



Dissipative Inflation via Scalar Production

Borna Salehian (ICTP)

based on 2305.07696, with **Paolo Creminelli**, **Soubhik Kumar** and **Luca Santoni**

Copernicus Webinar – 25 July 2023

Introduction

Look for **qualitative features** of the inflationary model, e.g. scale of inflation, speed of propagation etc.

Introduction

Look for **qualitative features** of the inflationary model, e.g. scale of inflation, speed of propagation etc.

Dissipation

Transfer energy from the inflaton to additional degrees of freedom.

Look for **qualitative features** of the inflationary model, e.g. scale of inflation, speed of propagation etc.

Dissipation

Transfer energy from the inflaton to additional degrees of freedom.

Cold inflation: scalar field with a potential. Coupling to other degrees of freedom becomes important only at the end, i.e. (pre)heating etc.

“Warm” inflation: class of models in which coupling to other particles are relevant all the time – Berera '95, Warm little inflation '16, Minimal warm inflation '19.

Look for **qualitative features** of the inflationary model, e.g. scale of inflation, speed of propagation etc.

Dissipation

Transfer energy from the inflaton to additional degrees of freedom.

Cold inflation: scalar field with a potential. Coupling to other degrees of freedom becomes important only at the end, i.e. (pre)heating etc.

“Warm” inflation: class of models in which coupling to other particles are relevant all the time – Berera '95, Warm little inflation '16, Minimal warm inflation '19.

They don't have to **thermalize!**, e.g. axion coupled to $U(1)$: $\phi F\tilde{F}$ by Anber and Sorbo '09.

Natural inflation, strong backreaction

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

One of the photon polarizations **grow exponentially** due to instability

$$\frac{d^2 A_{\pm}}{d\tau^2} + \left(k^2 \pm 2k\frac{\xi}{\tau}\right) A_{\pm} = 0, \quad \xi = \frac{\alpha\dot{\phi}_0}{2fH}.$$

Instability starts at $|k\tau| \simeq 2\xi$ and continues up to **superhorizon scales**.

Total amount of enhancement is $A \sim e^{\pi\xi}$. Inflaton equation of motion

$$\phi'' + 2aH\phi' - \nabla^2\phi + a^2V' = a^2\frac{\alpha}{f}\vec{E}\cdot\vec{B}.$$

Difficulties: Large power spectrum, non-locality of the response, resonant instability.

Anber and Sorbo '09, '12.

Domcke et al '20, Caravano et al '22, Peloso and Sorbo '22.

Natural inflation, strong backreaction

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

Complex Scalar field

One of the photon polarizations **grow exponentially** due to instability

$$\frac{d^2 A_{\pm}}{d\tau^2} + \left(k^2 \pm 2k\frac{\xi}{\tau}\right) A_{\pm} = 0, \quad \xi = \frac{\alpha\dot{\phi}_0}{2fH}.$$

Instability starts at $|k\tau| \simeq 2\xi$ and continues up to **superhorizon scales**.

subhorizon scales

Total amount of enhancement is $A \sim e^{\pi\xi}$. Inflaton equation of motion

$$\phi'' + 2aH\phi' - \nabla^2\phi + a^2V' = a^2\frac{\alpha}{f}\vec{E}\cdot\vec{B}.$$

Difficulties: Large power spectrum, non-locality of the response, resonant instability.

Remark: EFT with dissipation

Single field (clock) models

Single clock with dissipation

Multiple field models

Our model is an example of **Effective field theory of inflation with dissipation**, Nacir, Porto, Senatore and Zaldarriaga '11.

1. The Model
2. Linear Perturbations
3. Non-Gaussianity

The Model

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{Pl}}^2 R - \frac{1}{2} (\partial\phi)^2 - V(\phi) - |\partial\chi|^2 + M^2 |\chi|^2 - i \frac{\partial_\mu \phi}{f} (\chi \partial^\mu \chi^* - \chi^* \partial^\mu \chi) - \frac{1}{2} m^2 (\chi^2 + \chi^{*2}) \right].$$

- For $m = 0$ the action is $U(1)$ invariant. One can remove the current coupling by $\chi \rightarrow e^{-i\phi/f} \chi$, which changes $M^2 \rightarrow M^2 + (\partial\phi)^2/f^2$.
- We consider $M^2(X)$ and $m^2(X)$. With hindsight, M^2 is defined with the unconventional sign.
- The only shift-symmetry breaking term is the potential $V(\phi)$.

