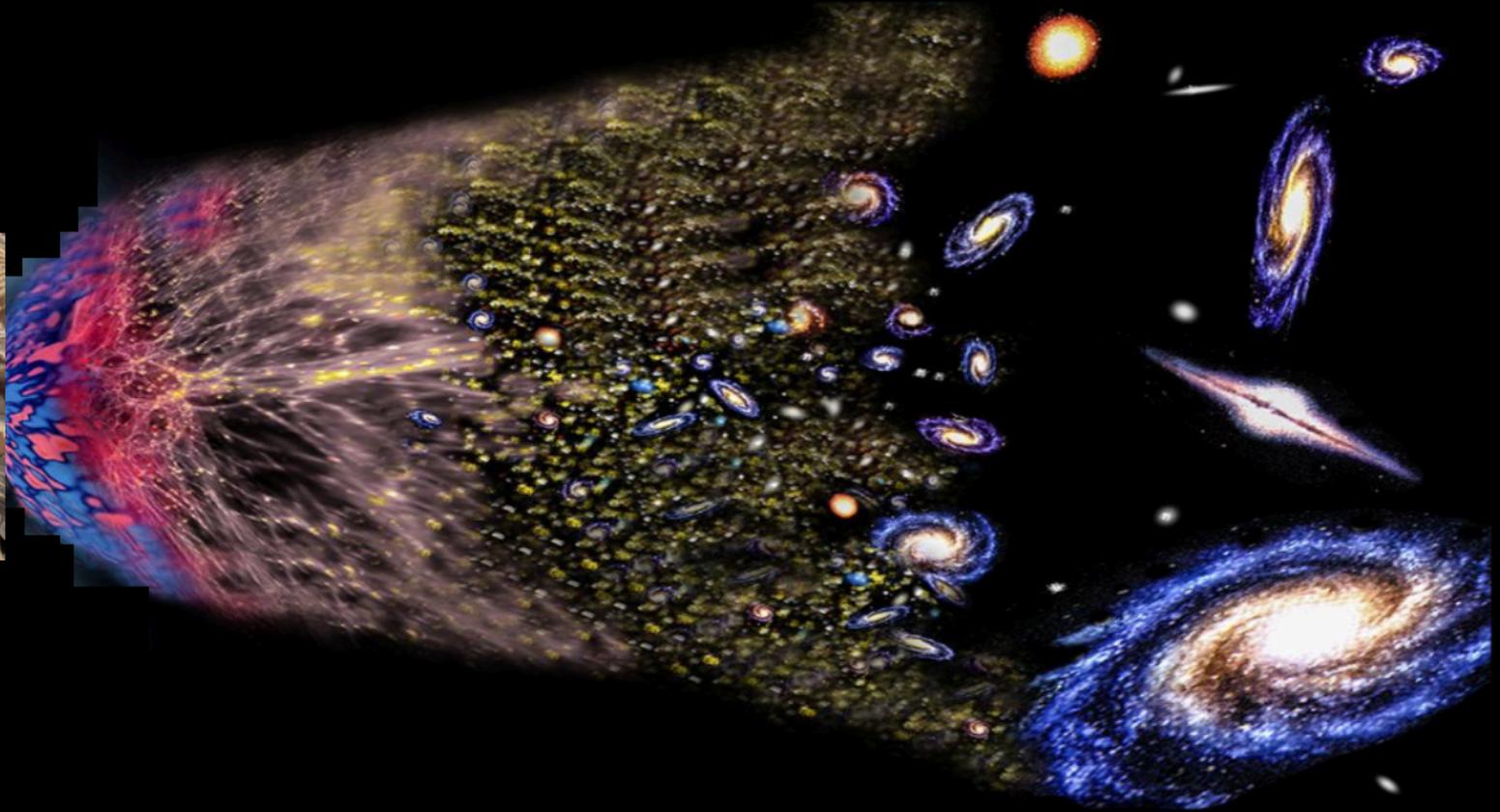


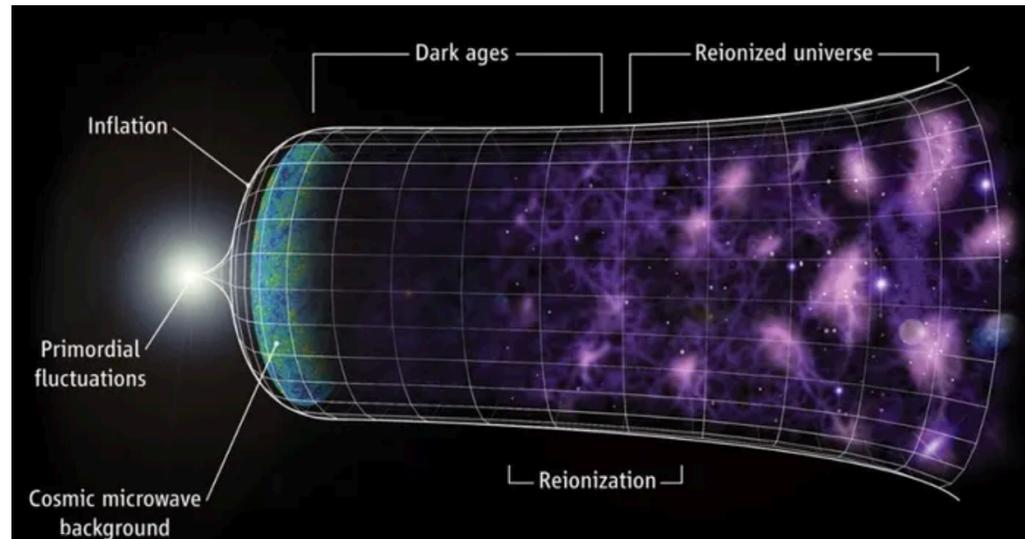
Inflation from Purgatory

Scott Watson (Syracuse University)

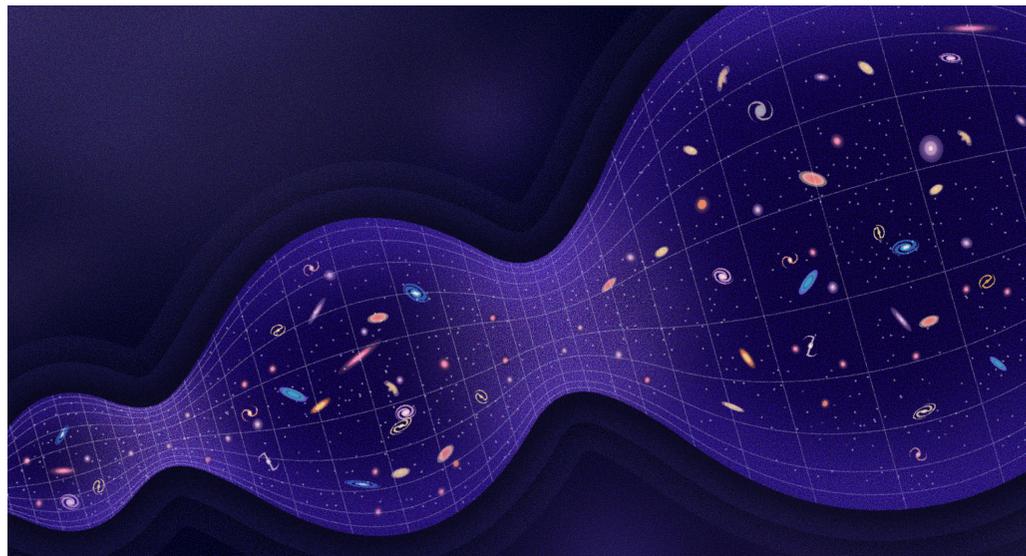


<http://scottwatsoncosmology.org/>

Did the universe begin with inflation and an initial singularity? (Past geodesic incompleteness)

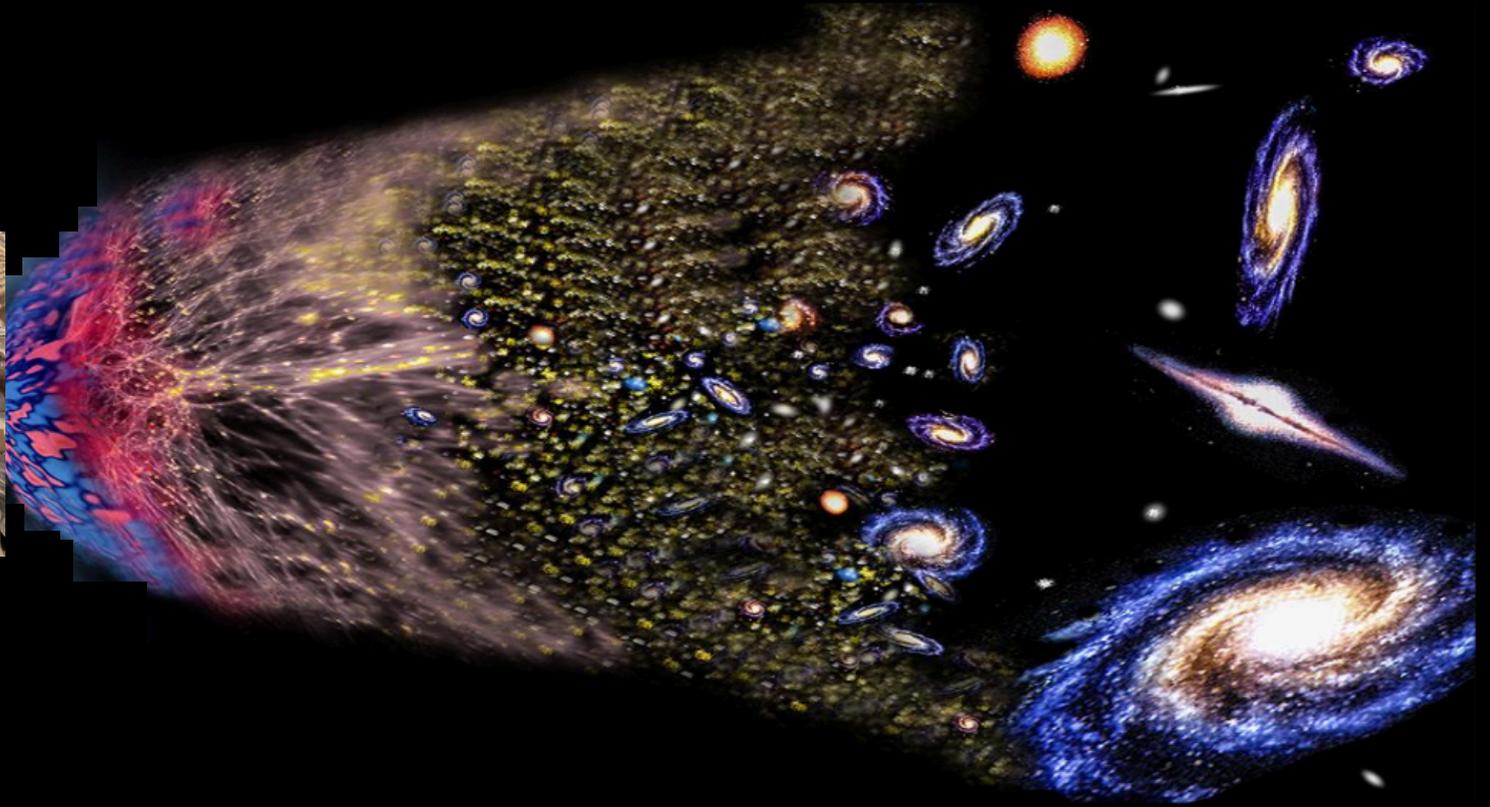


Or was there a bounce?



A third possibility?

A Lingering Universe



Waiting for Inflation: A New Initial State for the Universe
ArXiv: 2310.06019
Brandon Melcher, Arnab Pradhan (Syracuse), S.W.

