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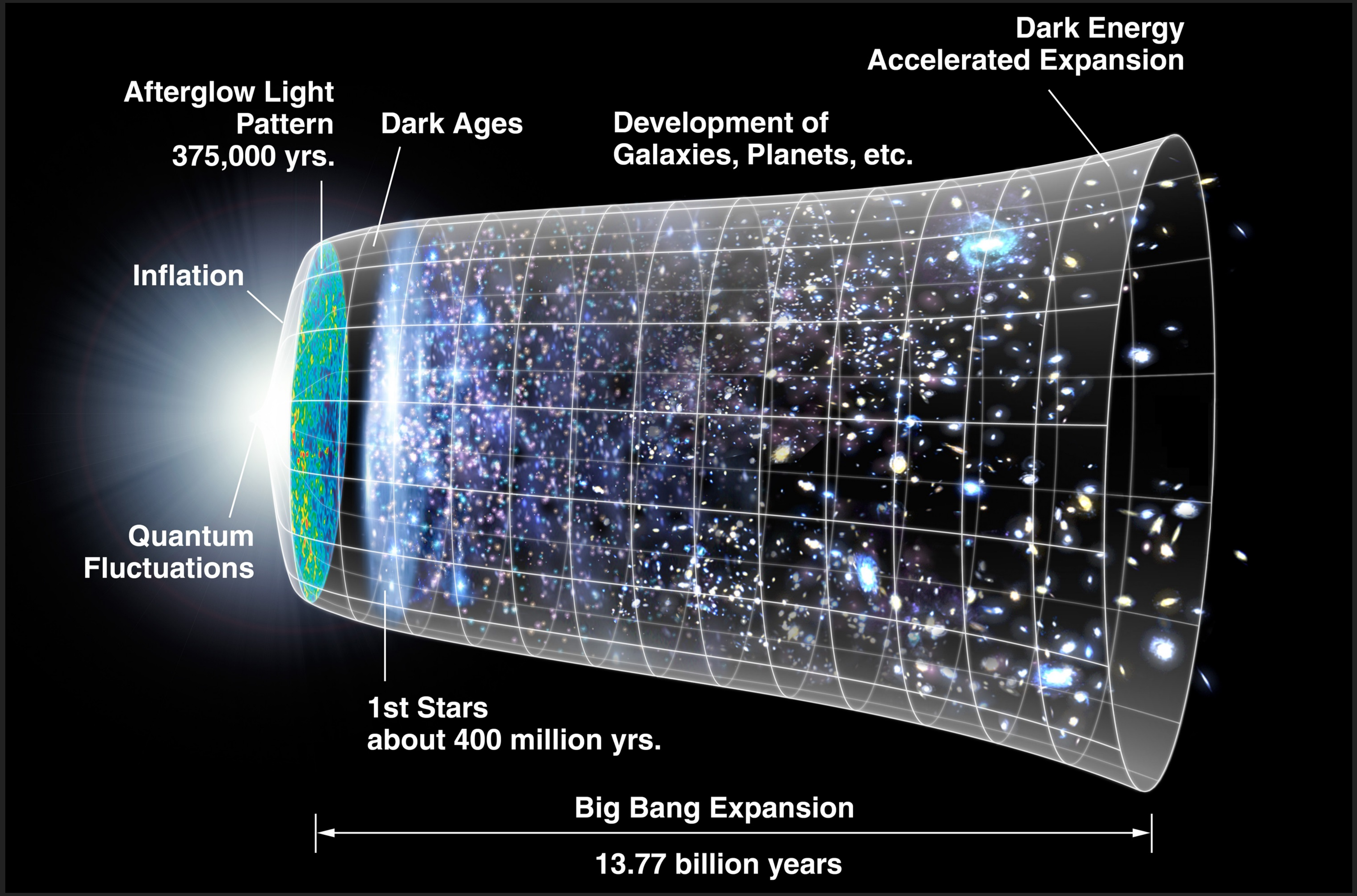
BACK-REACTION IN THE EARLY UNIVERSE

COPERNICUS WEBINAR, 23 APRIL 2024

COSMOLOGICAL INFLATION

- Period of accelerated expansion in the early universe

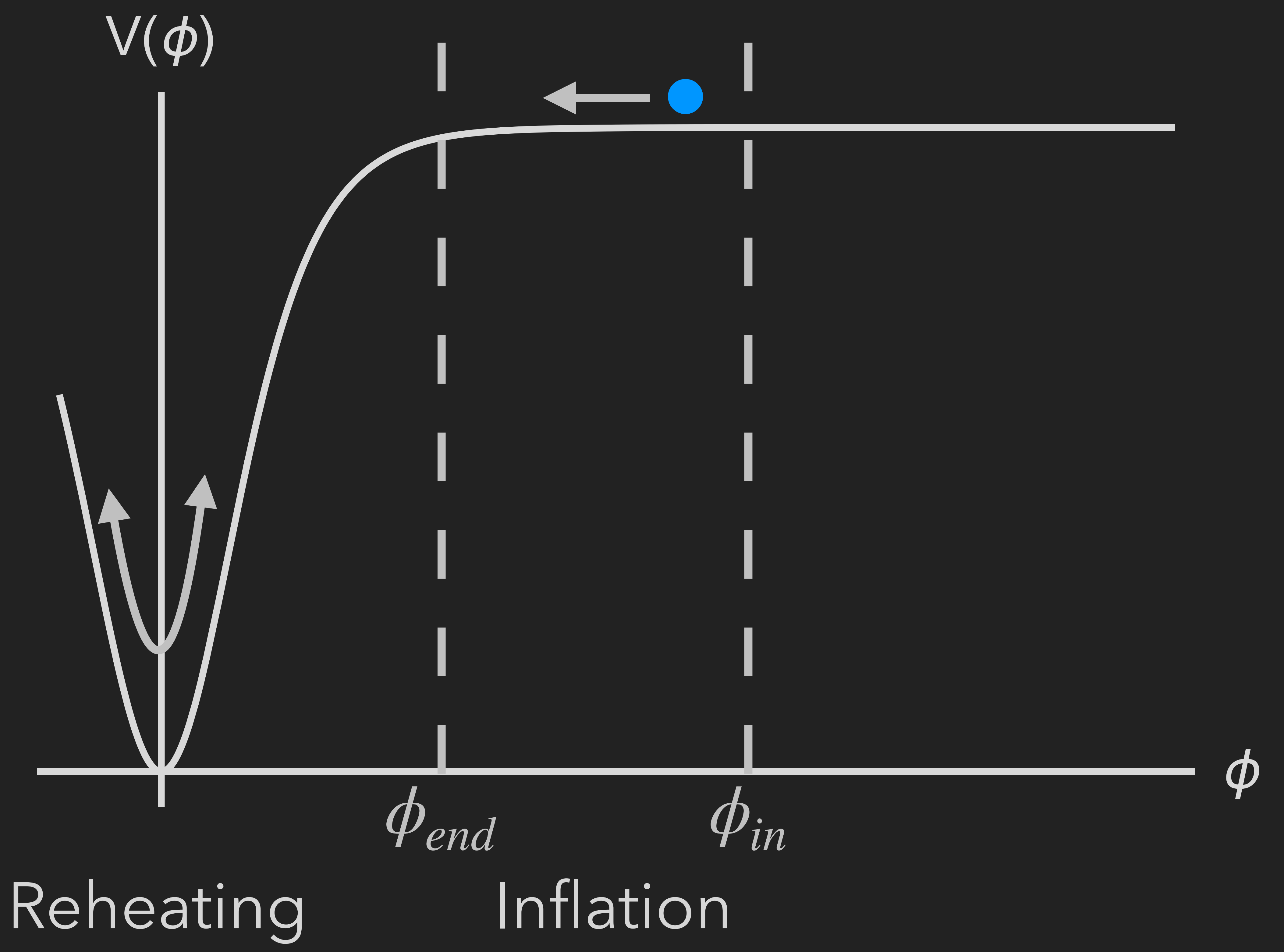


{see Guth; Linde; Starobinsky; Albrecht & Steinhardt}

SINGLE-FIELD SLOW-ROLL INFLATION

- $\mathcal{S}[\phi] = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{M_p^2}{2} \mathcal{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$
- $V(\phi) \sim$ flat, ϕ moving slowly

  $H^2 = \frac{1}{3M_p^2} \left(V(\phi) + \frac{1}{2} \dot{\phi}^2 \right) \simeq V_0$
- Acceleration: $H \equiv \frac{\dot{a}}{a} \simeq \mathrm{const}, a \simeq \exp \int \mathrm{d}t\, H(t) \equiv \exp N$
- Slow-roll parameters: $\epsilon_H = -\frac{\dot{H}}{H^2} \ll 1, \eta_H = \frac{\dot{\epsilon}}{H\epsilon} \ll 1$



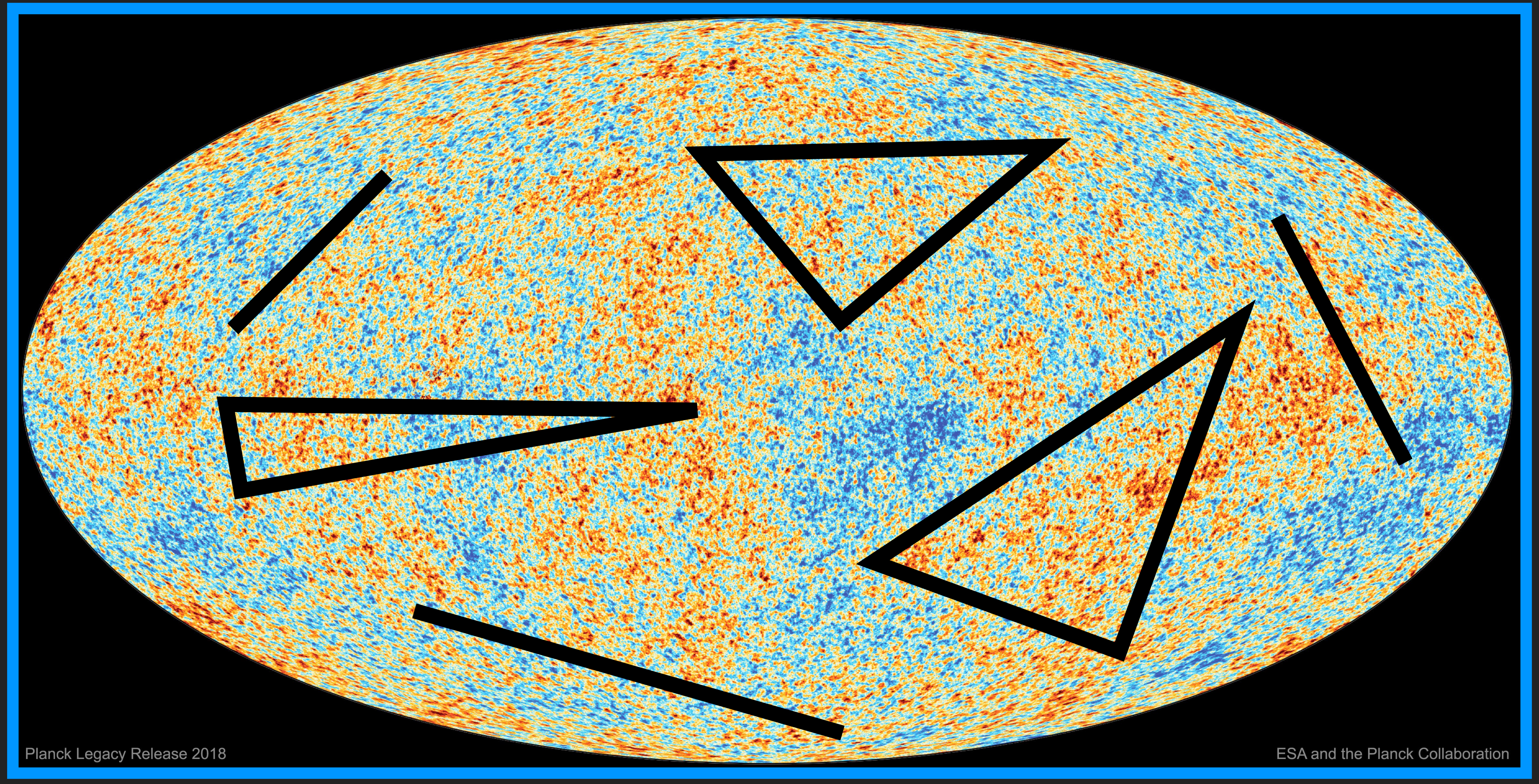
PERTURBATIONS DURING SINGLE-FIELD INFLATION (I)

- $\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2\,\delta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j \quad + \quad \phi = \bar{\phi}(t)$
- Perturb the metric and the inflaton, apply the constraints and choose a gauge
- Comoving gauge: $\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2\left[(1-2\zeta)\,\delta_{ij} + \gamma_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j$
- ζ: curvature perturbation, γ_{ij}: TT perturbation

{Mukhanov, Sasaki}

STATISTICS OF THE CURVATURE PERTURBATION

- 2- and 3-point statistics of the curvature perturbation
- $\langle \zeta(\mathbf{x}_1)\zeta(\mathbf{x}_2)\rangle \longrightarrow \langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)\frac{2\pi^2}{k^3}\mathscr{P}_\zeta(k)$
- $\langle \zeta(\mathbf{x}_1)\zeta(\mathbf{x}_2)\zeta(\mathbf{x}_3)\rangle \longrightarrow \langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\, B_\zeta(k_1,k_2,k_3) \Longrightarrow f_{NL}(k_1,k_2,k_3)$

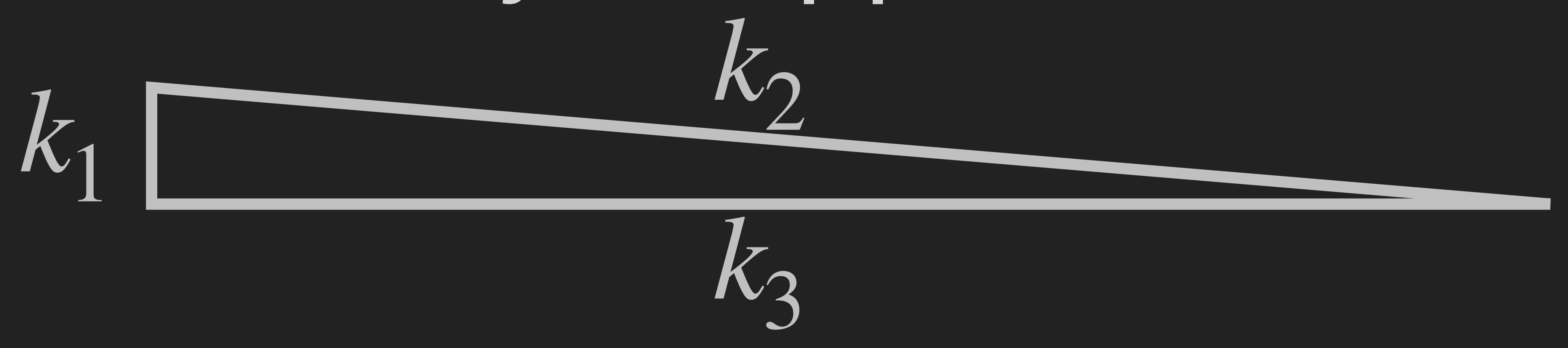


{1807.6211 - *Planck* collaboration}

PREDICTIONS FROM SFSR INFLATION

- $\mathscr{P}_\zeta(k) = \left.\frac{H^2}{8\pi^2\epsilon_H}\right|_{k=aH}$ and $n_s - 1 \equiv \frac{\mathrm{d}\log\mathscr{P}_\zeta(k)}{\mathrm{d}\log k} = -4\epsilon_H + 2\eta_H \ll 1$, almost scale-invariant power spectrum

