Entanglement from Superradiance and Hawking Radiation

Kerr BHs, rotating fluids and laboratory experiments

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Copernicus seminar series 26 Mar 2024

Based on arXiv:2307.06215, arXiv:2310.16031

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Superradiance and Hawking effect: history and examples

Klein ’29 Sauter 31’ Hund 41’

Klein Paradox
Large electrostatic potential step

Penrose 69’, Penrose-Floyd 71’

Penrose process

Zel’dovich ‘72

Amplification of waves by rotating bodies

Extraction of rotational energy from BH from particle or wave scattering

Rotational superradiance

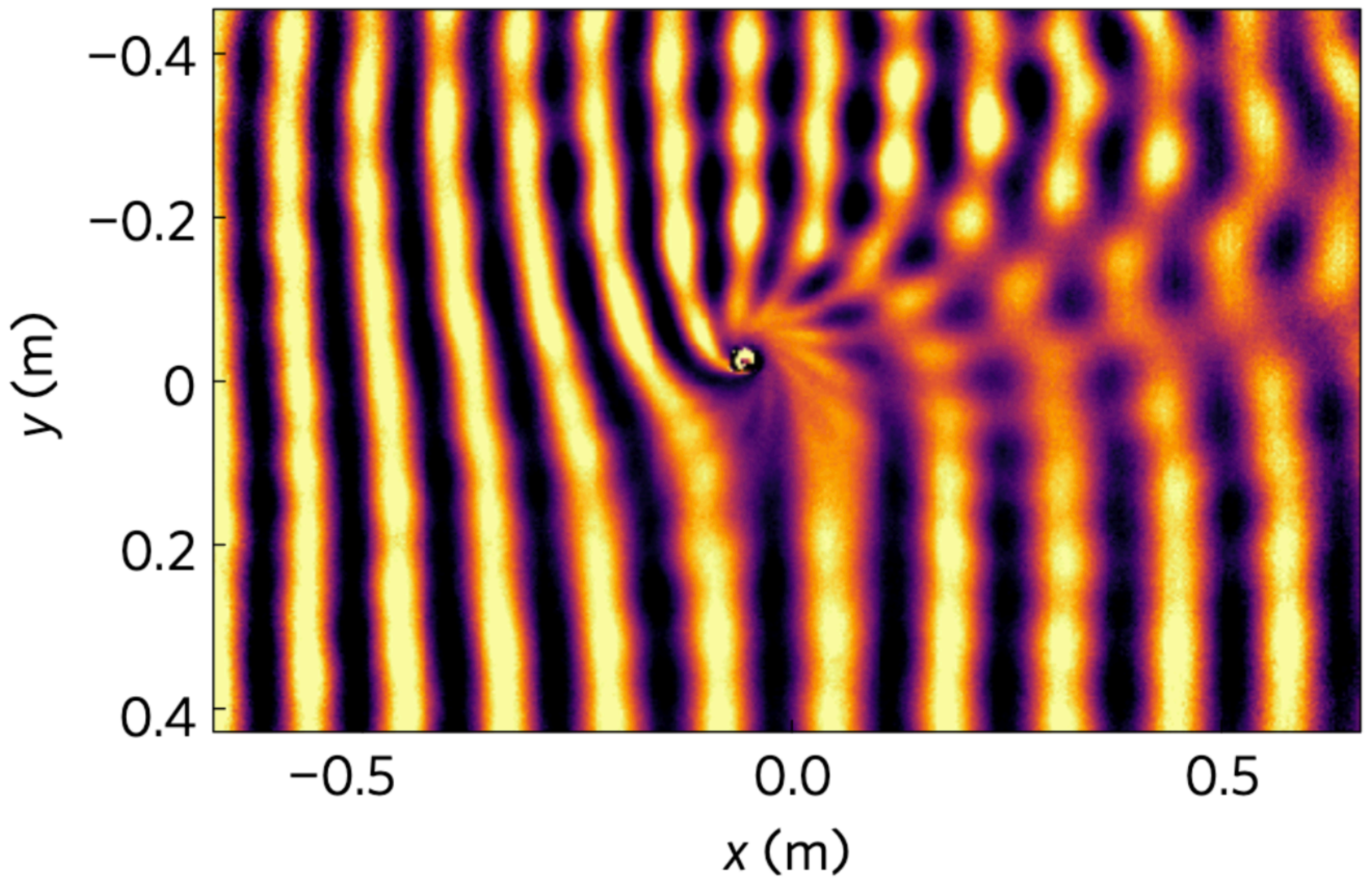
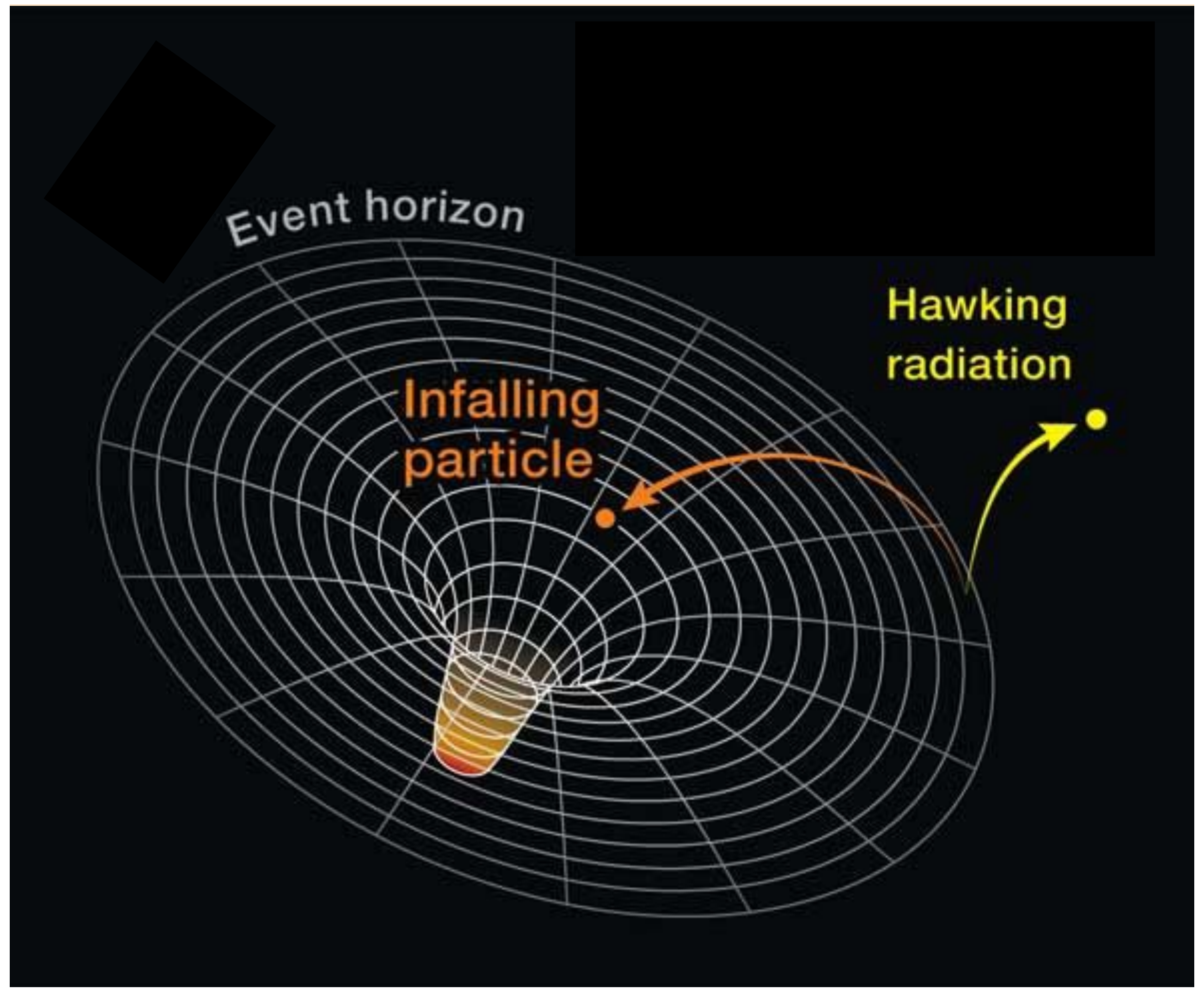


Figure from: T. Torres et. Al. *Nature Phys.* 13 (2017) 833-836

Examples of Superradiance: ubiquitous phenomenon in field theories

(also Cherenkhov, Landau critical velocity in spuerfluids, amplification by shock waves,…)

Superradiance and Hawking effect: history and examples



Hawking '74

Black holes radiate as black bodies

(Actually grey bodies)

$$T_{\rm H} = \frac{\hbar c^3}{8\pi G k_{\rm B} M}$$

Hawking temperature

Black Hole evaporation
+
Hawking radiation is entangled with radiation in the interior

→ Information loss

Superradiance and the Hawking effect: both are amplification processes in field theory

We know realistic BHs rotate: Interplay between Rotational SR and HE at the quantum level?