PLAN

Part One:

Lingering in a classical Universe

Part Two:

Lingering from String Theory

PLAN

Part One:

Lingering in a classical Universe

Part Two:

Lingering from String Theory

FLRW with positive spatial curvature

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right)$$

$$3H^2 = \kappa^2 \rho - \frac{3K}{a^2}$$

$$\dot{H} = -\frac{\kappa^2}{2} (\rho + p) + \frac{K}{a^2}$$

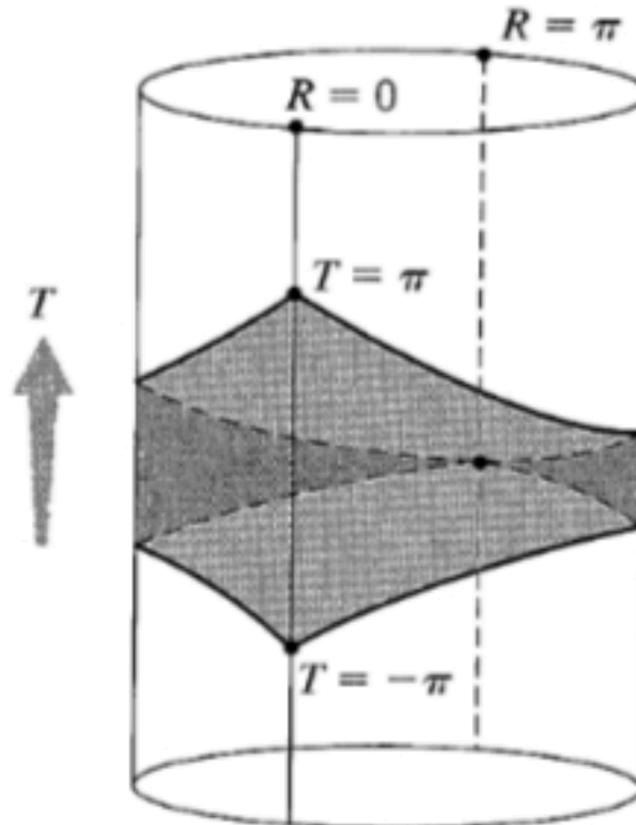
We will consider two fluids, one standard matter and the other sector obeying the Null Energy Condition.

$$\dot{\rho} + 3H(\rho + p) = 0 \quad \rho = \rho_m + \rho_e \quad -1 \leq w_e \leq -1/3.$$

Early models of cosmology

$a(t)$	$H(t)$	K	Λ	Ricci Scalar	Description
1	0	0	0	0	Minkowski (\mathcal{M}^4)
$\exp(H_0 t)$	$\sqrt{\Lambda/3}$	0	Λ	$4\Lambda = 12H_0^2$	De Sitter (dS)
a_0	0	+1	0	$(6K)/a_0^2$	Einstein Static Universe
a_0	0	+1	Λ	$(6K)/a_0^2$	Lemaître (a.k.a. lingering)

The Einstein static universe, $\mathbf{R} \times S^3$, portrayed as a cylinder. The shaded region is conformally related to Minkowski space



FLRW with positive spatial curvature

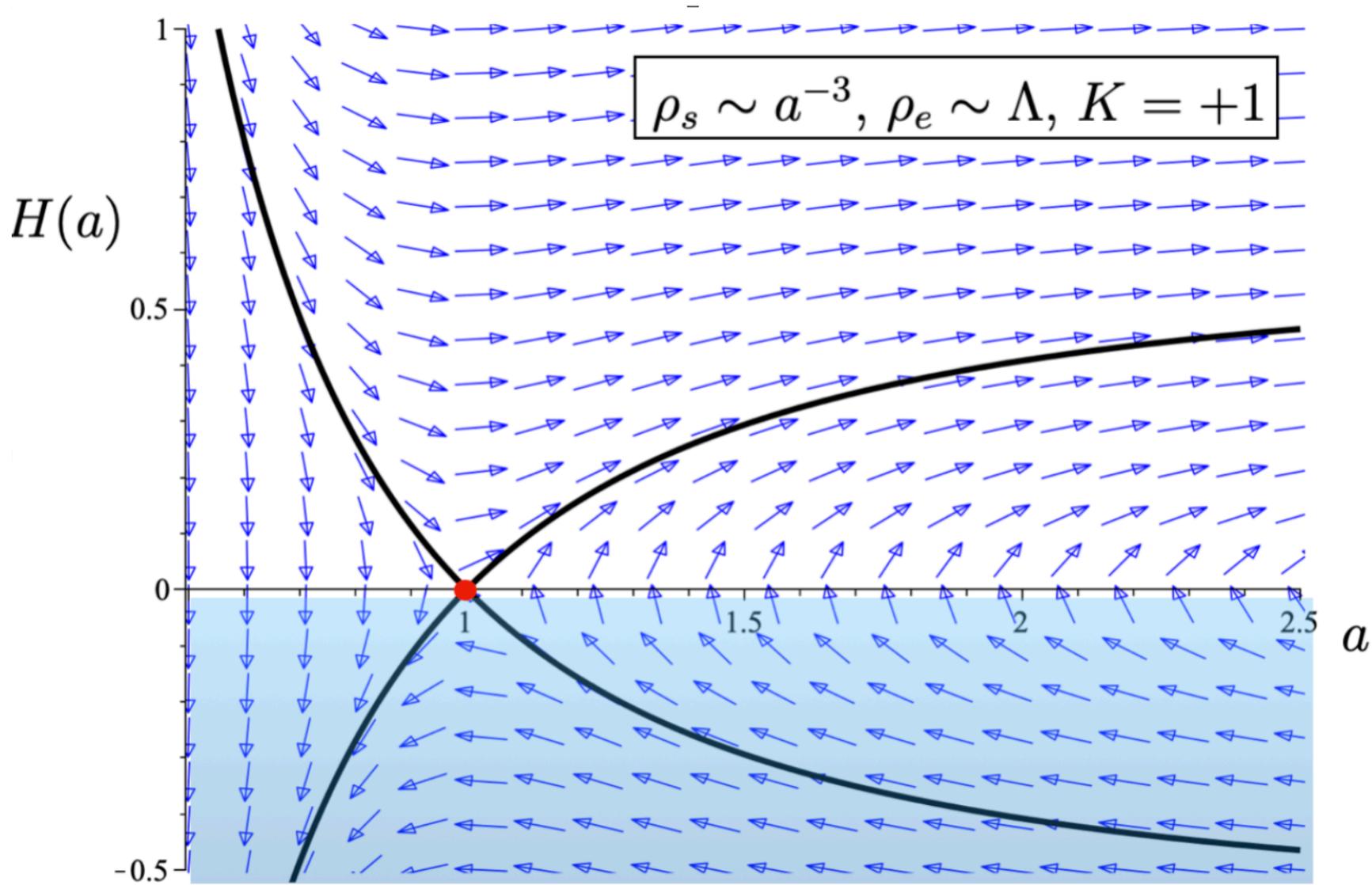
$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right)$$

$$3H^2 = \kappa^2 \rho - \frac{3K}{a^2} + \Lambda$$

$$\dot{H} = -\frac{\kappa^2}{2} (\rho + p) + \frac{K}{a^2}$$

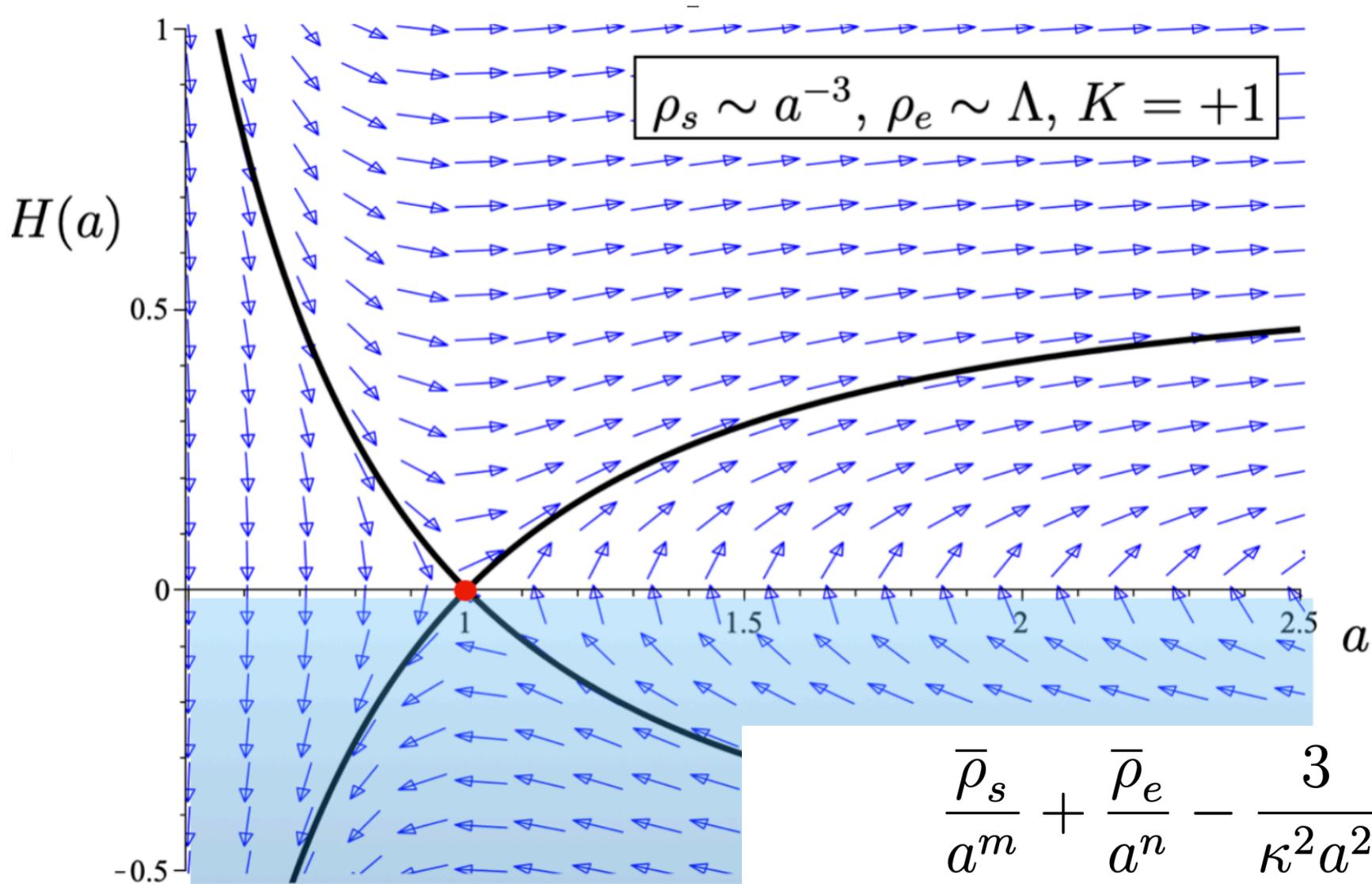
Spatial curvature? What about observations?

Linger leads to inflation (which wipes out spatial curvature).



Two fluid model

Linger leads to inflation (which wipes out spatial curvature).



