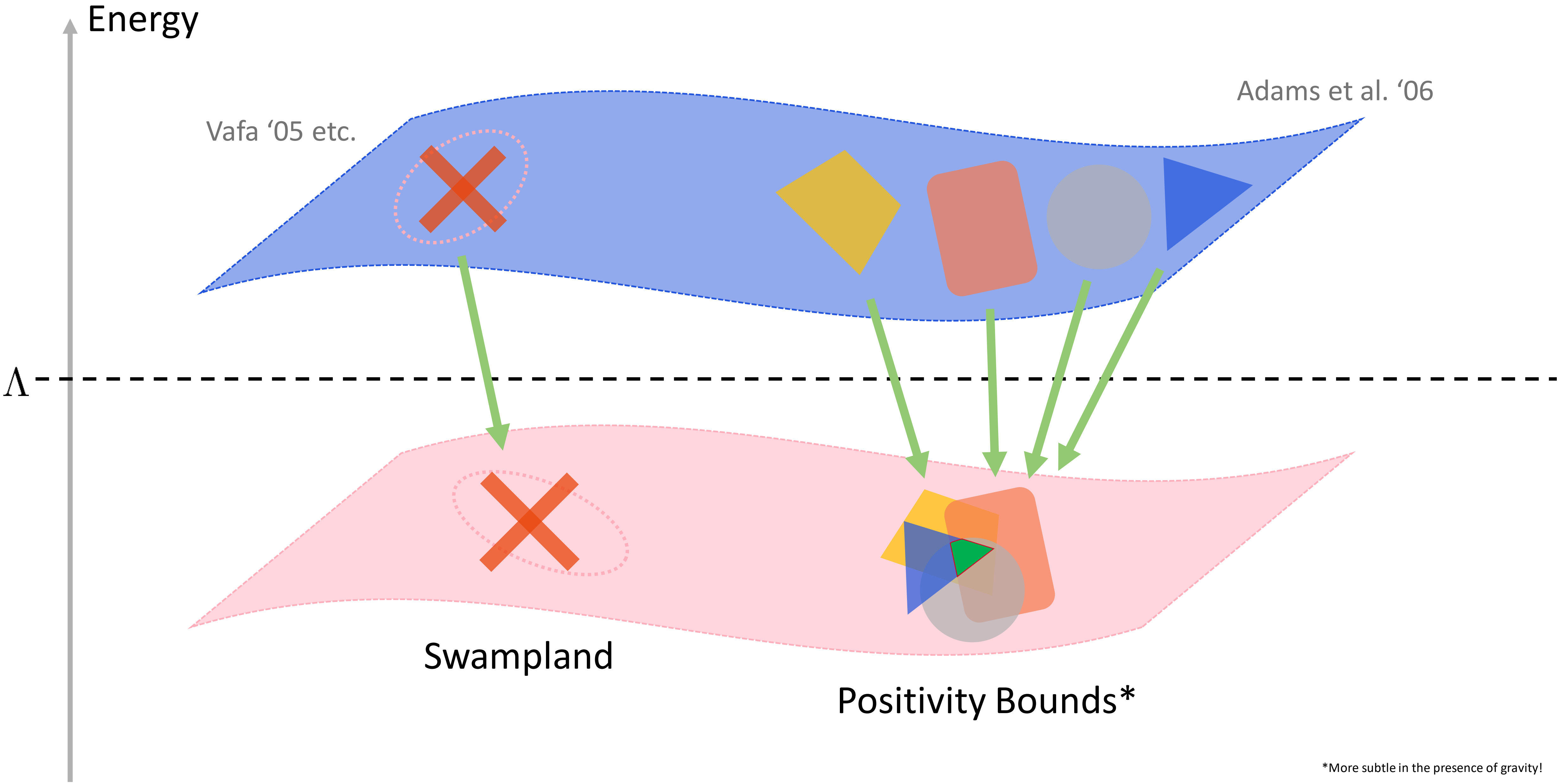
EFTs and RG flow



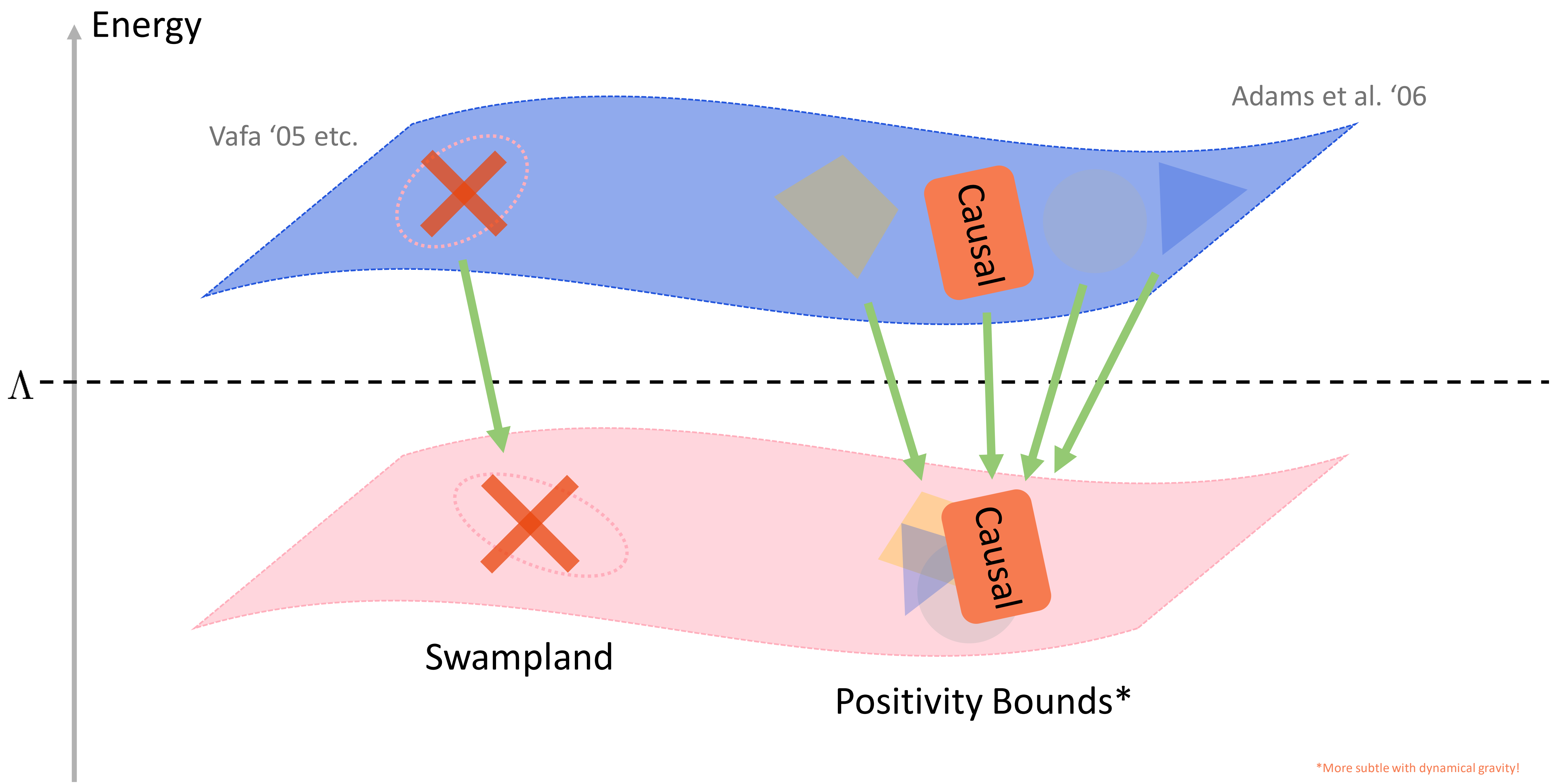
...but **reversing** RG flow is difficult!

UV imprints on IR



*More subtle in the presence of gravity!

Causality


Energy
Vafa '05 etc.
Adams et al. '06
Causal
Λ
Causal
Swampland
Positivity Bounds*
*More subtle with dynamical gravity!

Fine print?

*More subtle with dynamical gravity!

Cheung and Remmen ‘17
Alberte, de Rham, Jaitly, and Tolley ‘20
Tokuda, Aoki, and Hirano ‘20
etc.

More direct way of imposing causality: (Sub-)luminal propagation?