Goals

- Show that Superradiance generates entanglement in generic scenarios.
- Quantify entanglement radiated by rotating BHs.
- Assess entanglement degradation by CMB.
- Show how Gaussian quantum info. tools are extremely powerful in these contexts.
- Tame ergoregion instability in horizonless laboratory analogues.
- Quantify entanglement from horizonless rotational superradiance in the lab.
- Discuss analogues for the interplay HR - RSR.

Toolbox
Gaussian quantum information

Entanglement Quantification

Entanglement quantifiers from quantum info. theory: Entanglement entropy, Logarithmic Negativity, etc.

Each quantifier is valid only for a certain class of systems.

- Von Neumann entropy quantifies mixedness. Only equivalent to entanglement if state is pure.

- Logarithmic Negavity (based on the PPT criterion) is a convenient quantifier for our use:

  - For any Gaussian state and if either of the two subsystem is made of a single mode, LogNeg is a faithful quantifier.

  - Has an operational meaning: entanglement cost in "e-bits" to prepare a state (1 e-bit = entanglement in a Bell pair)

    [Wang, Wilde 18']

  - Measures distillable entanglement (can be zero if non-distillable entanglement is present)

- **N-dimensional quantum (bosonic) system:** $\hat{x}_1, \hat{p}_1; \hat{x}_2, \hat{p}_2; \cdots \hat{x}_N, \hat{p}_N \equiv \hat{r}^i$

C.C.R's: $[\hat{r}^i, \hat{r}^j] = i\,\hbar\,\Omega^{ij}$

$$\Omega^{ij} = \oplus_N \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

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- **Gaussian state** $\hat{\rho}$ **:** **Completely and uniquely determined by its first and second moments**

  $\mu^i \equiv \mathrm{Tr}[\hat{\rho}\,\hat{r}^i]$ **mean**

  $\mathrm{Tr}[\rho\,\hat{r}^i\,\hat{r}^j] \longrightarrow \sigma^{ij} = \mathrm{Tr}[\hat{\rho}\,\{(\hat{r}^i - \mu^i), (\hat{r}^j - \mu^j)\}]$ **covariance matrix**

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Examples:

Vacuum:	$\mu^i = 0$	$\sigma^{ij} = \mathbb{I}_{2N}$	**Pure**
Coherent state:	$\mu^i \neq 0$	$\sigma^{ij} = \mathbb{I}_{2N}$	**Pure**
Thermal:	$\mu^i = 0$	$\sigma^{ij} = \oplus_i^N (2\,n_i + 1)\,\mathbb{I}_2$	**Mixed**

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Thermal: $\mu^i = 0$ $\quad \sigma^{ij} = \oplus_i^N (2\,n_i + 1)\,\mathbb{I}_2$ **Mixed**

- **Linear time evolution and restriction to a subsystem produce another Gaussian state:**

  $(\mu^i_{\mathrm{in}}, \sigma^{ij}_{\mathrm{in}}) \longrightarrow (\mu^i_{\mathrm{out}}, \sigma^{ij}_{\mathrm{out}})$

  $S^i_{\ j}$ = **evolution matrix** $\in \mathrm{Sp}(2N)$

  $$\vec{\mu}_{\mathrm{out}} = S \cdot \vec{\mu}_{\mathrm{in}}$$
  $$\sigma_{\mathrm{out}} = S \cdot \sigma_{\mathrm{in}} \cdot S^{\top}$$

  $$\vec{\mu} = (\vec{\mu}_A^{\mathrm{red}}, \vec{\mu}_B^{\mathrm{red}}) \qquad \sigma = \begin{pmatrix} \sigma_A^{\mathrm{red}} & \sigma_{AB} \\ \sigma_{AB}^{\top} & \sigma_B^{\mathrm{red}} \end{pmatrix}$$

Example 1: Beam splitter

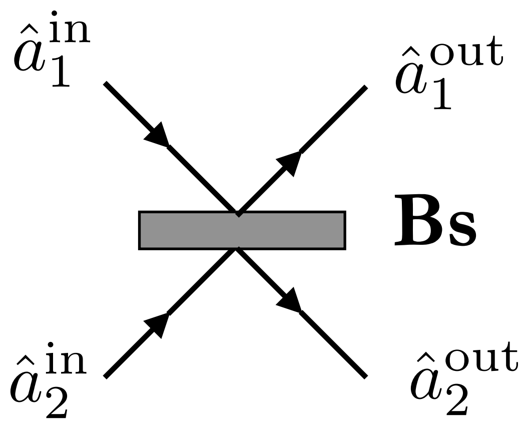
Evolution:

$$\hat{a}_1^{\text{in}} \to \hat{a}_1^{\text{out}} = \hat{a}_1^{\text{in}} \cos\theta + \hat{a}_2^{\text{in}} \sin\theta$$

$$\hat{a}_2^{\text{in}} \to \hat{a}_2^{\text{out}} = -\hat{a}_1^{\text{in}} \sin\theta + \hat{a}_2^{\text{in}} \cos\theta$$

$$\longrightarrow \quad r^i_{\text{out}} = S^i{}_j \, r^j_{\text{in}}$$

where $$S^i{}_j = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos r \end{pmatrix}$$



Entanglement quantifier after acting on vacuum:

$$\text{LogNeg} = 0$$

Pasive transformation: **Does not mix creation and annihilation operators.**
Divides **quanta and entanglement.**

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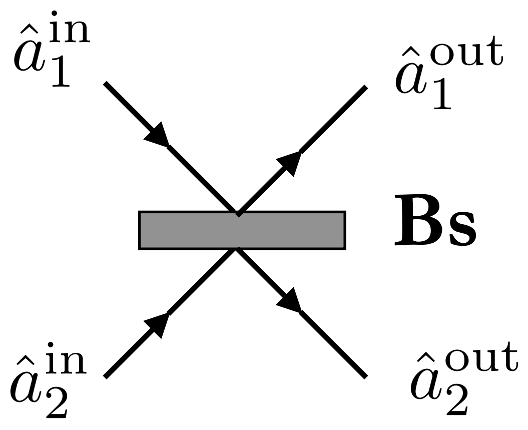


Entanglement quantifier after acting on vacuum:

$$\text{LogNeg} = 0$$

Pasive transformation: Does not mix creation and annihilation operators.
Divides quanta and entanglement.

Example 2: Two-mode squeezing

Evolution:

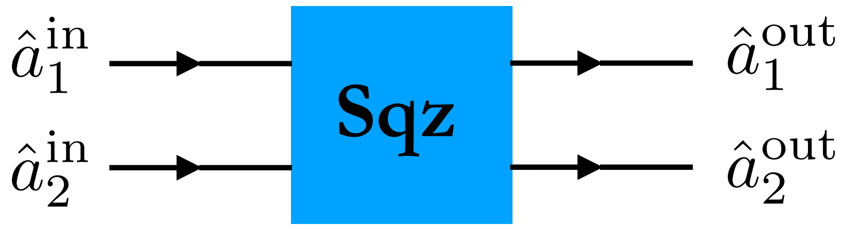
$$a_I = \frac{1}{\sqrt{2}}(x_I - i\,p_I)$$

$$\hat{a}_1^{\text{in}} \to \hat{a}_1^{\text{out}} = \hat{a}_1^{\text{in}}\,\cosh r + \hat{a}_2^{\text{in}\,\dagger}\,\sinh r$$

$$\hat{a}_2^{\text{in}} \to \hat{a}_2^{\text{out}} = \hat{a}_1^{\text{in}\,\dagger}\,\sinh r + \hat{a}_2^{\text{in}}\,\cosh r$$

$$\longrightarrow \quad \hat{r}^i_{\text{out}} = S^i{}_j\, \hat{r}^j_{\text{in}}$$

where $$S^i{}_j = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\cosh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}$$



Entanglement quantifier after acting on vacuum:

$$\text{LogNeg} = \ln_2 e^{2\,r} \quad \text{(e-bits)}$$

Active transformation: Mixes creation and annihilation operators (norm mixing).
Creates quanta and entanglement.

