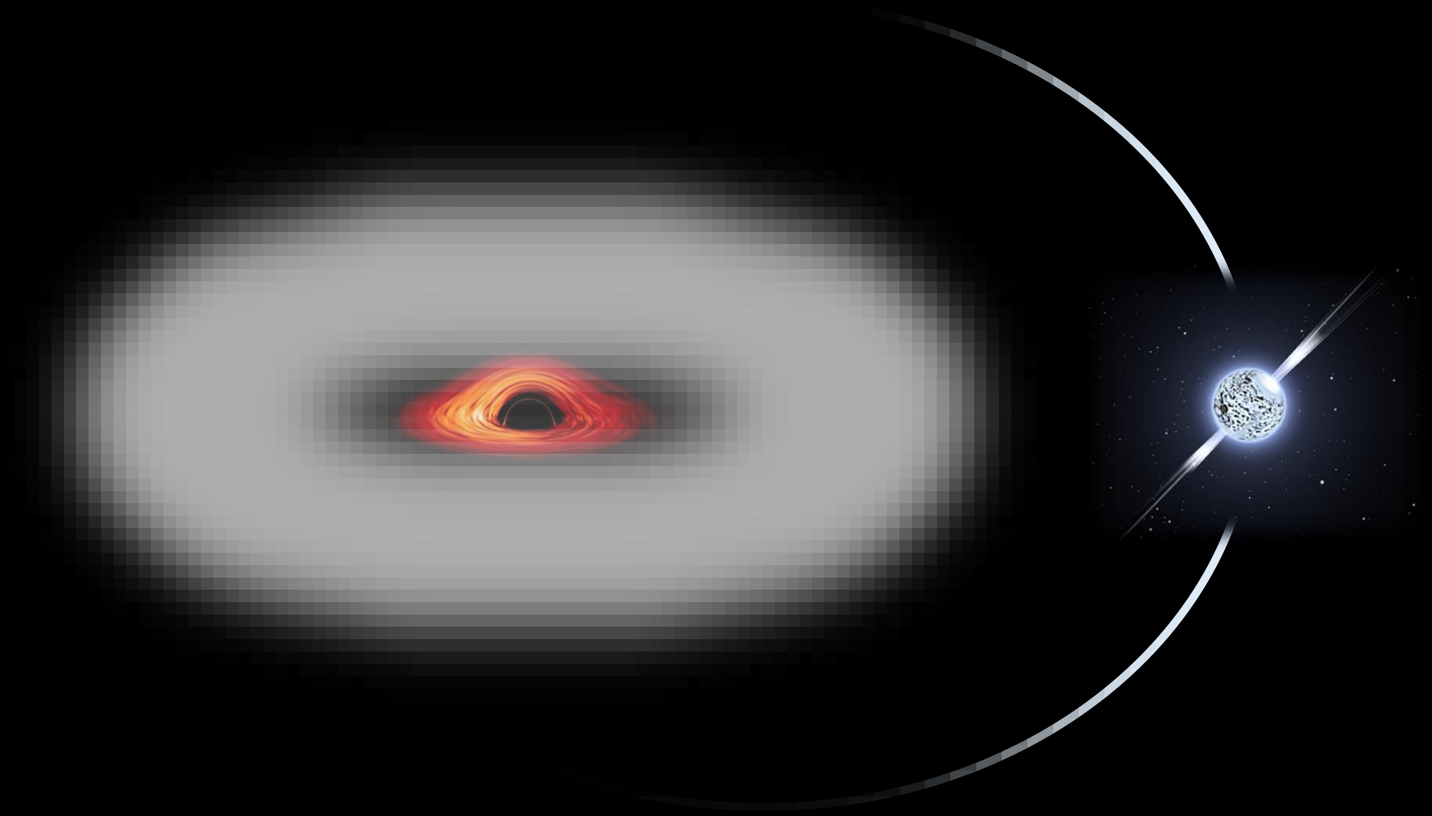
Termination of Black Hole Superradiance from a Binary Companion



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Copernicus Webinar

With
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Kaiyuan Fan

Based on
- 2205.10527
- 2311.17013

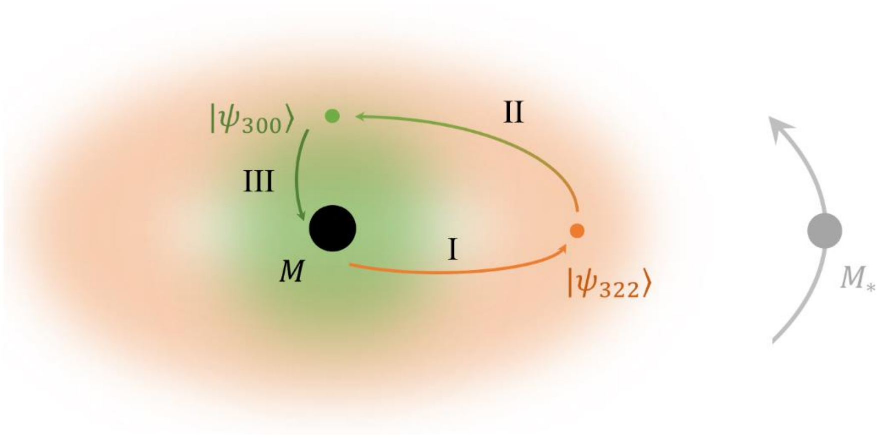
Outline

Black hole superradiance

The gravitational-atom phenomenology

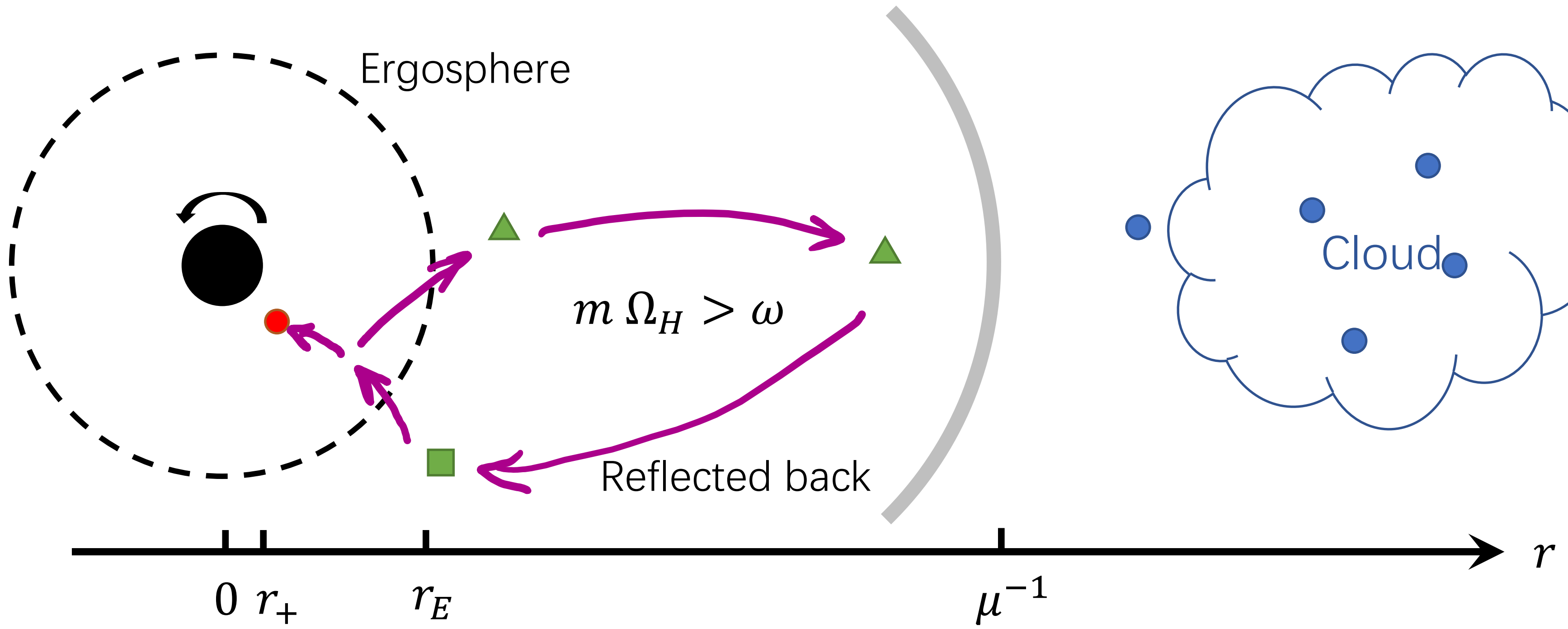
Superradiance termination

Observational consequences



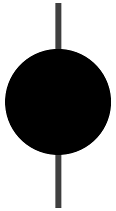
Superradiance

[Press & Teukolsky, 1972]
[Damour et al. 1976]



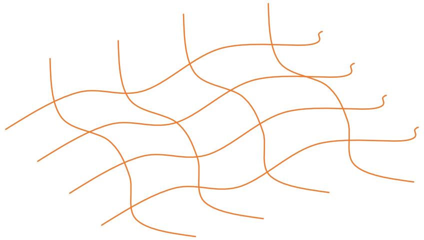
- Superradiant instability ⇨ Kerr BH grows a ultralight boson cloud

BH as a GA



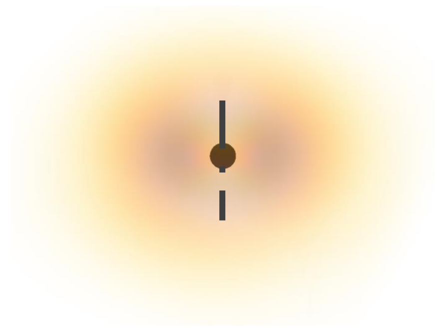
Kerr BH of mass M

+



Bosonic field of mass μ

=



Gravitational Atom

$$\mu^{-1} \sim GM$$

Compton wavelength

Black hole radius

BH as a GA

KG in Kerr:

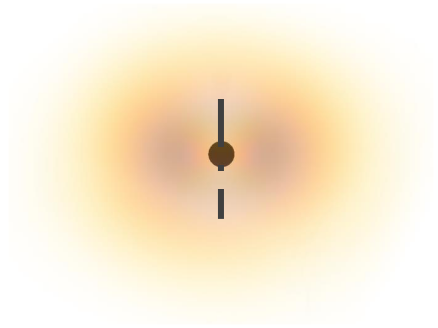
$$\left(g^{\alpha\beta}\nabla_\alpha\nabla_\beta - \mu^2\right)\Phi = 0$$