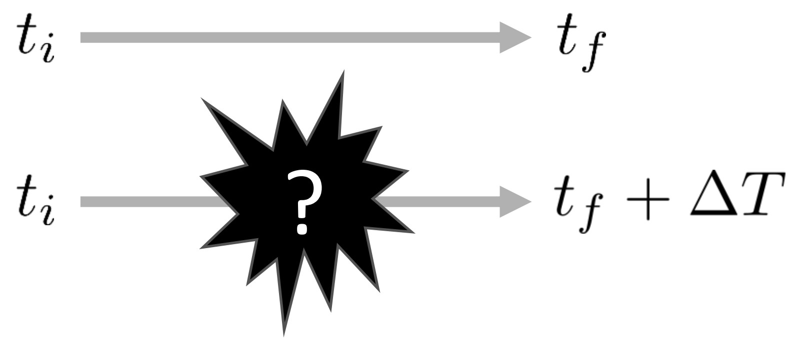
Question: How should we understand and use (sub-)luminality to study gravitational EFTs?

II. Causality in curved spacetime

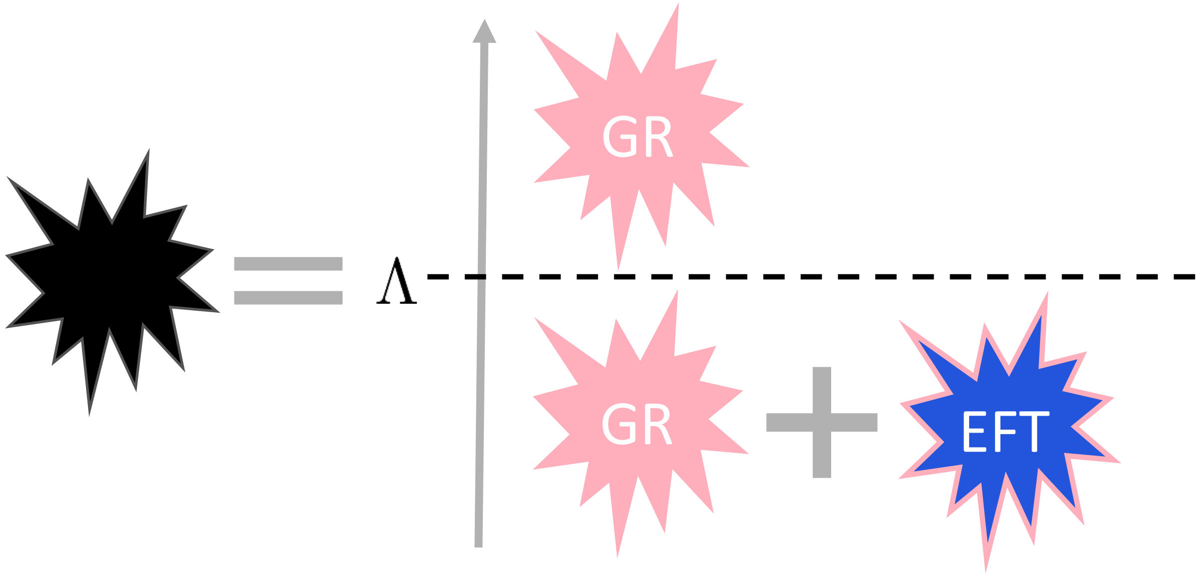
Infrared Causality

With **asymptotic flatness** – can define **S-Matrix**, which is invariant under field redefinitions!
Can then define **time delay**

$$\Delta\hat{T} = -iS^\dagger \frac{d\hat{S}}{d\omega} = \frac{d\hat{\delta}}{d\omega}$$



Causality should be related to positivity of time delay (asymptotic causality). With gravity, actually **infrared causality:**



reference for causal structure!

$$\Delta T = \underbrace{\Delta T^{\mathrm{GR}}}_{>0} + \Delta T^{\mathrm{EFT}}$$

positive for causality

Final ingredient: Resolvability

Uncertainty principle: Effects cannot be **observable** when scales probed are too large, i.e. **resolving power** too small!