Superradiance and entanglement

Definition (key properties)

1. Scattering on a stationary (time-independent) background

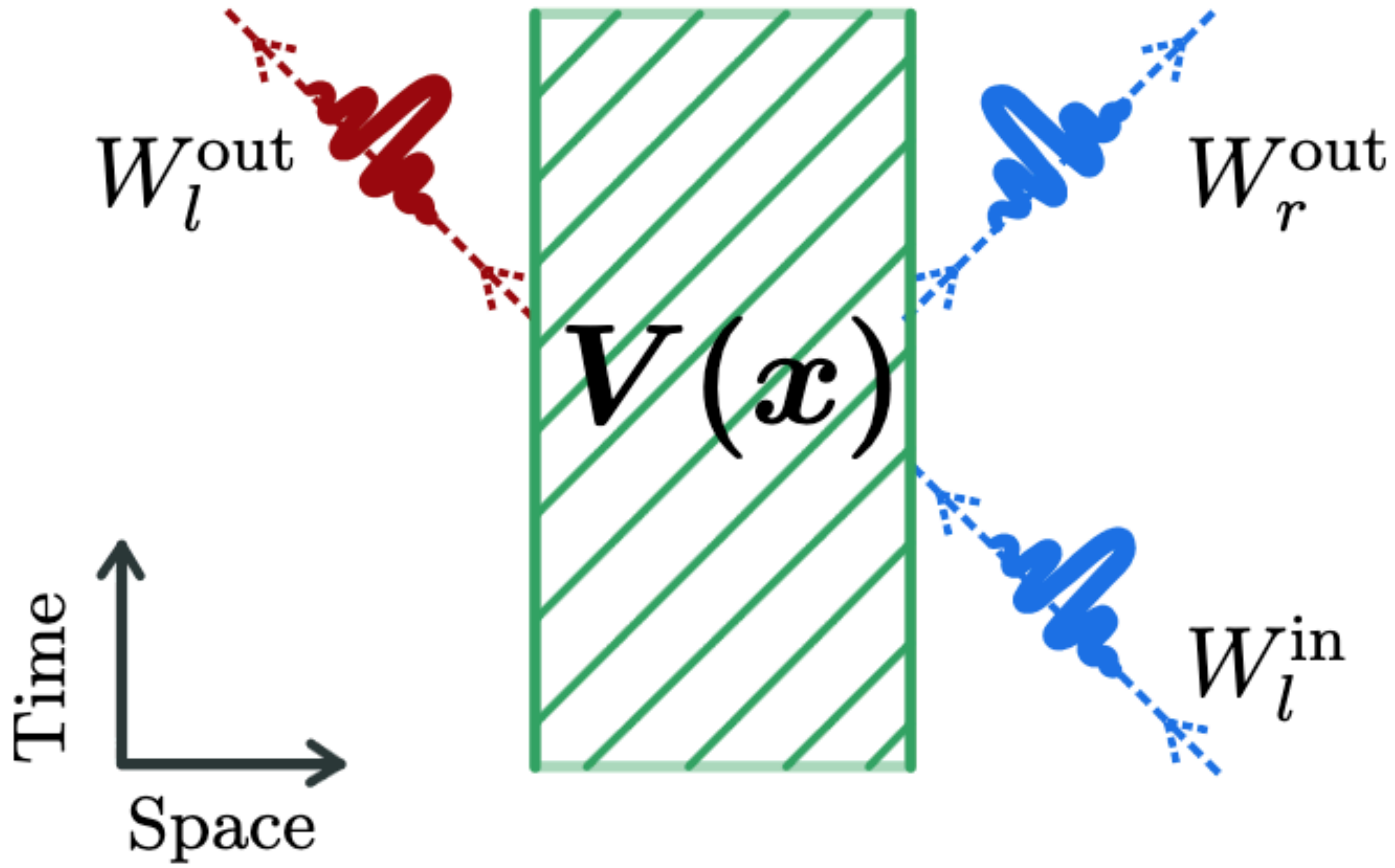
   Exists a Killing Vector Field that is asymptotically timelike

2. Scattered waves are amplified

1. Scattering on a stationary (time-independent) background

   Exists asymptotically timelike KVF, but not globally.

2. Scattered waves are amplified



ω is conserved

$$\Psi|_{t\to-\infty} = W_l^{\text{in}} \xrightarrow{\text{time}} \Psi|_{t\to\infty} = T\,W_l^{\text{out}} + R\,W_r^{\text{out}}$$

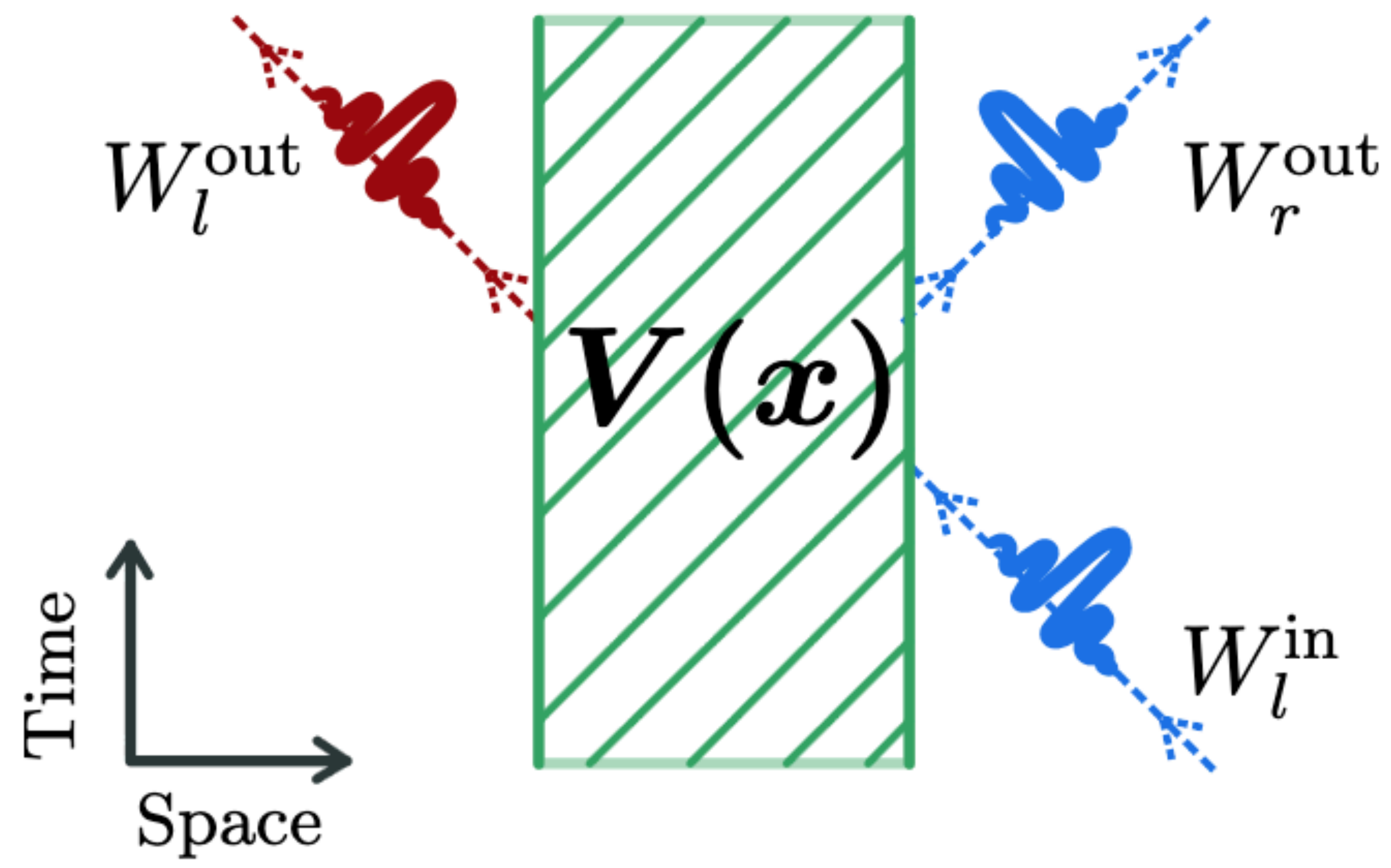
Superradiance iff $|R| > 1$

or/and $|T| > 1$

What theories admit superradiance?

Insight from conserved quantities!

What theories admit superradiance?