Gravitational fine structure constant

$$\alpha \equiv GM\mu \sim \frac{R_S}{\lambda_C} < 1$$

Factoring out the rest mass

$$\Phi \equiv \frac{1}{\sqrt{2\mu}} e^{-i\mu t}\psi + \text{c.c.}$$



Gravitational Atom

Hydrogen-like Schrodinger eq

$$i\partial_t\psi(t,\mathbf{r}) = H_0\psi(t,\mathbf{r})\ ,\ \ H_0 \equiv -\frac{1}{2\mu}\partial_{\mathbf{r}}^2 + V(r)$$

with $V(r) = -\frac{\alpha}{r} + \mathcal{O}(\alpha^2)$

[Press & Teukolsky, 1972]
[Damour et al., 1976]
[Detweiler, 1980]
[Baumann et al, 2019, 2020]

GA in a nutshell

[Press & Teukolsky, 1972]
[Damour et al., 1976]
[Detweiler, 1980]
[Baumann et al, 2019, 2020]

- Boson : Bose-Einstein condensate
- With in-going B.C. at the BH outer horizon

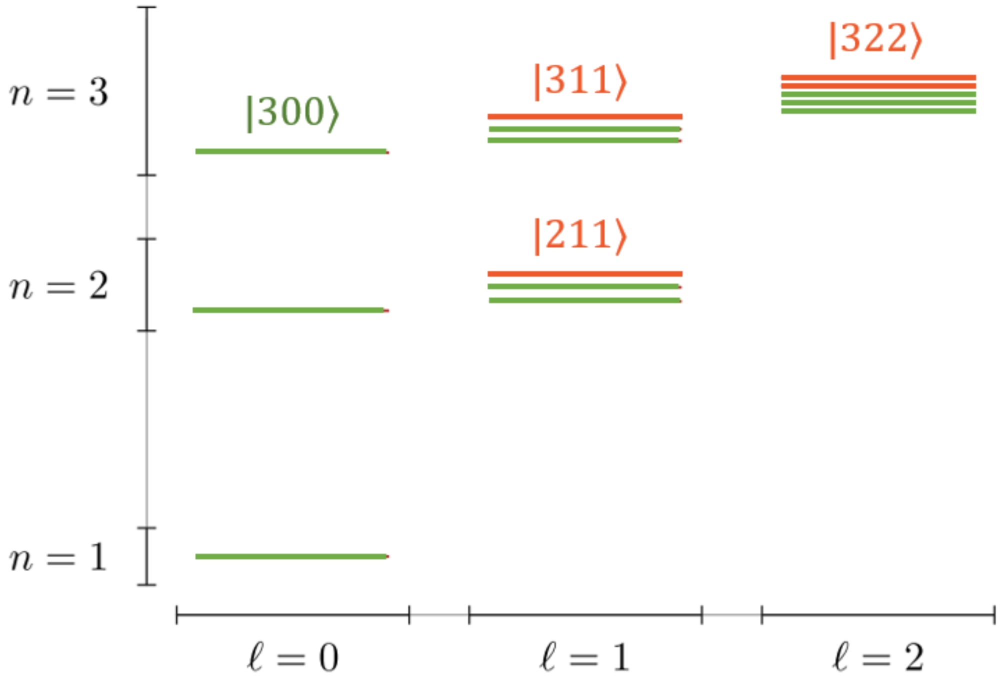
Solutions:

$|\psi_{nlm}\rangle$ with $\omega_{nlm} = E_{nlm} + \boxed{i\Gamma_{nlm}}$ ← By in-going B.C.

$$E_{nlm} = \mu\left(\underbrace{1}_{\text{Rest mass}} \underbrace{- \frac{\alpha^2}{2n^2}}_{\text{Bohr}} \underbrace{+ \alpha^4 A(n,l)}_{\text{Fine}} \underbrace{+ \alpha^5 \tilde{a} m\, B(n,l)}_{\text{Hyperfine}} + \cdots\right)$$

$$\Gamma_{nlm} \propto (m\Omega_H - \mu)\alpha^{4l+5} \begin{cases} > 0 & \text{Superradiance} \\ < 0 & \text{Absorption} \end{cases}$$

$$\psi_{nlm} \sim e^{-i\omega_{nlm}t} \sim e^{\Gamma_{nlm}t}$$



GA in a nutshell

[Press & Teukolsky, 1972]
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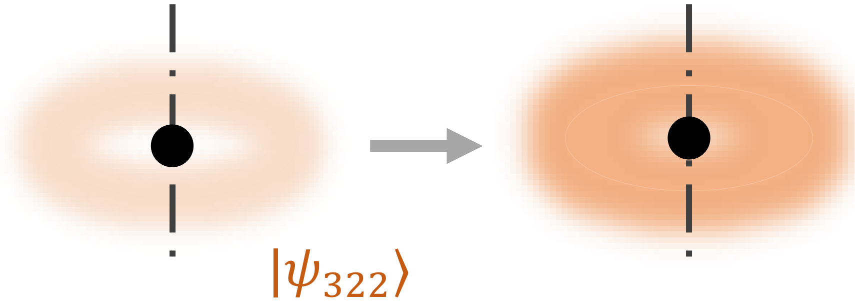
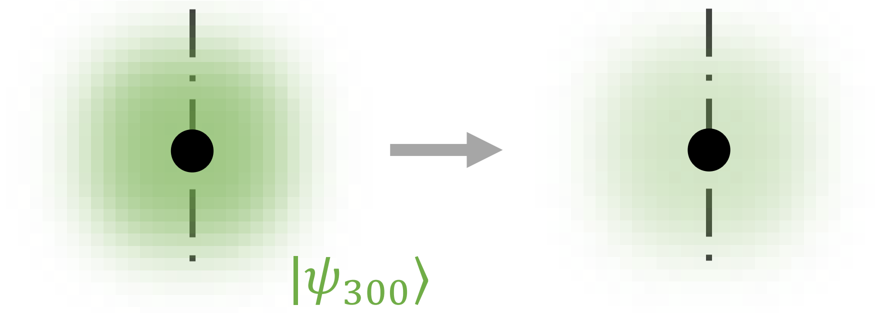
[Baumann et al, 2019, 2020]



$$T^{(grow)}_{nlm} = \Gamma^{-1}_{nlm}$$

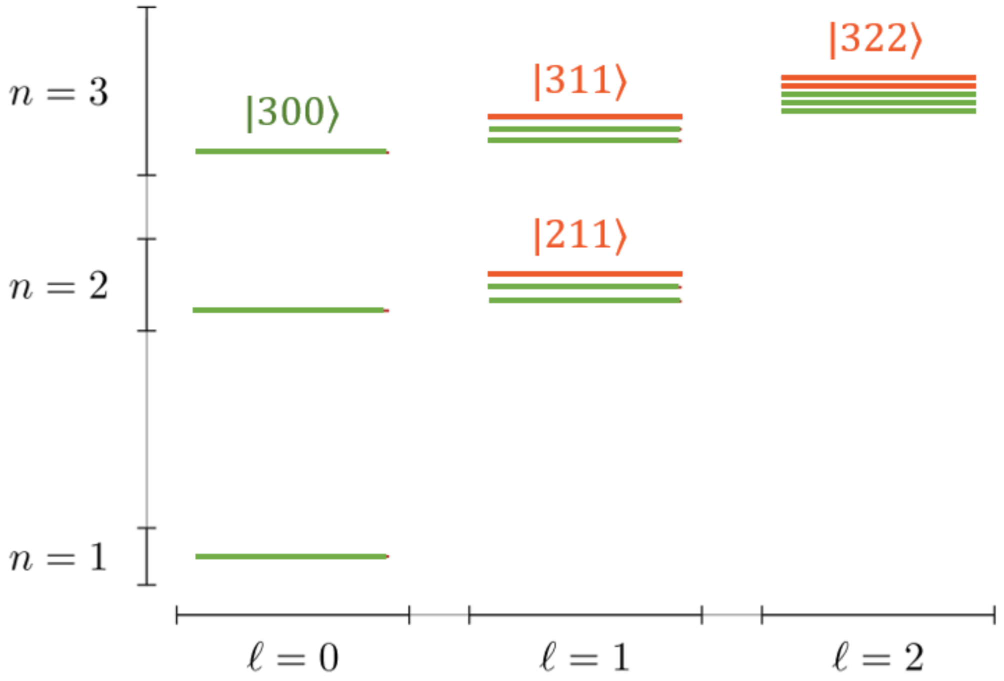
$$\Gamma_{322} \simeq 3 \times 10^{-13}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{13} \left(\frac{M}{10M_\odot}\right)^{-1}$$



$$T^{(absorb)}_{nlm} = \Gamma^{-1}_{nlm}$$

$$\Gamma_{300} \simeq -3 \times 10^{-3}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{5} \left(\frac{M}{10M_\odot}\right)^{-1}$$

time



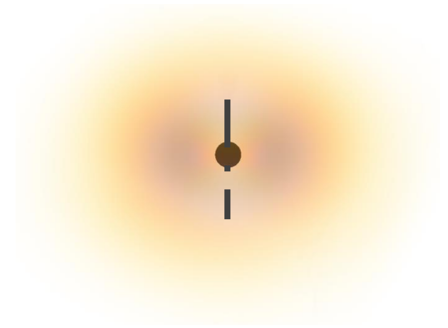
Q: What phenomena does Gravitational Atom have?

For an isolated gravitational atom:

- Cloud extracts the BH spin.
- Boson cloud emits monochromatic GW via pair annihilations.