Equation of motion for χ will be

$$\square\chi + \frac{2i}{f}\nabla^\mu\phi\nabla_\mu\chi + \left(M^2 + i\frac{\square\phi}{f}\right)\chi - m^2\chi^* = 0.$$

Equation of motion for χ will be

$$\square\chi + \frac{2i}{f}\nabla^\mu\phi\nabla_\mu\chi + \left(M^2 + i\frac{\square\phi}{f}\right)\chi - m^2\chi^* = 0.$$

We have $\phi = \phi_0$ with $\rho \equiv \dot{\phi}_0/f$, also define $\chi = (\sigma_1 + i\sigma_2)/\sqrt{2}a^{3/2}$

$$\ddot{\sigma}_1 - \frac{\vec{\nabla}^2\sigma_1}{a^2} - (M^2 - m^2)\sigma_1 - 2\rho\dot{\sigma}_2 = 0,$$

$$\ddot{\sigma}_2 - \frac{\vec{\nabla}^2\sigma_2}{a^2} - (M^2 + m^2)\sigma_2 + 2\rho\dot{\sigma}_1 = 0.$$

Equation of motion for χ will be

$$\square\chi + \frac{2i}{f}\nabla^\mu\phi\nabla_\mu\chi + \left(M^2 + i\frac{\square\phi}{f}\right)\chi - m^2\chi^* = 0.$$

We have $\phi = \phi_0$ with $\rho \equiv \dot{\phi}_0/f$, also define $\chi = (\sigma_1 + i\sigma_2)/\sqrt{2}a^{3/2}$

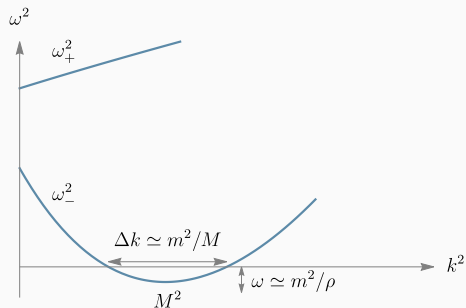
$$\ddot{\sigma}_1 - \frac{\vec{\nabla}^2\sigma_1}{a^2} - (M^2 - m^2)\sigma_1 - 2\rho\dot{\sigma}_2 = 0,$$

$$\ddot{\sigma}_2 - \frac{\vec{\nabla}^2\sigma_2}{a^2} - (M^2 + m^2)\sigma_2 + 2\rho\dot{\sigma}_1 = 0.$$

Neglecting expansion one can find the natural modes of the system assuming, in Fourier space, $\sigma \sim e^{-i\omega t}$ and obtains

$$\omega_\pm^2 = \left(\sqrt{k^2 + \rho^2 - M^2 + \frac{m^4}{4\rho^2}} \pm \rho \right)^2 - \frac{m^4}{4\rho^2},$$

Dynamics of ADOF (cont.)



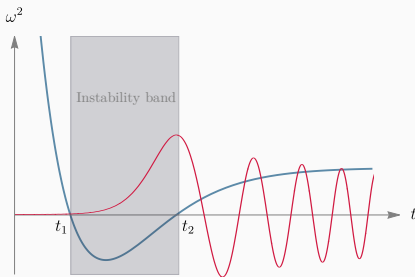
- Complex scalar field, two modes.
- ω_- has a minimum located at $k = M$ controlled by the value of m .
- For $m = 0$, $U(1)$ -invariant case, the band closes.
- The location of the band will be $M^2 - m^2 < k^2 < M^2 + m^2$.
- Very large and very small scales are healthy if

Figure 1: Dispersion relation

$$m \ll M \lesssim \rho$$

ADOF in expanding universe

Including expansion, momenta gets redshifted $k \rightarrow k/a$. Therefore, the instability is regulated by the limited amount of time spent in the band controlled by H .



- Length of the band

$$H\Delta t \sim \frac{m^2}{M^2} \ll 1.$$

- Total growth

$$\pi\xi \equiv \int_{t_1}^{t_2} dt |\omega_-| \sim \frac{m^4}{H\rho M^2}.$$

- Exponential enhancement of the fields $\chi \sim e^{\pi\xi}$.