- Squeezed non-Gaussianity is suppressed for almost-scale invariant power spectrum



{Sasaki, Mukhanov, Maldacena, …}

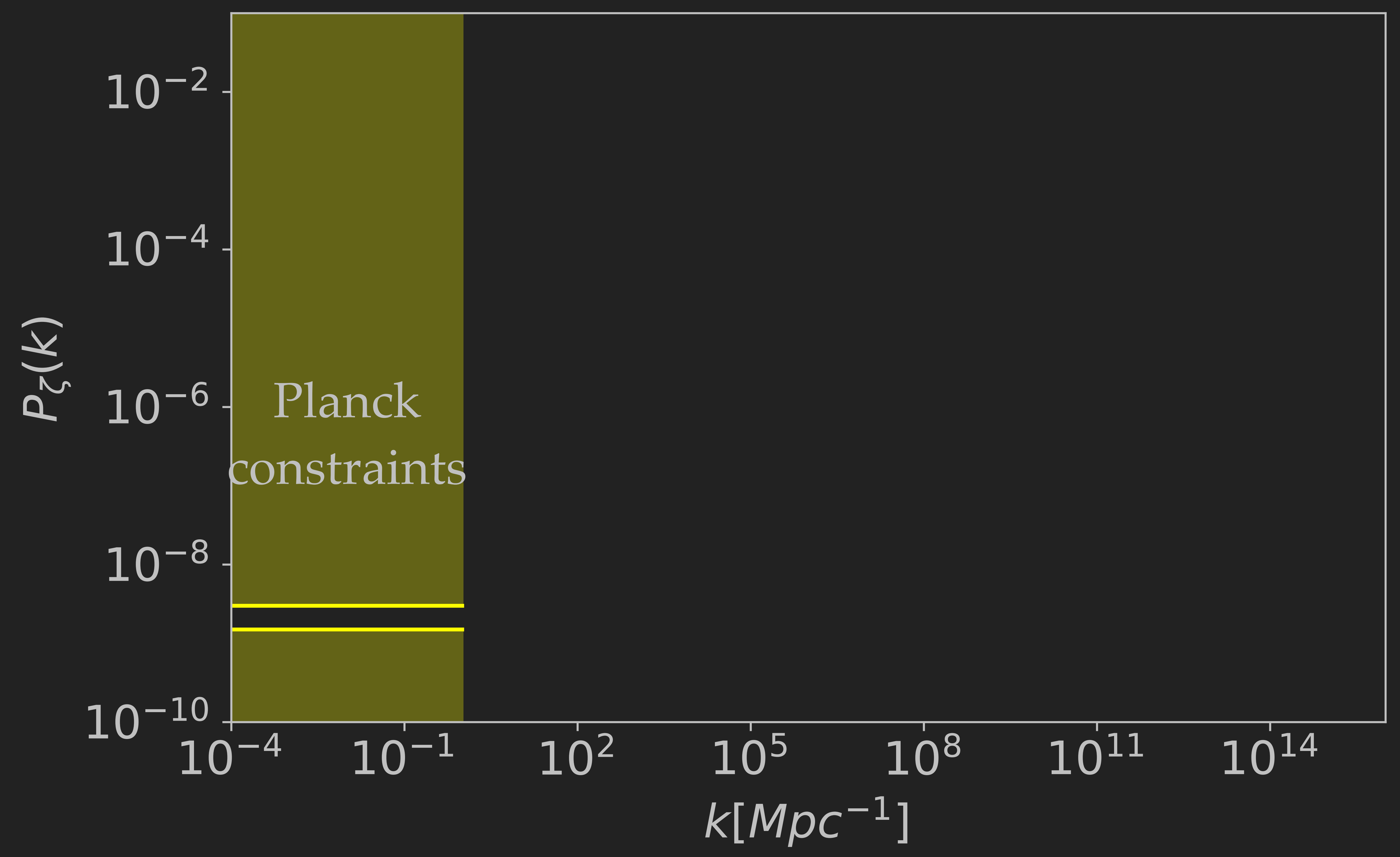
LARGE-SCALE OBSERVATIONS OF INFLATION

- CMB observations, $k_{CMB} = 0.05\,\text{Mpc}^{-1}$
- Modelling $\mathscr{P}_\zeta(k) = \mathscr{A}_s \left(\frac{k}{k_{CMB}}\right)^{n_s - 1 + \cdots} \rightarrow \mathscr{A}_s \sim 10^{-9}$ and $n_s = 0.9649 \pm 0.0042$ (68% C.L.)
- ${f_{NL}}^{sq} = -\,0.9 \pm 5.1$ (68% C.L.)
- Consistent with predictions from SFSR inflation

{1807.6211 - *Planck* Collaboration, 2110.0483 - *Planck*+BICEP/Keck}

SMALL-SCALE TESTS OF INFLATION

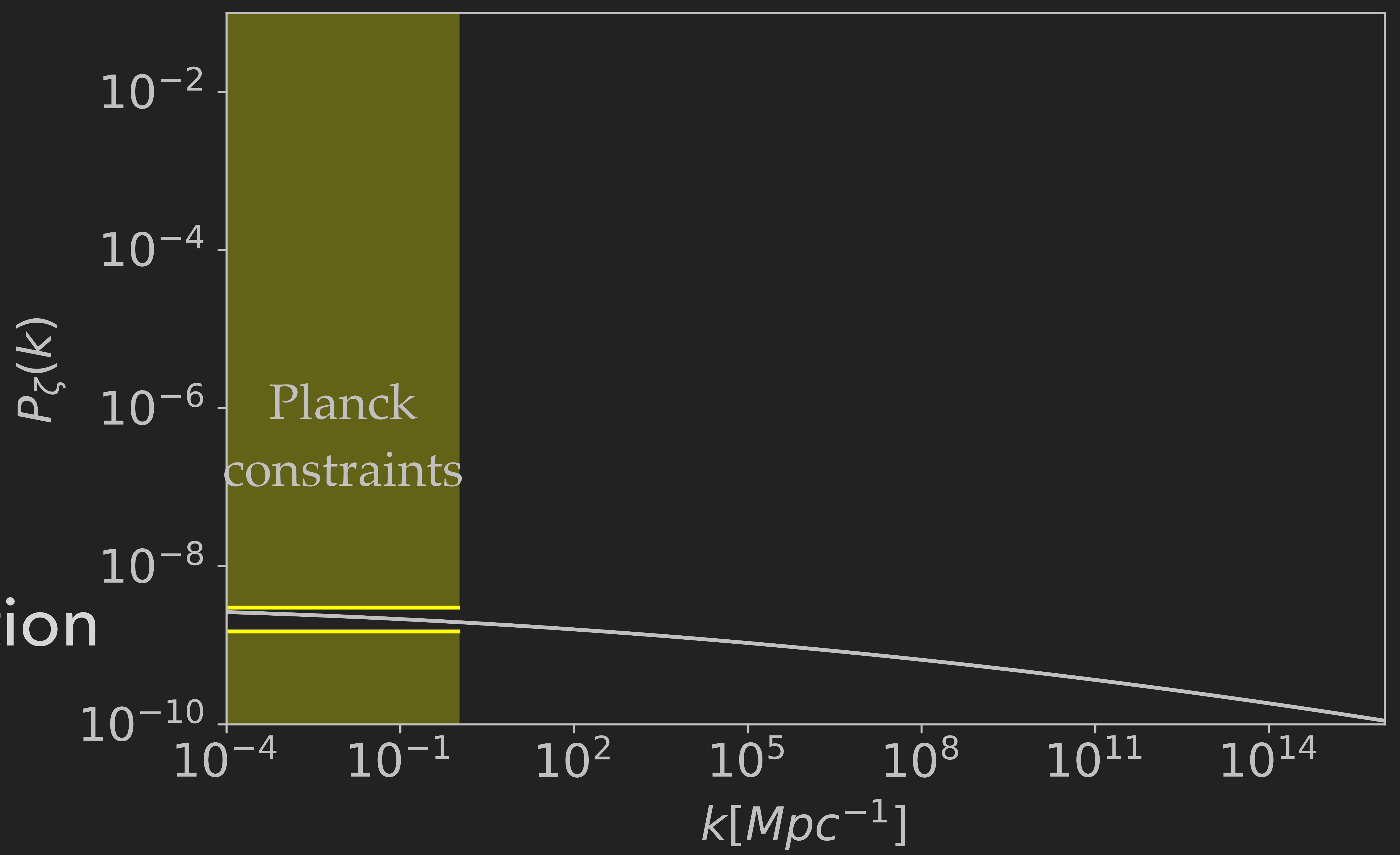
- $\mathscr{P}_\zeta(k)$ is well constrained on large scales, e.g. CMB



SMALL-SCALE TESTS OF INFLATION

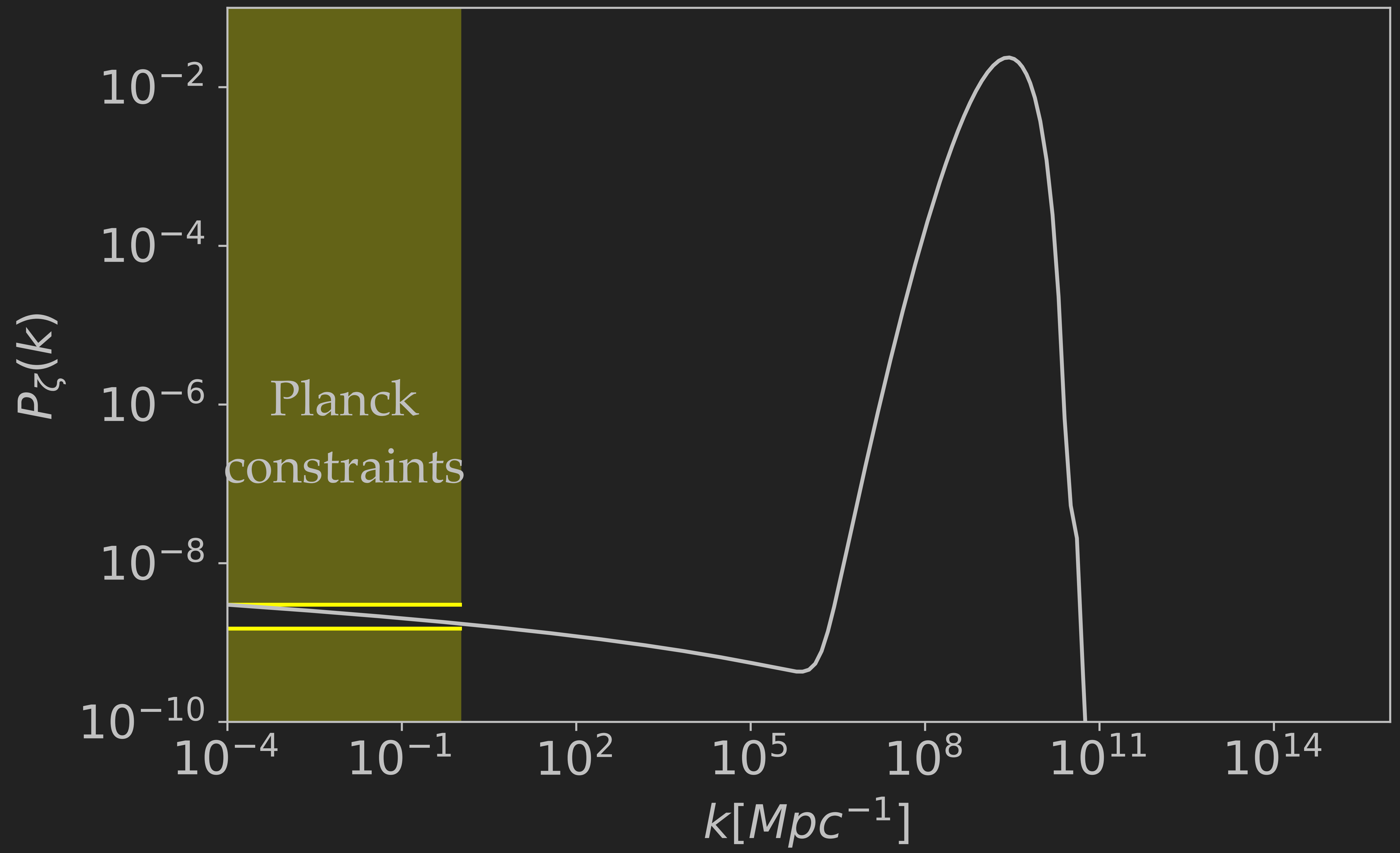
- $\mathscr{P}_\zeta(k)$ is well constrained on large scales, e.g. CMB