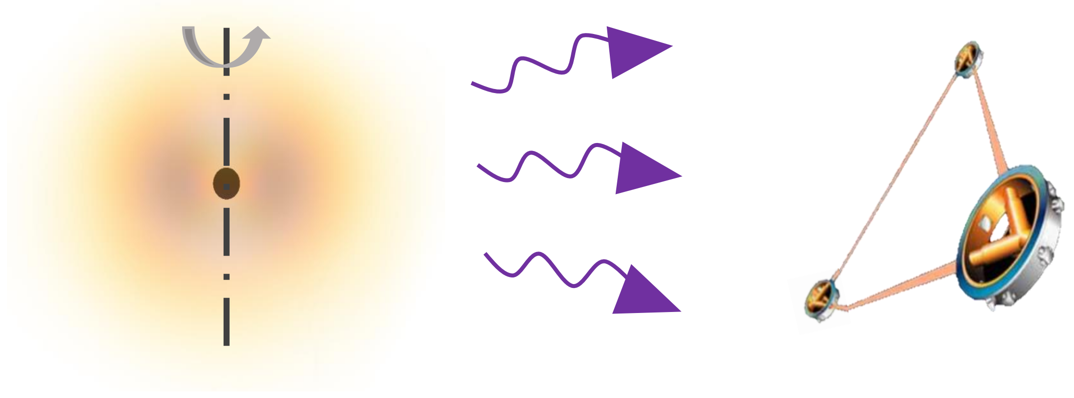
For binary systems:

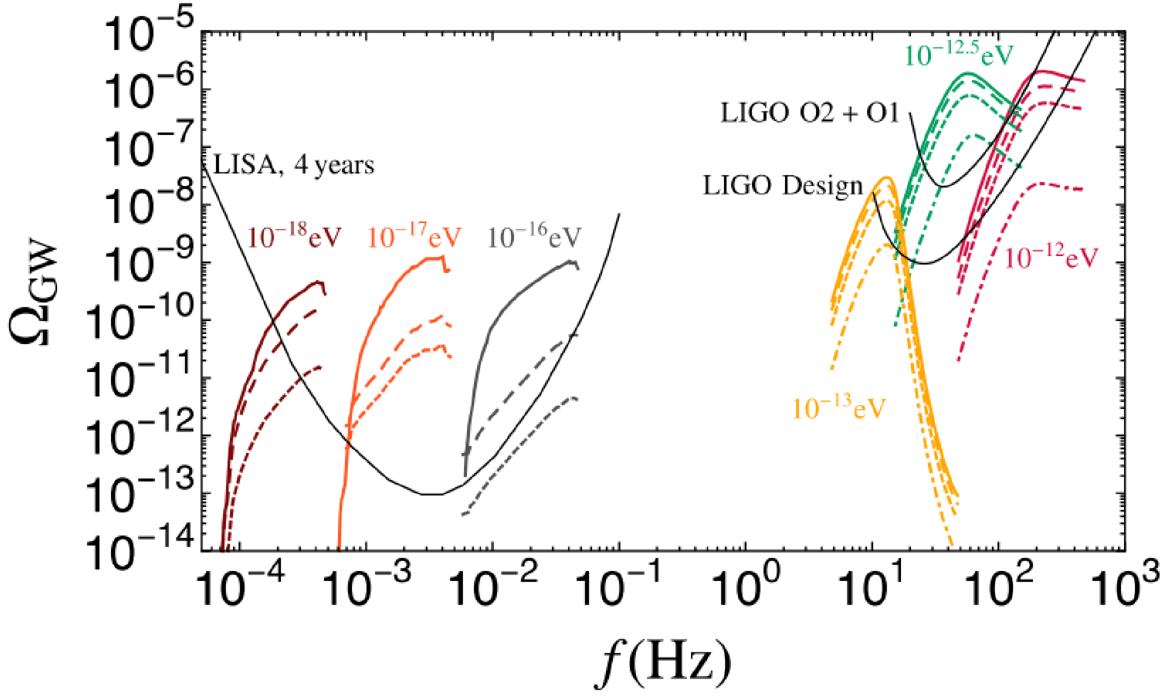
- Resonant transition triggered by orbital motion (GCP resonance transition), which can be detected by GW and Pulsar Timing.



GA phenomenology in isolation

- Near-monochromatic GW



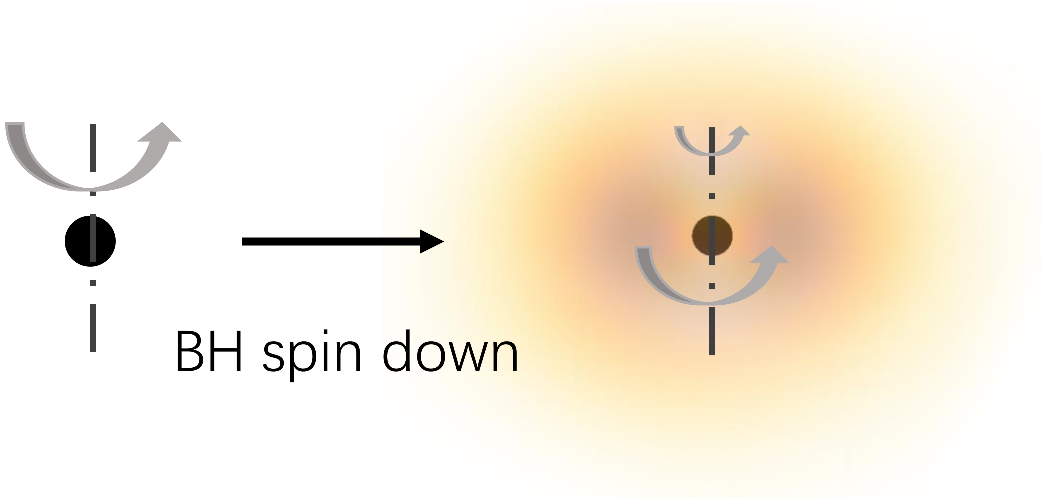


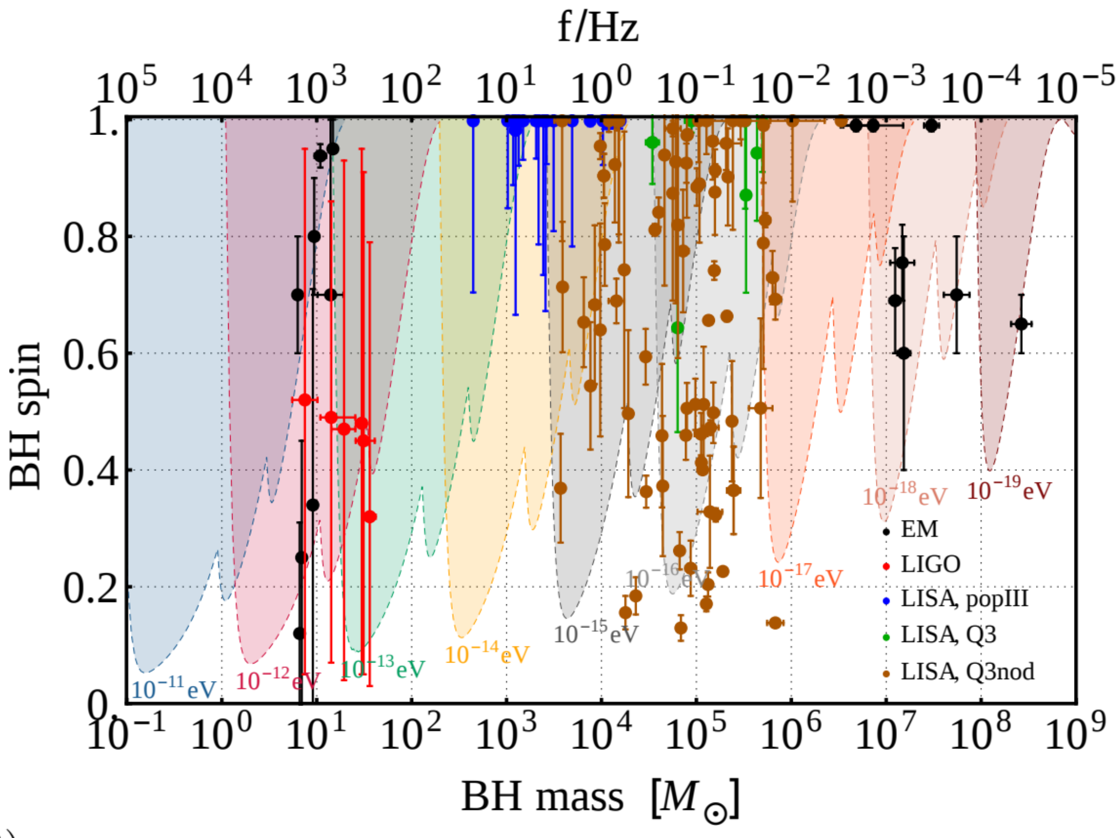
$$f_{\mathrm{GW}} \sim \omega_R/\pi \sim 5\,\mathrm{kHz}\left(\frac{\mu\hbar}{10^{-11}\mathrm{eV}}\right)$$

[Brito et al., 2017]

GA phenomenology in isolation

- Spin cutoff by superradiance





[Brito et al., 2017]

$$\mathrm{m}\Omega_{\mathrm{H}}\downarrow > \omega \sim \mu \Rightarrow \frac{a}{M} = \frac{4m(M\omega)}{m^2 + 4(M\omega)^2} = \frac{4\alpha}{m} + \mathcal{O}(\alpha^3)$$

Spin @ saturation

GA phenomenology in binaries

[Baumann et al, 2019]
[Baumann et al, 2020]
[Baumann et al, 2022]
[Ding, Tong & Wang, 2020]
[Tong, Wang & Zhu, 2021]

- Atomic transitions a.k.a. “Gravitational Collider Physics” (GCP)