Demanding $H \ll m \ll M \lesssim \rho$ we get $\xi = \mathcal{O}(1)$.

Canonical quantization of ADOF

Quantization of χ field

$$\sigma_i(t, \vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \left[(F_k(t))_{ij} \hat{a}_j(\vec{k}) + (F_k^*(t))_{ij} \hat{a}_j^\dagger(-\vec{k}) \right] .$$

The matrix F plays the role of mode functions. It has to be a matrix since the two fields are strongly coupled by presence of $\rho\dot{\sigma}$ term. Mode functions satisfy

$$\ddot{F}_k + \begin{pmatrix} 0 & -2\rho \\ 2\rho & 0 \end{pmatrix} \cdot \dot{F}_k + \begin{pmatrix} \frac{k^2}{a^2} - M^2 + m^2 & 0 \\ 0 & \frac{k^2}{a^2} - M^2 - m^2 \end{pmatrix} \cdot F_k = 0 .$$

Bunch–Davies initial condition implies

$$F_k(t \rightarrow -\infty) \rightarrow \frac{e^{-ik\tau}}{\sqrt{2k/a}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} .$$

WKB solution

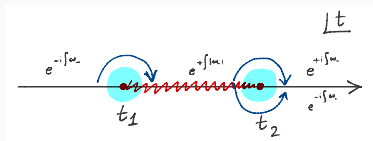
Focusing on each column

$$\vec{F}_{\text{column}} = \vec{Q}(t) \exp\left(-i \int dt \omega(t)\right),$$

with $D(\omega) \cdot \vec{Q} = 0$. For Nontrivial solutions $\det D(\omega_{\pm}) = 0$. In addition, \vec{Q} is the null vector of $D(\omega)$. Normalization is fixed by looking at NLO WKB

$$\frac{d}{dt} \left[\vec{Q}_{\pm}^{\dagger} \begin{pmatrix} \omega_{\pm} & -i\rho \\ i\rho & \omega_{\pm} \end{pmatrix} \vec{Q}_{\pm} \right] = 0.$$

General solution is addition of F_{\pm} and F_{\pm}^* . WKB is valid if $\frac{\dot{\omega}}{\omega^2} \ll 1$, therefore it breaks down at $\omega^2(t_{1,2}) = 0$. Need to do matching at $t_{1,2}$:



Weinberg 1961, Dufaux, et al '06, Landau QM

WKB solution (cont.)

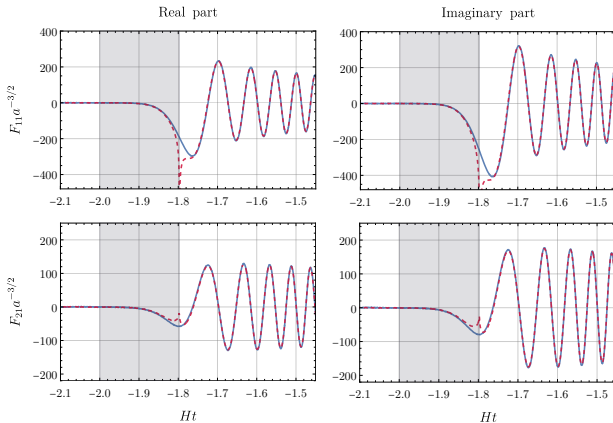


Figure 2: Comparison of numeric (solid) and analytic (dashed) solution. Gray region is the instability band.

Equation of motion for the inflaton is

$$\nabla_\mu \left[\left(1 + \frac{(M_X^2 - 2\rho^2)}{\rho^2 f^2} |\chi|^2 - \frac{m_X^2}{2\rho^2 f^2} (\chi^2 + \chi^{*2}) \right) \nabla^\mu \phi \right] - V'(\phi) + \frac{im^2}{f} (\chi^2 - \chi^{*2}) = 0.$$

Equation of motion for the inflaton is

$$\nabla_\mu \left[\left(1 + \frac{(M_X^2 - 2\rho^2)}{\rho^2 f^2} |\chi|^2 - \frac{m_X^2}{2\rho^2 f^2} (\chi^2 + \chi^{*2}) \right) \nabla^\mu \phi \right] - V'(\phi) + \frac{im^2}{f} (\chi^2 - \chi^{*2}) = 0.$$