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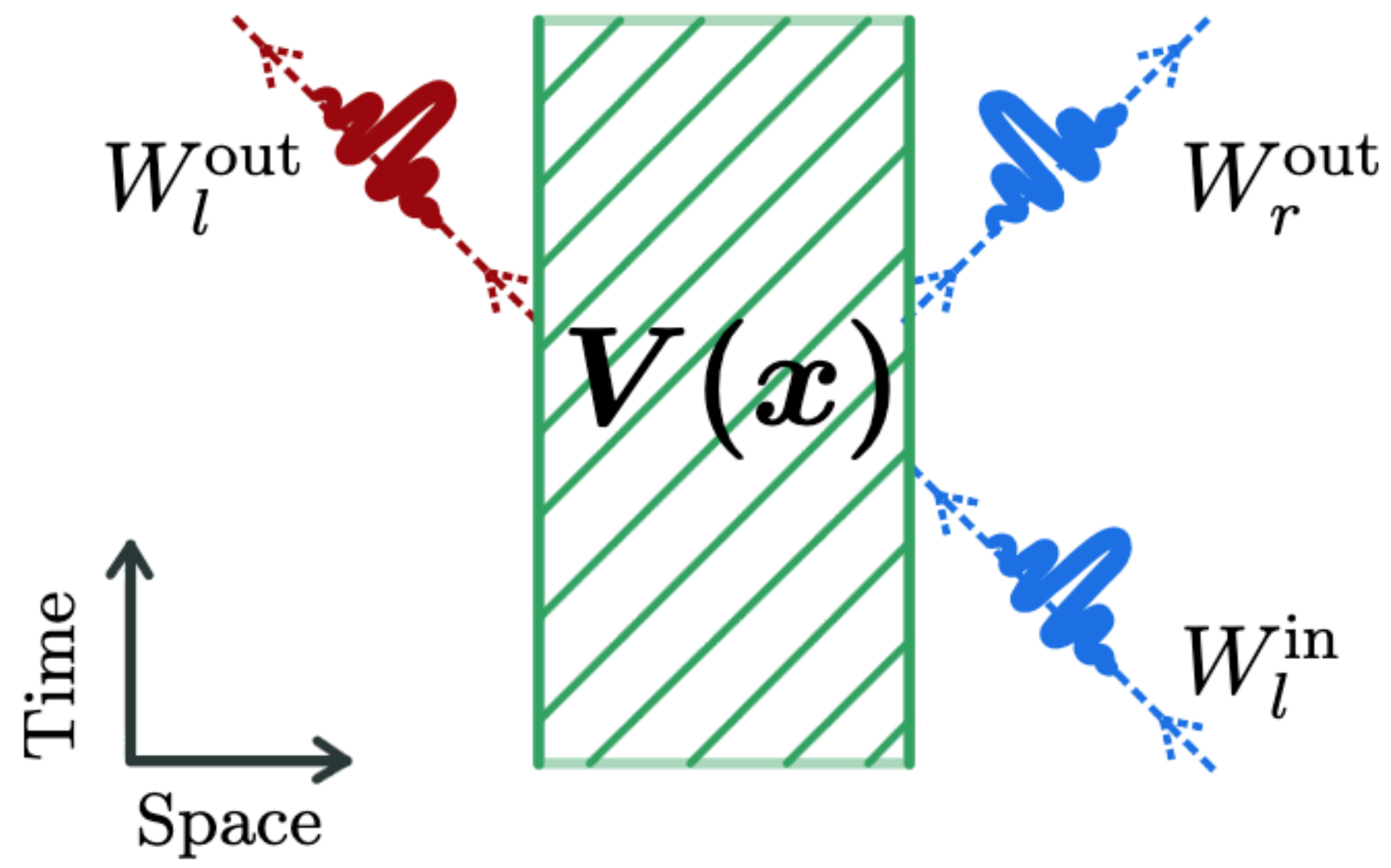

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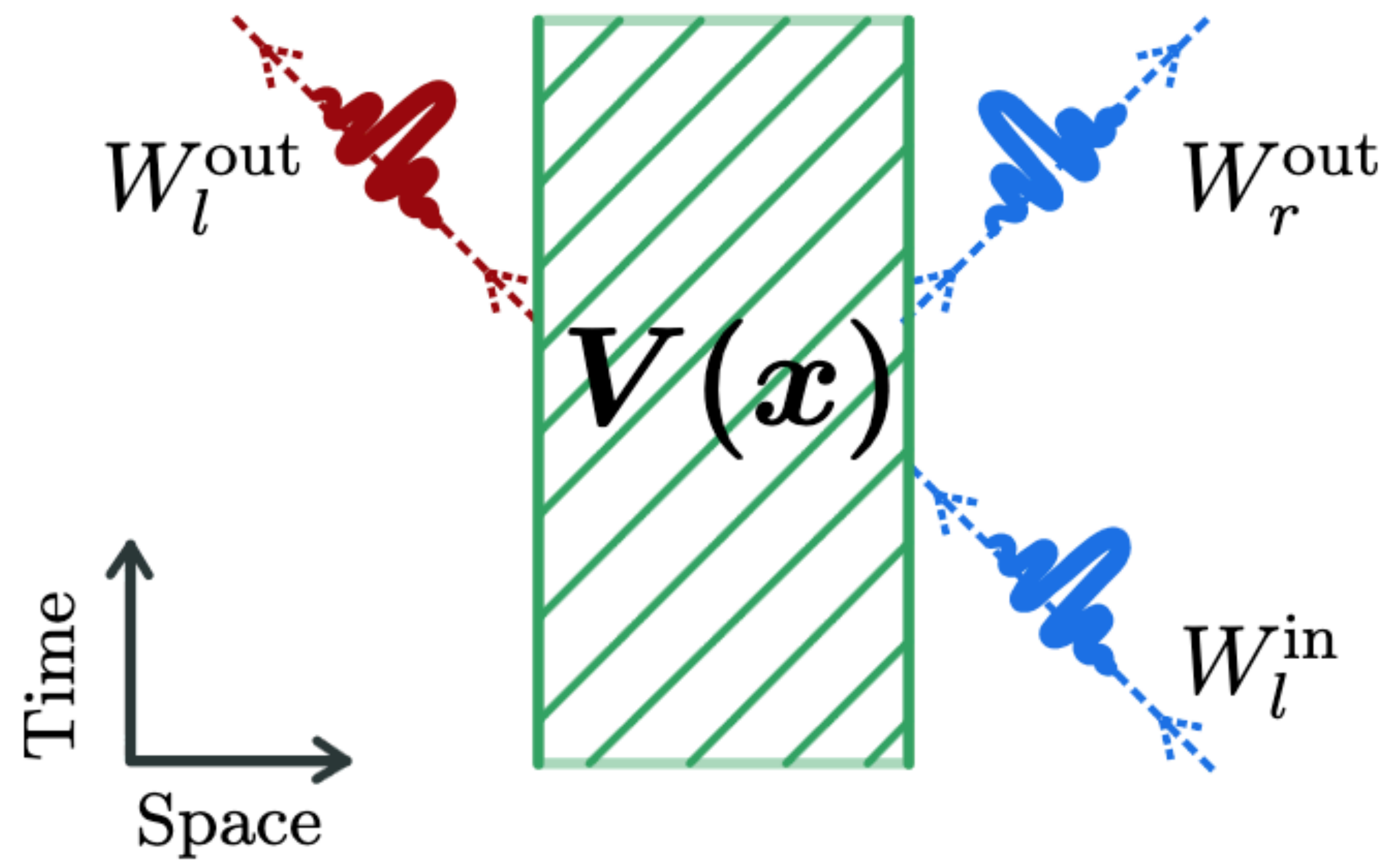
Examples:

Schrödinger/Dirac: $Q = \int_\Sigma |\Psi|^2 \geq 0$

$$1 = |T|^2 + |R|^2$$

Positivity $\longrightarrow$ No Superradiance

What theories admit superradiance?