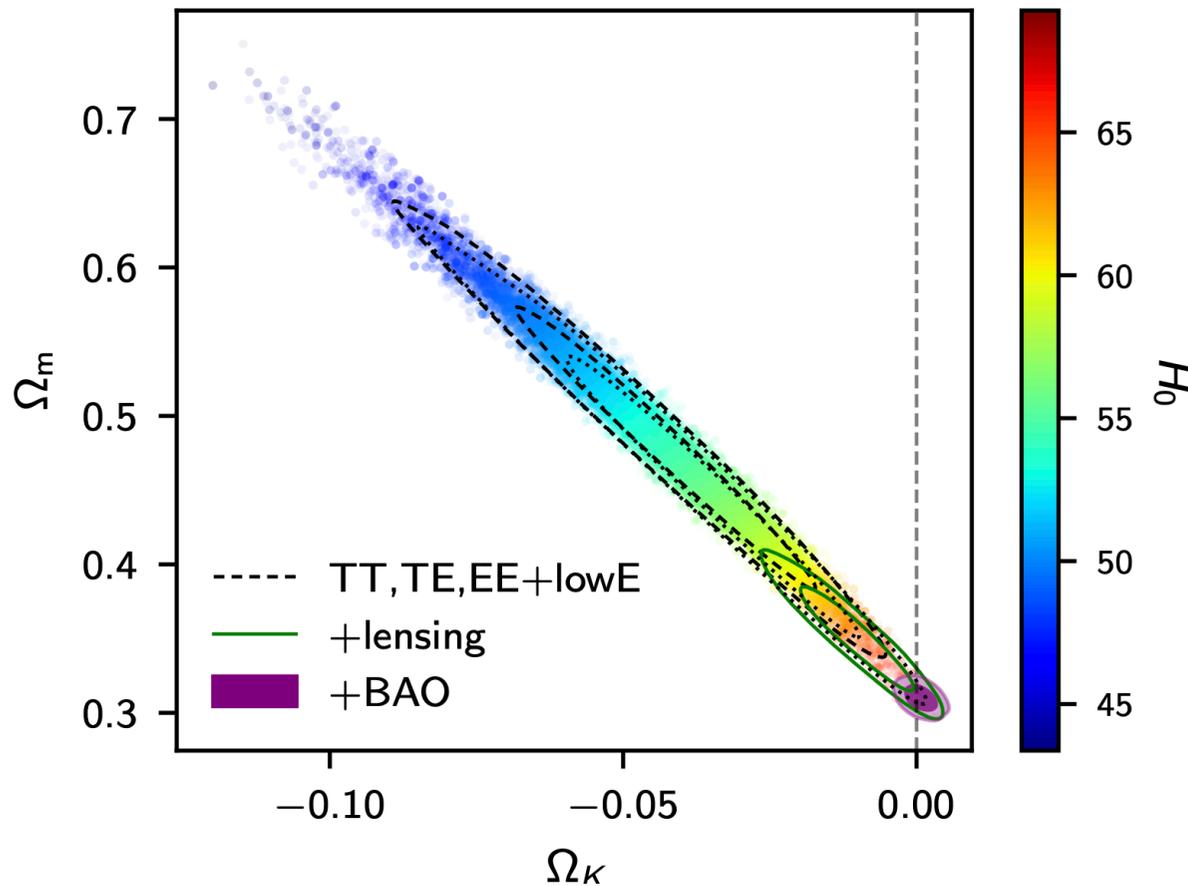
$$\frac{\bar{\rho}_s}{a^m} + \frac{\bar{\rho}_e}{a^n} - \frac{3}{\kappa^2 a^2} = 0$$

$$(m - 2) \frac{\bar{\rho}_s}{a^m} + (n - 2) \frac{\bar{\rho}_e}{a^n} = 0$$

However...

Depending on the duration of inflation there could be a small amount of spatial curvature observable today (recall Planck 2018 results)

<https://arxiv.org/abs/1807.06209>

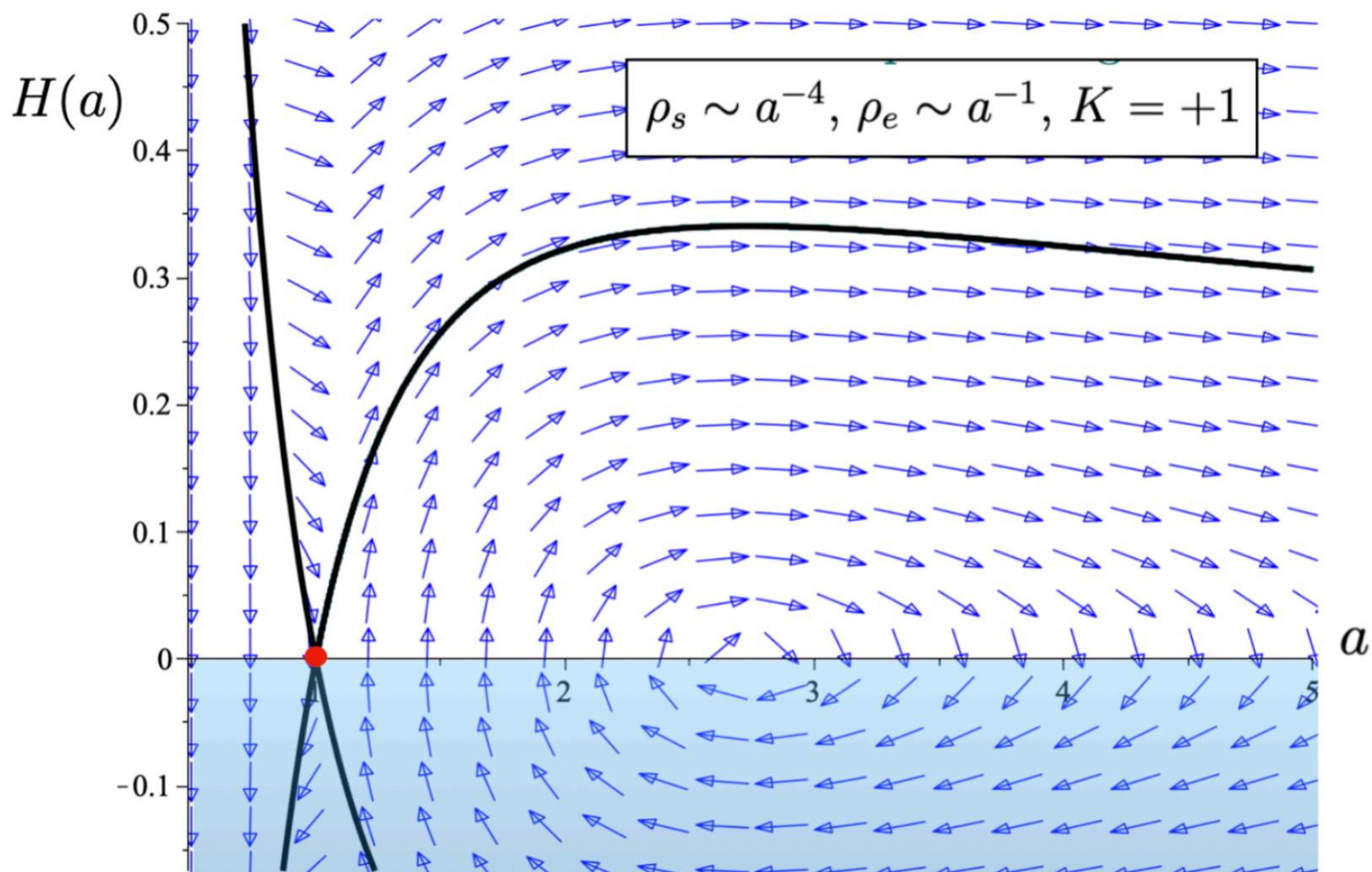


$\Omega_K = 0.0007 \pm 0.0019$ (68 %, TT,TE,EE+lowE +lensing+BAO).

Provocatively: Could this be a signal of a lingering universe?

Linger leads to inflation (which wipes out spatial curvature).

Lingering to Inflation with Radiation and String Network



Two fluid model

Many outstanding questions

Origin of initial conditions?

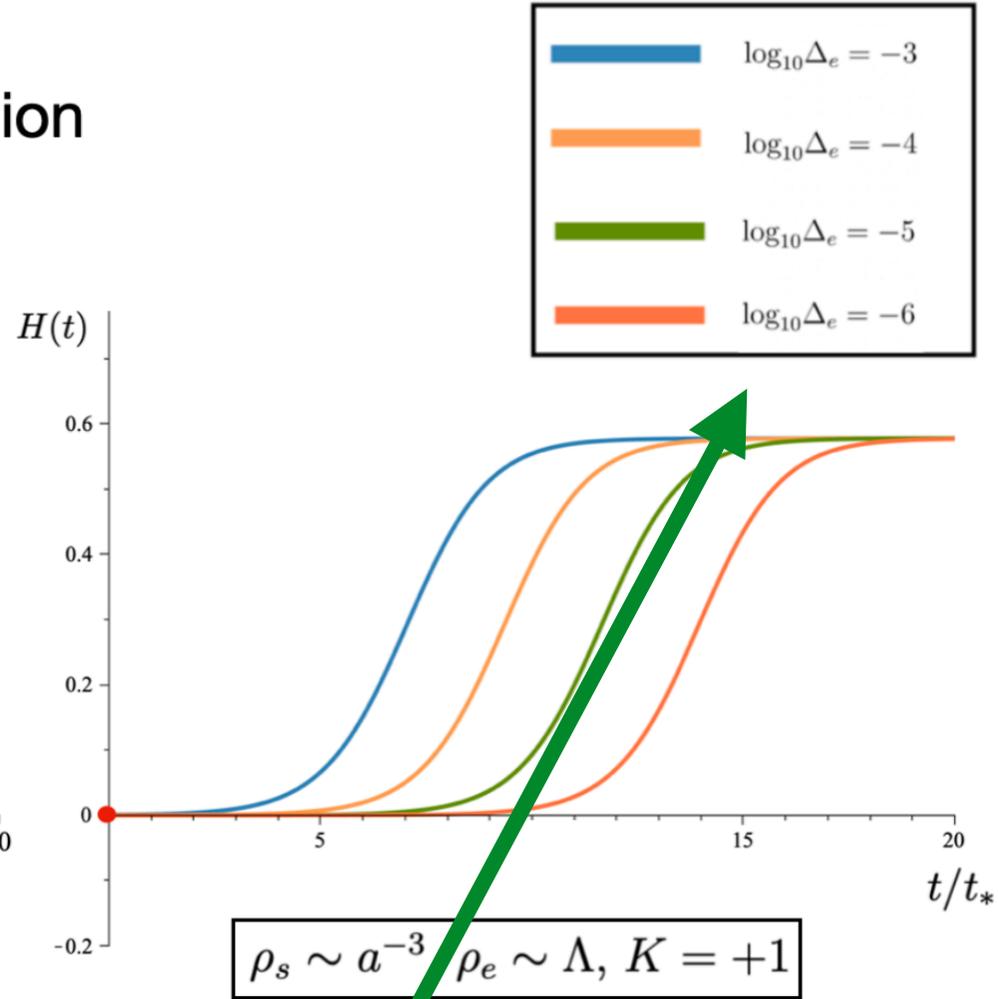
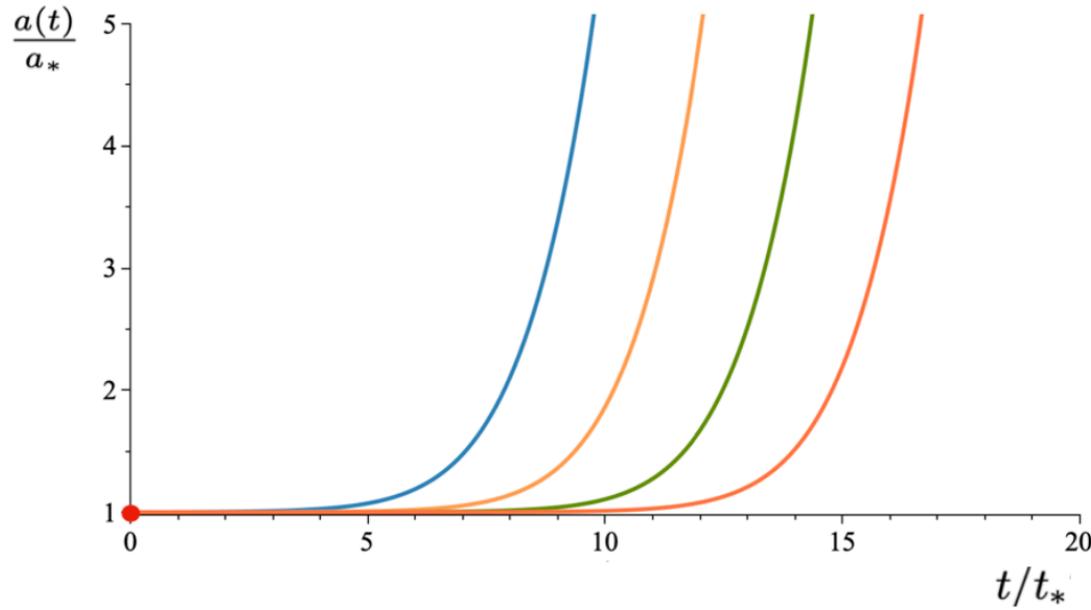
What determines the abundance of each fluid?