For causality: Waves with frequency ω cannot measure time delays with magnitude

$$|\Delta T| \lesssim \omega^{-1}$$

Together, our diagnostic for causality is:

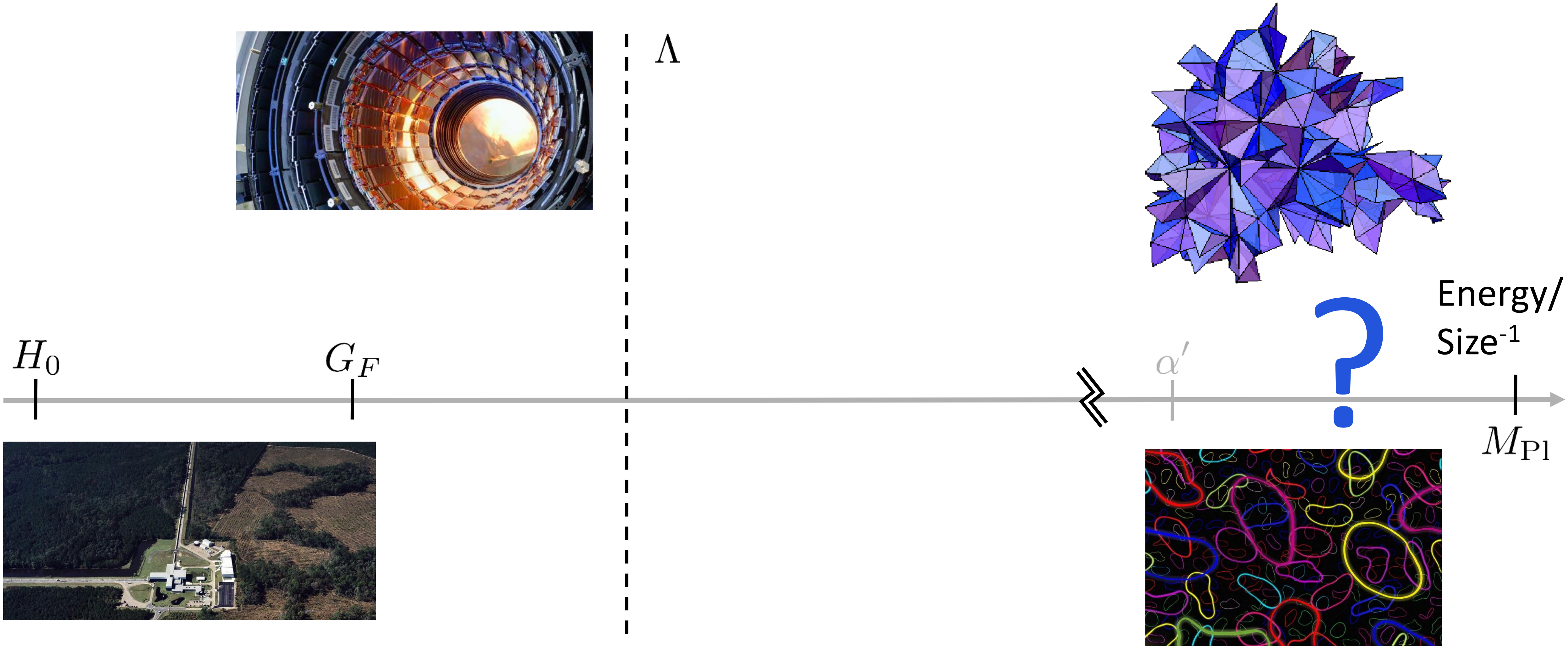
Acausality

$$\Updownarrow$$

$$\Delta T^{\mathrm{EFT}} < 0 \qquad\qquad |\Delta T^{\mathrm{EFT}}| \gtrsim \omega^{-1}$$

III. Causality and the EFT of gravity

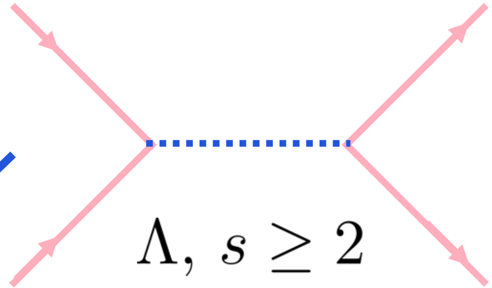
Effective field theory of gravity



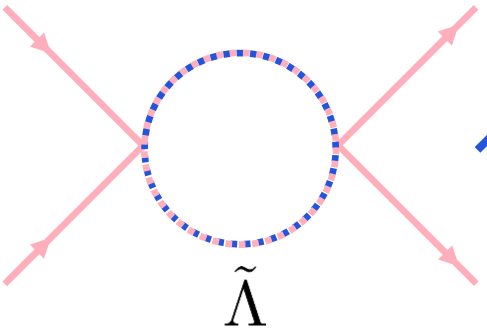
The **UV completion** of GR is unknown (if you have any information, please email!), but we can write down a **generic effective action**.