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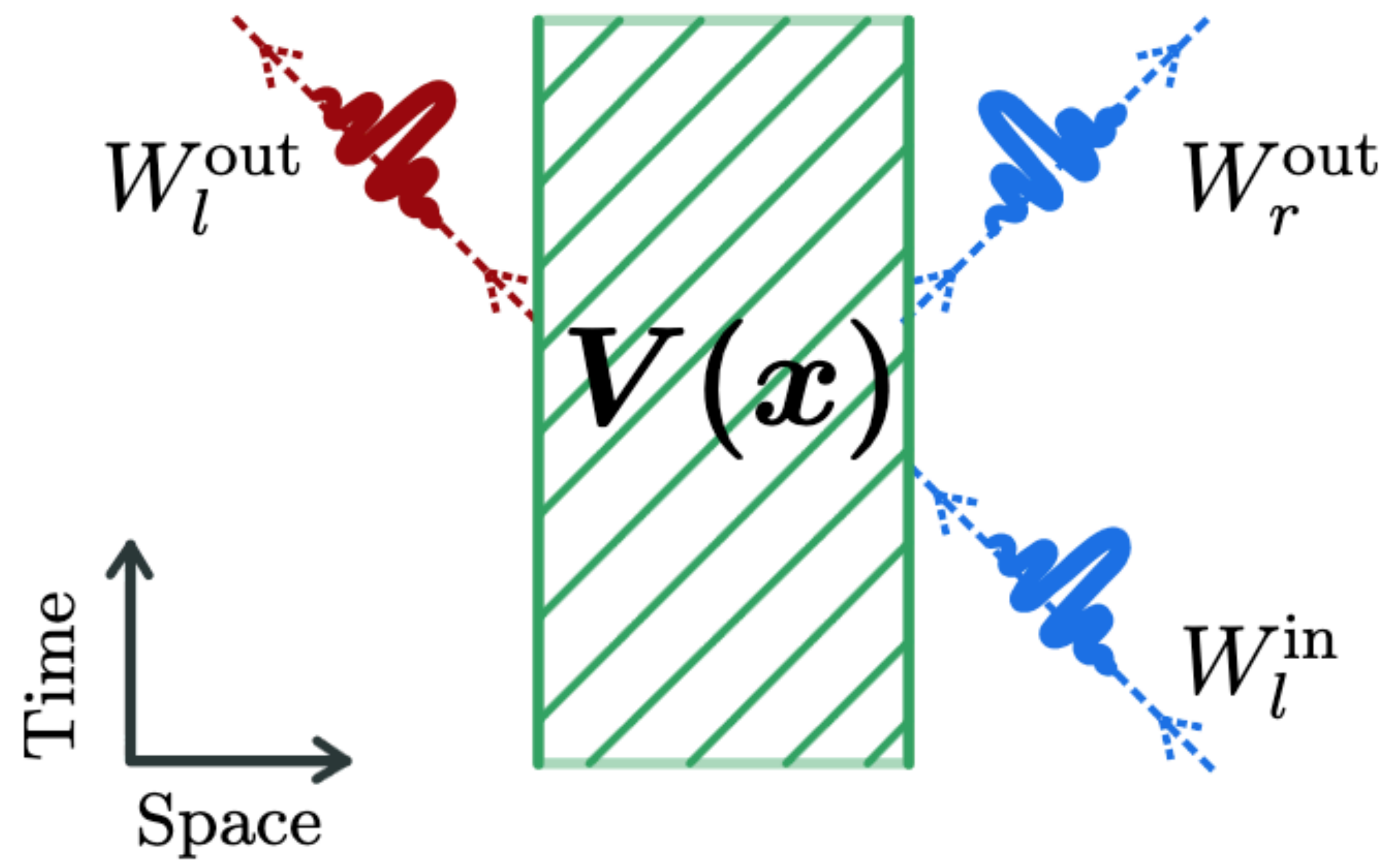
Positivity $\longrightarrow$ **No Superradiance**

Klein-Gordon $Q = \int_\Sigma \Phi^* \pi - \Phi \pi^*$

$$1 = |R|^2 - |T|^2 \qquad (Q(W_l^{out}) < 0)$$

Lack of positivity $\longrightarrow$ **Can have Superradiance**

What theories admit superradiance?



$$\Psi|_{t\to-\infty} = W_l^{\rm in} \xrightarrow{\text{time}} \Psi|_{t\to\infty} = T\,W_l^{\rm out} + R\,W_r^{\rm out}$$

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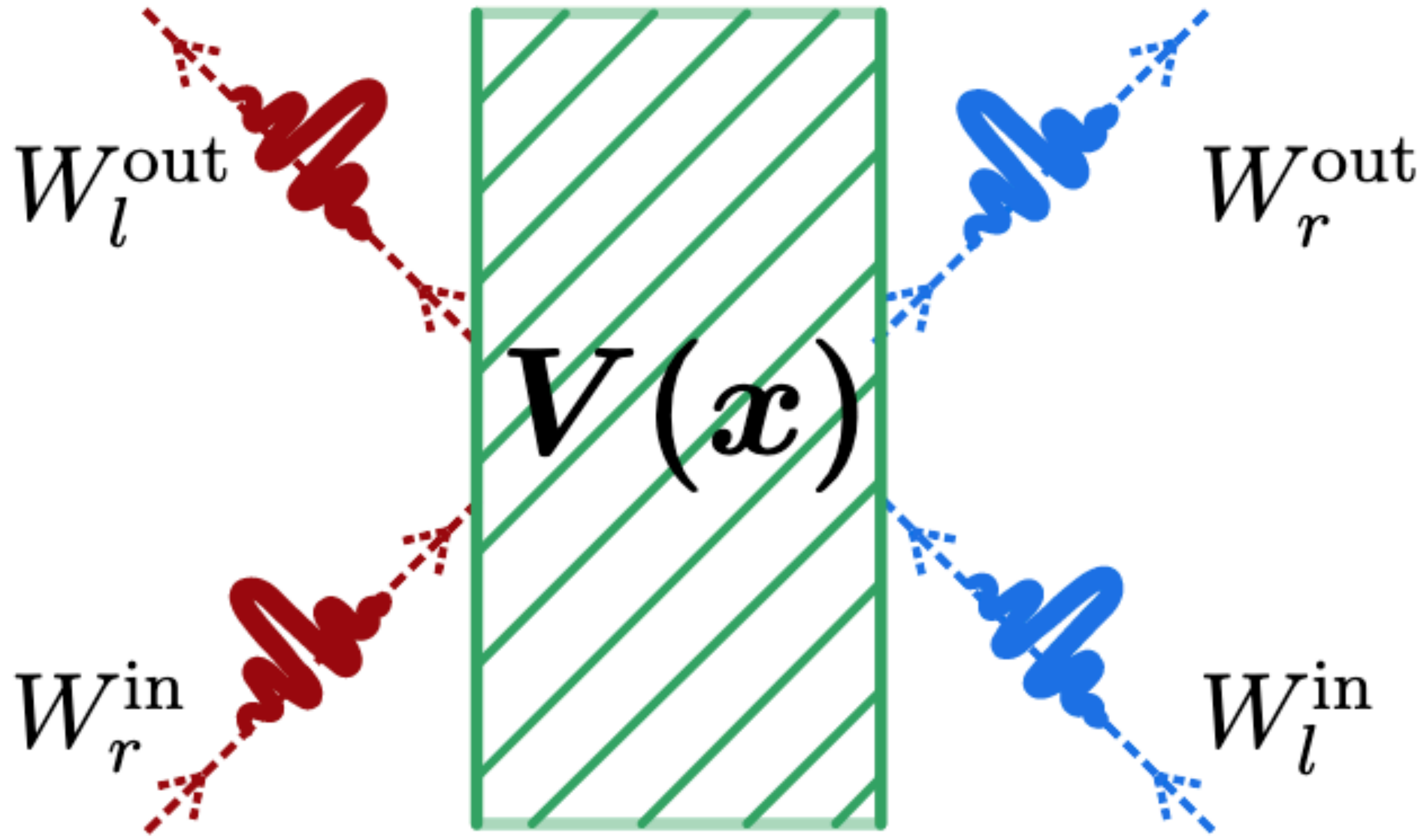
Linked to phase space invariant structure:
- Bosons → **Symplectic (non pos. Definite)**
- Fermions → **Metric (pos. Definite)**

General process (linear dispersion, 2 modes per ω)