$\mathscr{P}_\zeta(k)$ in single-field slow-roll inflation

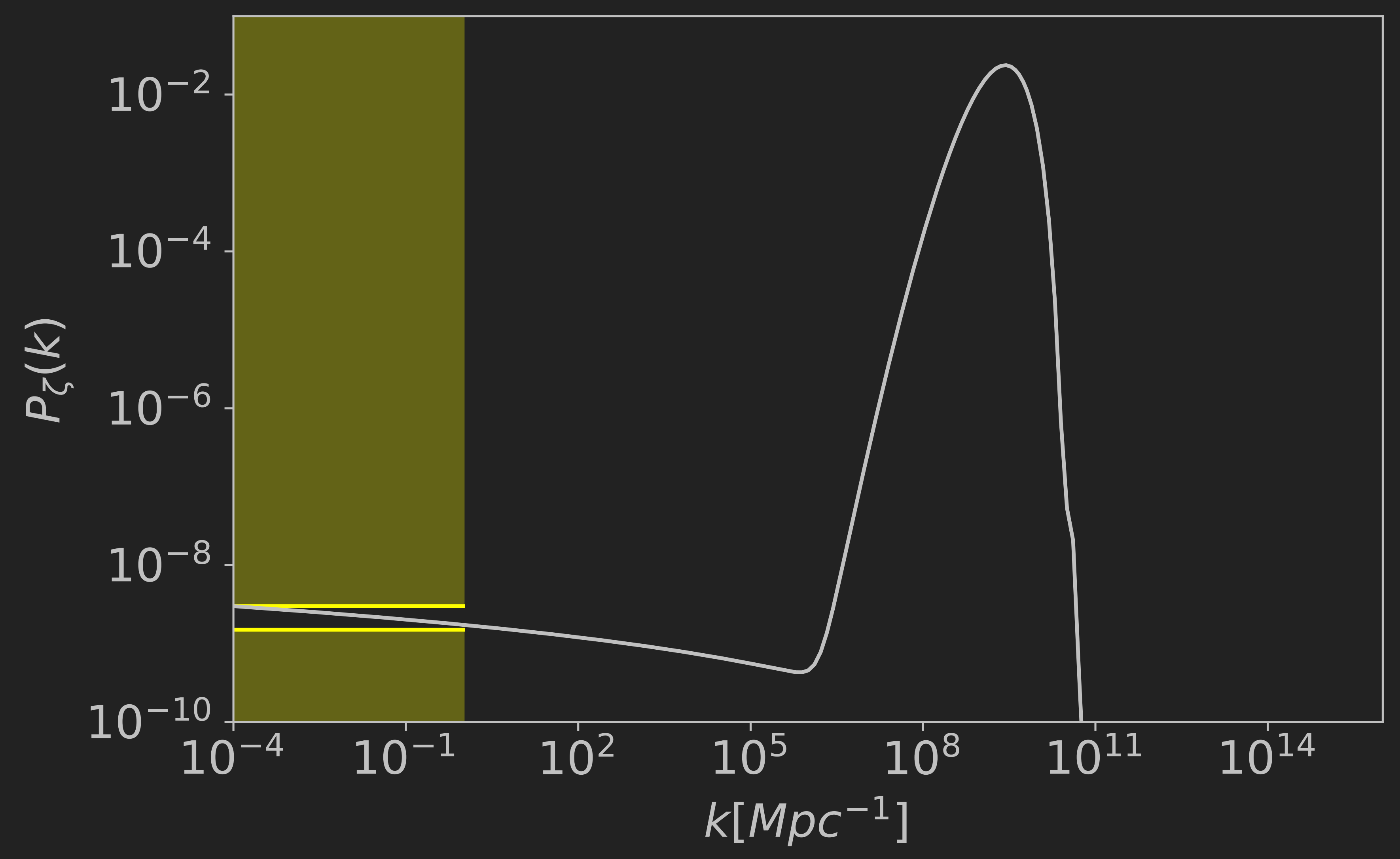


SMALL-SCALE TESTS OF INFLATION

- $\mathscr{P}_\zeta(k)$ is well constrained on large scales, e.g. CMB
- $\mathscr{P}_\zeta(k)$ could display a peak on small scales
- 2nd order GWs, $\Omega_{\rm GW} \sim \int {\mathscr{P}_\zeta}^2$
- $\mathscr{P}_\zeta(k) \simeq 0.01$, PBH production



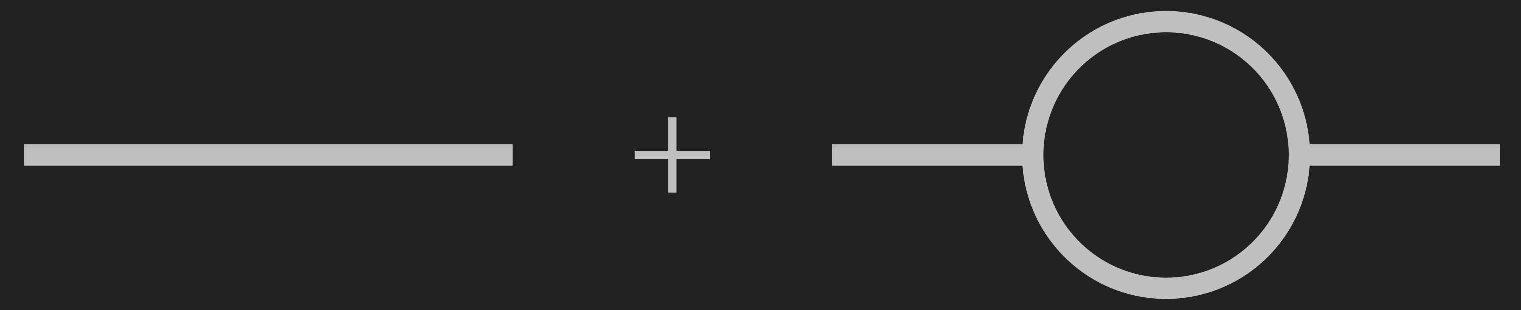
PEAK FROM SINGLE FIELD INFLATION



- **SF inflation:** need deviation from SR evolution
- $\zeta(N)_{k\ll aH} = c_1 + c_2 \int \mathrm{d}N\, e^{-\int \mathrm{d}N'(3-\epsilon+\eta)}$
- SR: $\epsilon, \eta \ll 1$, no super-horizon evolution $\zeta(N)_{k\ll aH} = c_1 + c_2 \int \mathrm{d}N e^{-3N} \sim c_1$
- USR: $\epsilon \ll 1, \eta \sim -6$, super-horizon growth $\zeta(N)_{k\ll aH} \sim \int \mathrm{d}N\, e^{3N}$
- E.g. almost-stationary inflection point, bump/dip

{Motohashi+, Kinney, Dimopoulos, Pattison+, Bellido+, Germani+, Ballesteros+, Cicoli+, Atal+, Mishra+, …}

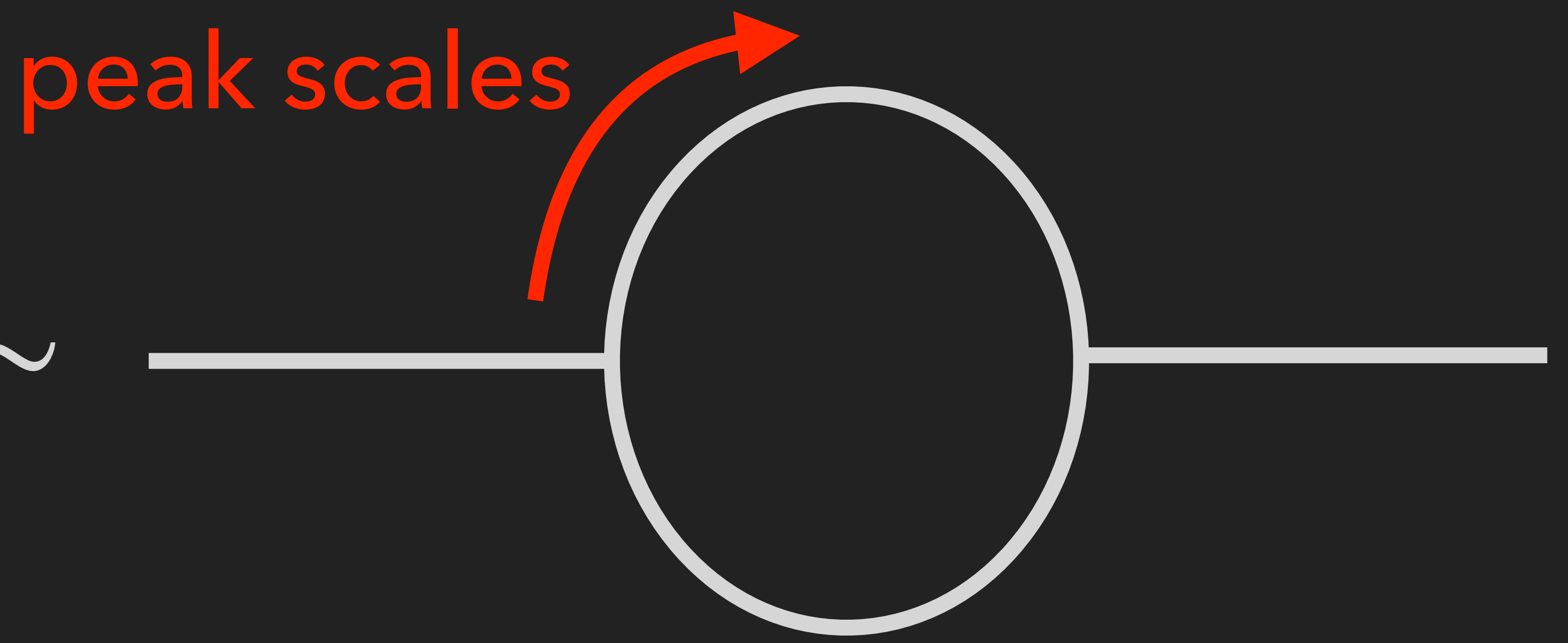
DO LARGE FLUCTUATIONS ON SMALL SCALES CAUSE TROUBLE?

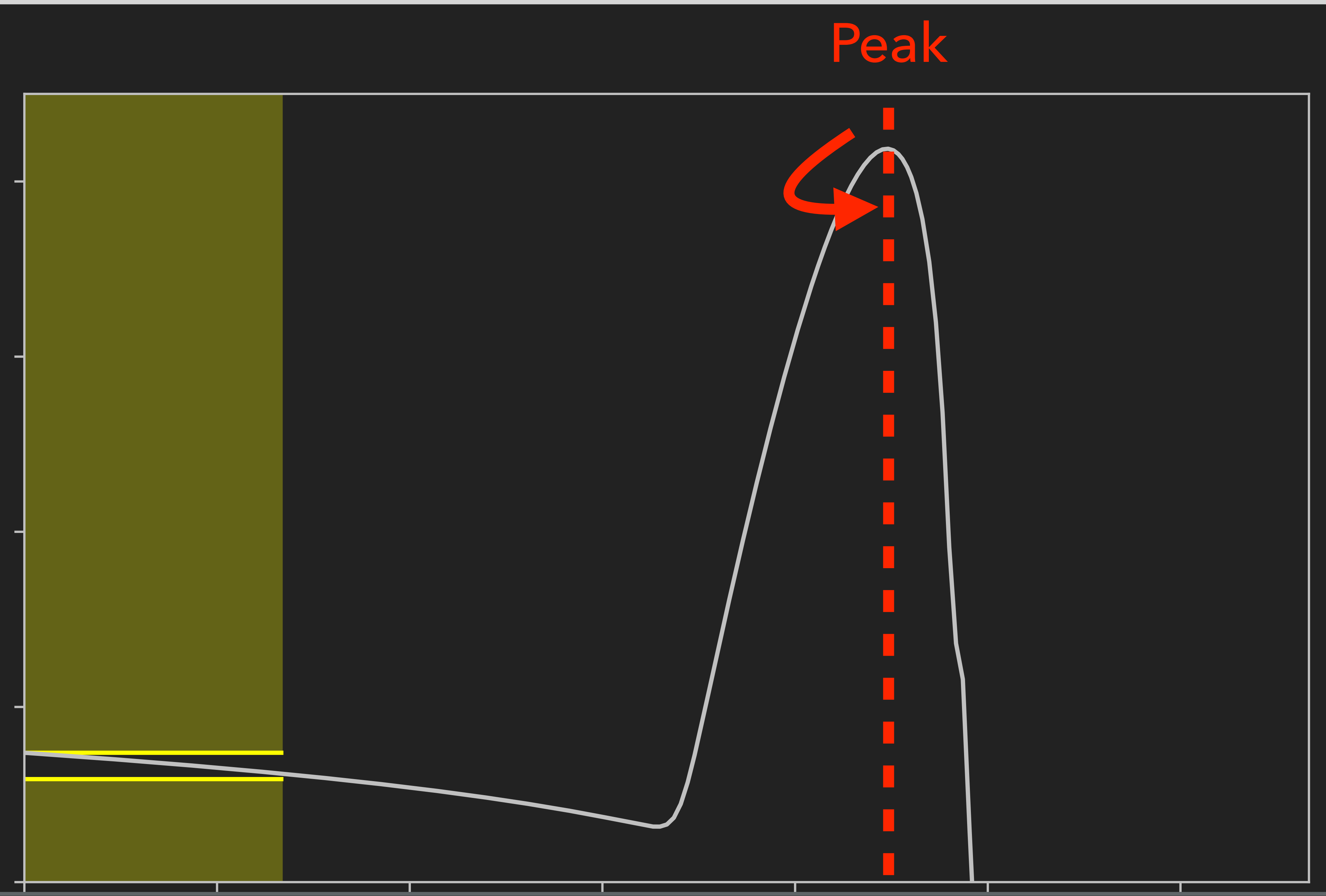
- $\mathscr{P}_\zeta(k) = \mathscr{P}_\zeta(k)_{\text{tree}} + \mathscr{P}_\zeta(k)_{\text{1-loop}} + \cdots + \mathscr{P}_\zeta(k)_{\text{n-loop}}$
- $\langle\zeta\zeta\rangle \sim$ 
- A (first) perturbativity criterion: $\mathscr{P}_\zeta(k)_{\text{1-loop}} \ll \mathscr{P}_\zeta(k)_{\text{tree}}$
- 1-loop correction involves integral over all scales, how do enhanced modes contribute?