Including spatial curvature leads to new models, but
where is the fundamental motivation?

Can you have a lingering universe without spatial
curvature?

**Summary: such solutions are possible
And can lead to inflation without initial singularity
(kinda of — we need to explore further).**

Transition from Lingering to Inflation

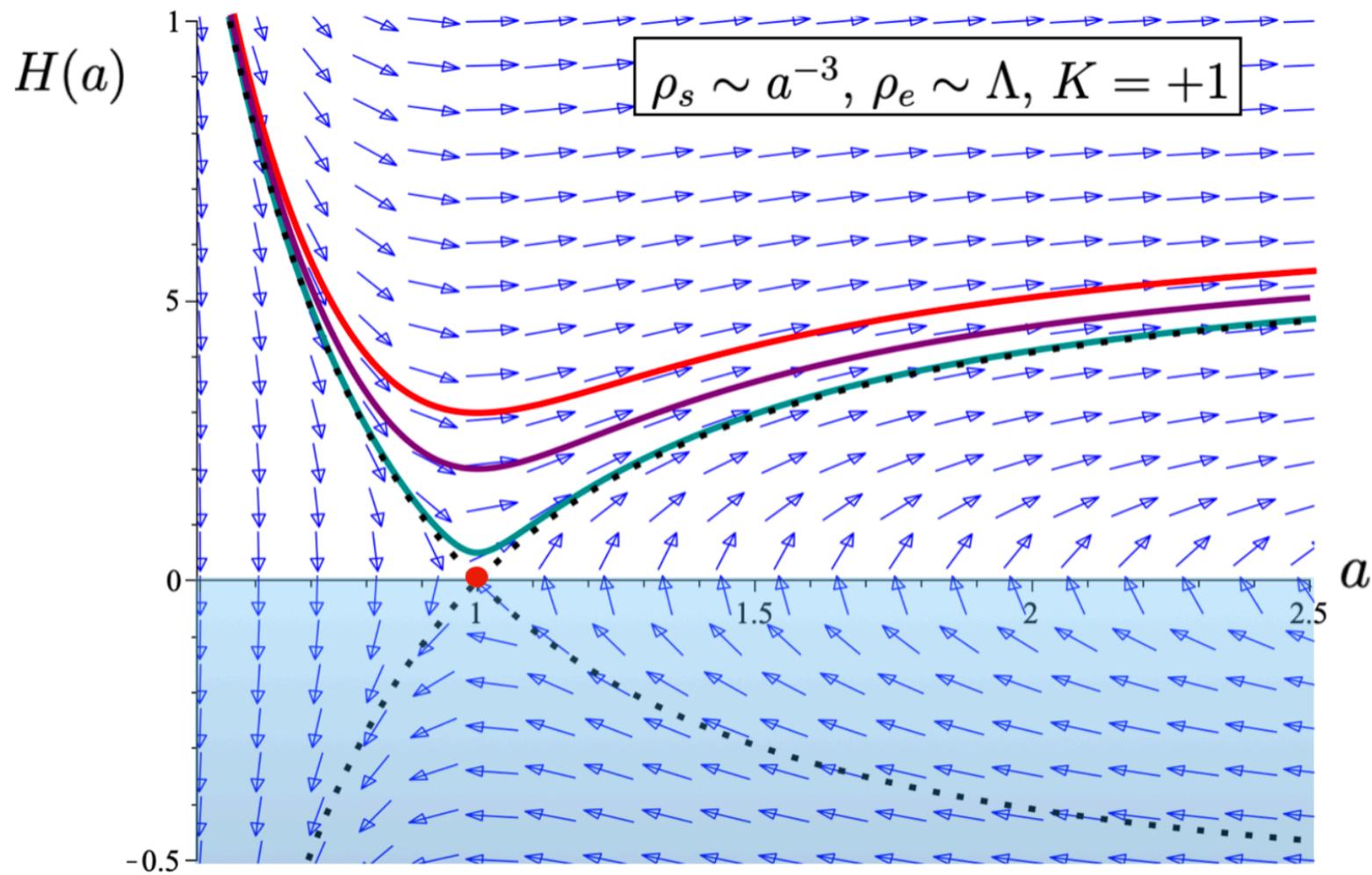


Lingering Initial Conditions

$$\frac{\bar{\rho}_s}{a^m} + \frac{\bar{\rho}_e}{a^n} - \frac{3}{\kappa^2 a^2} = 0$$

$$(m - 2) \frac{\bar{\rho}_s}{a^m} + (n - 2) \frac{\bar{\rho}_e}{a^n} = 0$$

**Departure from lingering
Initial conditions**



$$\frac{\bar{\rho}_s}{a^m} + \frac{\bar{\rho}_e}{a^n} - \frac{3}{\kappa^2 a^2} = 0$$

$$(m-2)\frac{\bar{\rho}_s}{a^m} + (n-2)\frac{\bar{\rho}_e}{a^n} = 0$$

Lingering Initial Conditions

PLAN

Part One:

Lingering in a classical Universe

Part Two:

Lingering from String Theory

Some of the motivation for this part:

Loitering phase in brane gas cosmology

Robert Brandenberger (Brown U.), Damien A. Easson (Brown U. and McGill U.), Dagny Kimberly (Brown U.) (Sep, 2001)

Published in: *Nucl.Phvs.B* 623 (2002) 421-436 • e-Print: [hep-th/0109165](https://arxiv.org/abs/hep-th/0109165) [hep-th]

String gas cosmology

Thorsten Battefeld (Brown U.), Scott Watson (Toronto U.) (Oct, 2005)

Published in: *Rev.Mod.Phys.* 78 (2006) 435-454 • e-Print: [hep-th/0510022](https://arxiv.org/abs/hep-th/0510022) [hep-th]

Geometric precipices in string cosmology

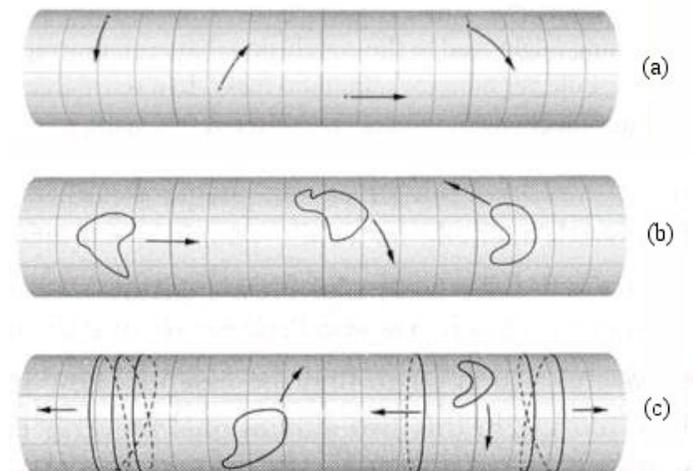
Nemanja Kaloper (UC, Davis), Scott Watson (Michigan U., MCTP) (Dec, 2007)

Published in: *Phys.Rev.D* 77 (2008) 066002 • e-Print: [0712.1820](https://arxiv.org/abs/0712.1820) [hep-th]

Waiting for Inflation: A New Initial State for the Universe

ArXiv: [2310.06019](https://arxiv.org/abs/2310.06019)

Brandon Melcher, Arnab Pradhan (Syracuse), S.W.



Model independent String Cosmology

String frame action

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-G} e^{-\phi_s} \left[R + (\partial\phi_s)^2 - \frac{1}{12} H_3^2 \right] + \frac{1}{2} \int d^4x \mathcal{L}_m(\phi_s, G_{\mu\nu}, \dots)$$

$$e^{-\phi_s} = \frac{\text{Vol } \mathcal{M}_6}{(2\pi\sqrt{\alpha'})^6} \frac{1}{g_s^2}, \quad 2\kappa_4^2 = 2\pi\alpha' \equiv \frac{1}{M_s^2},$$

$$ds_s^2 \rightarrow ds_E^2 = e^{-\phi_s} ds_s^2, \quad G_{\mu\nu} \rightarrow g_{\mu\nu} = e^{-\phi_s} G_{\mu\nu}, \quad \phi_s \rightarrow \phi_E = \phi_s$$

Einstein frame action

$$S = \int d^4x \sqrt{g} \left[\frac{R[g_{\mu\nu}]}{2\kappa^2} - \frac{1}{2} (\partial\phi_E)^2 - \frac{1}{2} e^{2\phi_E} (\partial\chi)^2 \right] + \frac{1}{2} \int d^4x \tilde{\mathcal{L}}_m(\phi_E, g_{\mu\nu} e^{\phi_E}, \dots)$$

Model independent String Cosmology

WHAT I AM NOT TALKING ABOUT

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-G} e^{-\phi_s} \left[R + (\partial\phi_s)^2 - \frac{1}{12} H_3^2 \right] + \frac{1}{2} \int d^4x \mathcal{L}_m(\phi_s, G_{\mu\nu}, \dots)$$
$$e^{-\phi_s} = \frac{\text{Vol } \mathcal{M}_6}{(2\pi\sqrt{\alpha'})^6 g_s^2}, \quad 2\kappa_4^2 = 2\pi\alpha' \equiv \frac{1}{M_s^2},$$

$$ds_s^2 \rightarrow ds_E^2 = e^{-\phi_s} ds_s^2, \quad G_{\mu\nu} \rightarrow g_{\mu\nu} = e^{-\phi_s} G_{\mu\nu}, \quad \phi_s \rightarrow \phi_E = \phi_s$$

These equations lead to cosmological solutions that have been used to argue for alternatives to inflation.

Examples: String Gas Cosmology, Pre-Big Bang

String gas cosmology

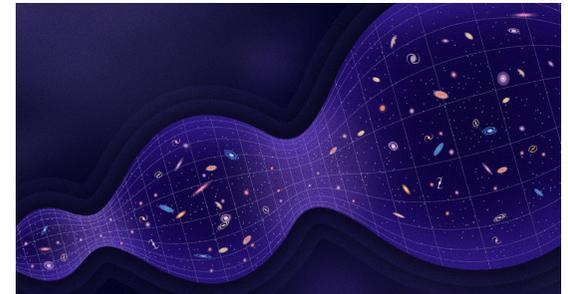
Thorsten Battefeld (Brown U.), Scott Watson (Toronto U.) (Oct, 2005)

Published in: *Rev.Mod.Phys.* 78 (2006) 435-454 • e-Print: [hep-th/0510022](https://arxiv.org/abs/hep-th/0510022) [hep-th]

The Pre - big bang scenario in string cosmology

M. Gasperini (Bari U. and INFN, Bari), G. Veneziano (CERN) (Jul, 2002)

Published in: *Phys.Rept.* 373 (2003) 1-212 • e-Print: [hep-th/0207130](https://arxiv.org/abs/hep-th/0207130) [hep-th]



Such approaches violate the Null Energy Condition (NEC), resulting in catastrophic instabilities and fine-tuning of initial conditions

T_H Hagedorn Cosmology

Key point: There is a limiting temperature in string theory
And therefore limiting energy.