Define $\mathcal{O} \equiv -i(\chi^2 - \chi^{*2})$, neglect $\ddot{\phi}_0$, \dot{H} at the background level

$$3H\dot{\phi}_0 + V' + \frac{m^2}{f} \langle \mathcal{O} \rangle \simeq 0.$$

Equation of motion for the inflaton is

$$\nabla_\mu \left[\left(1 + \frac{(M_X^2 - 2\rho^2)}{\rho^2 f^2} |\chi|^2 - \frac{m_X^2}{2\rho^2 f^2} (\chi^2 + \chi^{*2}) \right) \nabla^\mu \phi \right] - V'(\phi) + \frac{im^2}{f} (\chi^2 - \chi^{*2}) = 0.$$

Define $\mathcal{O} \equiv -i(\chi^2 - \chi^{*2})$, neglect $\ddot{\phi}_0$, \dot{H} at the background level

$$3H\dot{\phi}_0 + V' + \frac{m^2}{f} \langle \mathcal{O} \rangle \simeq 0.$$

- Backreaction could be large since $\langle \mathcal{O} \rangle \simeq \frac{m^2}{2\pi^2} e^{2\pi\xi}$.
- For moderate values of f ($\gg M$) we get $2\pi\xi \sim \log fV'/m^4$
- For $\dot{H}/H^2 \ll 1$ we require $V \gg$ kinetic of ϕ and χ and therefore, $3M_{\text{Pl}}^2 H^2 \approx V$.

Inflaton dynamics (cont.)

- We can neglect the other terms in the equation

$$\frac{m_X^2 \langle \chi^2 + \chi^{*2} \rangle}{(M_X^2 - 2\rho^2) \langle |\chi|^2 \rangle} \simeq \frac{m^4}{\rho^4} \ll 1, \quad \frac{\frac{H\dot{\phi}_0}{f^2} \langle |\chi|^2 \rangle}{\frac{im^2}{f} \langle \chi^2 - \chi^{*2} \rangle} \simeq \frac{H\rho^3}{m^4} \simeq \frac{1}{8\xi} \lesssim 1.$$

- The sign of the backreaction term is correct

$$\dot{\phi}_0 > 0 \implies -i \langle \chi^2 - \chi^{*2} \rangle > 0.$$

- Require an attractor solution: $\frac{d\xi}{d\phi_0} > 0$. We have seen that

$$\xi \simeq \frac{m(\dot{\phi}_0)^4}{8H \left(\frac{\dot{\phi}_0}{f} \right) M(\dot{\phi}_0)^2}.$$

- Without $M^2(X)$ and $m^2(X)$ tends to move away from the desired solution. Sign of M^2 can be a consequence of inflating background.

Linear Perturbations

Much easier to perturb the equations of motion. Parametrize deviations $\phi = \phi_0 + \delta\phi$ and $\mathcal{O} = \langle \bar{\mathcal{O}} \rangle + \delta\mathcal{O}$ and assume decoupling limit.

It is single-clock inflation and the main observable is $\zeta = -H\delta\phi/\dot{\phi}_0$.

Much easier to perturb the equations of motion. Parametrize deviations $\phi = \phi_0 + \delta\phi$ and $\mathcal{O} = \langle \bar{\mathcal{O}} \rangle + \delta\mathcal{O}$ and assume decoupling limit.

It is single-clock inflation and the main observable is $\zeta = -H\delta\phi/\dot{\phi}_0$.

For any operator \mathcal{O} , deviations from $\langle \bar{\mathcal{O}} \rangle$ can be decomposed into intrinsic **noise** and induced **response** fluctuations

$$\delta\mathcal{O} = \delta\mathcal{O}_S + \delta\mathcal{O}_R.$$

Much easier to perturb the equations of motion. Parametrize deviations $\phi = \phi_0 + \delta\phi$ and $\mathcal{O} = \langle \bar{\mathcal{O}} \rangle + \delta\mathcal{O}$ and assume decoupling limit.

It is single-clock inflation and the main observable is $\zeta = -H\delta\phi/\dot{\phi}_0$.