{Inomata+, Kristiano+, Riotto, Firouzjahi, Choudhury+, Motohashi+, Tasinato+, Franciolini+, Fumagalli, Fumagalli+, Tada+…}

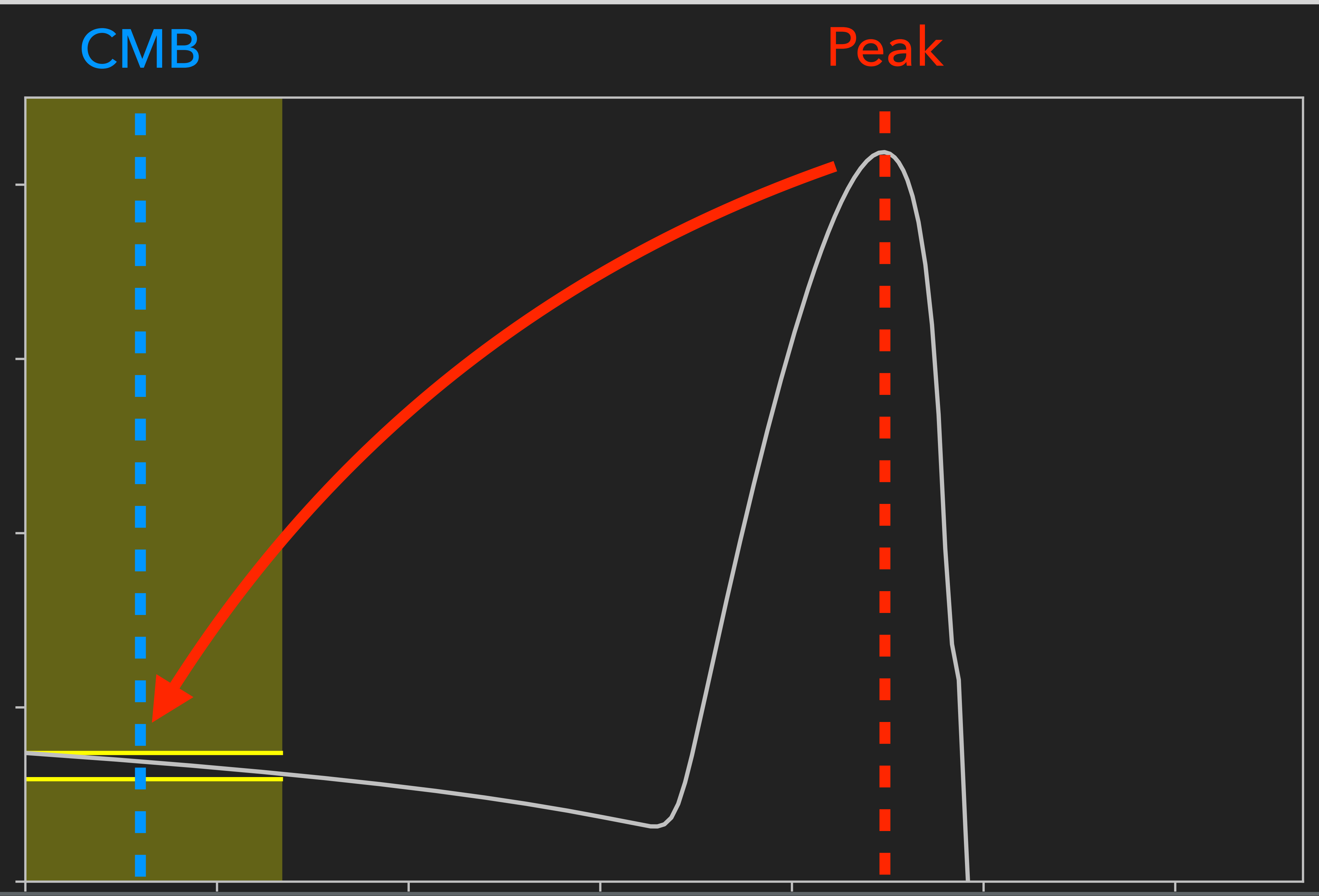
1LC EFFECTS AT DIFFERENT SCALES

$\mathscr{P}_\zeta(k)_{\text{1-loop}} \sim$





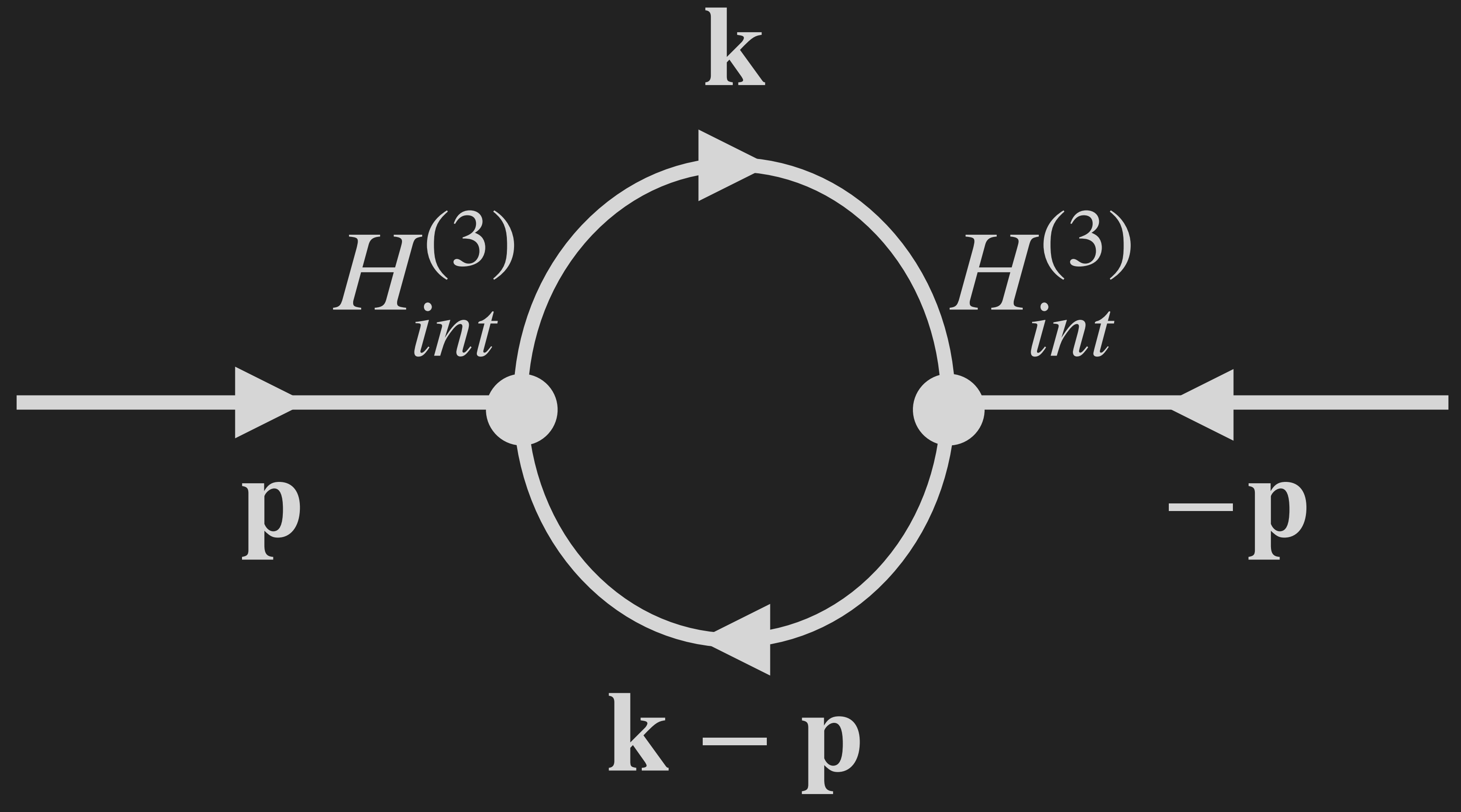
{Inomata+; **2304.14260: LI, D.Mulryne**; Fumagalli+}

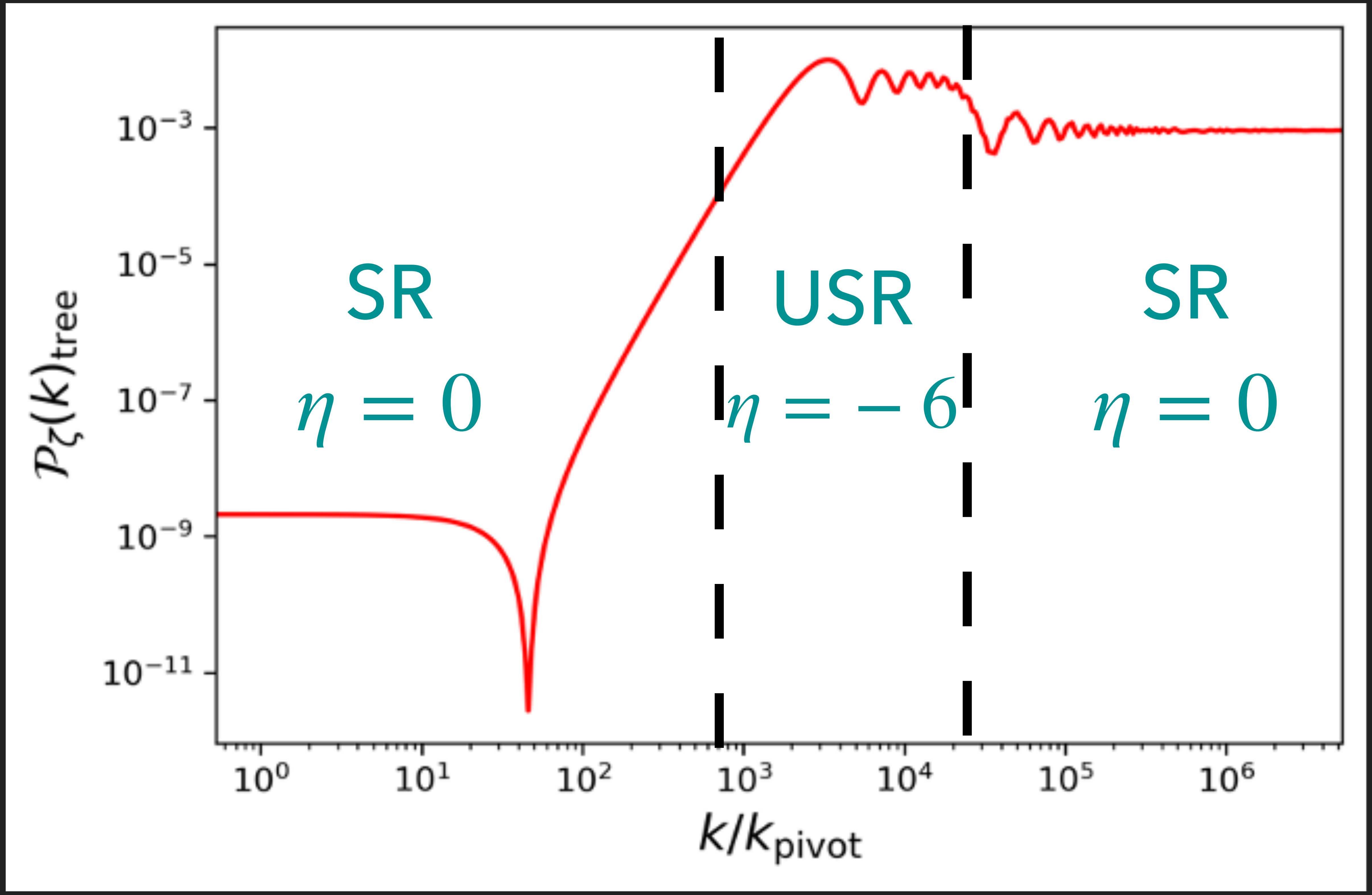


{Kristiano+; ...; **2312:05694, M.Davies, LI, D.Mulryne**; **2312:12424, LI, D.Mulryne, D.Seery**; ...}

1LC ON LARGE SCALES BY ENHANCED USR MODES (I)

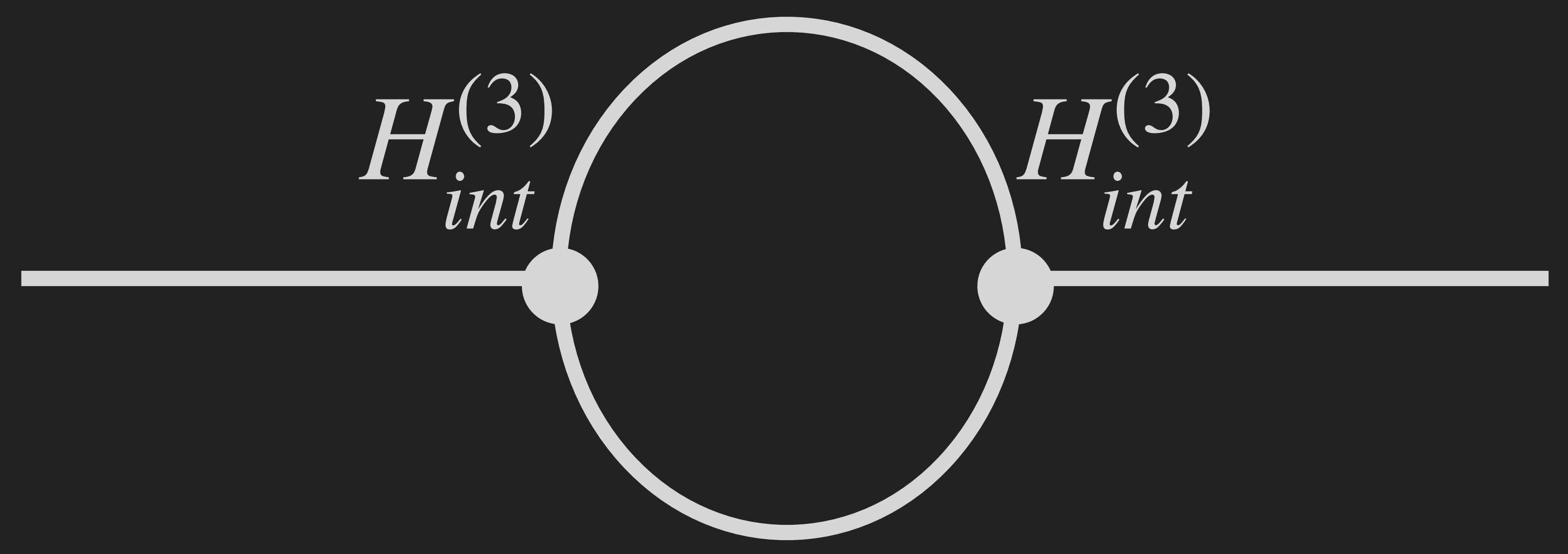
- SR/USR/SR inflationary dynamics
- Instant transitions
- Calculate 1LC due to cubic interactions $H^{(3)}_{int} \propto \eta'$ using in-in formalism





{2211.03395 - Kristiano & Yokoyama}

1LC ON LARGE SCALES BY ENHANCED USR MODES (II)



- Imposing $\mathscr{P}_\zeta(p)_{\text{1-loop}} \ll \mathscr{P}_\zeta(p)_{\text{tree}}$ yields $\mathscr{P}_\zeta(k_{peak}) \ll 0.03$
- PBH production ($\mathscr{P}_\zeta(k_{peak}) \sim 0.01$) in tension with perturbativity requirement