Cosmological Dynamics

$$ds^2 = n(t)^2 dt^2 + \sum_{i=1}^N e^{\lambda_s^i(t)} dx_i^2$$

Shifted dilaton
(Helps manifest dualities)

$$\varphi_s = \phi_s - 3\lambda_s$$

$$\dot{\phi}_s = 3H_s \pm \sqrt{3H_s^2 + e^{\phi_s} \rho_s},$$

$$\dot{H}_s = \pm H_s \sqrt{3H_s^2 + e^{\phi_s} \rho_s} + \frac{1}{2} e^{\phi_s} (p_s + \Delta_\phi \mathcal{L}_m),$$

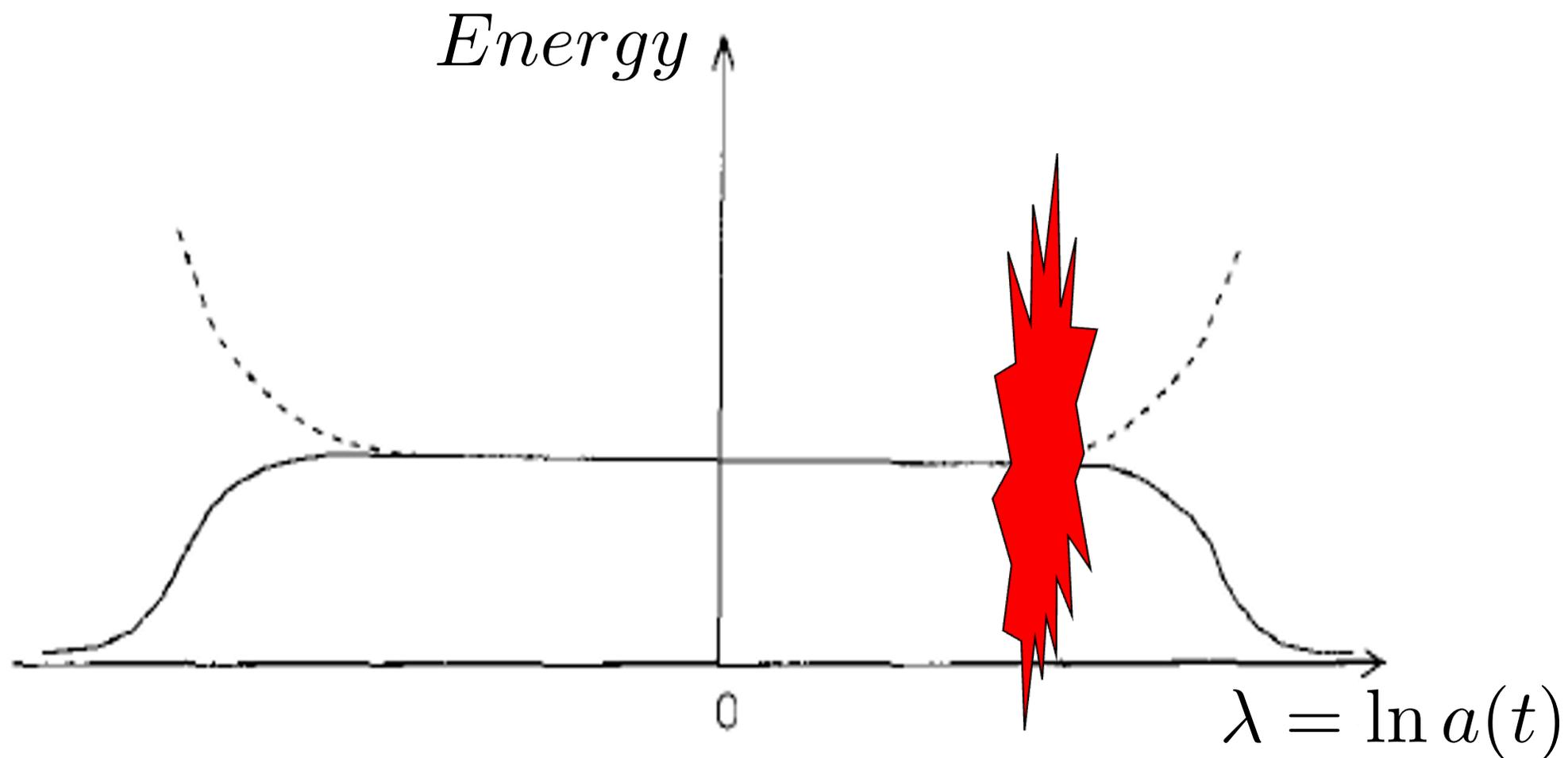
$$\dot{\rho}_s + 3H_s (\rho_s + p_s) = -\dot{\phi}_s \Delta_\phi \mathcal{L}_m,$$

Elements of string cosmology

Arkady A. Tseytlin (Johns Hopkins U.), C. Vafa (Harvard U.) (Sep, 1991)

Published in: *Nucl.Phys.B* 372 (1992) 443-466 • e-Print: [hep-th/9109048](https://arxiv.org/abs/hep-th/9109048) [hep-th]

Hagedorn Cosmology



T_H Hagedorn Cosmology

Particles behave as strings at high temperature (early universe)

Cosmological Dynamics

$$E_s = -(F + \beta \partial_\beta F) \quad P = \partial_{\lambda_s} F = -\partial_{\lambda_s} E \big|_{\beta=\text{const}}$$

$$dx = \left(\frac{E}{E_0} \right) dt.$$

$$\begin{aligned} \varphi_s'^2 - 3\lambda_s'^2 &= E_0 e^{\varphi_s + 3\gamma\lambda_s}, \\ \varphi_s'' - 3\gamma\varphi_s'\lambda_s' - 3\lambda_s'^2 &= \frac{1}{2} E_0 e^{\varphi_s + 3\gamma\lambda_s}, \\ \lambda_s'' - 3\gamma\lambda_s'^2 - \varphi_s'\lambda_s' &= \frac{1}{2} \gamma E_0 e^{\varphi_s + 3\gamma\lambda_s}, \end{aligned}$$

T_H Hagedorn Cosmology

One can find exact solutions

$$\begin{aligned}\lambda_s &= \lambda_{s0} + \frac{\gamma}{\alpha} \ln [(x - x_-)(x - x_+)] + \frac{1}{\alpha\sqrt{3}} \ln \left(\frac{x - x_+}{x - x_-} \right), \\ \varphi_s &= \varphi_{s0} - \frac{1}{\alpha} \ln [(x - x_-)(x - x_+)] - \frac{\gamma\sqrt{3}}{\alpha} \ln \left(\frac{x - x_+}{x - x_-} \right), \\ \phi_s &= \phi_{s0} - \ln [(x - x_-)(x - x_+)] - \frac{(\gamma - 1)\sqrt{3}}{\alpha} \ln \left(\frac{x - x_+}{x - x_-} \right),\end{aligned}$$

$$dx = \left(\frac{E}{E_0} \right) dt.$$

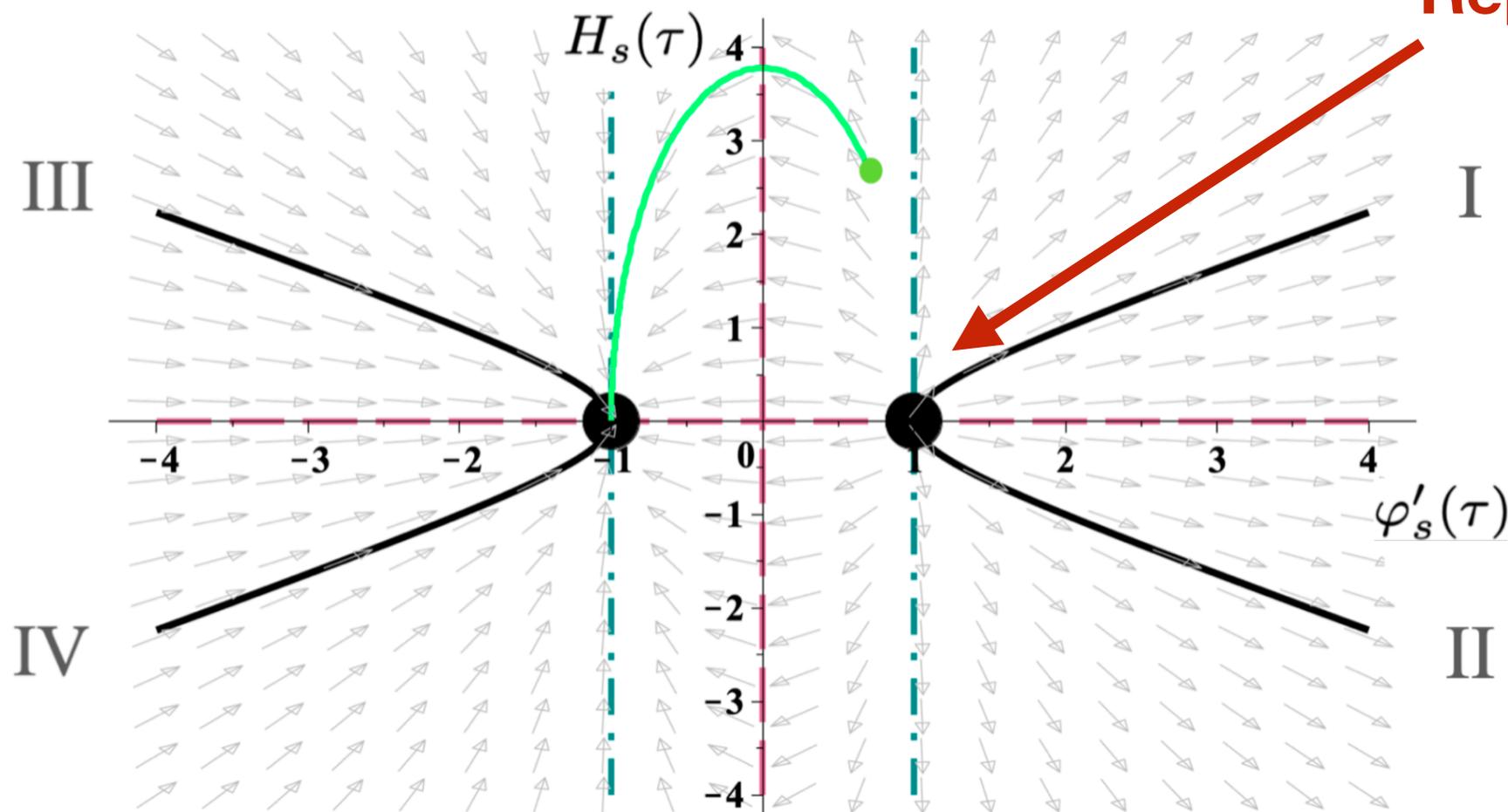
T_H Hagedorn Cosmology

Region	Branch	Expansion	Shifted Dilaton	Dilaton
I	(+)	$H_s > 0$	$\dot{\phi}_s > 0$	$\dot{\phi}_s > 0$
II	(+)	$H_s < 0$	$\dot{\phi}_s > 0$	$\dot{\phi}_s > 0$ or $\dot{\phi}_s < 0$
III	(-)	$H_s > 0$	$\dot{\phi}_s < 0$	$\dot{\phi}_s > 0$ or $\dot{\phi}_s < 0$
IV	(-)	$H_s < 0$	$\dot{\phi}_s < 0$	$\dot{\phi}_s < 0$