Einstein-Hilbert +

Full **effective action** (redundantly parameterised):

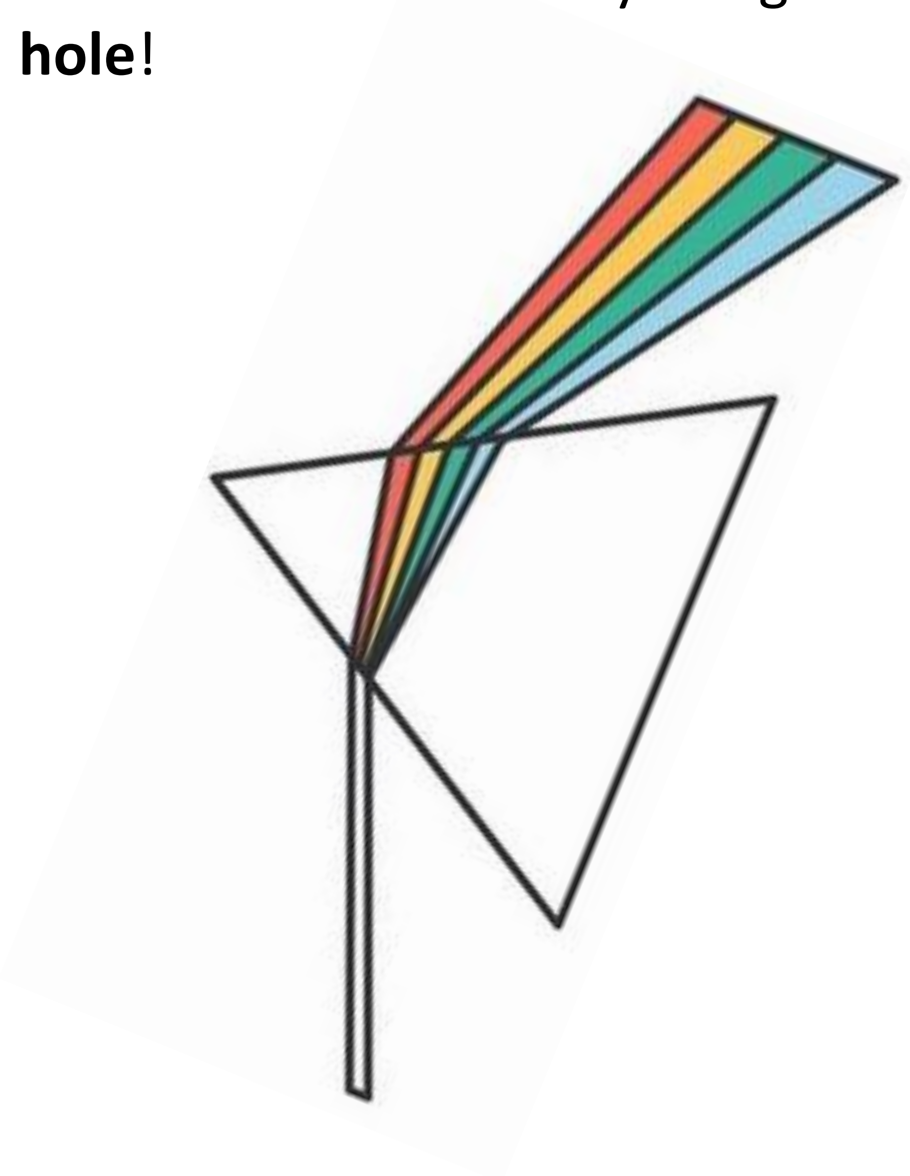


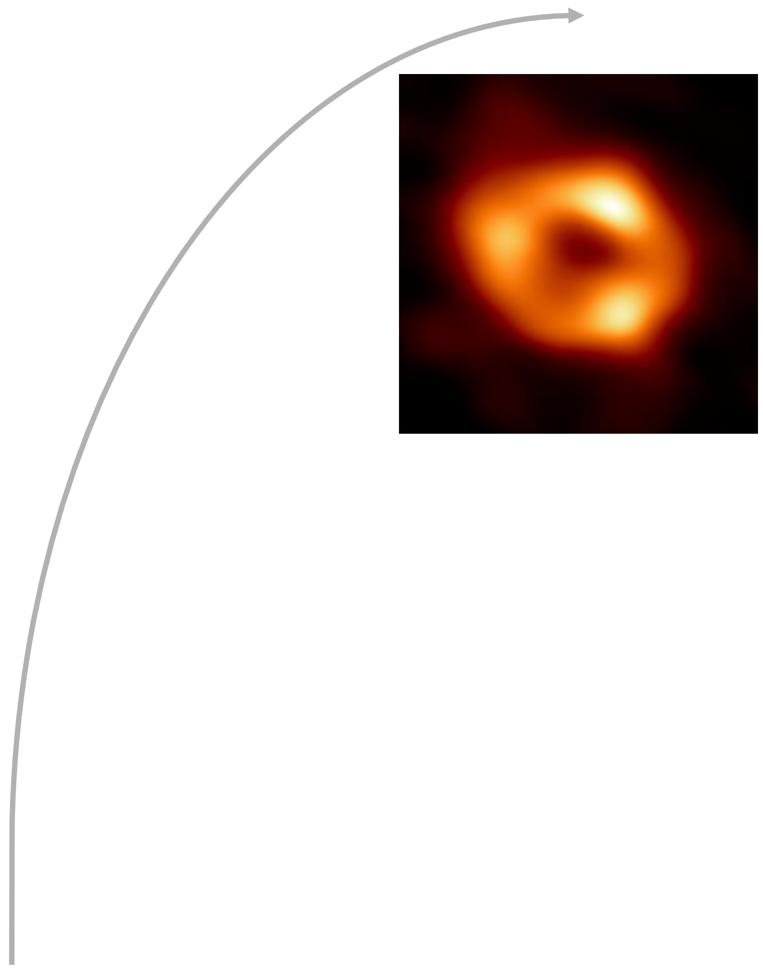
$$S_{\rm EFT} = \int d^D x \sqrt{-g} \Big[M_{\rm Pl}^{D-2} \Big(\frac{1}{2} R + \Lambda^2 \sum_{m \geq 0, n \geq 2} c_{mn} \left(\frac{\nabla}{\Lambda} \right)^m \left(\frac{\text{Riemann}}{\Lambda^2} \right)^n \Big) + \tilde{\Lambda}^D \sum_{m \geq 0, n \geq 2} d_{mn} \left(\frac{\nabla}{\tilde{\Lambda}} \right)^m \left(\frac{\text{Riemann}}{\tilde{\Lambda}^2} \right)^n \Big]$$



Fourth test of GR: Shapiro time delay

What do we do to study things? We smash them into other things: scatter gravitons off **black hole**!