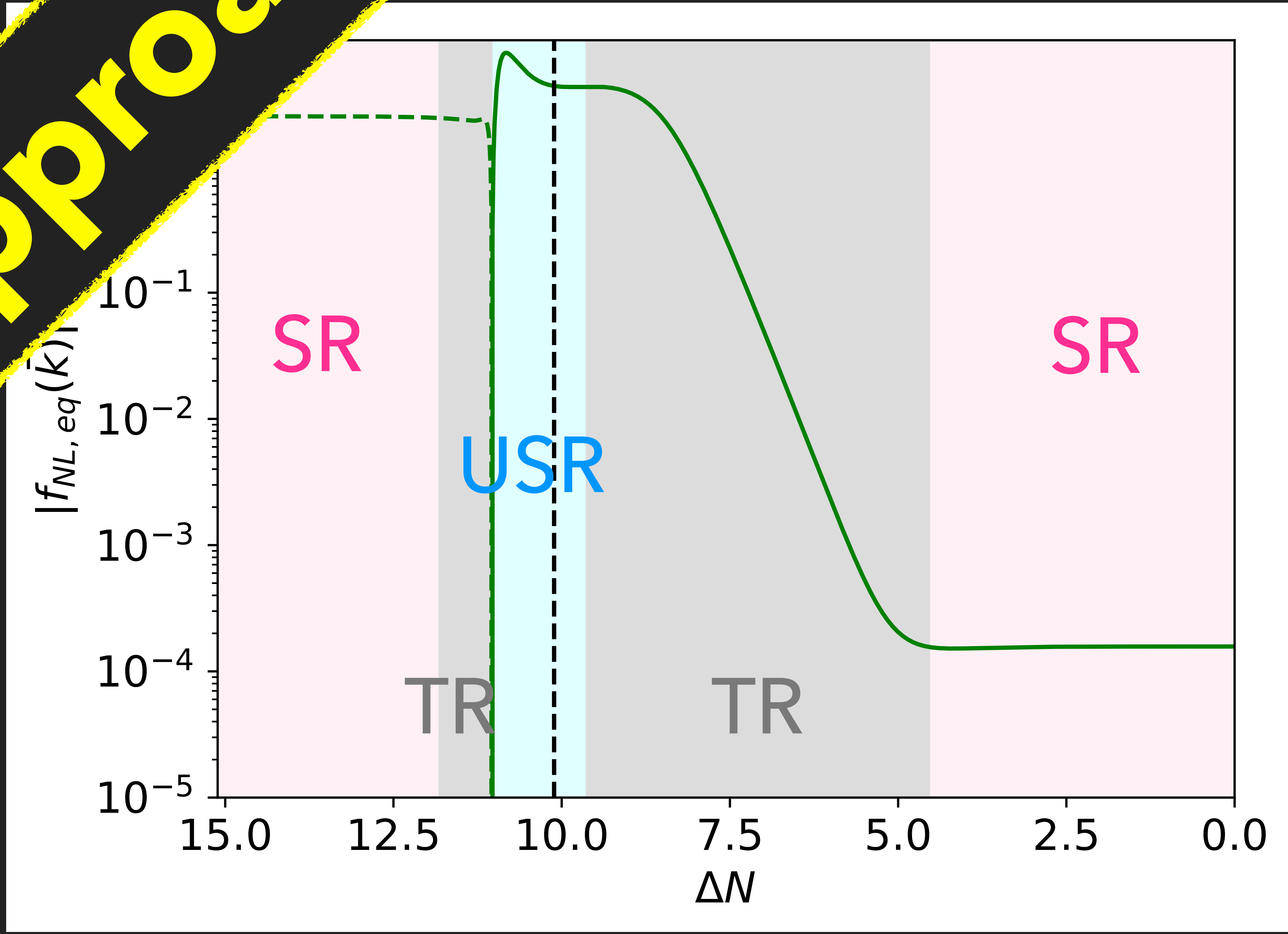
{2211.03395 - Kristiano & Yokoyama}

CAVEATS/POINTS OF CONTENTION

- ($H^{(4)}_{int}$ {Firouzjahi} , regularisation and renormalisation of the divergent integral)
- Did we include all the relevant interactions (boundary terms)? {Fumagalli, Firouzjahi Braglia+}
- E.g. $f_{NL} = \mathcal{O}(1)$ {Firouzjahi+,…} p during USR becomes SR sup realistic, smooth USR→SR {Cai+}

Does smoothing off th ons wipe off the large 1LC? See {Francio , 2312.05694 Davies,LI,Mulryne}

Alternative approach



LOOPS IN THE SEPARATE UNIVERSE PICTURE

{arXiv:2312.12424 - LI, Mulryne, Seery}

LOOPS IN THE SEPARATE UNIVERSE PICTURE

- Provide general framework to calculate loops in the separate universe picture (not limited to SF USR case, could be used e.g. for MFI)
- In this talk:

1. Classical back-reaction model to understand the physical origin of the 1LC
2. Specialise to case of 1LC in presence of USR
3. Sketch of the separate universe calculation (see also {2304.07081 - Riotto, Firouzjahi})
4. Results & Future directions

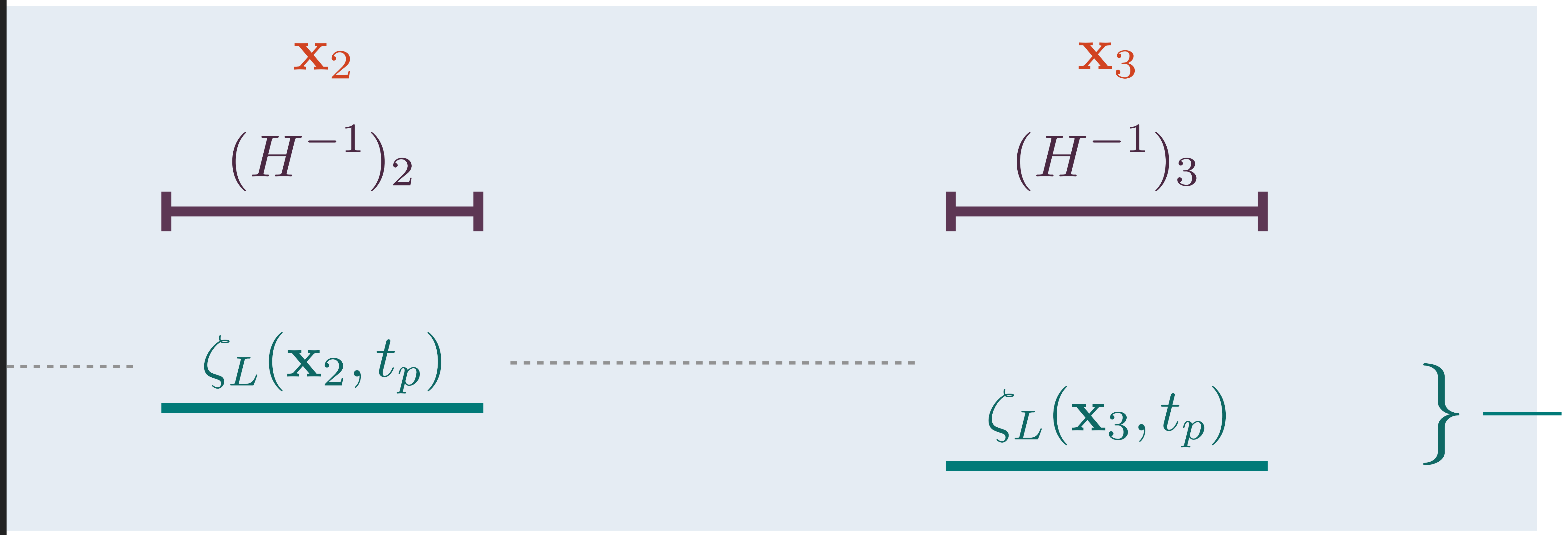
{2312.12424 - LI, Mulryne, Seery}

$L-$patches

$(L \sim 1/p)$, $\zeta_L(\mathbf{x})$

average of ζ in

each patch

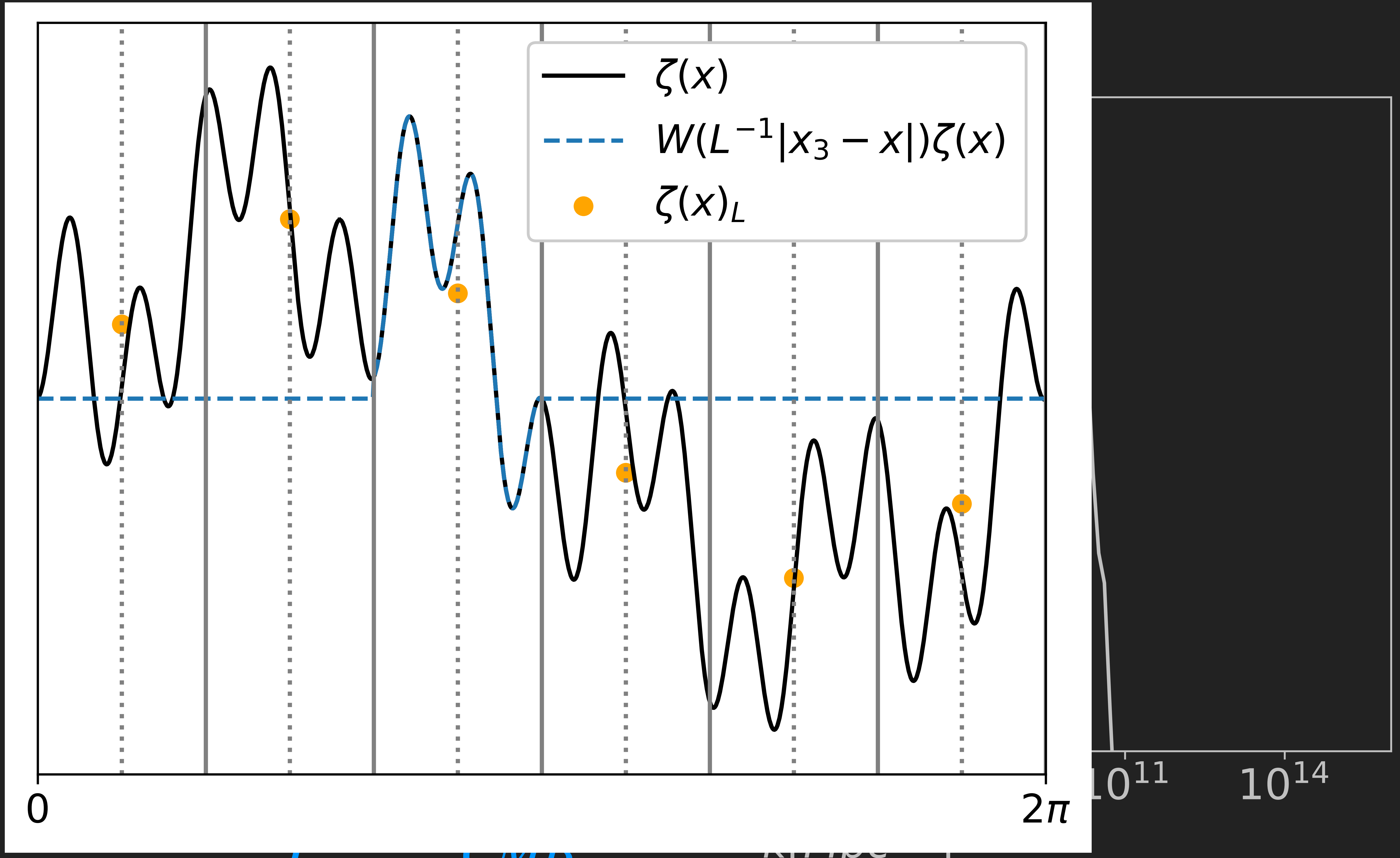


scatter in

values of $\zeta_L(t_p)$

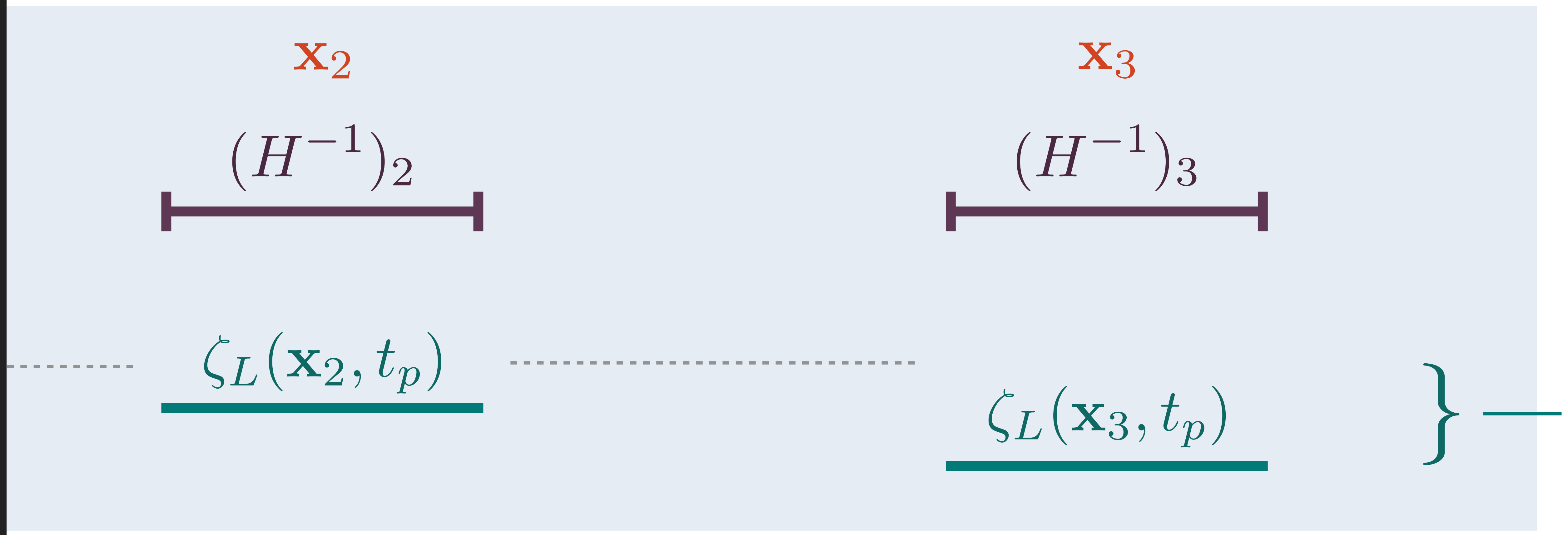
$\zeta_L(t_p) \sim N[0, \sigma_L^2(t_p)]$