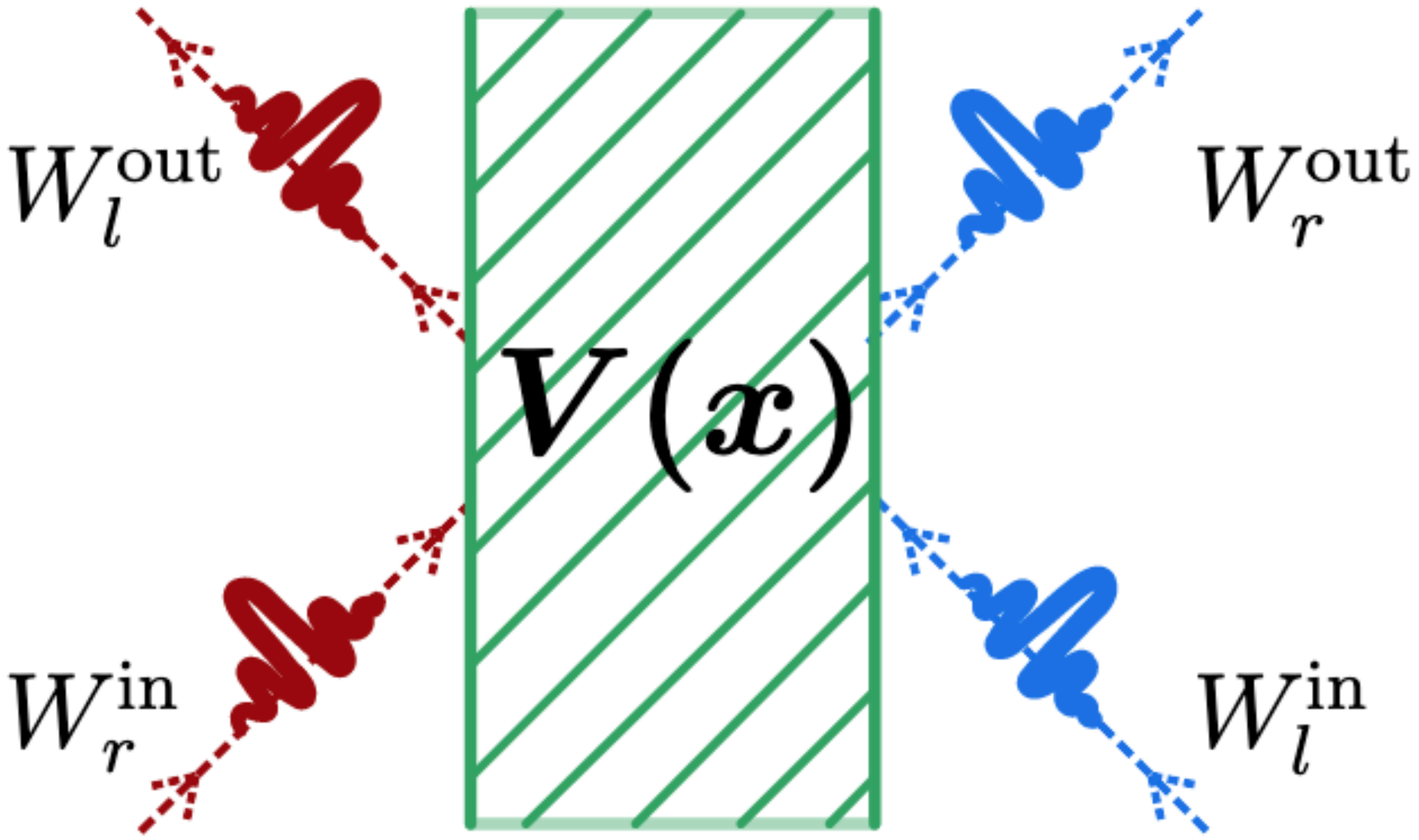


$$a_1 W_r^{\text{in}} + a_2 W_l^{\text{in}} \xrightarrow{\textit{time}} b_1 W_r^{\text{out}} + b_2 W_l^{\text{out}}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \boldsymbol{B} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad \textbf{with} \quad \boldsymbol{B} = \begin{pmatrix} T & r \\ R & t \end{pmatrix}$$

Characterization of superradiant scattering

General process (linear dispersion, 2 modes per ω)



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Theorem: Stationary scattering is superradiant iff B is non-unitary

arXiv:2310.16031

Classical or Quantum?

Spontaneous emission from SR predicted by Zeldovich (72') and Unruh (74')

(Stimulated) superradiant amplification commonly regarded classical amplification…

Quantization of KG field: $$\mathrm{i}/\hbar \int \left(W_i^* \, \partial_t \hat{\Phi} - \partial_t W_i^* \, \hat{\Phi} \right) \begin{cases} \hat{a}_i & \textbf{if } Q(W_i) = 1 \\ -\hat{a}_i^\dagger & \textbf{if } Q(W_i) = -1 \end{cases}$$

Yields right CCR: $$\left[\hat{a}_i, \hat{a}_j^\dagger \right] = |Q(W_i)| \delta_{ij}$$

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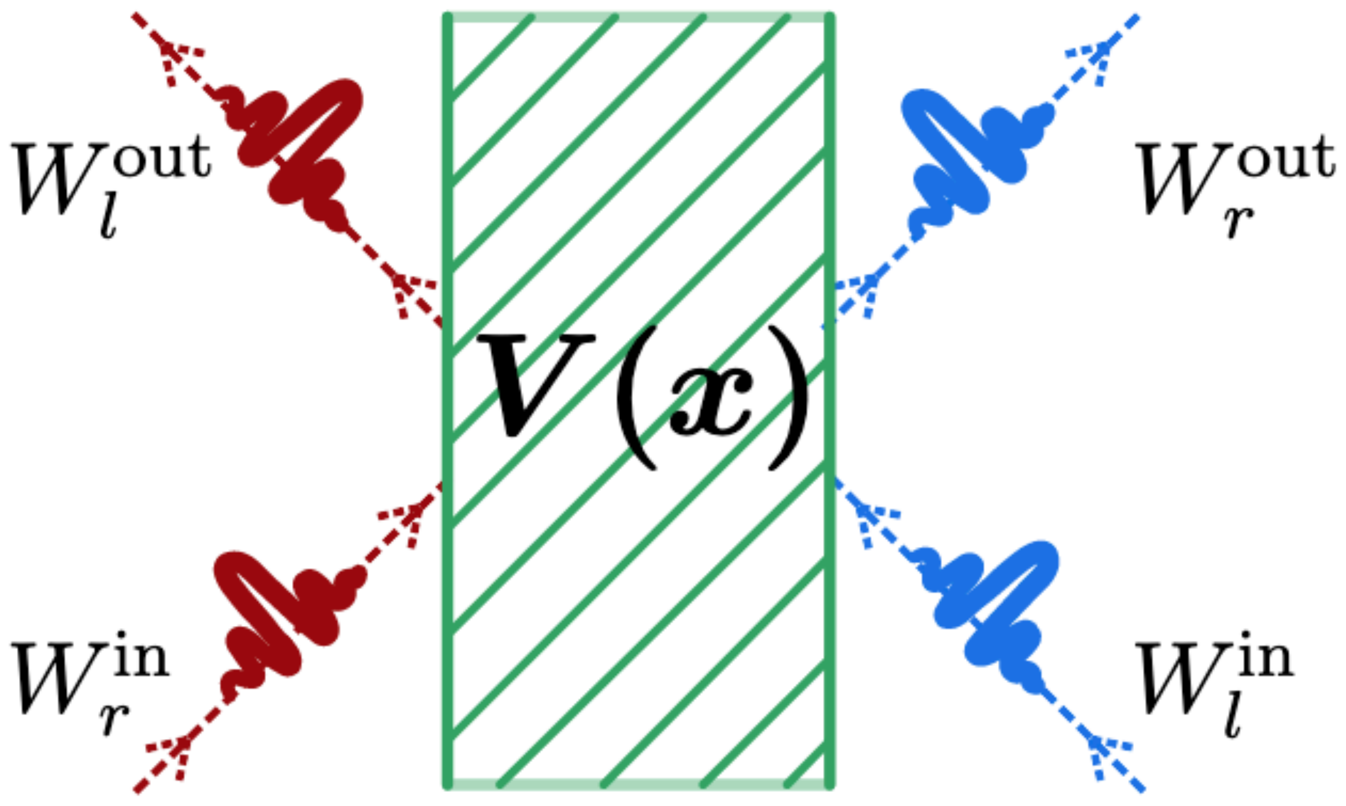
classical scattering $B = \begin{pmatrix} T & R \\ r & t \end{pmatrix}$



Quantum scattering:

$$\begin{pmatrix} \hat{a}_r^{\text{out}} \\ \hat{a}_l^{\text{out}} \\ \hat{a}_r^{\text{out}\dagger} \\ \hat{a}_l^{\text{out}\dagger} \end{pmatrix} = \boldsymbol{S} \cdot \begin{pmatrix} \hat{a}_r^{\text{in}} \\ \hat{a}_l^{\text{in}} \\ \hat{a}_r^{\text{in}\dagger} \\ \hat{a}_l^{\text{in}\dagger} \end{pmatrix}$$

$$\boldsymbol{S}_{\text{NSR}} = \begin{pmatrix} T & r & 0 & 0 \\ R & t & 0 & 0 \\ 0 & 0 & T^* & r^* \\ 0 & 0 & R^* & t^* \end{pmatrix}$$

$$\boldsymbol{S}_{\text{SR}} = \begin{pmatrix} 0 & r & T & 0 \\ R^* & 0 & 0 & t^* \\ T^* & 0 & 0 & r^* \\ 0 & t & R & 0 \end{pmatrix}$$