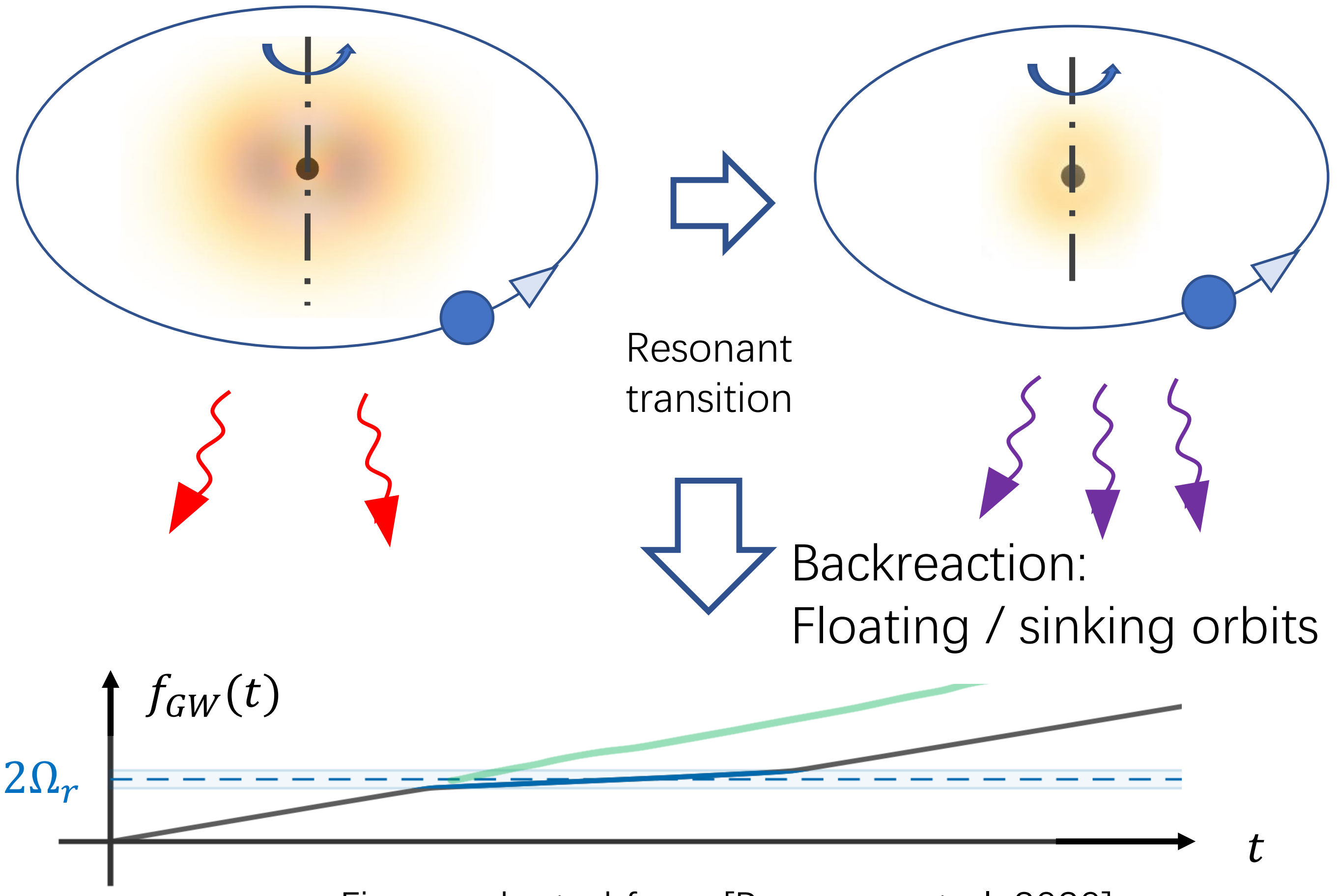
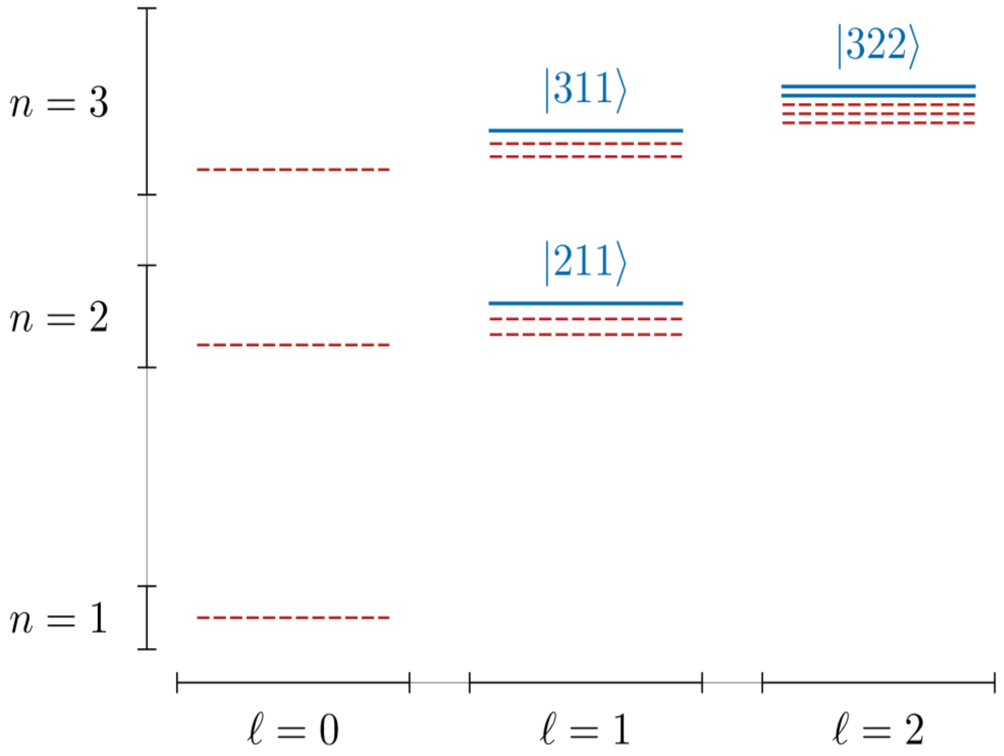


Figure adapted from [Baumann et al, 2020]



A hidden assumption: there ***is*** a boson cloud

Effect of a binary companion: State mixture

GA in isolation

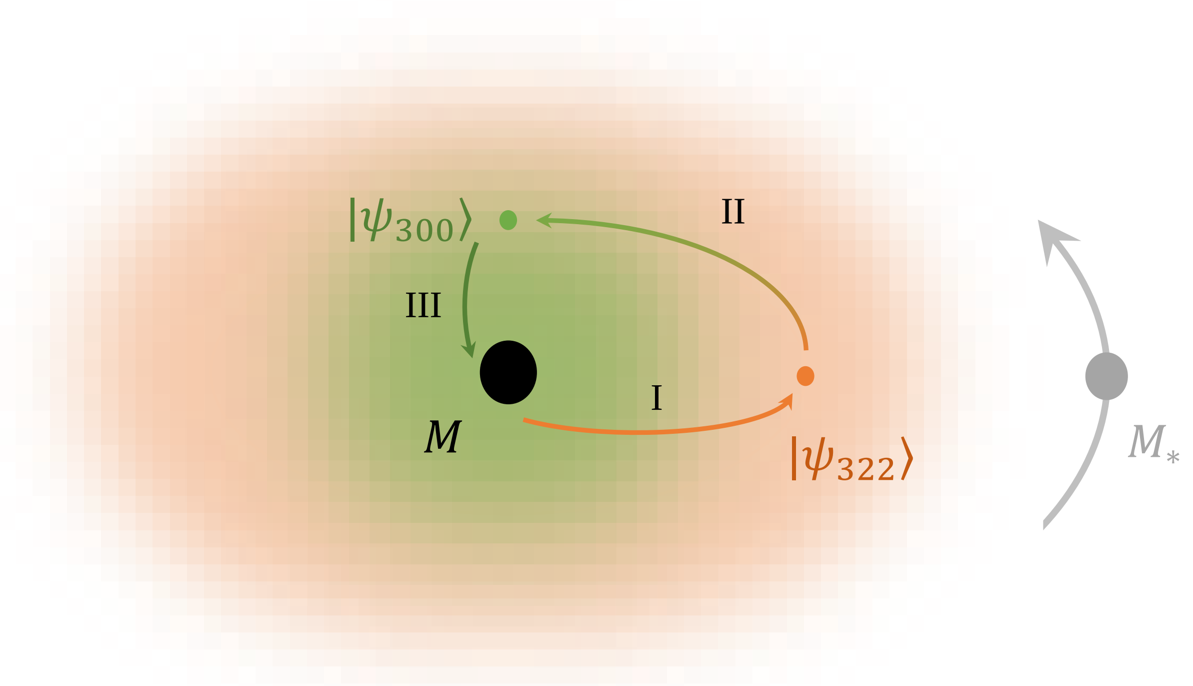
$|\psi_{322}\rangle$

$$\Gamma_{322} \simeq 3 \times 10^{-13}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{13} \left(\frac{M}{10M_\odot}\right)^{-1}$$

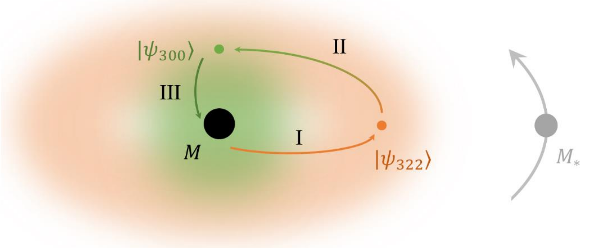
$|\psi_{300}\rangle$

$$\Gamma_{300} \simeq -3 \times 10^{-3}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{5} \left(\frac{M}{10M_\odot}\right)^{-1}$$

GA in a binary



Superradiance Termination (ST)



- GA isolated: $i\partial_t\psi(t,\mathbf{r}) = H_0\psi(t,\mathbf{r})\ ,\ H_0 \equiv -\frac{1}{2\mu}\partial_{\mathbf{r}}^2 + V(r)$
- GA in a binary: $i\partial_t\psi(t,r) = H\psi(t,r)$, with $H = H_0 + V_*(t)$

$$V_* = -\alpha\, q \sum_{l_*\geqslant 2}\sum_{|m_*|\leqslant l_*} \mathcal{E}_{l_* m_*}(\iota_*,\varphi_*)\, Y_{l_* m_*}(\theta,\phi) \times \left(\frac{r^{l_*}}{R_*^{l_*+1}}\,\Theta(R_*-r) + \frac{R_*^{l_*}}{r_*^{l_*+1}}\,\Theta(r-R_*)\right)$$

Tidal perturbation

- E.g., consider a two-state subspace $\{|1\rangle, |2\rangle\}$

$|1\rangle$ is superradiant with $\Gamma_1 > 0$, (e.g., $|322\rangle$)
$|2\rangle$ is absorptive with $\Gamma_2 < 0$, (e.g., $|300\rangle$)

"Free" cloud

$$H = \boxed{\begin{pmatrix} E_1 + i\Gamma_1 & 0 \\ 0 & E_2 + i\Gamma_2 \end{pmatrix}} + \boxed{\begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}}$$
$$= \begin{pmatrix} \bar{E}_1 + i\Gamma_1 & \eta^* \\ \eta & \bar{E}_2 + i\Gamma_2 \end{pmatrix}$$

$$V_{ij} \equiv \langle i|V_*|j\rangle$$
$$\bar{E}_i \equiv E_i + V_{ii}$$
$$\eta \equiv V_{21}$$

Superradiance termination

[Tong, Wang & **Zhu**, 2022]

- Schrodinger eq: $i\partial_t|\psi\rangle = \begin{pmatrix} \bar{E}_1 + i\Gamma_1 & \eta^* \\ \eta & \bar{E}_2 + i\Gamma_2 \end{pmatrix} |\psi\rangle$

- WKB solution $\langle i|\psi\rangle = C_{i+}e^{-i\int \lambda_+ dt} + C_{i-}e^{-i\int \lambda_- dt}\,,\ i = 1, 2$

Eigenfrequencies: $\lambda_\pm \simeq \begin{cases} \bar{E}_1 + \frac{|\eta|^2}{\bar{E}_1 - \bar{E}_2} + i\left[\Gamma_1 \boxed{- \frac{\Gamma_1 - \Gamma_2}{(\bar{E}_1 - \bar{E}_2)^2}|\eta|^2}\right], & + \\ \bar{E}_2 + \frac{|\eta|^2}{\bar{E}_2 - \bar{E}_1} + i\left[\Gamma_2 - \frac{\Gamma_2 - \Gamma_1}{(\bar{E}_1 - \bar{E}_2)^2}|\eta|^2\right], & - \end{cases}$

$\Delta\Gamma_1$: Correction to the superradiance rate

- Significant correction if the binary is close:

At a binary separation $R_* = 10^5 M$

$$\Gamma_{322} \simeq 3 \times 10^{-13}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{13} \left(\frac{M}{10M_\odot}\right)^{-1}$$

$$\Delta\Gamma_{322} \simeq -7 \times 10^3 \frac{q^2}{\alpha^{10}} \frac{M^5}{R_*^6}$$

$$\simeq -0.6 \times 10^{-13}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{-10} \left(\frac{q}{0.2}\right)^2 \left(\frac{M}{10M_\odot}\right)^{-1}$$

20% reduction!