$\langle \zeta_L(\mathbf{x})^2 \rangle = \sigma_L^2$



{2312.12424 - LI, Mulryne, Seery}

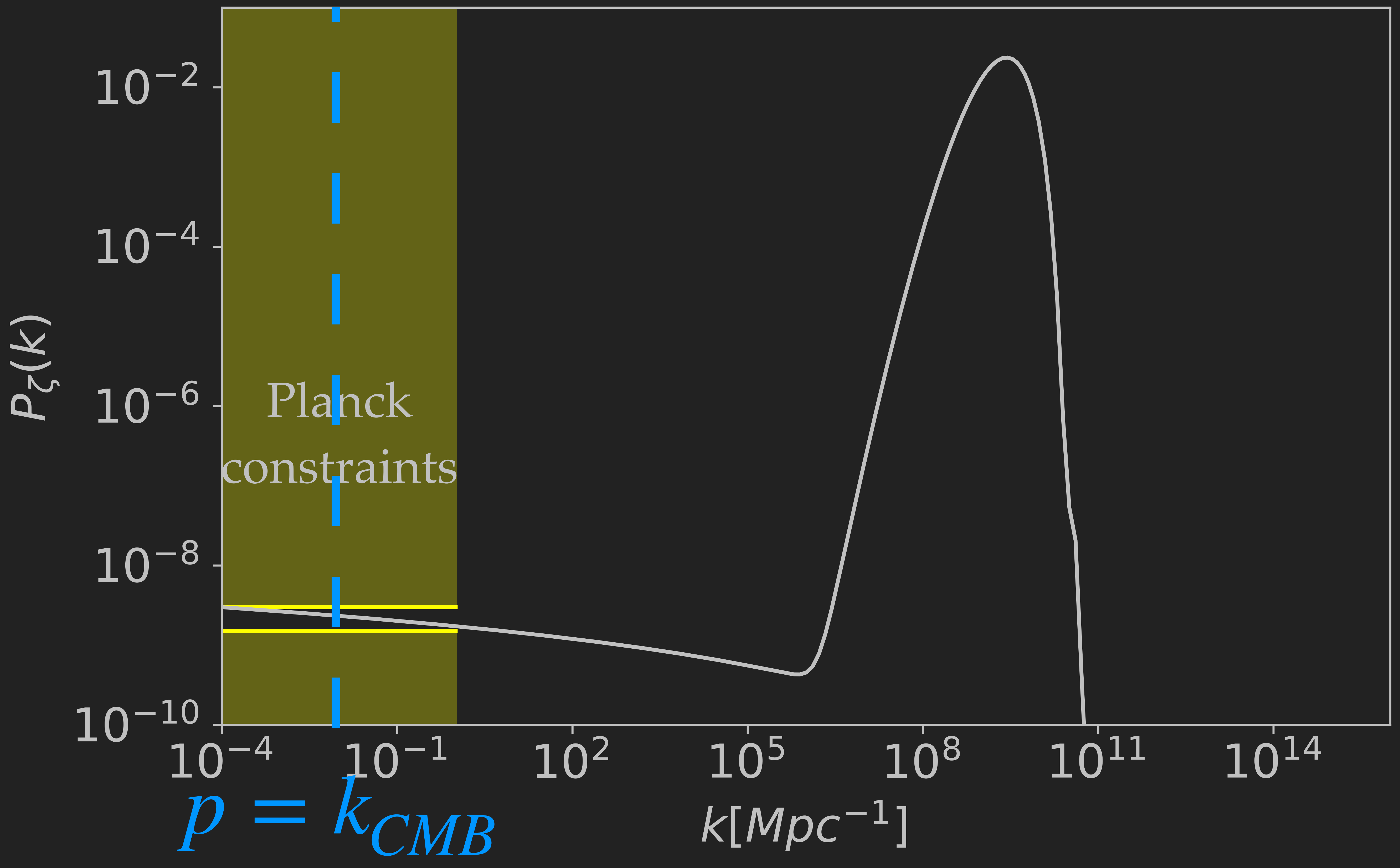
$L-$patches ($L \sim 1/p$), $\zeta_L(\mathbf{x})$ average of ζ in each patch



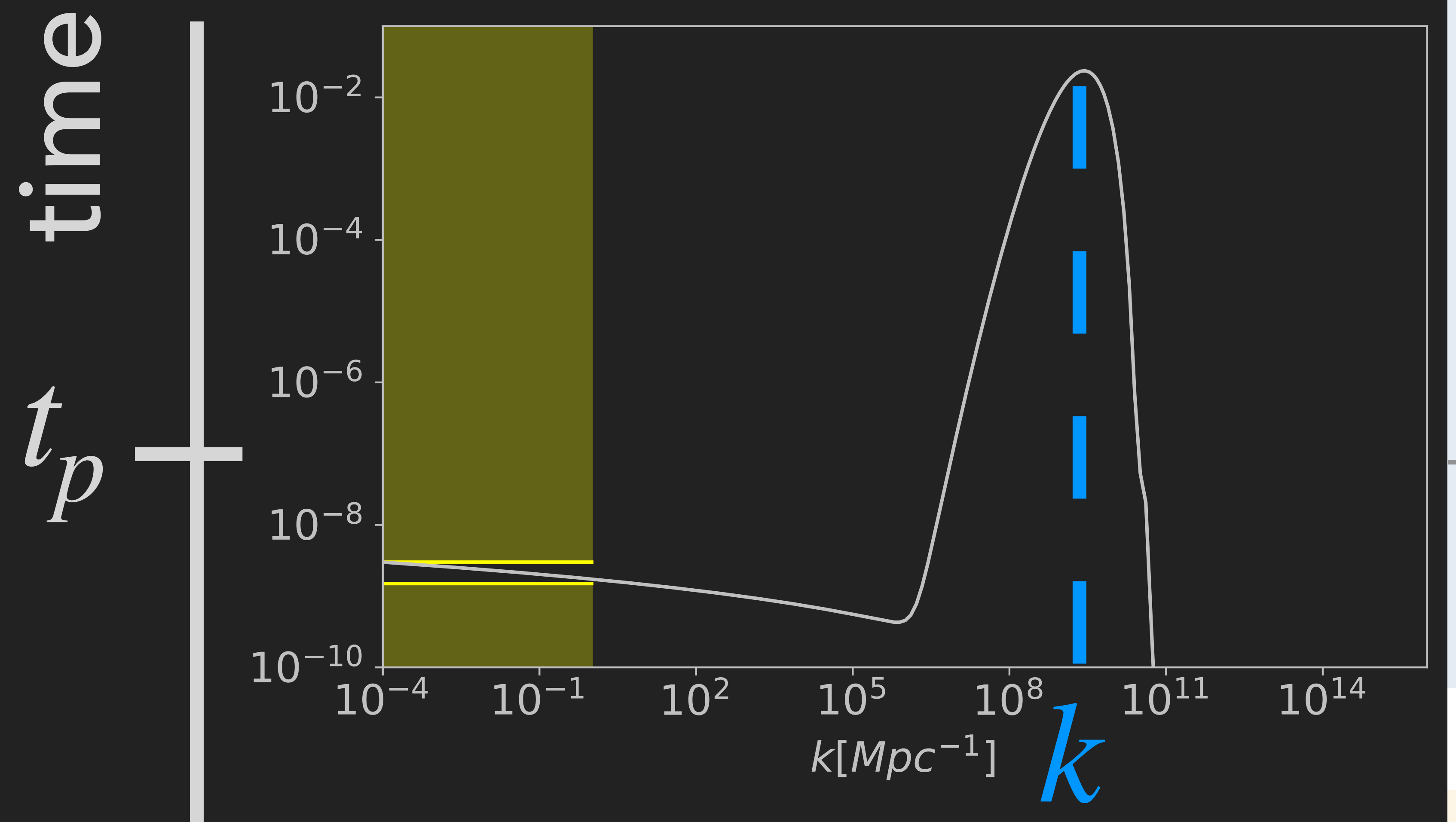
scatter in values of $\zeta_L(t_p)$

$\zeta_L(t_p) \sim N[0, \sigma_L^2(t_p)]$

$\langle \zeta_L(\mathbf{x})^2 \rangle = \sigma_L^2$

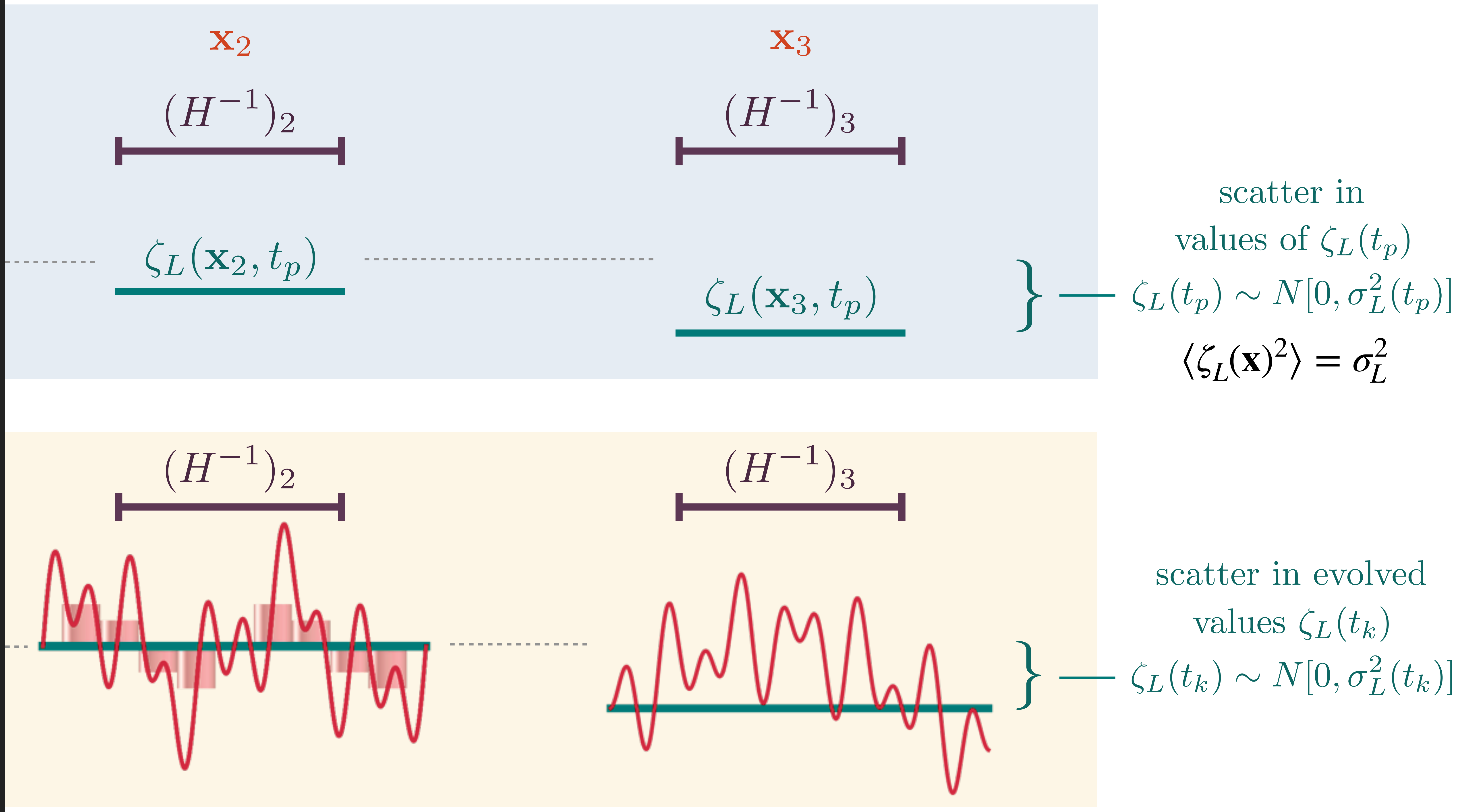


{2312.12424 - Ll, Mulryne, Seery}



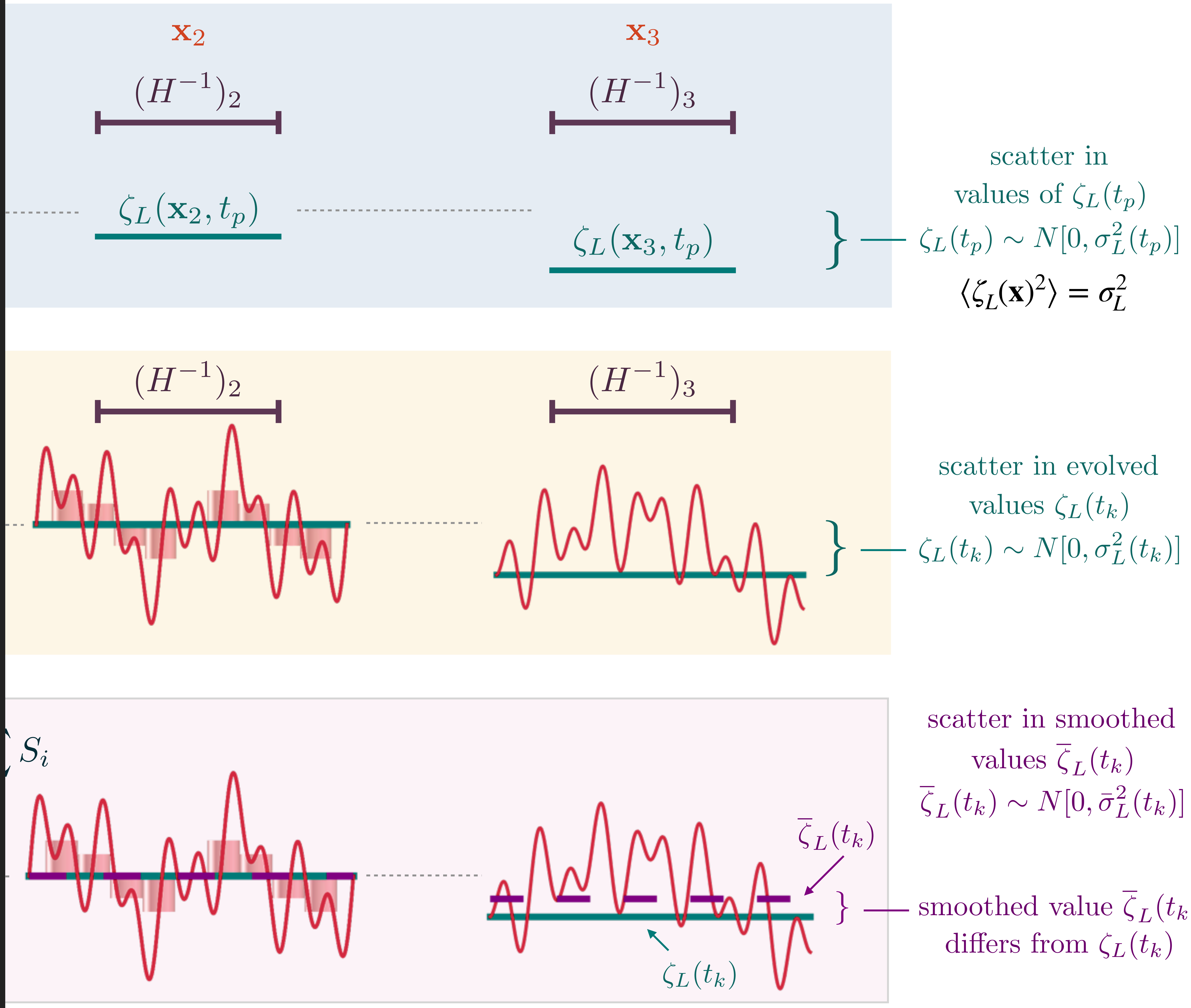
t_k

overlay on $L-$patch
$N = (L/\ell)^3$ $\ell-$boxes
$(\ell \sim 1/k)$, with random
perturbation S_ℓ



S_ℓ might change behaviour
depending on $\zeta_L(\mathbf{x})$ (*reaction*)