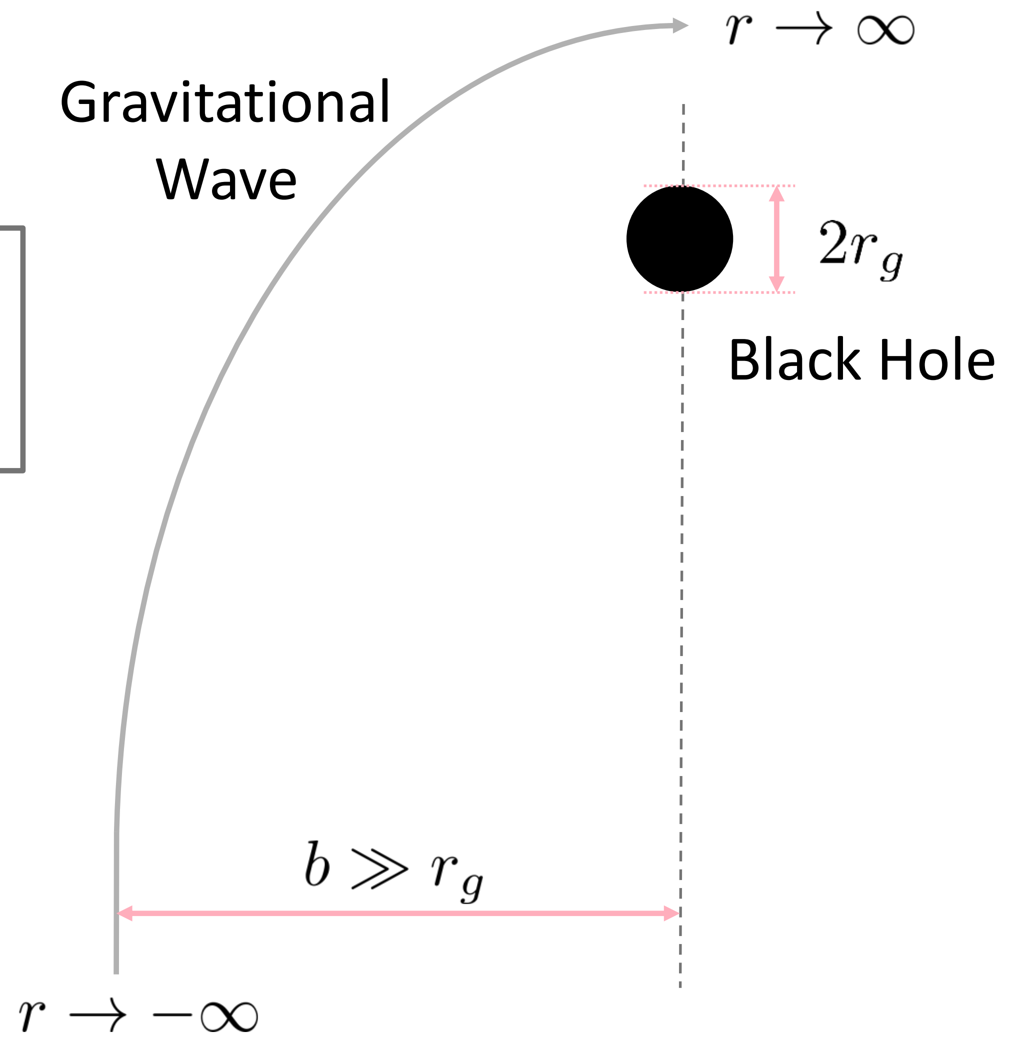
Fourth test of GR: Shapiro time delay

What do we do to study things? We smash them into other things: scatter gravitons off **black hole**!

FOURTH TEST OF GENERAL RELATIVITY

Irwin I. Shapiro
Lincoln Laboratory,* Massachusetts Institute of Technology, Lexington, Massachusetts
(Received 13 November 1964)

$$\Delta T^{\rm GR} = \frac{(D-2)\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{D-4}{2}\right)}{\Gamma\left(\frac{D-3}{2}\right)}\left(\frac{r_g}{b}\right)^{D-3} b$$



Leading-order EFT

Want to look at scattering in **black hole spacetime**, so try to find static and spherically symmetric **vacuum** solution! In $D \geq 5$:

In vacuum:

$$R_{\mu\nu} = 0 + \mathcal{O}\left(\Lambda^{-2}\right) \longrightarrow S_{\text{EFT}} = \int d^D x \sqrt{-g} M_{\text{Pl}}^{D-2} \left(\frac{1}{2} R + \frac{c_{\text{GB}}}{\Lambda^2} R_{\text{GB}}^2 + \mathcal{O}\left(\Lambda^{-4}\right) \right)$$

$$R_{\text{GB}}^2 = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

i.e. **Einstein-Gauss-Bonnet gravity!**

We want to solve everything **perturbatively** in the parameter $\epsilon = \dfrac{1}{\Lambda^2 r_g^2}$.

Background and Perturbations

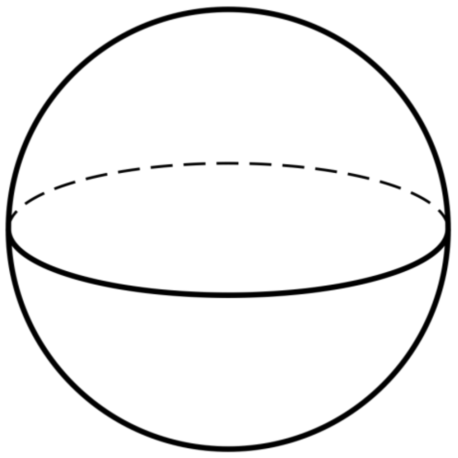
Static and **spherically symmetric** background solution:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{D-2}^2,$$

$$f(r) = 1 - \left(\frac{r_g}{r}\right)^{D-3} + 2(D-3)(D-4)c_{\text{GB}}\epsilon\left(\frac{r_g}{r}\right)^{2D-4} + \mathcal{O}\left(\epsilon^2\right)$$

Physical degrees of freedom in **perturbations** $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ can be SVT-decomposed:

Kodama, Ishibashi, and Soda '00
Kodama and Ishibashi '03
Takahashi and Soda '11



Spherical symmetry

$$h_{\mu\nu} = \left(\begin{array}{c|c} \text{scalar} & \begin{array}{c}\text{vector}\\ \text{vector}\end{array} \\ \hline & \text{tensor}\end{array}\right) \begin{array}{c} t \\ r \\ \text{angles}\end{array}$$

Scalar-Vector-Tensor decomposition

Time delay

Compute **time delay** from effective potential.

$$\boxed{\Delta T^{\mathrm{GR}} \sim \left(\frac{r_g}{b}\right)^{D-3} b}$$

$$\left.\begin{aligned}
\Delta T_T &= \Delta T^{\mathrm{GR}} \left[1 - \frac{8(D-1)}{D-3} c_{\mathrm{GB}} \epsilon \left(\frac{r_g}{b}\right)^2\right] \\
\Delta T_V &= \Delta T^{\mathrm{GR}} \left[1 + \frac{4(D-1)(D-4)^2}{D-3} c_{\mathrm{GB}} \epsilon \left(\frac{r_g}{b}\right)^2\right] \\
\Delta T_S &= \Delta T^{\mathrm{GR}} \left[1 + \frac{8(D-1)(D-4)^2}{D-3} c_{\mathrm{GB}} \epsilon \left(\frac{r_g}{b}\right)^2\right]
\end{aligned}\right\} \quad |\Delta T^{\mathrm{EFT}}| \sim c_{\mathrm{GB}} \epsilon \left(\frac{r_g}{b}\right)^{D-1} b$$

For sufficiently small impact parameter $b \sim r_g$ and low cut-off $\epsilon \gg 1$, *could* make this large, so that for **causality**

$$c_{\mathrm{GB}} \stackrel{?}{=} 0$$

Camanho et al. '14
Reall, Tanahashi, and Way '14
Papallo and Reall '15

EFT is **derivative expansion**, so control of series is lost when curvature and derivatives become large.

Regime of validity

EFT breaks down when probed at length scales that are too small and when probed by particles with energies that are too high.

We want to put bounds on **Lorentz scalars** towards infinity.

$$\text{“}\lim_{p,m,n\to\infty}\left(\frac{\nabla}{\Lambda}\right)^p\left(\frac{\text{Riemann}}{\Lambda^2}\right)^m\left(\frac{k}{\Lambda}\right)^{2n+p}\ll 1\text{”}$$

Evaluate at $r \sim b$ to find lower bound on impact parameter:

$$(k^\mu\nabla_\mu)^p\,[\ldots]\ll\Lambda^{2p}\quad\xrightarrow[p\to\infty]{}\quad\omega\ll\Lambda^2 b$$

Note that this is distinct from the naïve guess:

$$k^2=k^\mu k_\mu=0\neq\omega^2\quad\longrightarrow\quad\cancel{\omega\ll\Lambda}$$

Casually causal or cautiously acausal?

Putting everything together:

$$\omega \ll \Lambda^2 b$$

$$\omega\left|\Delta T^{\mathrm{EFT}}\right| \sim |c_{\mathrm{GB}}|\,\frac{1}{r_g^2\Lambda^2}\,\omega\left(\frac{r_g}{b}\right)^{D-1} b \ll |c_{\mathrm{GB}}|\,\frac{1}{r_g^2\Lambda^2}\,\Lambda^2 b\left(\frac{r_g}{b}\right)^{D-1} b$$

$$\sim |c_{\mathrm{GB}}|\left(\frac{r_g}{b}\right)^{D-3} \ll |c_{\mathrm{GB}}|$$

$$r_g \ll b$$

Infrared causality is respected as long as $|c_{\mathrm{GB}}| \lesssim 1$.

This is consistent with **gravitational positivity bounds**!

Conclusion

Summary

- Causality in EFTs can help us learn about UV physics and possible EFTs.
- In curved spacetime, **infrared causality** is the correct notion of causality in EFTs.
- Properly identifying the **regime of validity** shows that the leading-order EFT of gravity, i.e. EGB gravity, is **not acausal**.
- Complementary understanding of **positivity bounds.**

Outlook

- Use infrared causality on different (less symmetric?) **backgrounds** to bound different EFT operators?
- Alternative when positivity bounds fail?
- **Shockwaves**: Can look at more complicated configurations of sources to accumulate time delay. Stay tuned for arXiv:2XXX.XXXXX (spoiler: this is not possible)!

Thanks for your attention!

Any questions?