Superradiance termination

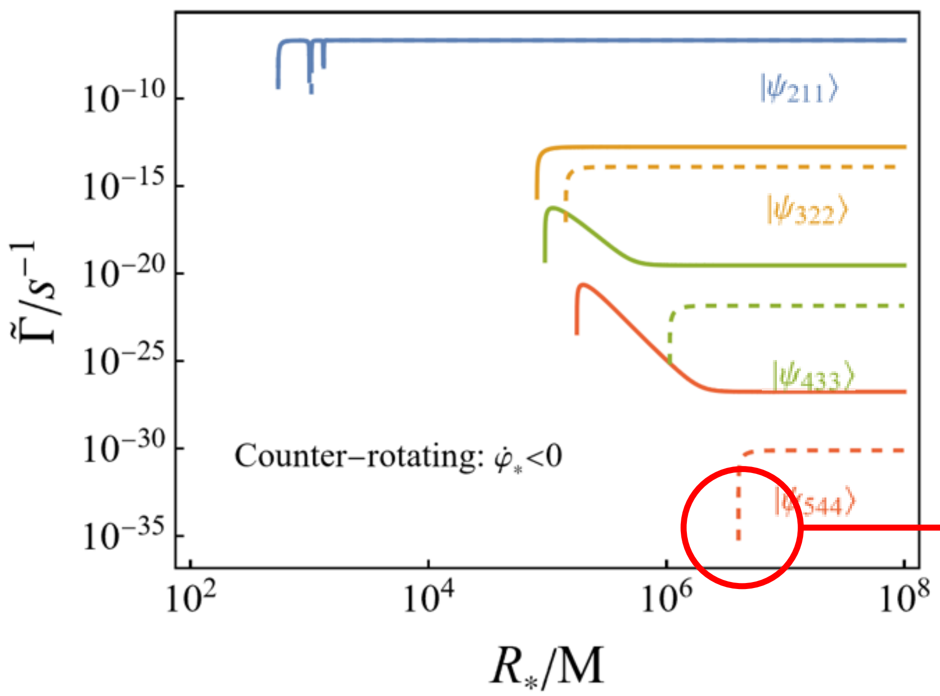
[Tong, Wang & **Zhu**, 2022]

- More generally, considering multiple states and rotation effects

Corrected superradiance rate $\tilde{\Gamma}_1 = \Gamma_1 + \Delta\Gamma_1$

$$\Delta\Gamma_1 \simeq \sum_{i=n'l'm'} \frac{\Gamma_1 - \Gamma_i}{[\bar{E}_1 - \bar{E}_i - (m_1 - m_i)\dot{\phi}_*(R_*)]^2} |\eta_{1i}(R_*)|^2$$

with $\eta_{ij} \equiv V_{ij} = \langle i|V_*|j\rangle$



$\tilde{\Gamma}$ drops to 0 at a finite binary separation, ***terminating*** superradiance

A critical distance

Mass ratio: $q = \frac{M_*}{M}$

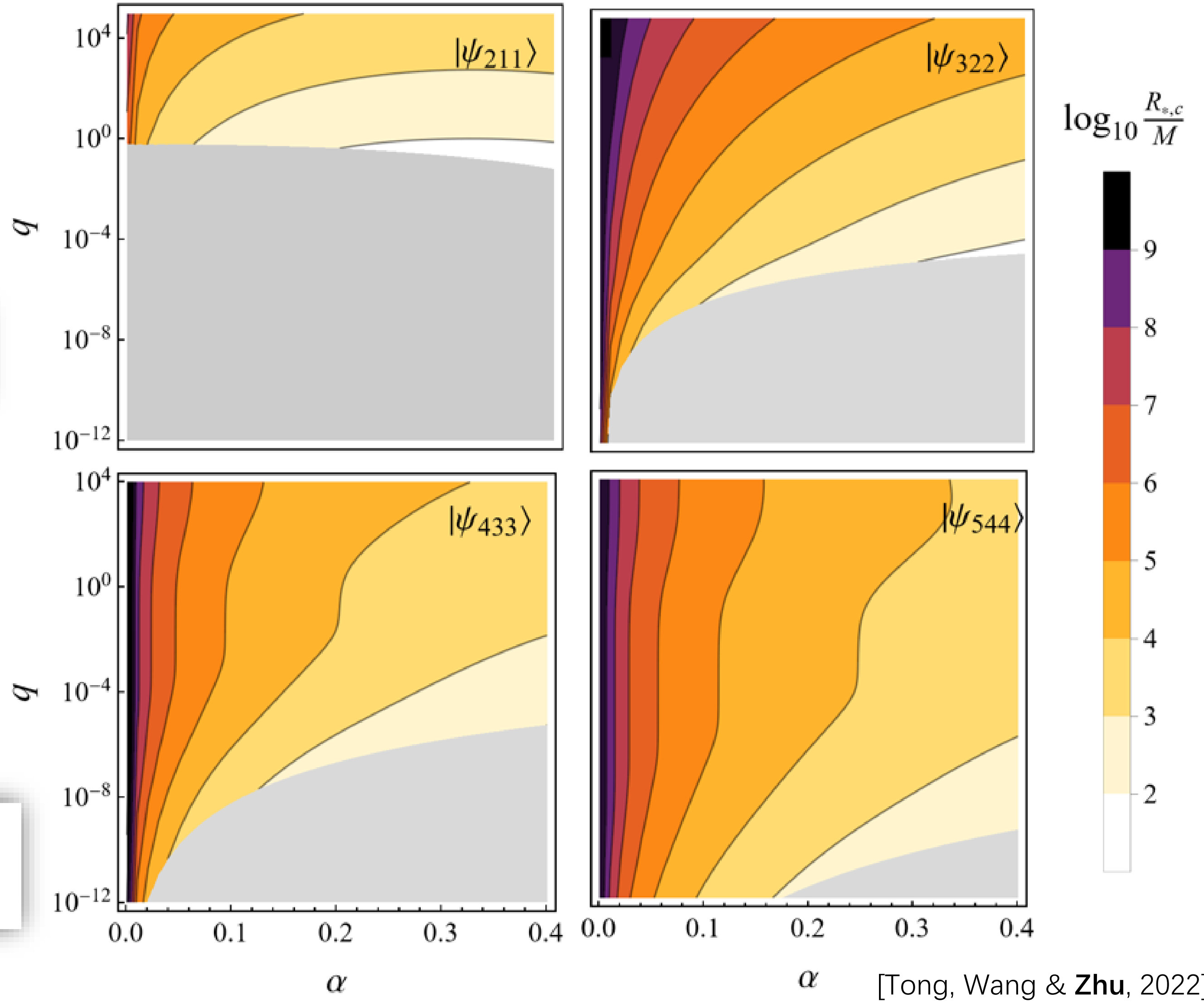
Fine structure const: $\alpha \equiv GM\mu$

- The critical distance $R_{*,c}$ of $|\psi_{nlm}\rangle$ is defined as

$$\tilde{\Gamma}_{nlm}(R_{*,c}) = \Gamma_{nlm} + \Delta\Gamma_{nlm}(R_{*,c}) \equiv 0$$

- $R_{*,c}(nlm)$ is the distance below which ***no superradiance*** can happen

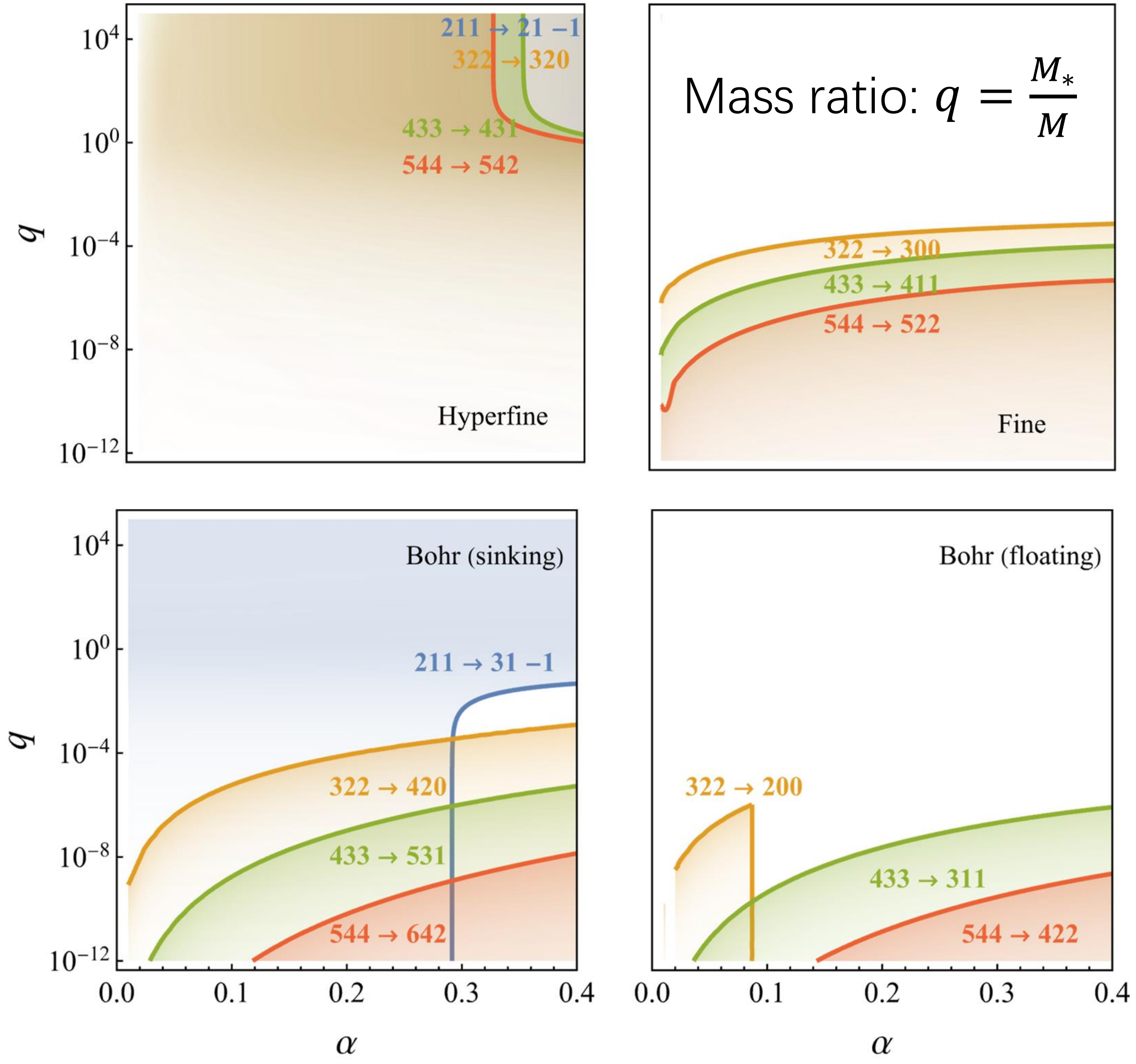
$$R_{*,c}(322) \simeq 10^6 \text{ km} \left(\frac{\alpha}{0.1}\right)^{-23/6} \left(\frac{q}{0.2}\right)^{1/3} \frac{M}{10M_\odot}$$