For any operator \mathcal{O} , deviations from $\langle \bar{\mathcal{O}} \rangle$ can be decomposed into intrinsic **noise** and induced **response** fluctuations

$$\delta\mathcal{O} = \delta\mathcal{O}_S + \delta\mathcal{O}_R.$$

By suitable assumptions it is enough to focus on $\mathcal{O} = -i(\chi^2 - \chi^{*2})$ in the equation of motion

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{\vec{\nabla}^2\delta\phi}{a^2} + V''\delta\phi = -\frac{m^2}{f}(\delta\mathcal{O}_S + \delta\mathcal{O}_R),$$

while other operators like $|\chi|^2$, $\chi^2 + \chi^{*2}$ etc. could be neglected.

Response and Locality

At leading order, response is the change in $\langle \mathcal{O} \rangle$ as a result of perturbation $\delta\phi$, i.e. $\delta\mathcal{O}_R = \langle \mathcal{O} \rangle_\phi - \langle \mathcal{O} \rangle_{\phi_0}$.

- **Hierarchy of scales** variation of $\delta\phi$ is much slower/longer than χ , WKB solution can be extended to include $\delta\phi$.
- **Local operator** certain class of operators that $\langle \mathcal{O} \rangle$ is dominated by modes around the instability band.

The response in this case is **local**

$$\delta\mathcal{O}_R \simeq \frac{\partial \langle \mathcal{O} \rangle}{\partial \dot{\phi}_0} \delta\dot{\phi}.$$

Response and Locality

At leading order, response is the change in $\langle \mathcal{O} \rangle$ as a result of perturbation $\delta\phi$, i.e. $\delta\mathcal{O}_R = \langle \mathcal{O} \rangle_\phi - \langle \mathcal{O} \rangle_{\phi_0}$.

- **Hierarchy of scales** variation of $\delta\phi$ is much slower/longer than χ , WKB solution can be extended to include $\delta\phi$.
- **Local operator** certain class of operators that $\langle \mathcal{O} \rangle$ is dominated by modes around the instability band.

The response in this case is **local**

$$\delta\mathcal{O}_R \simeq \frac{\partial \langle \mathcal{O} \rangle}{\partial \dot{\phi}_0} \delta\dot{\phi}.$$

The equation will become

$$\ddot{\delta\phi} + (3H + \gamma)\dot{\delta\phi} - \frac{\vec{\nabla}^2 \delta\phi}{a^2} + V''\delta\phi = -\frac{m^2}{f}\delta\mathcal{O}_S,$$

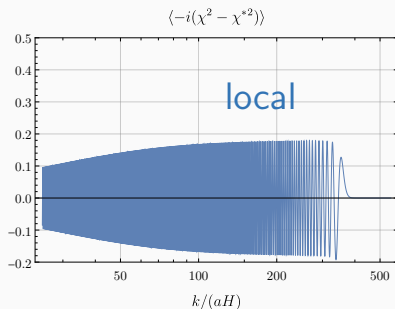
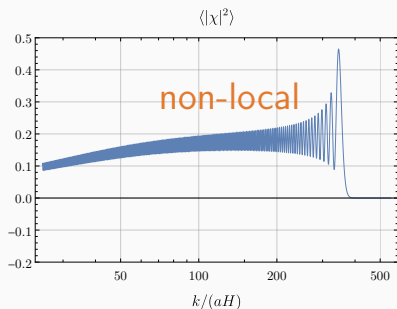
with $\gamma/H \sim \xi^2 e^{2\pi\xi} M^2/f^2 \gg 1$.

Local vs Non-Local

For a generic operator of the form $\mathcal{O} = \frac{1}{a^3} A_{ij} \sigma_i \sigma_j$ we have

$$\langle \mathcal{O} \rangle = \frac{1}{a^3} \int \frac{d^3 \vec{k}}{(2\pi)^3} \text{Tr} \left(A^T F_k F_k^\dagger \right).$$

For a homogeneous perturbation, each mode is mostly sensitive to the value $\dot{\phi}$ at the moment of instability.