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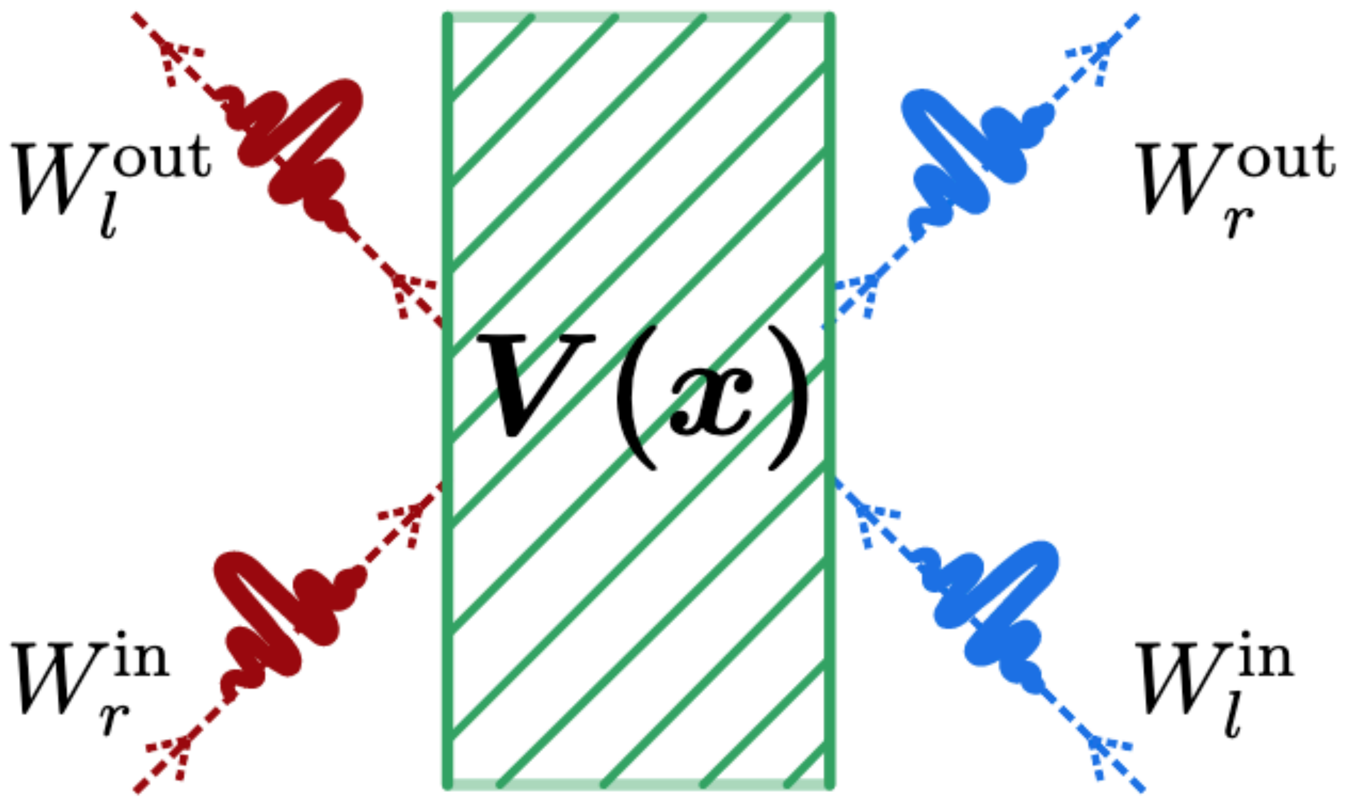
classical scattering

$B = \begin{pmatrix} T & R \\ r & t \end{pmatrix}$



Quantum scattering:

$$\begin{pmatrix} \hat{a}_r^{\text{out}} \\ \hat{a}_l^{\text{out}} \\ \hat{a}_r^{\text{out}\dagger} \\ \hat{a}_l^{\text{out}\dagger} \end{pmatrix} = S \cdot \begin{pmatrix} \hat{a}_r^{\text{in}} \\ \hat{a}_l^{\text{in}} \\ \hat{a}_r^{\text{in}\dagger} \\ \hat{a}_l^{\text{in}\dagger} \end{pmatrix}$$

$$S_{\text{NSR}} = \begin{pmatrix} T & r & 0 & 0 \\ R & t & 0 & 0 \\ 0 & 0 & T^* & r^* \\ 0 & 0 & R^* & t^* \end{pmatrix}$$

$$S_{\text{SR}} = \begin{pmatrix} 0 & r & T & 0 \\ R^* & 0 & 0 & t^* \\ T^* & 0 & 0 & r^* \\ 0 & t & R & 0 \end{pmatrix}$$

S describes superradiant scatering iff non-unitary

Non-superradiant: Unitary S $\longrightarrow N_r^{\text{in}} + N_l^{\text{in}} = N_r^{\text{out}} + N_l^{\text{out}}$

$$\boldsymbol{S}_{\text{NSR}} = \begin{pmatrix} T & r & 0 & 0 \\ R & t & 0 & 0 \\ 0 & 0 & T^* & r^* \\ 0 & 0 & R^* & t^* \end{pmatrix}$$

Beam-splitter

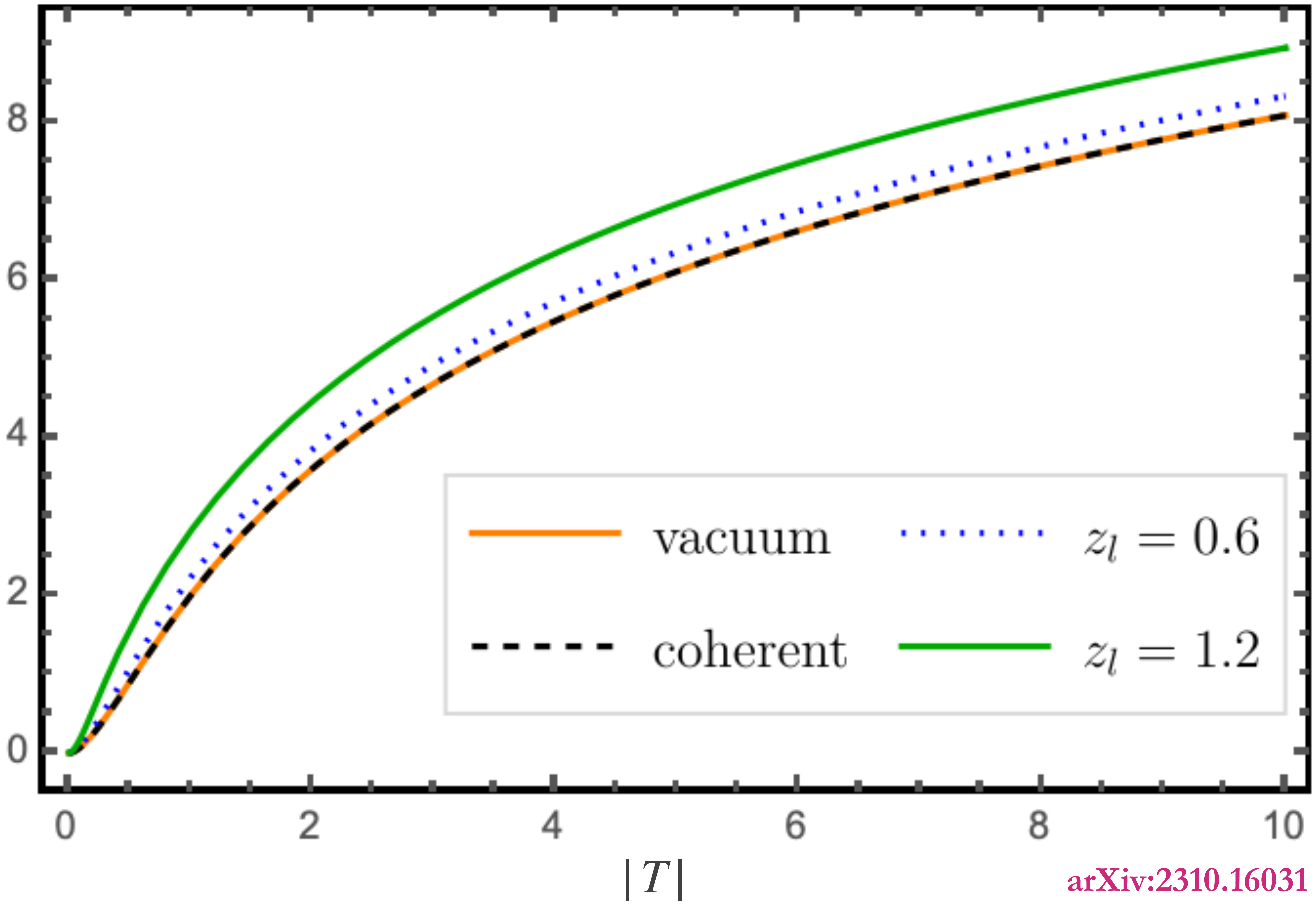
Superradiant: Non-unitary S $\longrightarrow N_r^{\text{in}} - N_l^{\text{in}} = N_r^{\text{out}} - N_l^{\text{out}}$

$$\boldsymbol{S}_{\text{SR}} = \begin{pmatrix} 0 & r & T & 0 \\ R^* & 0 & 0 & t^* \\ T^* & 0 & 0 & r^* \\ 0 & t & R & 0 \end{pmatrix}$$

Two-mode squeezer

Generates and amplifies entanglement!!