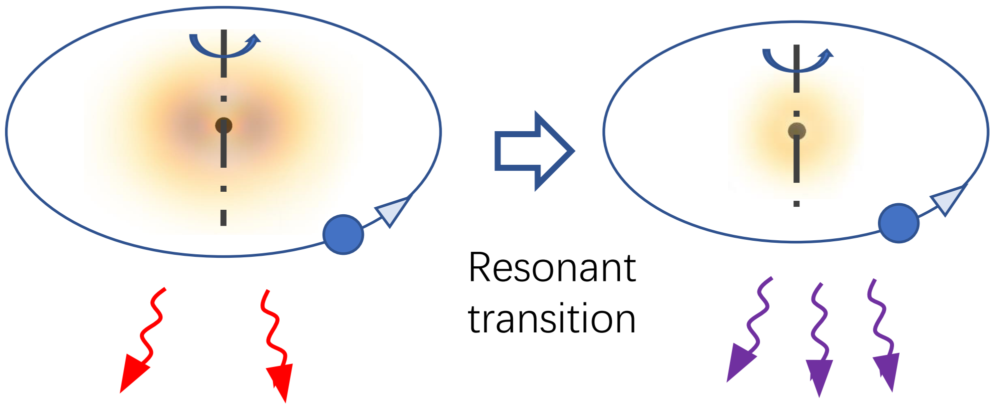


[Tong, Wang & **Zhu**, 2022]

- Superradiance is terminated below a critical binary distance
- What are the phenomenological consequences?

Consequences of ST: Impact on GCP





- Viable GCP transition requires

$$R_{*,r}(nlm \to n'l'm') > R_{*,c}(nlm)$$

[Tong, Wang & **Zhu**, 2022]

ST backreaction: Orbital flow of EMRIs ($q \ll 1$)

[Fan, Tong, Wang & **Zhu**, 2023]

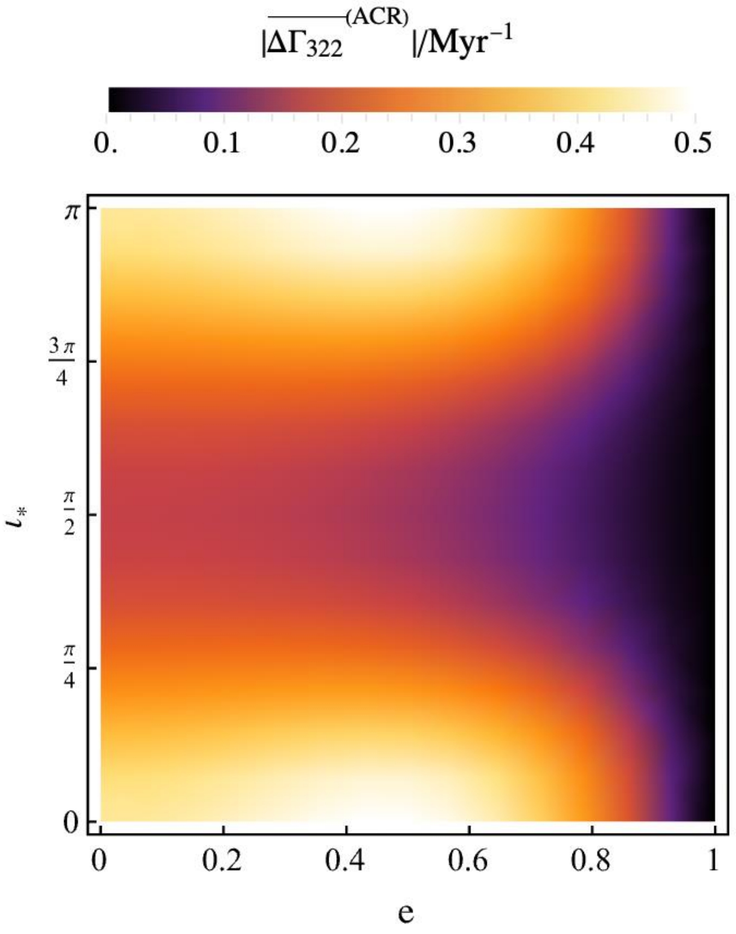
- General binary orbits: $\{p(t), e(t), \iota(t)\} \cup \{S_c(t)\}$

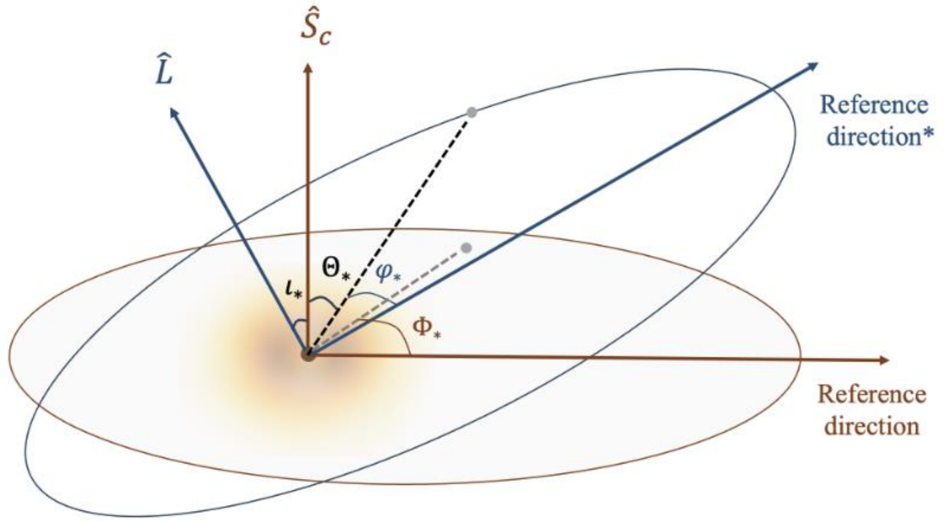
Cloud angular momentum

Inclination angle

Eccentricity

Semi-latus rectum





ST backreaction: Orbital flow of EMRIs ($q \ll 1$)

[Fan, Tong, Wang & **Zhu**, 2023]

- General binary orbits: $\{p(t), e(t), \iota(t)\} \cup \{S_c(t)\}$

Cloud angular momentum

Inclination angle

Eccentricity

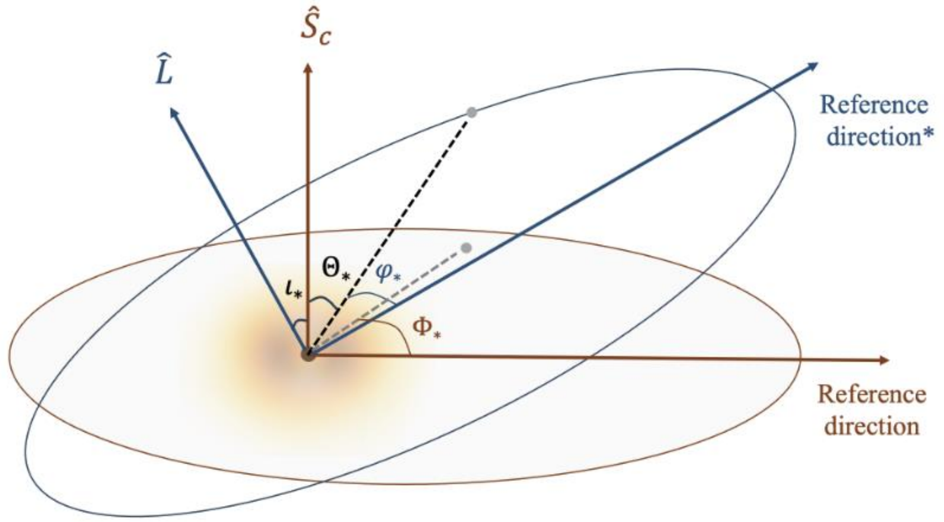
Semi-latus rectum

$$\frac{\mathrm{d}}{\mathrm{d}t}[L(t)\,\cos\iota_*(t)] = \tau_\mathrm{c} + \tau_\mathrm{bGW}\,\cos\iota_*(t)\ ,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[L(t)\,\sin\iota_*(t)] = \tau_\mathrm{bGW}\,\sin\iota_*(t)\ .$$

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = P_\mathrm{c} + P_\mathrm{bGW}\ .$$