Hagedorn Phase Space

$$T = T_{\text{Hagedorn}}$$

Repeller



One solution is asymptotic inflation driven by dilaton

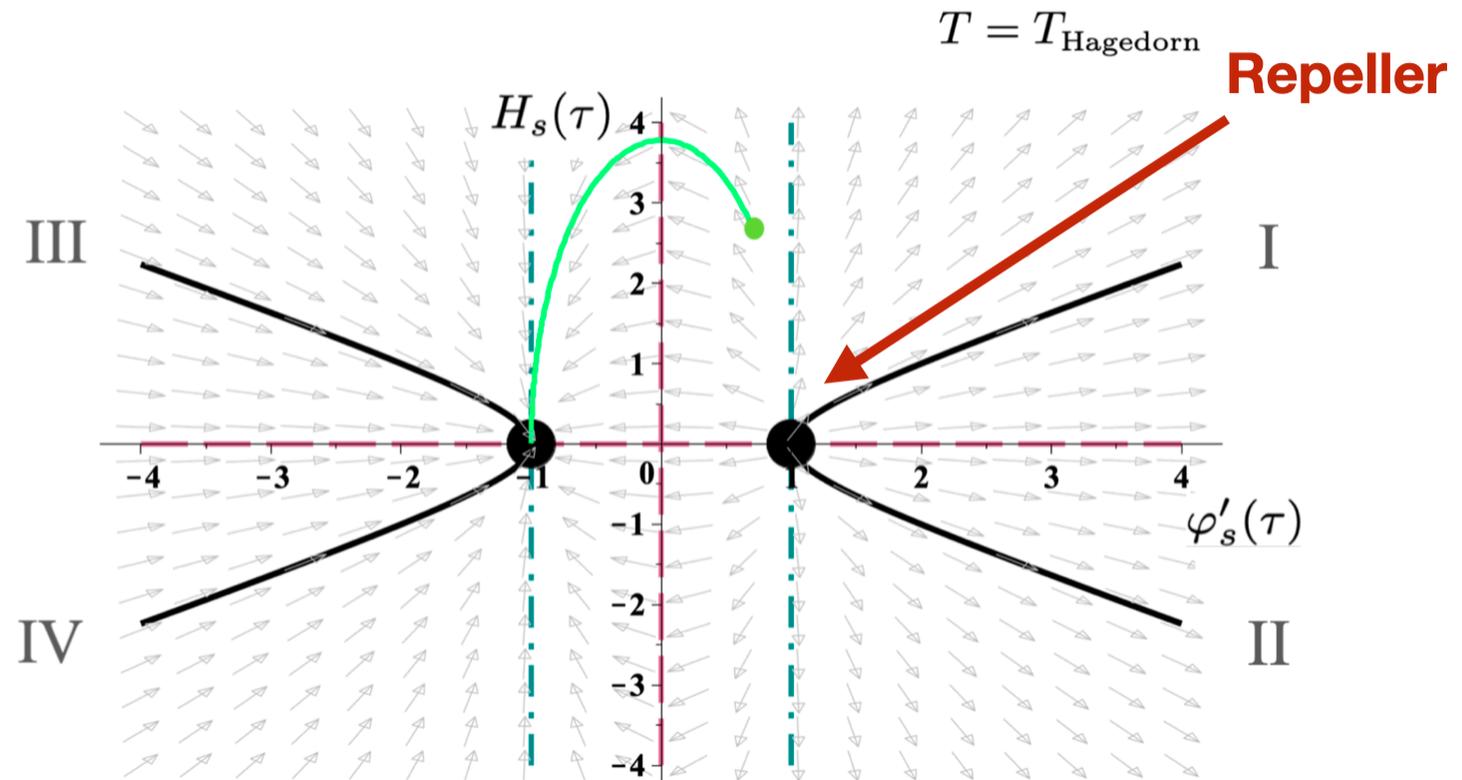
Dynamics come with “branches” (super-selection sectors)

$$\dot{\phi}_s = 3H_s \left(\pm \sqrt{3H_s^2 + e^{\phi_s} \rho_s} \right),$$

$$\dot{H}_s = \left(\pm H_s \sqrt{3H_s^2 + e^{\phi_s} \rho_s} + \frac{1}{2} e^{\phi_s} (p_s + \Delta_\phi \mathcal{L}_m) \right),$$

$$\dot{\rho}_s + 3H_s (\rho_s + p_s) = -\dot{\phi}_s \Delta_\phi \mathcal{L}_m,$$

Hagedorn Phase Space



One solution is asymptotic inflation driven by dilaton

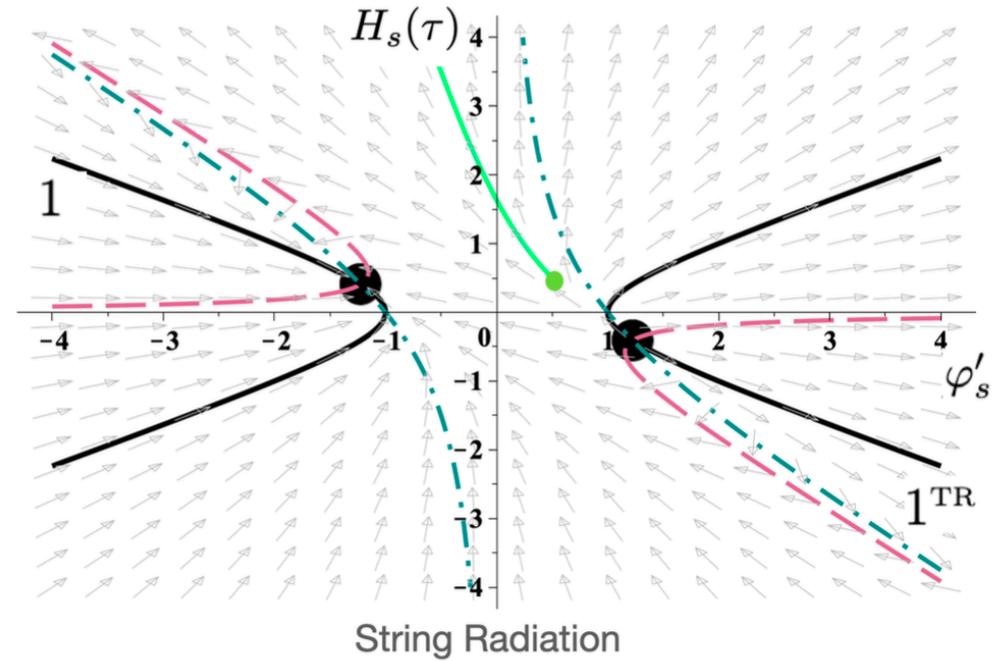
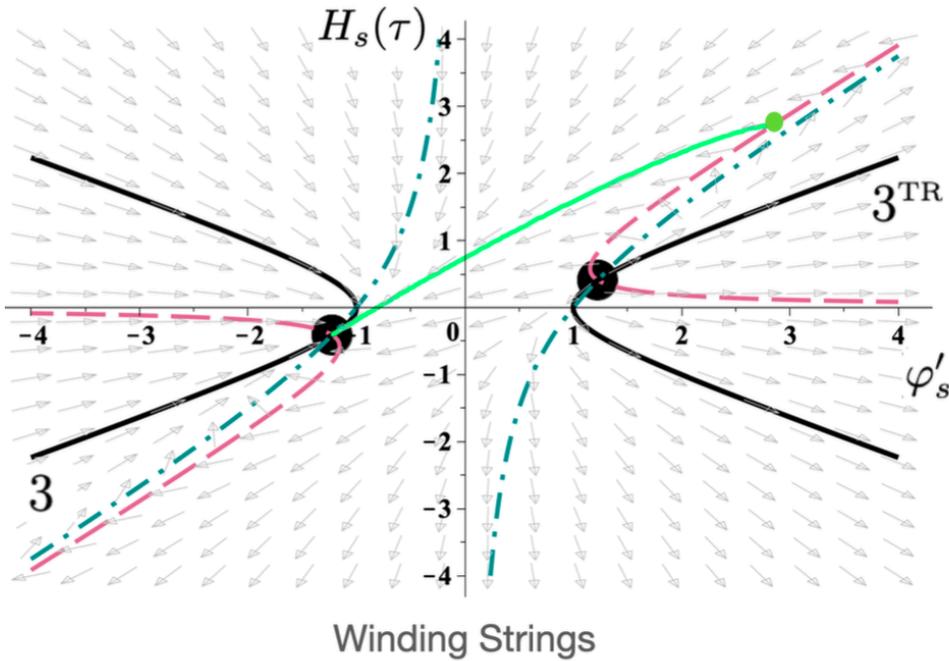
Post Hagedorn Cosmology

$$T \lesssim T_H$$

Region	Branch	Expansion	Shifted Dilaton
1	(-)	$H_s > 0$	$\dot{\varphi}_s < 0$
1^{TR}	(+)	$H_s < 0$	$\dot{\varphi}_s > 0$
3	(-)	$H_s < 0$	$\dot{\varphi}_s < 0$
3^{TR}	(+)	$H_s > 0$	$\dot{\varphi}_s > 0$

Post-Hagedorn Phase Spaces

$$T < T_{\text{Hagedorn}}$$



“Stringy matter phases”

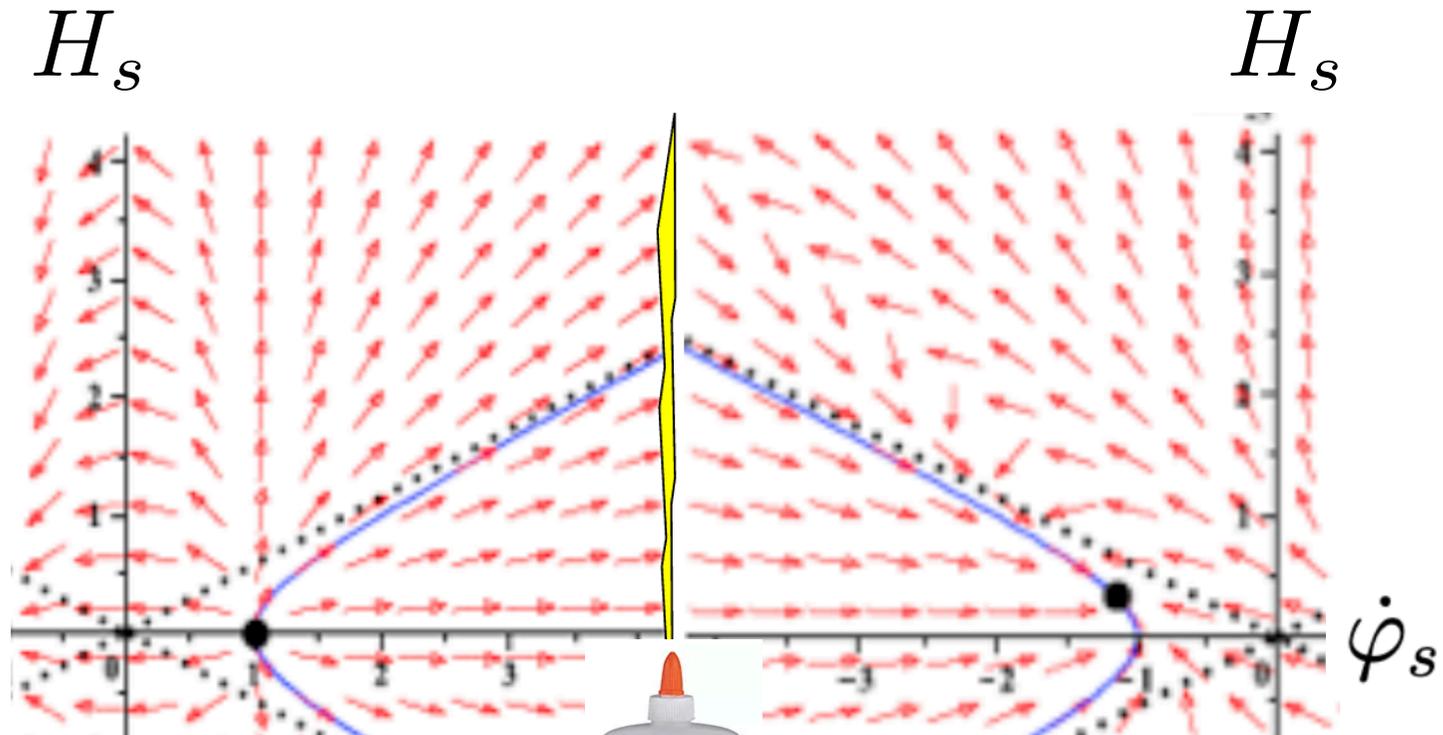
The Problem with String Gas Cosmology is NEC violation