{2312.12424 - LI, Mulryne, Seery}



Mode p then *back-reacts* to the presence of k modes

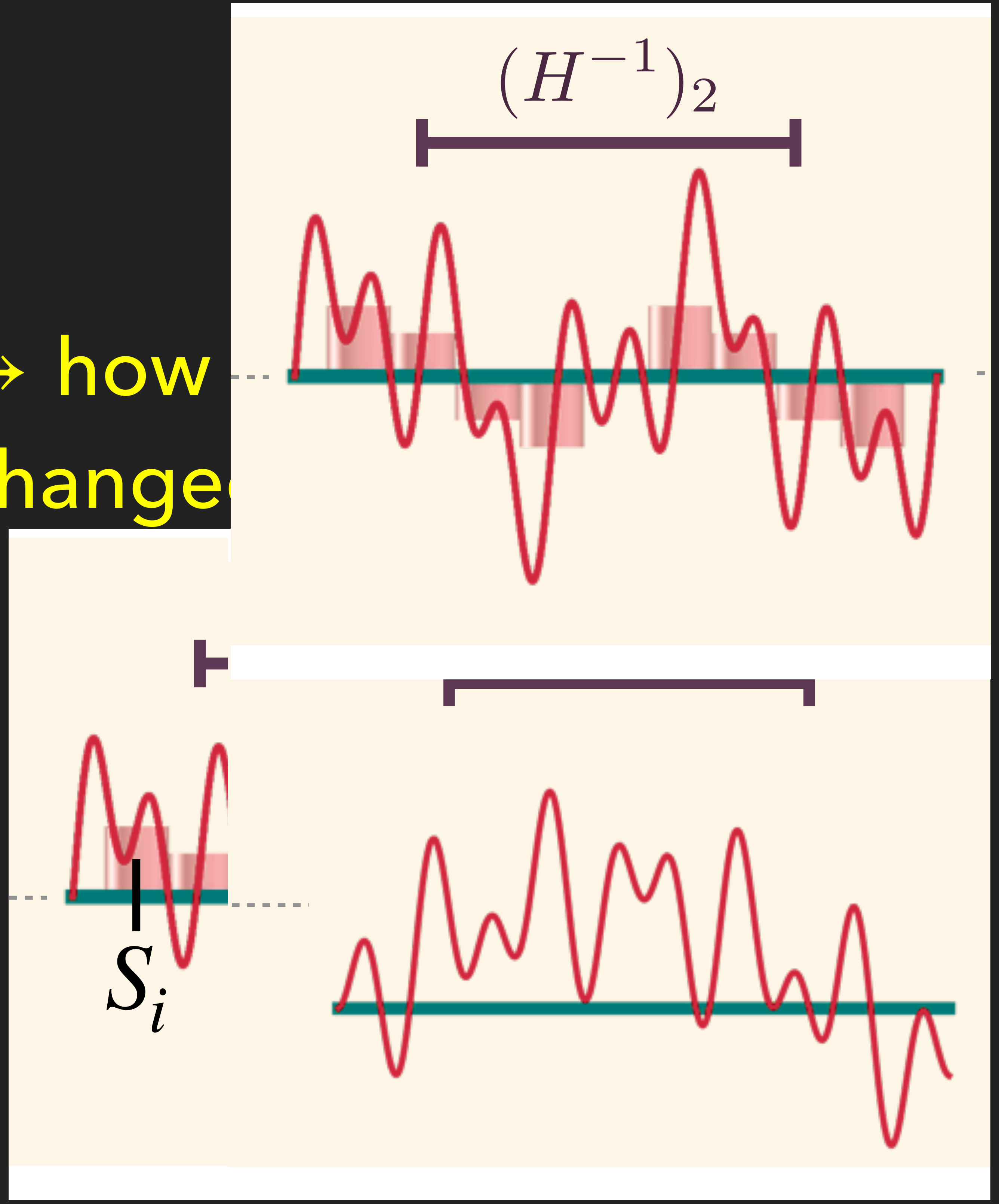
$$\bar{\zeta}_L = \zeta_L + \frac{1}{N}\sum_i S_i$$

{2312.12424 - LI, Mulryne, Seery}

A CLASSICAL BACKREACTION MODEL

- Dressed long wavelength perturbation: $\bar{\zeta}_L = \zeta_L + \frac{1}{N}\sum_i S_i$
- New variance: $\bar{\sigma}_L{}^2 = \langle \bar{\zeta}_L{}^2 \rangle \supseteq \frac{1}{N}\sigma_\ell^2 + \frac{1}{N^2}\sum_i \sum_{i \neq j} \langle S_i S_j \rangle + \langle \zeta_L S_\ell \rangle$

$\bar{\sigma}_L^2 \rightarrow$ how has change[d]



A. $\frac{1}{N}\sigma_\ell^2 = \left(\frac{p}{k}\right)^3 \sigma_\ell^2$, only contribution for IID random variable (N finite,

B. $\frac{1}{N^2}\sum_i \sum_{i \neq j} \langle S_i S_j \rangle \neq 0$ if long-range correlation between small boxes

C. $\langle \zeta_L S_\ell \rangle \neq 0$ if small-scale S_ℓ perturbation correlated with large-scale ζ_L perturbation (no $1/N$ suppression)

{2312.12424 - LI, Mulryne, Seery}

INTRODUCTION TO SEPARATE UNIVERSE METHOD

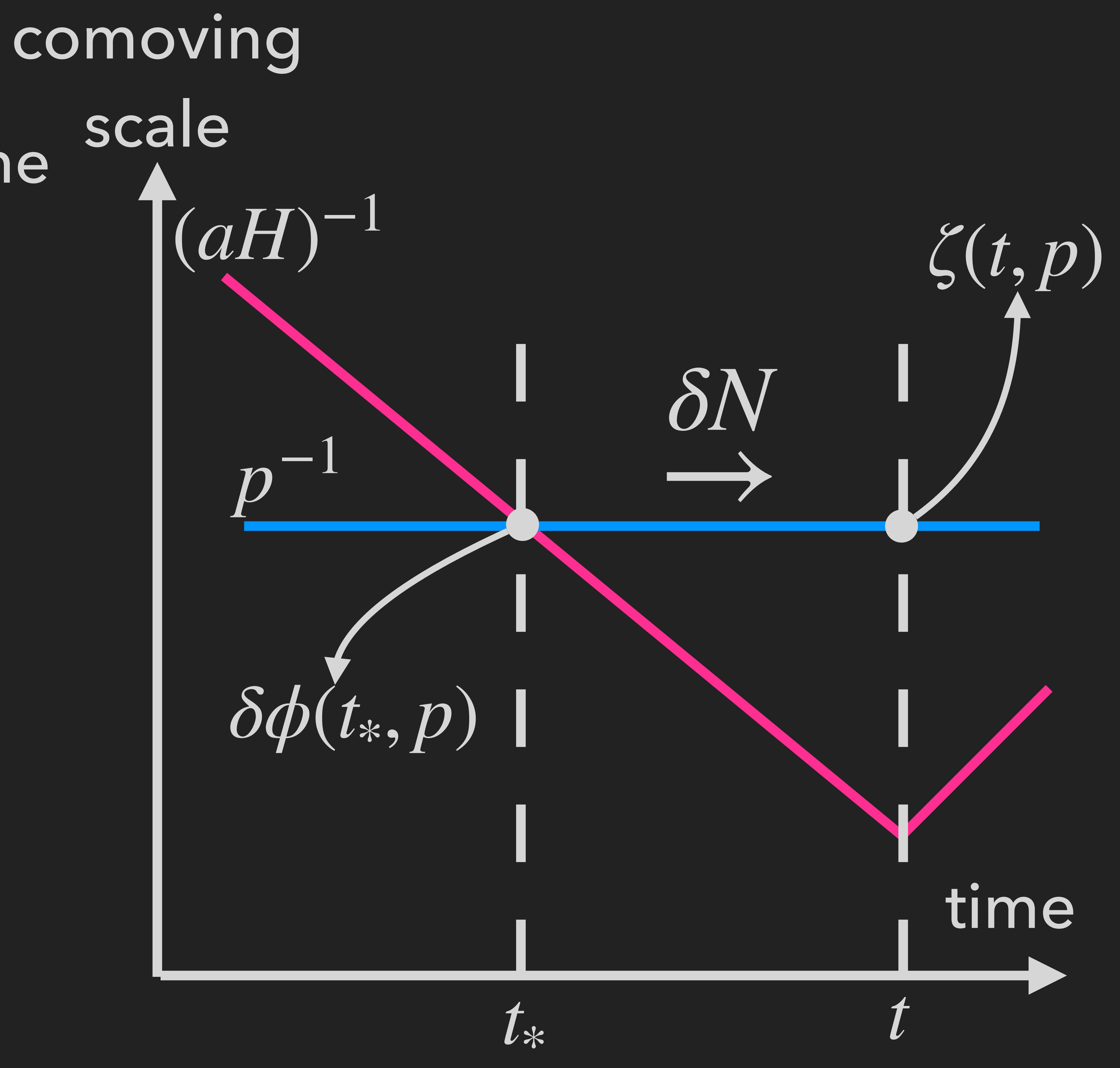
- Separate universe (δN) method: super-horizon evolution of the curvature perturbation

- Example: SFSR inflation $\dot{\phi}(\phi)$

- Taylor-expansion of the δN formula:
  $\zeta(t,x) = N_{\phi}^{(t_*,t)}\delta\phi(t_*,x) + 1/2\, N_{\phi\phi}^{(t_*,t)}\delta\phi(t_*,x)^2 + \cdots$

- t_*: initialisation time; $\delta\phi(t_*,x)$: initial condition; $N^{(t_*,t)}$: e-folds between t_* and t

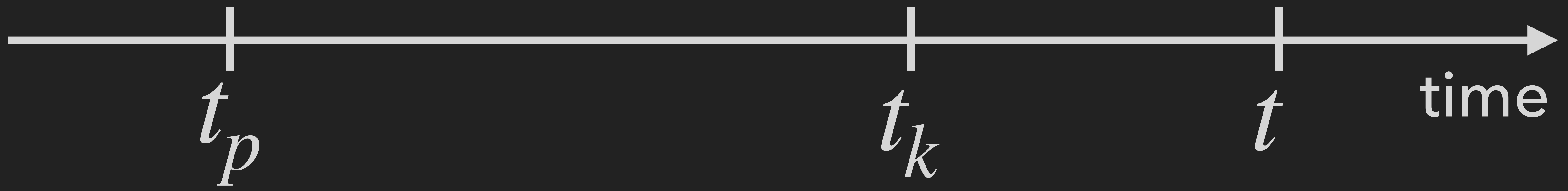
- Linear term: $\langle\zeta_p\zeta_{-p}\rangle_{\text{tree}} = N_\phi^2\langle\delta\phi\delta\phi\rangle \to \mathscr{P}_\zeta(p;t) = \dfrac{H(t_p)^2}{8\pi^2\epsilon(t_p)}$



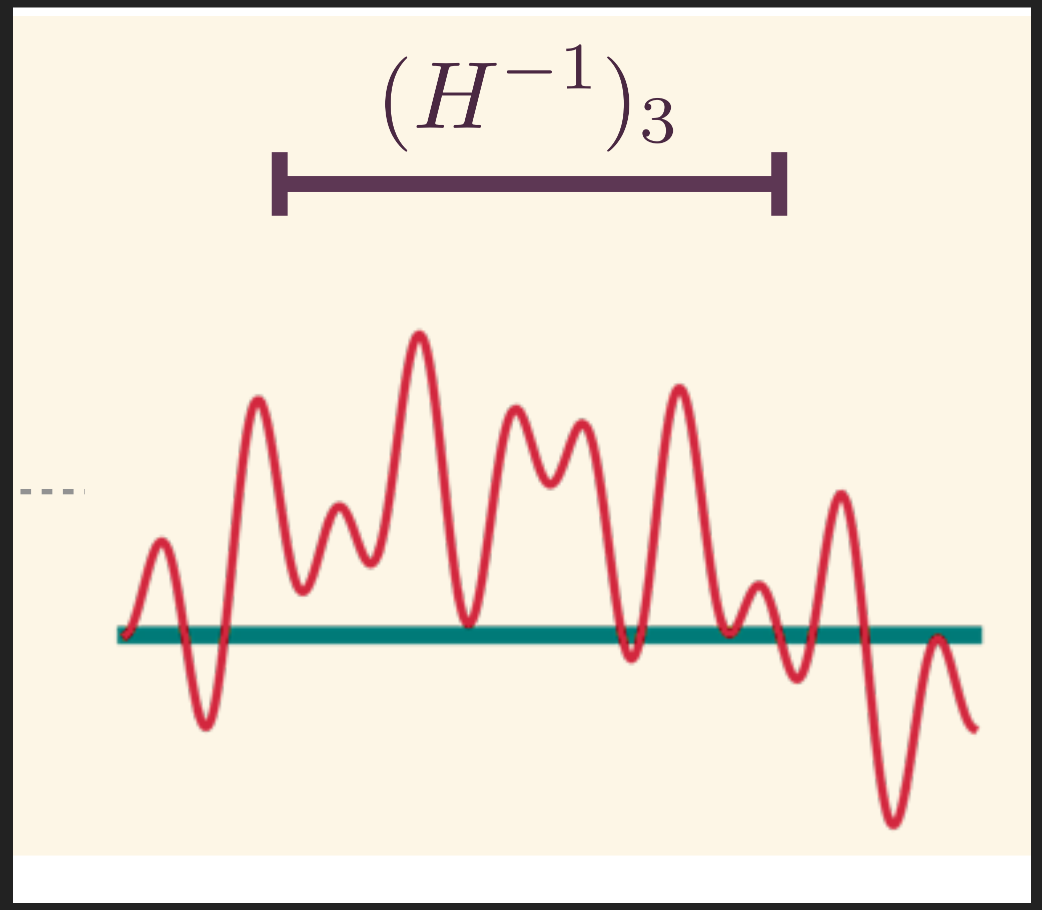
(t_* doesn't have to be h-c time)

{2304.07081 - Riotto, Firouzjahi ; 2312.12424 - LI, Mulryne, Seery}
{Starobinsky, Wands, …}

SET-UP FOR δN CALCULATION



- At time t_k smooth over scale $\lambda_k \sim 1/k$: $\zeta_{\lambda_k}(\mathbf{x}, t_k)$. In Fourier space this reads $\zeta_{\lambda_k}(\mathbf{x}, t_k) \sim \int_{q \lesssim k} \mathrm{d}^3 q \, \zeta(\mathbf{q}, t_k) \, e^{i\mathbf{q}\cdot\mathbf{x}}$ (contribution of peak scales is included *explicitly*)



- Apply δN formalism to evolve $\zeta_{\lambda_k}(\mathbf{x}, t_k) \to \zeta_{\lambda_k}(\mathbf{x}, t)$

- From $\zeta_{\lambda_k}(\mathbf{x}, t)$ extract the long wavelength mode $\zeta_p(\mathbf{x}, t)$

$$\zeta(t)_p = N_I^{(t_k,t)} \delta X_p^I(t_k) + 1/2 \, N_{IJ}^{(t_k,t)} \int_{q \lesssim k} \mathrm{d}^3 q \; \delta X_q^I(t_k) \delta X_{p-q}^J(t_k) + \cdots$$