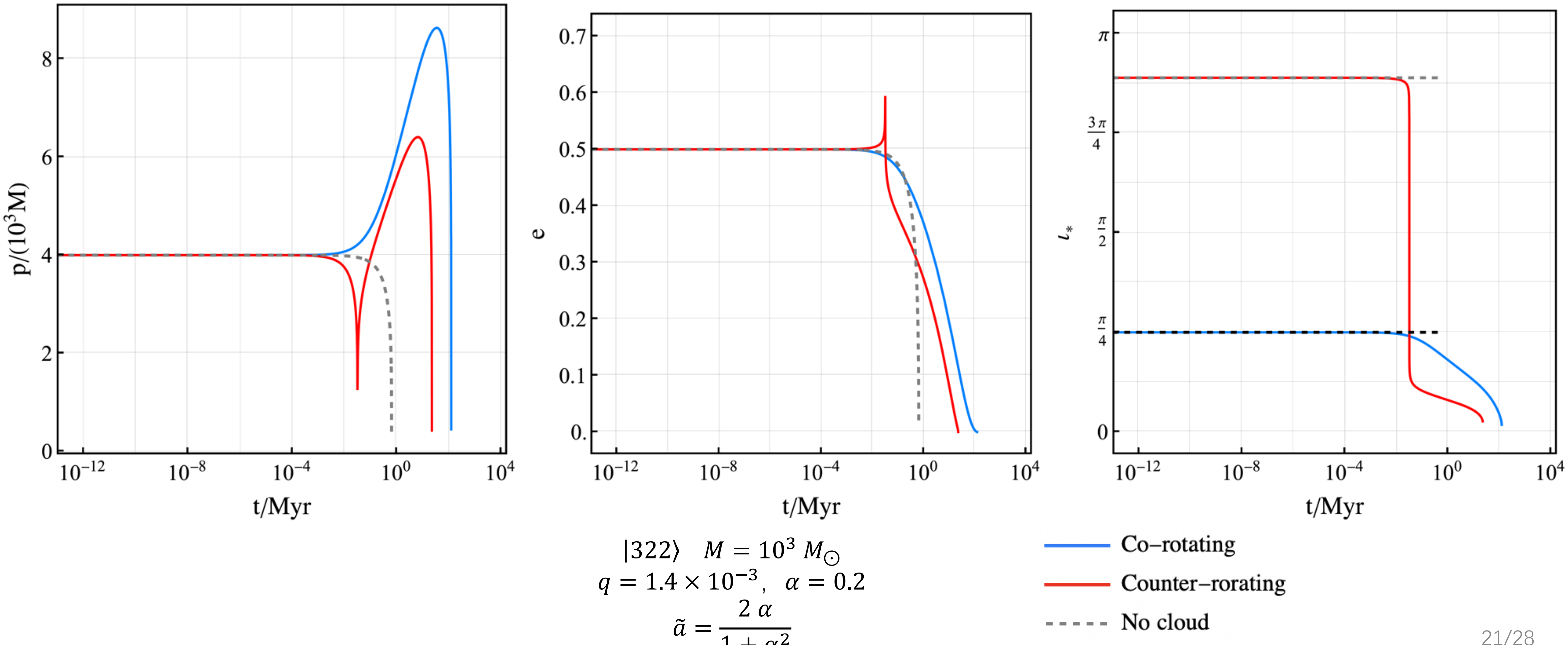
$$\frac{\mathrm{d}S_\mathrm{c}(t)}{\mathrm{d}t} = \left(\frac{\mathrm{d}S_\mathrm{c}(t)}{\mathrm{d}t}\right)_\mathrm{ST} + \left(\frac{\mathrm{d}S_\mathrm{c}(t)}{\mathrm{d}t}\right)_\mathrm{cGW}$$



ST backreaction: Orbital flow of EMRIs

[Fan, Tong, Wang & **Zhu**, 2023]

- Orbital evolution

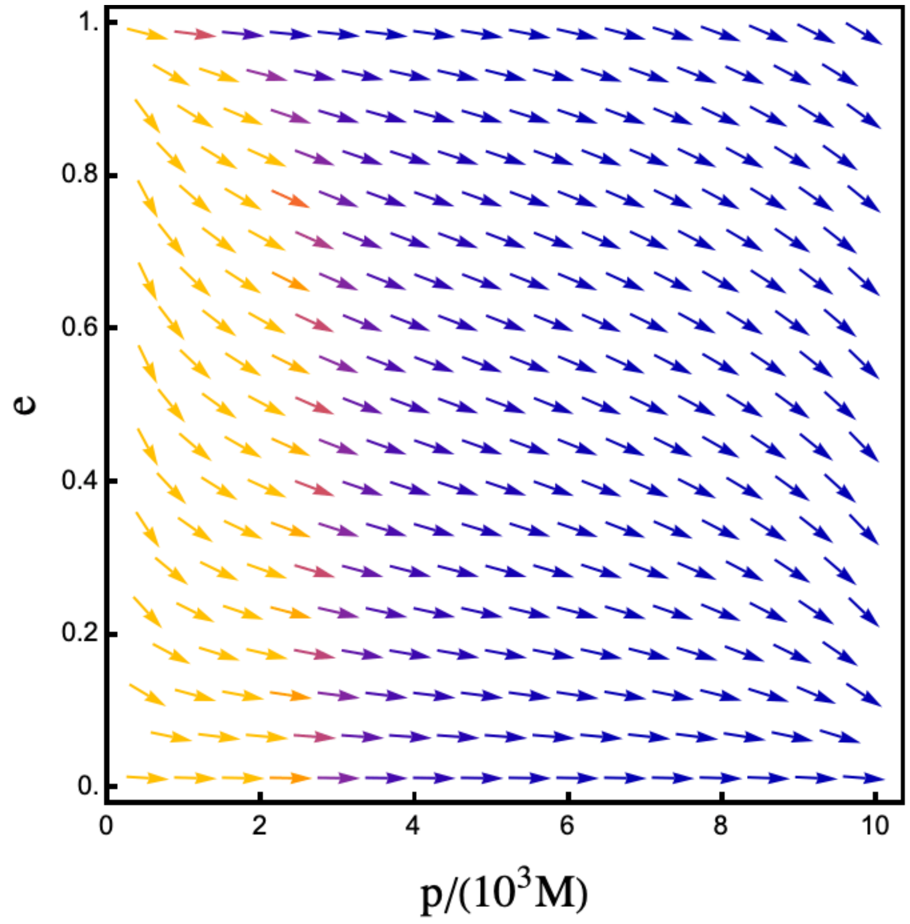


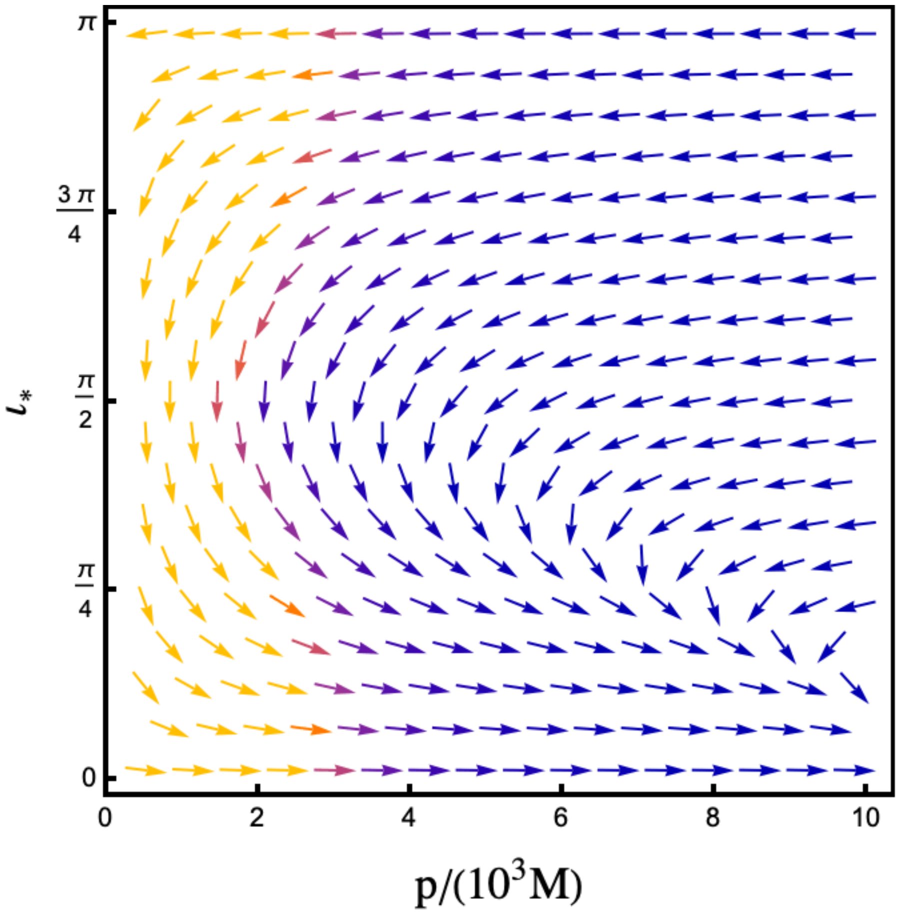
$$|322\rangle \quad M = 10^3\, M_\odot$$
$$q = 1.4\times 10^{-3}, \quad \alpha = 0.2$$
$$\tilde{a} = \frac{2\,\alpha}{1+\alpha^2}$$

ST backreaction: Orbital flow of EMRIs

[Fan, Tong, Wang & **Zhu**, 2023]