We get rid of non-locality for $\xi \gtrsim 1$, fine tuning, etc.

$$\delta\ddot{\phi} + (3H + \gamma)\delta\dot{\phi} - \frac{\vec{\nabla}^2\delta\phi}{a^2} + V''\delta\phi = -\frac{m^2}{f}\delta\mathcal{O}_S$$

Statistics of the Noise

Noise is quantum mechanical fluctuation $\delta\mathcal{O}_S = \mathcal{O} - \langle\mathcal{O}\rangle$. Eventually we are interested in correlation functions

$$\left\langle \delta\mathcal{O}_S(t, \vec{k}) \delta\mathcal{O}_S(t', \vec{k}') \right\rangle' = \int \frac{2 d^3\vec{p}}{(2\pi)^3 a^3 a'^3} \text{Tr} F_q^\dagger(t) A F_p(t) F_p^\dagger(t') A F_q(t'),$$

in which $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $q = |\vec{k} - \vec{p}|$.

The integrand is dominated by the instability band. We are interested in long distance correlations $k \ll p \sim q$. This is delta function in real space.

In addition, the correlation decrease for large temporal separations, $t - t' \gg m^{-1}$, due to oscillations after the instability band.

$$\left\langle \delta\mathcal{O}_S(t, \vec{k}) \delta\mathcal{O}_S(t', \vec{k}') \right\rangle \simeq (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{\delta(t - t')}{a^3} \nu_{\mathcal{O}}$$

with $\nu_{\mathcal{O}} = M e^{4\pi\xi} / 4\pi^2 m$.

$$\ddot{\delta\phi} + (3H + \gamma)\dot{\delta\phi} + \left(\frac{k^2}{a^2} + V''\right)\delta\phi = -\frac{m^2}{f}\delta\mathcal{O}_S.$$

The generic solution is a linear combination of homogeneous and the sourced part. In the limit that $\gamma \gtrsim H$, vacuum fluctuations becomes exponentially suppressed. Therefore, the main source for fluctuations come from the noise

$$\delta\phi(\tau, \vec{k}) = -\frac{m^2}{f} \int d\tau' a'^2 G_k(\tau, \tau') \delta\mathcal{O}_S(\tau', \vec{k}).$$

Eventually power spectrum can be written

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{H^2 m^4}{\rho^2 f^4} \left(\frac{e^{4\pi\xi}}{4\pi^2} \frac{M}{m} \right) \int d\tau' G_k(0, \tau')^2.$$

The amplitude

$$\Delta_s^2 \simeq \frac{1}{32\xi^2} \left(\frac{\gamma}{\pi H} \right)^{3/2} \frac{M H^4}{m^5} \sim 10^{-9}$$

Non-Gaussianity

Genuine test of the model is provided by the non-Gaussian features of perturbations. We need to expand the e.o.m beyond linear order.

Two types of non-Gaussianities:

- Non-Gaussian **statistics** of the noise term $\delta\mathcal{O}_S$. It can be shown
- Non-linear **dynamics** of the system, i.e. quadratic terms in the e.o.m. The relevant contribution is the non-linear response: $\delta\mathcal{O}_R$ up to quadratic order.

Similar to the two point function we get

$$\langle \delta \mathcal{O}_S(1) \delta \mathcal{O}_S(2) \delta \mathcal{O}_S(3) \rangle \simeq (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \delta(\tau_1 - \tau_2) \delta(\tau_1 - \tau_3) H^8 \tau_1^8 \nu_{\mathcal{O}^3},$$

with $\nu_{\mathcal{O}^3} \simeq e^{6\pi\xi} / \pi^2 m^2$. The three point function of the inflaton will be

$$\langle \delta \phi_{\vec{k}_1} \delta \phi_{\vec{k}_2} \delta \phi_{\vec{k}_3} \rangle = - \left(\frac{m^2}{f} \right)^3 \int \left(d\tau_i a_i^2 G_{k_i}(0, \tau_i) \right)^3 \langle \delta \mathcal{O}_S(1) \delta \mathcal{O}_S(2) \delta \mathcal{O}_S(3) \rangle$$

which leads to

$$f_{\text{NL}}^{\text{eq}} = \frac{5}{18} \frac{\int dy y^2 \tilde{G}(0, y)^3}{\left(\int dy \tilde{G}(0, y)^2 \right)^2} \frac{\nu_{\mathcal{O}^3} H^2}{\frac{H}{\rho f} \frac{m^2}{f} \nu_{\mathcal{O}}} \simeq \boxed{\frac{40\pi}{9} \xi \frac{m^2}{M^2}}.$$

Non-linear Response

We expect that local approximation remains valid up to higher orders.