Geometric precipices in string cosmology

Nemanja Kaloper (UC, Davis), Scott Watson (Michigan U., MCTP) (Dec, 2007)

Published in: *Phys.Rev.D* 77 (2008) 066002 • e-Print: [0712.1820](https://arxiv.org/abs/0712.1820) [hep-th]

Hagedorn I \rightarrow Momentum 1.



$$T \lesssim T_H$$



Region	Branch	Expansion	Shifted Dilaton
1	(-)	$H_s > 0$	$\dot{\varphi}_s < 0$
1 ^{TR}	(+)	$H_s < 0$	$\dot{\varphi}_s > 0$
3	(-)	$H_s < 0$	$\dot{\varphi}_s < 0$
3 ^{TR}	(+)	$H_s > 0$	$\dot{\varphi}_s > 0$

Null Energy Condition

To make this issue quantitative we can introduce the “egg function”

(Let me explain how my graduate thesis was wrong — kinda)

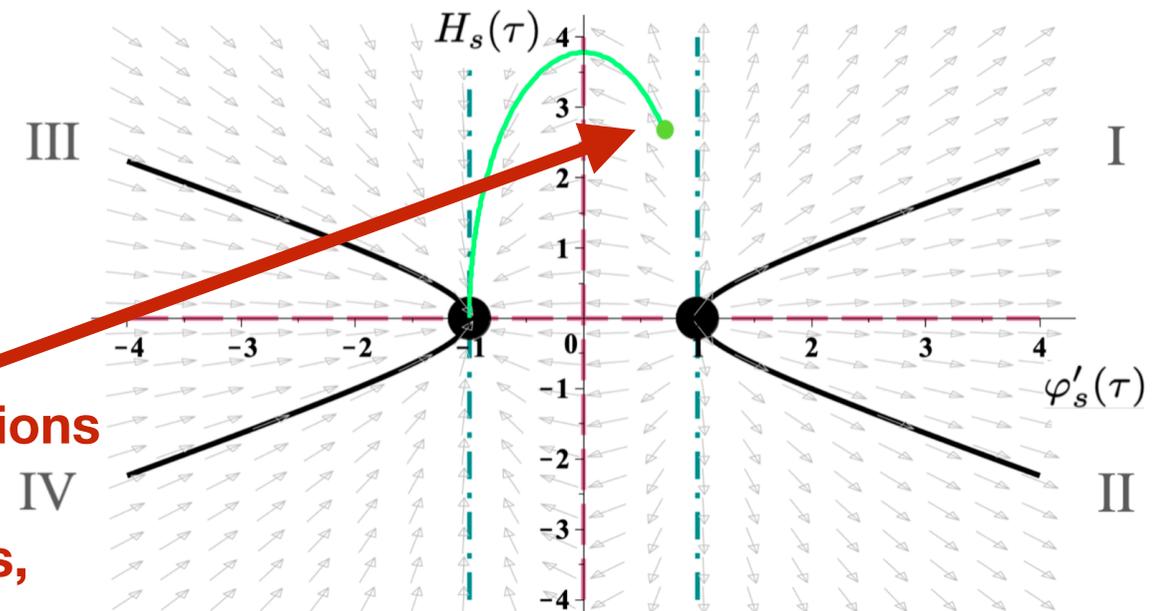
$$\dot{\phi}_s = 3H_s \pm \sqrt{3H_s^2 + e^{\phi_s} \rho_s},$$

$$\dot{H}_s = \pm H_s \sqrt{3H_s^2 + e^{\phi_s} \rho_s} + \frac{1}{2} e^{\phi_s} (p_s + \Delta_\phi \mathcal{L}_m),$$

$$\dot{\rho}_s + 3H_s (\rho_s + p_s) = -\dot{\phi}_s \Delta_\phi \mathcal{L}_m$$

Hagedorn Phase Space

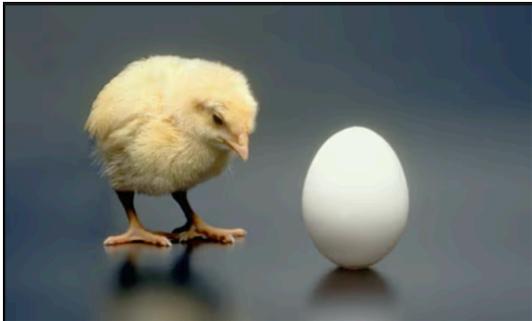
$T = T_{\text{Hagedorn}}$



You can not change branches!
(Without NEC violation and violations
of conservation of Energy,
a.k.a. the Hamiltonian constraints,
Such solutions are not viable.)

Null Energy Condition

$$T_{\mu\nu}n^\mu n^\nu \geq 0 \quad \longrightarrow \quad \rho + p \geq 0$$



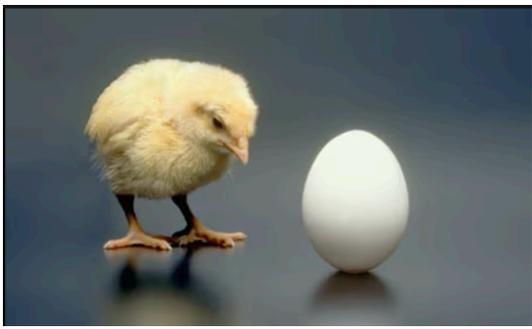
Egg Function

$$\dot{\phi}_s = \pm \sqrt{3\dot{\lambda}_s^2 + e^{\phi_s} \rho_s},$$

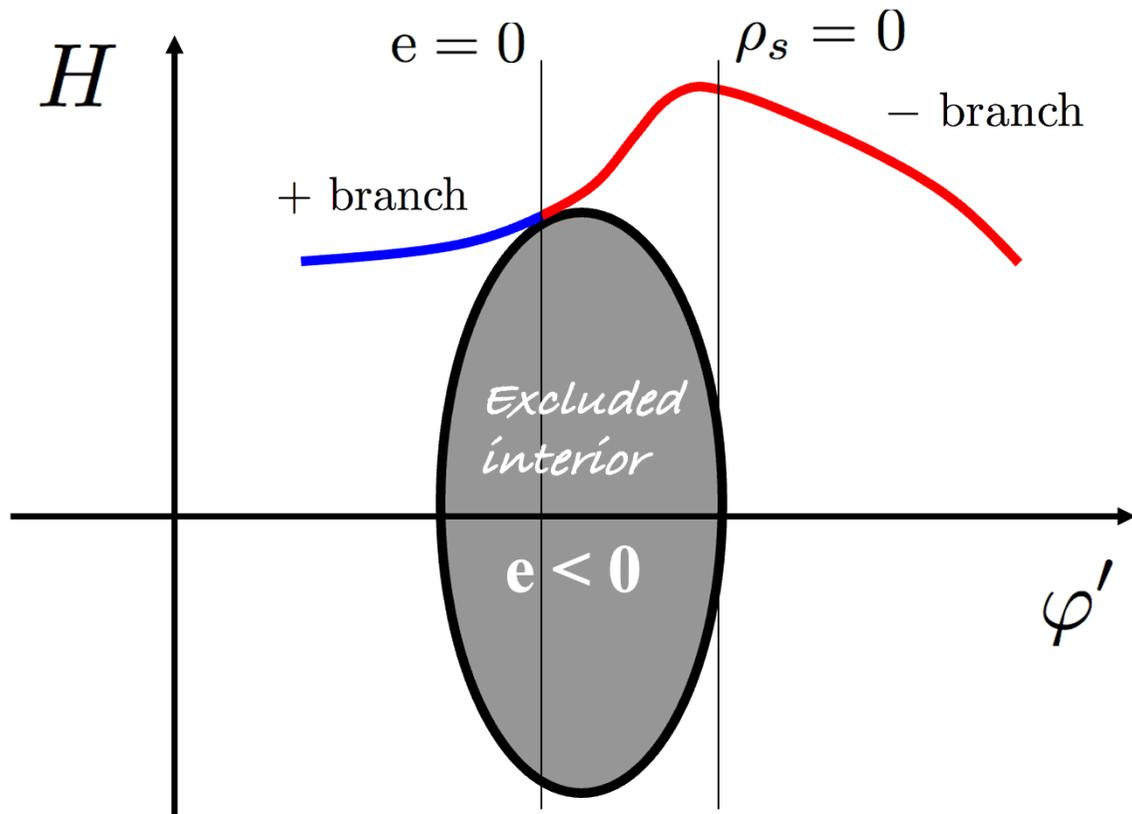
$$\dot{H}_s = \pm \dot{\lambda}_s \sqrt{3\dot{\lambda}_s^2 + e^{\phi_s} \rho_s} + \frac{1}{2} e^{\phi_s} (p_s + \Delta_\phi \mathcal{L}_m),$$

$$\dot{\rho}_s = -3\dot{\lambda}_s (\rho_s + p_s) - \dot{\phi} \Delta_\phi \mathcal{L}_m,$$

$$e = 3\dot{\lambda}_s^2 + e^{\phi_s} \rho_s$$

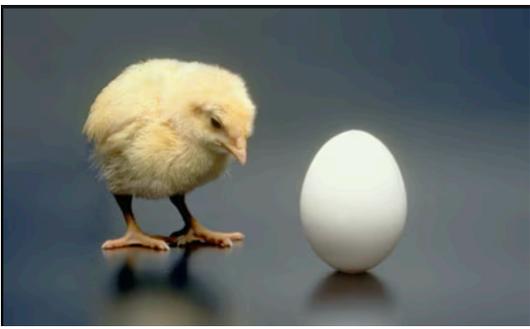


Egg Function



Violating the NEC is not enough.

The path in the phase space must violate it long enough and escape



Egg Function

$$e = 3\dot{\lambda}_s^2 + e^{\phi_s} \rho_s$$

Branch choice

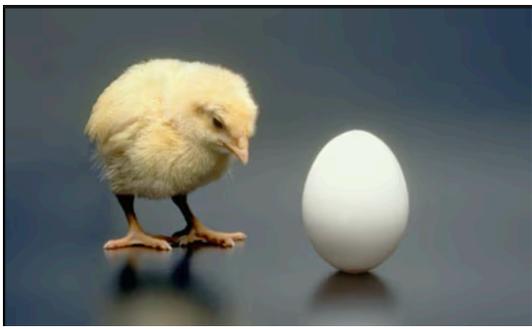
$$\pm \frac{d}{dt} (\sqrt{e}) = -\ddot{\lambda}_s + \dot{\lambda}_s \dot{\phi}_s + \frac{1}{2} e^{\phi_s} (\rho_s + p_s)$$

Escape

Negative Branch to positive

$$\int_{t_h}^{t_e} dt \left[\frac{d}{dt} \left(-\sqrt{e} + \dot{\lambda}_s \right) - \dot{\lambda}_s \dot{\phi}_s \right] = \frac{1}{2} \int_{t_h}^{t_e} dt e^{\phi} (\rho_s + p_s)$$

Branch change



Egg Function

Escape

Negative Branch to positive

$$\int_{t_h}^{t_e} dt \left[\frac{d}{dt} \left(-\sqrt{e} + \dot{\lambda}_s \right) - \dot{\lambda}_s \dot{\phi}_s \right] = \frac{1}{2} \int_{t_h}^{t_e} dt e^{\phi} (\rho_s + p_s)$$

Branch change

$$\left(\sqrt{3} - 1 \right) \dot{\lambda}_s(t_e) + \dot{\lambda}_s(t_h) + \mathcal{A} = -\frac{1}{2} \int_{t_h}^{t_e} dt e^{\phi} (\rho_s + p_s)$$

$$\mathcal{A} = \int \dot{\lambda}_s d\phi > 0 \quad \dot{\lambda} = H_s \quad \text{Always positive}$$

Positive, definite volume
of the phase space.