{2312.12424 - LI, Mulryne, Seery}

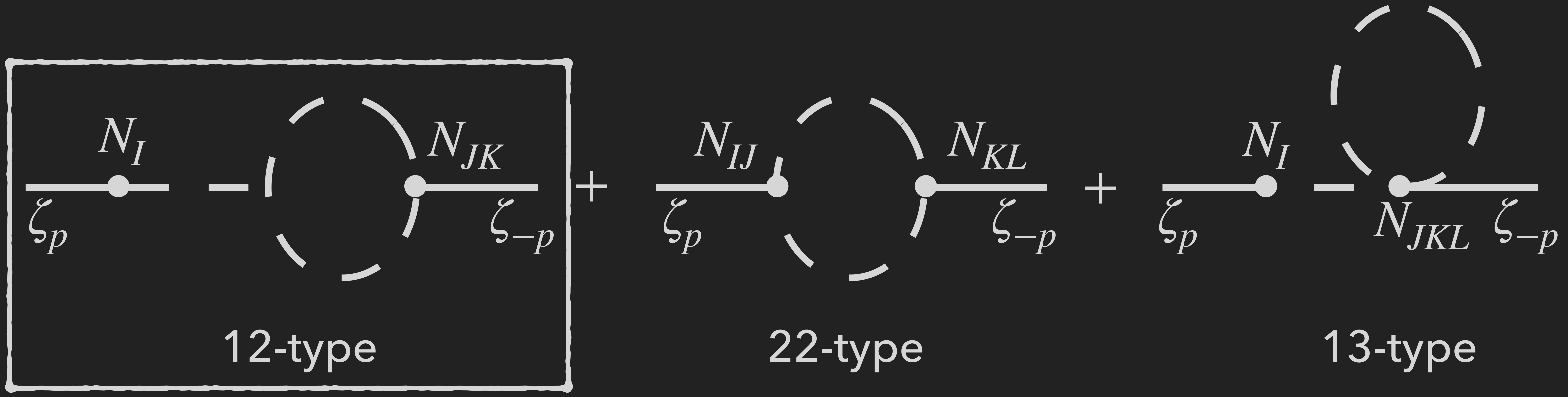
1LC WITH SEPARATE UNIVERSE APPROACH

USR model: potentially both $\delta\phi$ and $\delta\phi'$ contribute, $\delta\phi \to \delta X = \{\delta\phi, \delta\phi'\}$

$$\zeta(t)_p = N_I^{(t_k,t)}\delta X_p^I(t_k) + 1/2\, N_{IJ}^{(t_k,t)} \int_{q \lesssim k} \mathrm{d}^3 q\ \delta X_q^I(t_k)\delta X_{p-q}^J(t_k) + \cdots$$

linear — over peak scales — quadratic

$\langle \zeta_p \zeta_{-p} \rangle$1-loop ~



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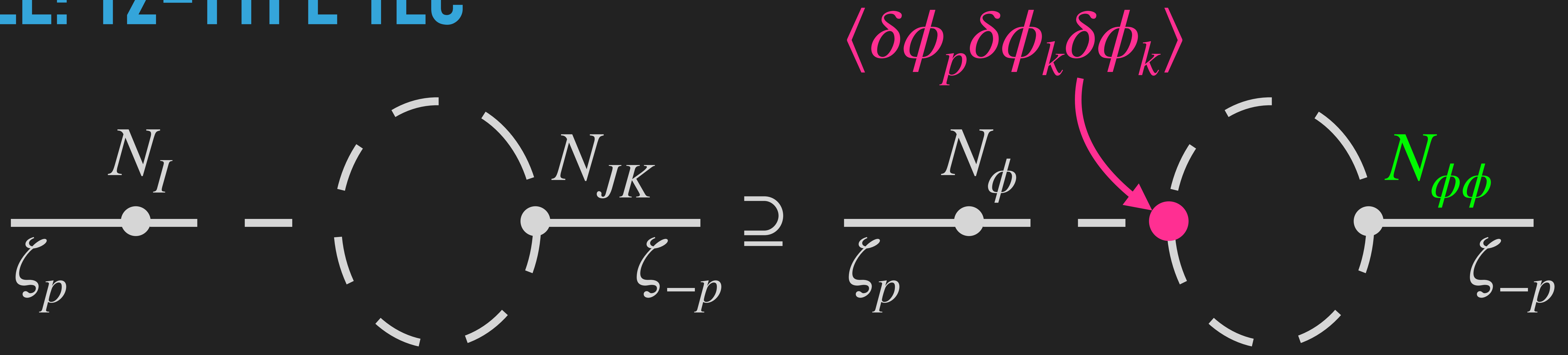
EXAMPLE: 12-TYPE 1LC



Ingredients:

- $N^{(t_k,t)}_{\phi\phi} \neq 0$: non-linearity in the mapping between $\zeta_p(t)$ and $\delta\phi_k(t_k)$
- $\lim_{p \ll k} \langle \delta\phi_p \delta\phi_k \delta\phi_k \rangle^{(t_k)} \sim P^{\phi\phi}{}_{,I}(k; t_k)$ {Kenton&Mulryne}: reaction, long-short mode coupling

$$\bar{\sigma}_L{}^2 = \langle \bar{\zeta}_L{}^2 \rangle \supseteq \frac{1}{N}\sigma_\ell^2 + \frac{1}{N^2}\sum_i \sum_{i \neq j} \langle S_i S_j \rangle + \langle \zeta_L S_\ell \rangle$$

$$\Delta \mathscr{P}_\zeta(p; t)_{12} = \frac{2}{5}\left(\frac{\eta}{6}\right)^2 \epsilon(t_s) f_{NL,eq}^{(N^{\phi\phi})}(\bar{k}; t)\, \mathscr{P}_\zeta(\bar{k})_{\text{tree}}$$

{2312.12424 - LI, Mulryne, Seery}

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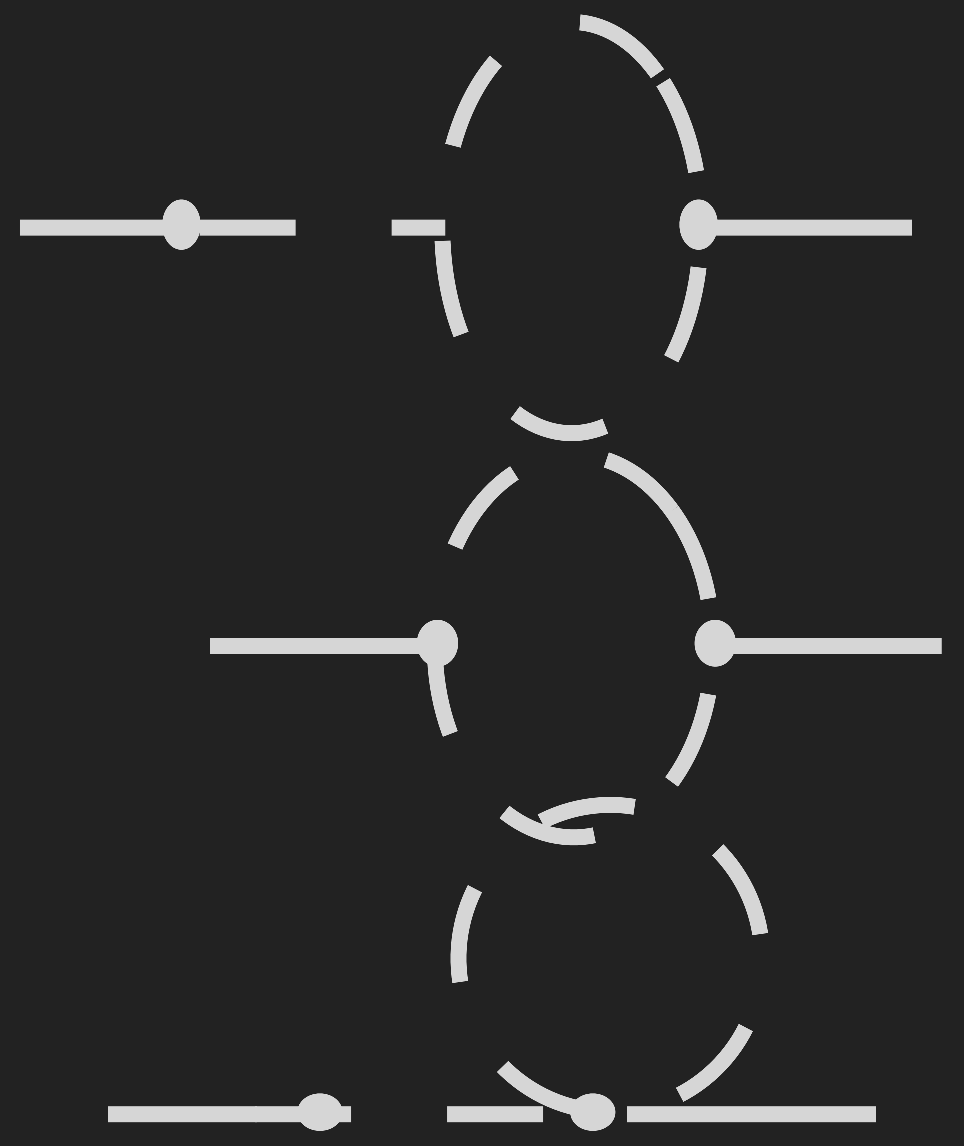
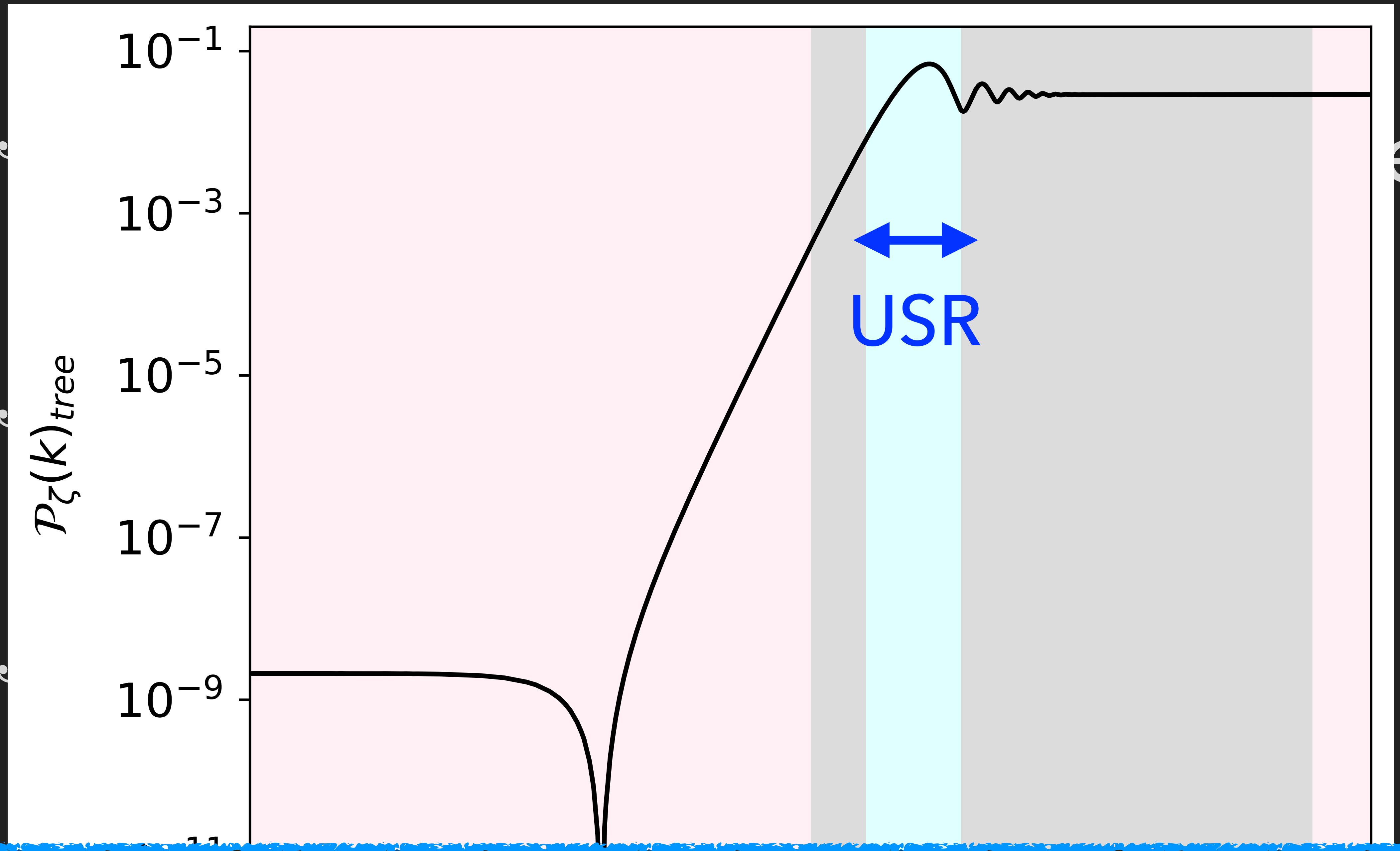
1LC FOR SMOOTH AND INSTANTANEOUS TRANSITIONS

- USR evolution
- Effect of peak scales only (not rise and fall of a realistic spectrum)





$$\Delta\mathscr{P}_\zeta(p;t)_{1\text{loop}} = \Delta\mathscr{P}_\zeta(p;t)_{12} + \Delta\mathscr{P}_\zeta(p;t)_{22} + \Delta\mathscr{P}_\zeta(p;t)_{13} \ll 1$$

1LC IN THE SEPARATE UNIVERSE PICTURE

- General framework to calculate back-reaction at 1-loop from enhanced small-scale modes on large scales using separate universe picture
- Classical back-reaction model: explicit understanding of the physical origin of the 1LC
- Specialise our general framework to one example: transient USR phase
- 12-type and 13-type are *not* volume suppressed: requires non-linearity on small scales + long-short mode coupling
- In absence of long-short mode couplings: 22-type, volume suppressed
- Our results do not depend on the quality of the the USR→SR transition
- Separate universe picture *vs* in-in formalism: physics is more transparent

{2312.12424 - LI, Mulryne, Seery}

THE END?

- Smoothing scale ($\Lambda^{-1} \sim \lambda_{\text{peak scales}}$ in our set up) works as EFT cut-off: effect of UV modes $k \gg \Lambda \sim k_{\text{peak}}$ is included in couterterms
- Counterterms that renormalise $\dot{\zeta}^2$ operator produce same momentum dependence as 12- and 13-type loops
- 1LC is degenerate with counterterms which include the effect of UV modes
- Up to now *not* found genuine (not degenerate) loop correction $\mathscr{P}_\zeta(p)_{\text{1loop}} \sim \log p$
- *How does this calculation depends on assumption of plateau of peak scales?*
- *Renormalisation, …*

{2312.12424 - LI, Mulryne, Seery}