In the Gaussian approximation, two parameters that can change influenced by $\delta\phi$: **mean** and **variance**

$$\delta\mathcal{O}_R \simeq \frac{\partial \langle \mathcal{O} \rangle}{\partial \dot{\phi}_0} \left(\delta\dot{\phi} - \frac{(\partial_i \delta\phi)^2}{2\dot{\phi}_0 a^2} \right) + \frac{1}{2} \frac{\partial^2 \langle \mathcal{O} \rangle}{\partial \dot{\phi}_0^2} \delta\dot{\phi}^2 + \frac{1}{2\nu_{\mathcal{O}}} \frac{\partial \nu_{\mathcal{O}}}{\partial \dot{\phi}_0} \delta\dot{\phi} \delta\mathcal{O}_S + \dots,$$

The first two terms: $\delta \langle \mathcal{O} \rangle (\sqrt{\partial_\mu \phi \partial^\mu \phi})$, the last term is the change in $\langle \delta\mathcal{O}^2 \rangle$.

Non-linear Response

We expect that local approximation remains valid up to higher orders.

In the Gaussian approximation, two parameters that can change influenced by $\delta\phi$: **mean** and **variance**

$$\delta\mathcal{O}_R \simeq \frac{\partial \langle \mathcal{O} \rangle}{\partial \dot{\phi}_0} \left(\delta\dot{\phi} - \frac{(\partial_i \delta\phi)^2}{2\dot{\phi}_0 a^2} \right) + \frac{1}{2} \frac{\partial^2 \langle \mathcal{O} \rangle}{\partial \dot{\phi}_0^2} \delta\dot{\phi}^2 + \frac{1}{2\nu_{\mathcal{O}}} \frac{\partial \nu_{\mathcal{O}}}{\partial \dot{\phi}_0} \delta\dot{\phi} \delta\mathcal{O}_S + \dots,$$

The first two terms: $\delta \langle \mathcal{O} \rangle (\sqrt{\partial_\mu \phi \partial^\mu \phi})$, the last term is the change in $\langle \delta\mathcal{O}^2 \rangle$.

Therefore, one would obtain

$$\begin{aligned} \delta\ddot{\phi} + (3H + \gamma)\delta\dot{\phi} - \frac{\vec{\nabla}^2 \delta\phi}{a^2} + V''\delta\phi = & \frac{\gamma}{2\rho f} \left[\frac{(\vec{\nabla} \delta\phi)^2}{a^2} - 2\pi\xi \delta\dot{\phi}^2 \right] \\ & - \frac{m^2}{f} \left(1 + 2\pi\xi \frac{\delta\dot{\phi}}{\rho f} \right) \delta\mathcal{O}_S. \end{aligned}$$

$$\delta\phi^{\text{NLO}}(\tau, \vec{k}) = - \int d\tilde{\tau} G_k(\tau, \tilde{\tau}) \int \frac{d^3\vec{p}}{(2\pi)^{3/2}} \left[\frac{\gamma}{2\rho f} (\vec{p} \cdot \vec{q} \delta\phi_p \delta\phi_q + 2\pi\xi \delta\phi'_p \delta\phi'_q) \right. \\ \left. + 2\pi\xi \tilde{a}^2 \frac{m^2}{\rho f^2} \delta\phi'_q \delta\mathcal{O}_S(\tilde{\tau}, p) \right],$$

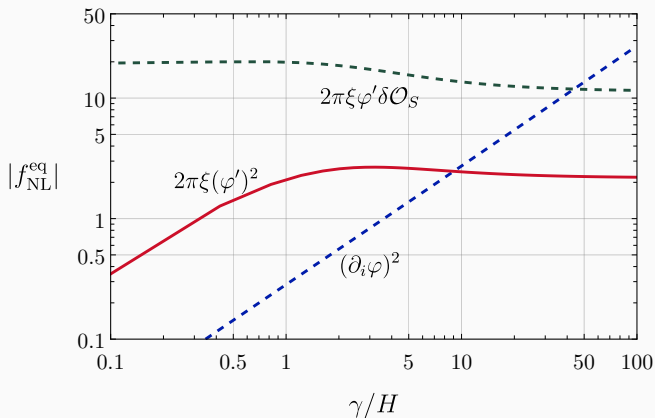
with $\vec{q} = \vec{k} - \vec{p}$ and $\delta\phi$ the is linear order solution, i.e. $\delta\phi \sim \int G \delta\mathcal{O}_S$.