Egg Function

$$\left(\sqrt{3} - 1\right) \dot{\lambda}_s(t_e) + \dot{\lambda}_s(t_h) + \mathcal{A} = -\frac{1}{2} \int_{t_h}^{t_e} dt e^{\phi} (\rho_s + p_s)$$

$$\mathcal{A} = \int \dot{\lambda}_s d\phi > 0 \quad \dot{\lambda} = H_s \quad \text{Always positive}$$

Positive, definite volume
of the phase space.

Must have NEC violation and for long enough to escape the egg

$$\rho_s + p_s < 0$$

Is this bad?

Must have **NEC violation** and for long enough to escape the egg

$$\rho_s + p_s < 0$$

There are several reasons this is problematic.

NEC violation implies instabilities in GR.

$$T_{\mu\nu}n^\mu n^\nu \geq 0 \quad \longrightarrow \quad R_{\mu\nu}n^\mu n^\nu \geq 0$$

**Null convergence condition
(Geodesic completeness of light)**

Fine-tuning of initial conditions and perturbations.

- Problems with causality of observers (S-matrix)
- Problems for cosmological perturbations
- Attractor behavior of the solutions

PLAN

Part One:

Lingering in a classical Universe

Part Two:

Lingering from String Theory

**In both models we consider
we do not violate the NEC.**

Null Energy Condition

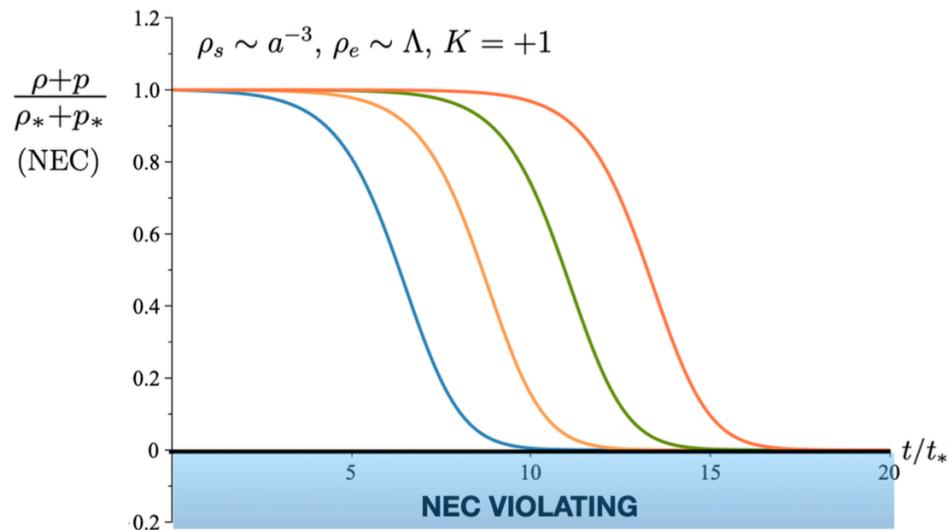
$$T_{\mu\nu}n^\mu n^\nu \geq 0 \quad \longrightarrow \quad \rho + p \geq 0$$

In **PART ONE** (classical GR),
we didn't violate the NEC due to **SPATIAL CURVATURE**

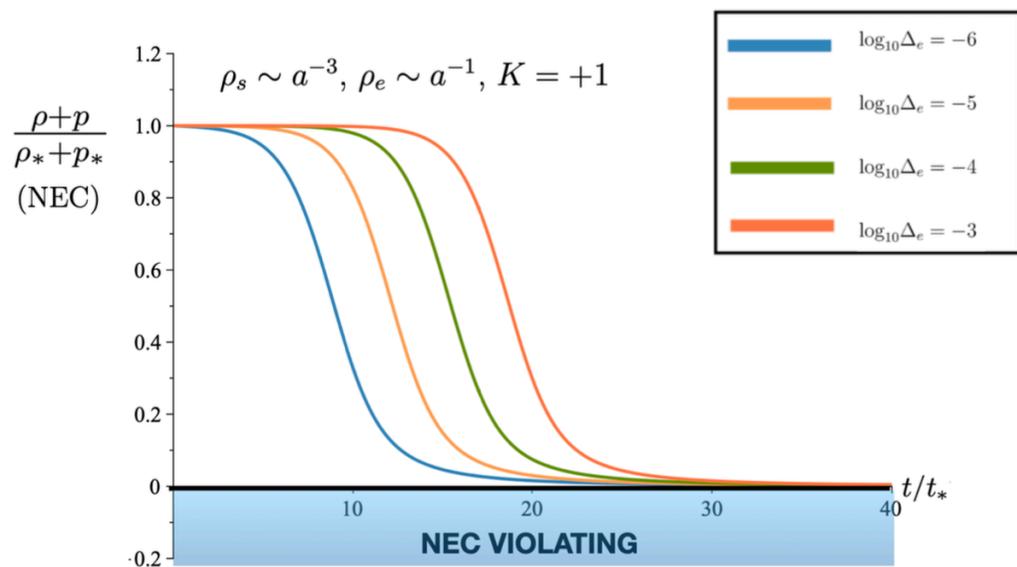
$$\frac{\bar{\rho}_s}{a^m} + \frac{\bar{\rho}_e}{a^n} - \frac{3}{\kappa^2 a^2} = 0$$

$$(m - 2)\frac{\bar{\rho}_s}{a^m} + (n - 2)\frac{\bar{\rho}_e}{a^n} = 0$$

Transition to Inflation with Matter and Vacuum Energy



Transition to Inflation with Matter and String Network



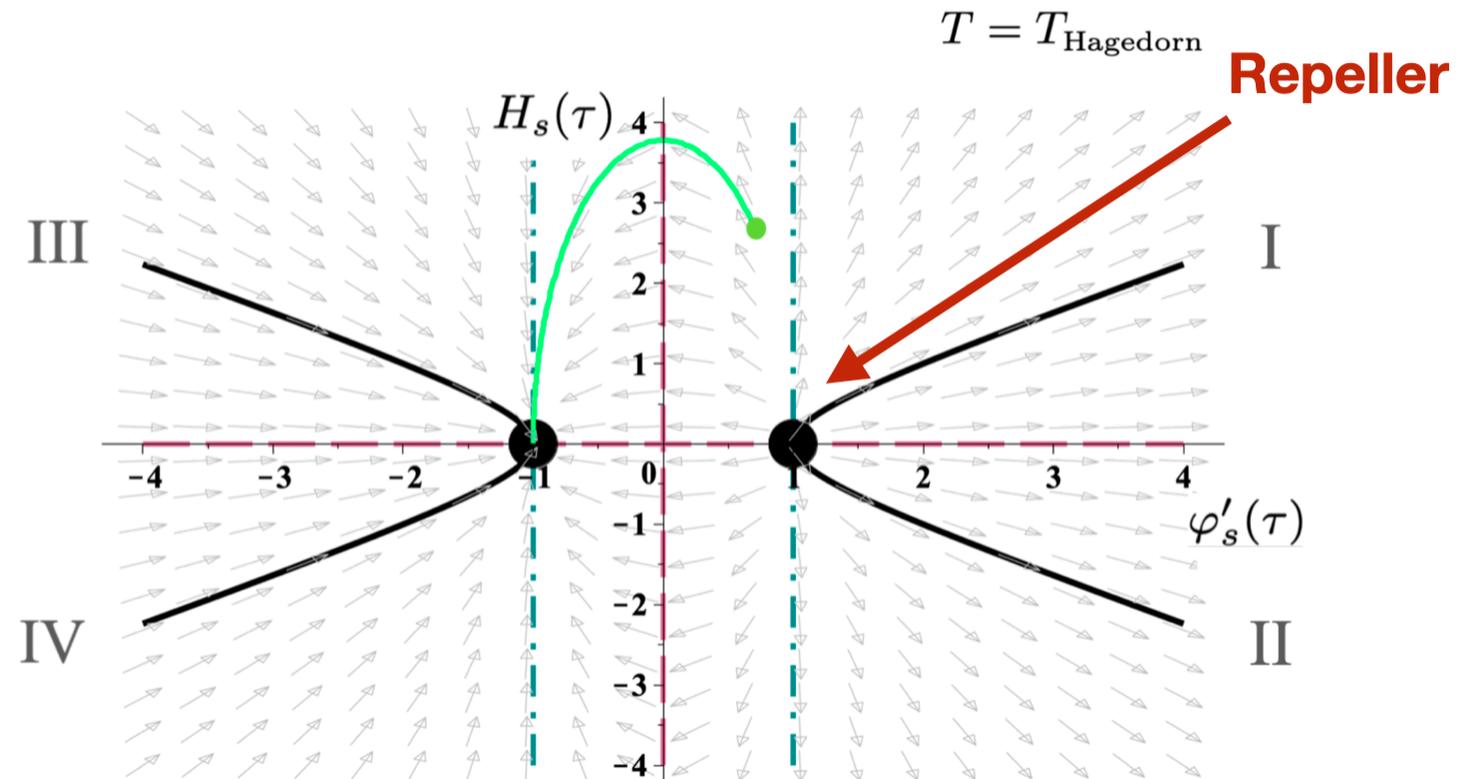
PART TWO: We simply remain on the same branch

$$\dot{\phi}_s = 3H_s \left(\pm \sqrt{3H_s^2 + e^{\phi_s} \rho_s} \right),$$

$$\dot{H}_s = \left(\pm H_s \sqrt{3H_s^2 + e^{\phi_s} \rho_s} + \frac{1}{2} e^{\phi_s} (p_s + \Delta_\phi \mathcal{L}_m) \right),$$

$$\dot{\rho}_s + 3H_s (\rho_s + p_s) = -\dot{\phi}_s \Delta_\phi \mathcal{L}_m,$$

Hagedorn Phase Space



One solution is asymptotic inflation driven by dilaton

Challenges for PART TWO

How long does lingering last?

(Determined by string dynamics and dilaton evolution)

Nature of the Hagedorn phase? Is there a phase transition?

The Hagedorn Transition and the Number of Degrees of Freedom of String Theory

Joseph J. Atick (Princeton, Inst. Advanced Study), Edward Witten (Princeton, Inst. Advanced Study) (Apr, 1988)

Published in: *Nucl.Phys.B* 310 (1988) 291-334

How does inflation end? And how long does that last?

Motivation for the approach

A lingering universe gives a new cosmological paradigm.
No initial singularity (unlike inflation... to be discussed with Damien).

We can address initial conditions for inflation (can avoid subtleties of dS).

Can one address the initial singularity and issues with the S-matrix via this approach (even the classical model)?

The approach can also address the landscape / multiverse. During the Hagedorn phase the motion of fields is expected to be ergodic (hyperbolic moduli space). Sampling the phase space leads to preferred values of the fields (localized at enhanced symmetry points). This dynamics can then determine the size of extra dimensions, the nature of inflation, etc...



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Will Kinney @WKCosmo

Looks interesting. (Ping @StartsWithABang)



arxiv.org

Waiting for Inflation: A New Initial State for the Universe
We propose a cosmological lingering phase for the initial state prior to inflation which would help address the ...

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Carolyn Porco @carolynporco · Oct 13

I laughed out loud at 'There are many open questions that remain.'

No sxxx, Sherlock!



Will Kinney @WKCosmo · Oct 13

Hey, at least people are trying to get a handle on the problem! It's a really interesting question.

Thank you

Waiting for Inflation: A New Initial State for the Universe

ArXiv: 2310.06019

Brandon Melcher, Arbab Pradhan (Syracuse), S.W.