- Flow of orbital parameters





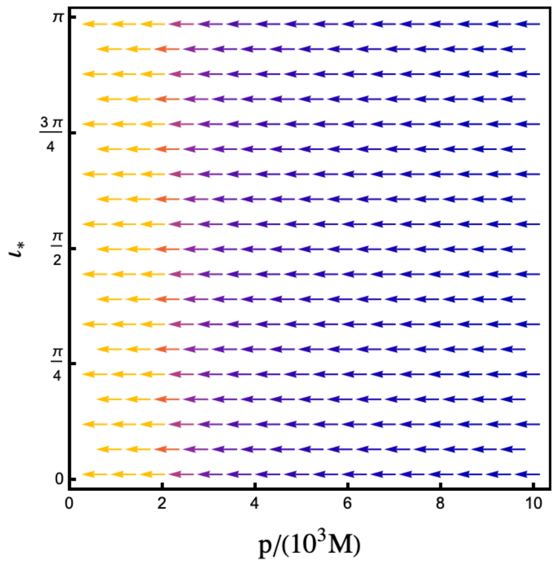
$|322\rangle$

$M = 10^3\, M_\odot$

$q = 1.4 \times 10^{-3}$

$\alpha = 0.2$

$$\tilde{a} = \frac{2\,\alpha}{1+\alpha^2}$$



Consequences of ST: Observing backreaction

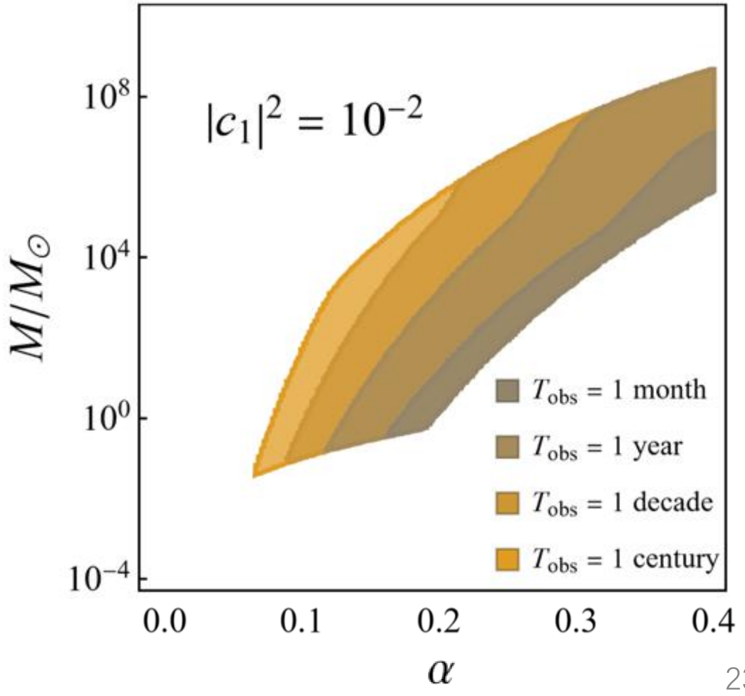
- Mixing of cloud states backreacts on the binary orbit

$$\frac{(\dot{P})_{c}}{(\dot{P})_{\rm GR}} \simeq -15|c_{322}|^2 \left(\frac{\alpha}{0.1}\right)^{-9} \frac{q}{(1+q)^{7/3}} \left(\frac{M}{10M_\odot}\right)^{5/3} \left(\frac{P}{1\ \mathrm{hr}}\right)^{-5/3}$$

Extra period derivative due to the cloud

[Tong, Wang & **Zhu**, 2022]

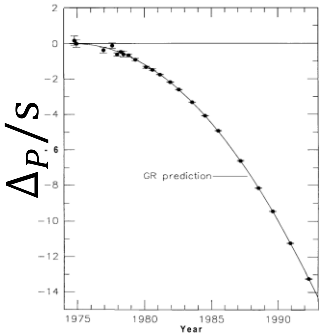
(Pulsar-black hole binary)



- Measuring the orbital derivative à la Hulse & Taylor

$$\Delta_P \equiv t - P(0)\int_0^t \frac{\mathrm{d}t'}{P(t')} \approx \frac{1}{2}\frac{\dot{P}}{P}t^2$$

Russel Alan Hulse　　Joseph Hooton Taylor



[Hulse & Taylor 1975]

Summary and outlook

- ✓ BH superradiance instability
- ✓ GA enjoys a rich phenomenology
- ✓ Yet a binary companion can destabilize the cloud
- ✓ This leads to ST at a critical distance
- ✓ ST poses tight constraints on possible GCP transitions
- ✓ Orbital backreactions observable from pulsar timing

- ☐ Alleviate the boson mass bound (To what extent)?
- ☐ Fully relativistic treatment?
- ☐ Self gravity? Self interaction?

Thank you for listening!

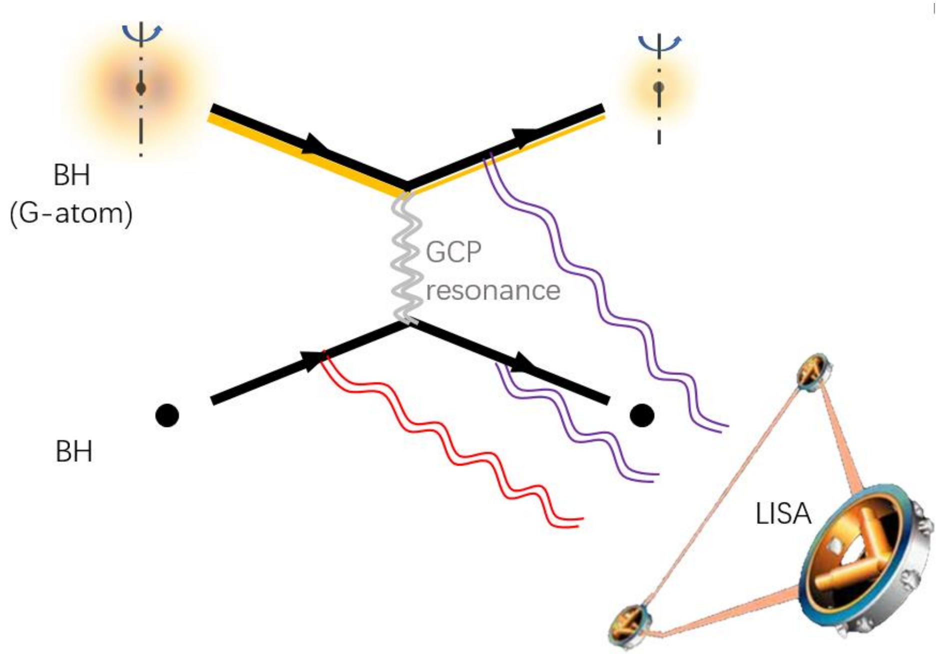
Backup slides

Appendix: Pulsar Timing Accuracy

- Suppose we observe the pulsar for t_{obs} every day, and the pulse period τ.
- We can measure t_{obs}/P periods every day.
- The error for every single continuous measurement is $\tau/[\min(t_{obs}, t)/P]$.
- If we observe for $0 < t \leq T_{obs}$, where T_{obs} is the longest observation time. Then the uncertainty for Periastron time shift is

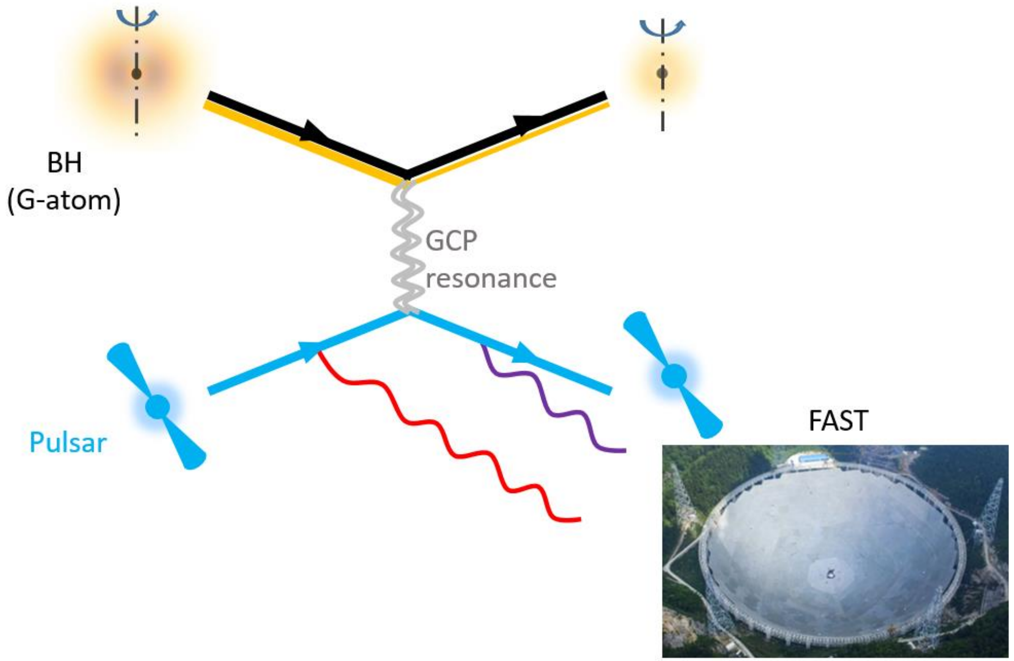
$$\sigma_{\Delta P} = \frac{1}{\sqrt{\left[\frac{t}{1day}\right]}} \frac{\tau}{\min(t_{obs}, t)\,/P}$$

GCP channels



GCP: The BH-BH-GW channel

[Baumann et al,2019,2020]



GCP: The BH-PSR-Radio channel

[Tong et al, 2021]

The GA spectrum

$$E_{nlm} = \mu\left(-\frac{\alpha^2}{2n^2} - \frac{\alpha^4}{8n^4} - \frac{(3n-2l-1)\alpha^4}{n^4(l+1/2)}\right) + \frac{2\tilde{a}m\alpha^5}{n^3 l(l+1/2)(l+1)} + \mathcal{O}(\alpha^6).$$

$$\Gamma_{n00} = -\frac{4}{n^3}\left(1+\sqrt{1-\tilde{a}^2}\right)\mu\alpha^5,$$

$$\Gamma_{nlm} = 2\tilde{r}_+ C_{nl} g_{lm}(\tilde{a},\alpha,\omega)(m\Omega_H - \omega_{nlm})\alpha^{4l+5}.$$

$$C_{nl} \equiv \frac{2^{4l+1}(n+l)!}{n^{2l+4}(n-l-1)!}\left[\frac{l!}{(2l)!(2l+1!)}\right]^2,$$

$$g_{lm}(\tilde{a},\alpha,\omega) \equiv \prod_{k=1}^{l}\left(k^2(1-\tilde{a}^2) + (\tilde{a}m - 2\tilde{r}_+ M\omega)^2\right).$$

Going to the co-rotating frame

$$H = \begin{pmatrix} \omega_1 + V_{11} & V_{12} \\ V_{21} & \omega_2 + V_{22} \end{pmatrix} \equiv \begin{pmatrix} \bar{E}_1 + i\Gamma_1 & \eta^* \\ \eta & \bar{E}_2 + i\Gamma_2 \end{pmatrix},$$

$$\Big\downarrow$$

$$H_D = U(t)^\dagger (H(t) - i\partial_t) U(t),$$

$$\text{with} \quad U(t) \equiv e^{-i\varphi_*(t) L_z},$$

$$H_D = \begin{pmatrix} \bar{E}_1 + i\Gamma_1 - m_1\dot{\varphi}_* & |\eta| \\ |\eta| & \bar{E}_2 + i\Gamma_2 - m_2\dot{\varphi}_* \end{pmatrix}.$$

"Wick" rotation

$$\frac{1}{[\bar{E}_1 - \bar{E}_i - (m_1 - m_i)\dot{\varphi}_*(R_*)]^2} \to \frac{1}{(\bar{E}_1 - \bar{E}_i)^2 + [(m_1 - m_i)\dot{\varphi}_*(R_*)]^2}.$$