The 3-point function of curvature perturbation is given by

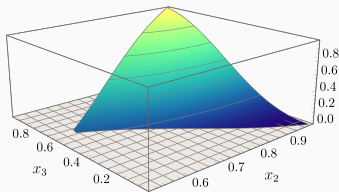
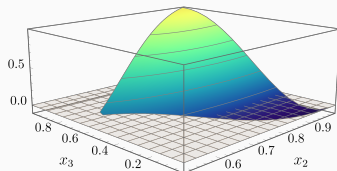
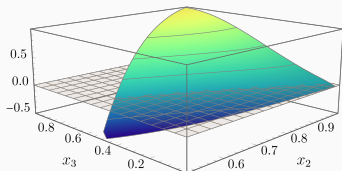
$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle_{\text{NL}} = - \left(\frac{H}{\rho f} \right)^3 \left[\langle \delta\phi_{\vec{k}_1}^{\text{NLO}} \delta\phi_{\vec{k}_2} \delta\phi_{\vec{k}_3} \rangle + \vec{k}_1 \leftrightarrow \vec{k}_2 + \vec{k}_1 \leftrightarrow \vec{k}_3 \right] \\ \equiv (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B(k_1, k_2, k_3).$$

We parametrize the 3-point function with the magnitude at equilateral triangle

$$B(k, k, k) = \frac{1}{k^6} \frac{18}{5} f_{\text{NL}} (2\pi^2 \Delta_s^2)^2.$$



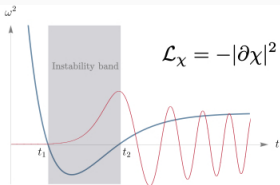
- The coefficient of $(\vec{\nabla}\delta\phi)^2$ is fixed by nonlinear realization of Lorentz symmetry and $f_{\text{NL}}^{\text{eq}} \simeq -\gamma/4H$. Same sign as the reduced speed of sound contribution.
- In the limit of small friction the only remaining term is $\dot{\delta\phi}\delta\mathcal{O}_S$ with $f_{\text{NL}}^{\text{eq}} \simeq -5.7\xi$.



Shapes corresponding to (from left to right) terms $(\nabla\delta\phi)^2$, $\xi\dot{\delta\phi}^2$ and $\xi\dot{\delta\phi}\delta\mathcal{O}_S$.

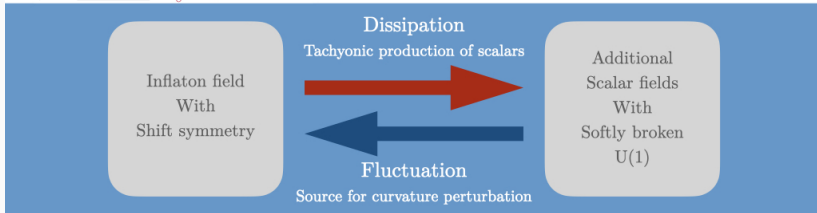
- The peak is at the equilateral configuration.
- Squeezed limit vanishes since the model is single clock.
- Partial enhancement in the collinear configuration.

Summary



$$\mathcal{L}_\chi = -|\partial\chi|^2 + M^2|\chi|^2 - i\frac{\partial_\mu\phi}{f}(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi) - \frac{1}{2}m^2(\chi^2 + \chi^{*2})$$

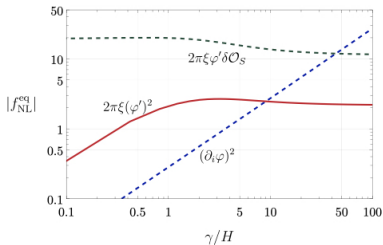
Tachyonic instability is triggered by **coupling to the inflaton** which causes particle production due to **U(1) breaking**.



Curvature perturbation is sourced by the **stochastic fluctuation** of the additional scalars. Power spectrum is the evolution of the noise power

$$\Delta^2 \sim \frac{H^4 M}{m^5}$$

Non-linear evolution of the Gaussian noise is the source of non-Gaussianities. The shape is equilateral with amplitude shown in the figure.



- Gravitational Waves
- Thermalization
- Fermions (AdShead, et. al. 18)

Thank you