Entanglement Entropy



arXiv:2310.16031

Stimulated Superradiance "amplifies" entanglement!

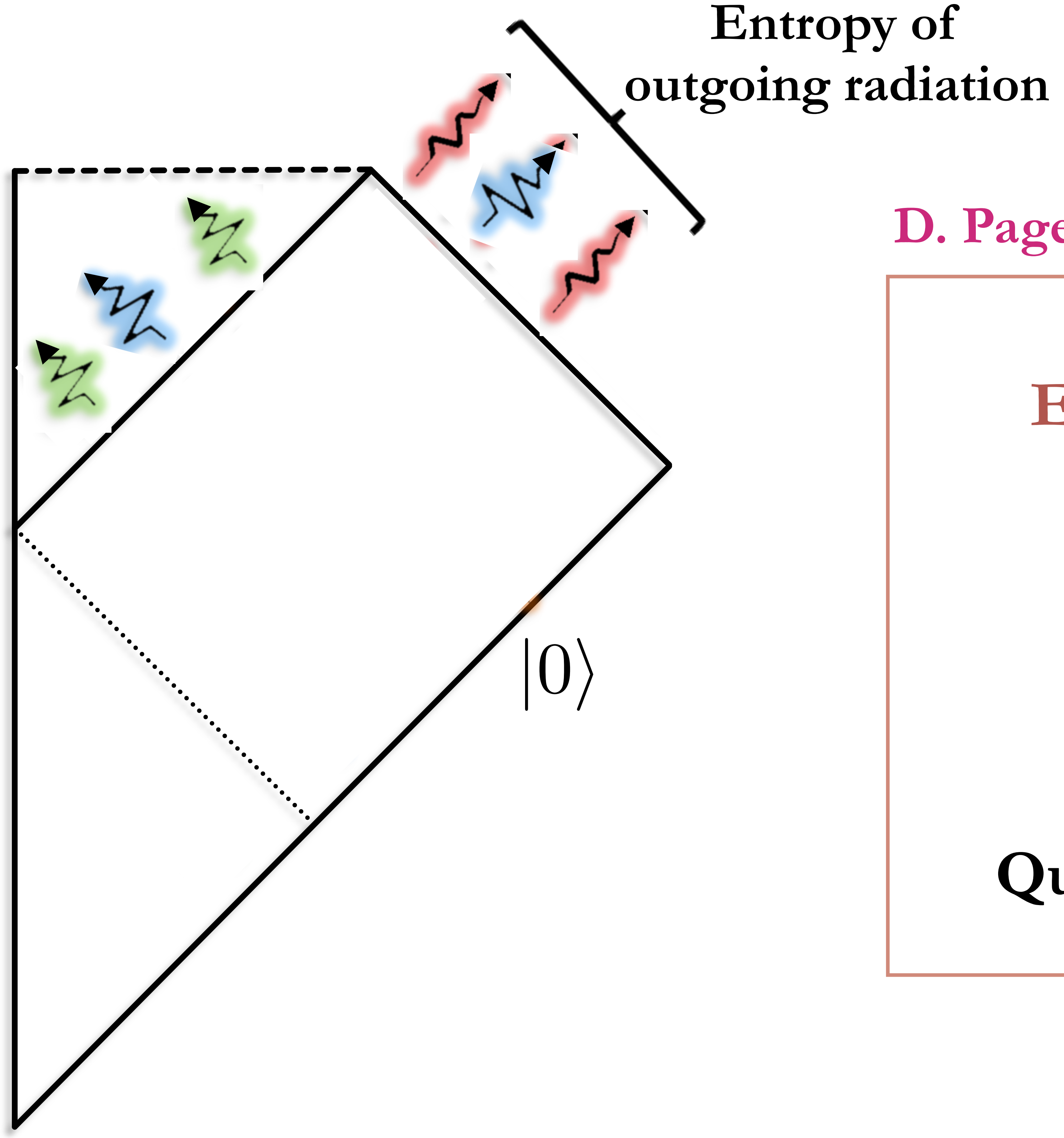
Superradiance is Inherently quantum

also stimulated!

Entanglement from realistic BHs

(rotating and thermally illuminated)

Previous work



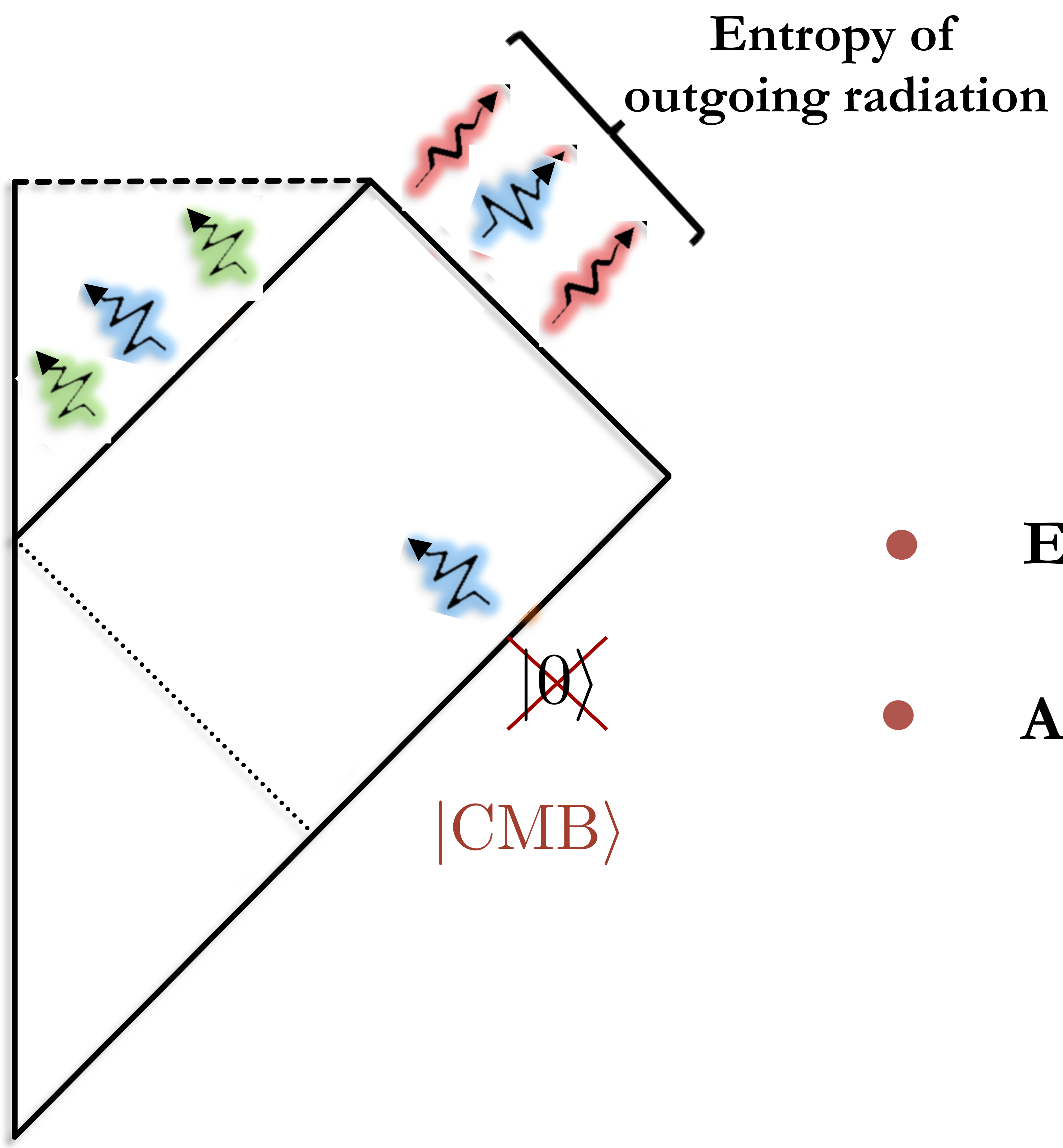
D. Page 2013

Entanglement entropy is a quantifier of Hawking-generated entanglement

Entropy of the radiation reaching infinity = Entanglement entropy

↓

Quantify generated entanglement as entropy of Hawking radiation at infinity



Problem:

- **Entanglement entropy quantifies entanglement only if state is pure.**
- **Astrophysical black holes are immersed in a thermal bath: the CMB**
  Known cases where thermal inputs destroy all entanglement.

Entanglement entropy is not a quantifier in realistic Hawking emission

Use Logarithmic Negativity

Can we apply Gaussian tools to compute LogNeg?

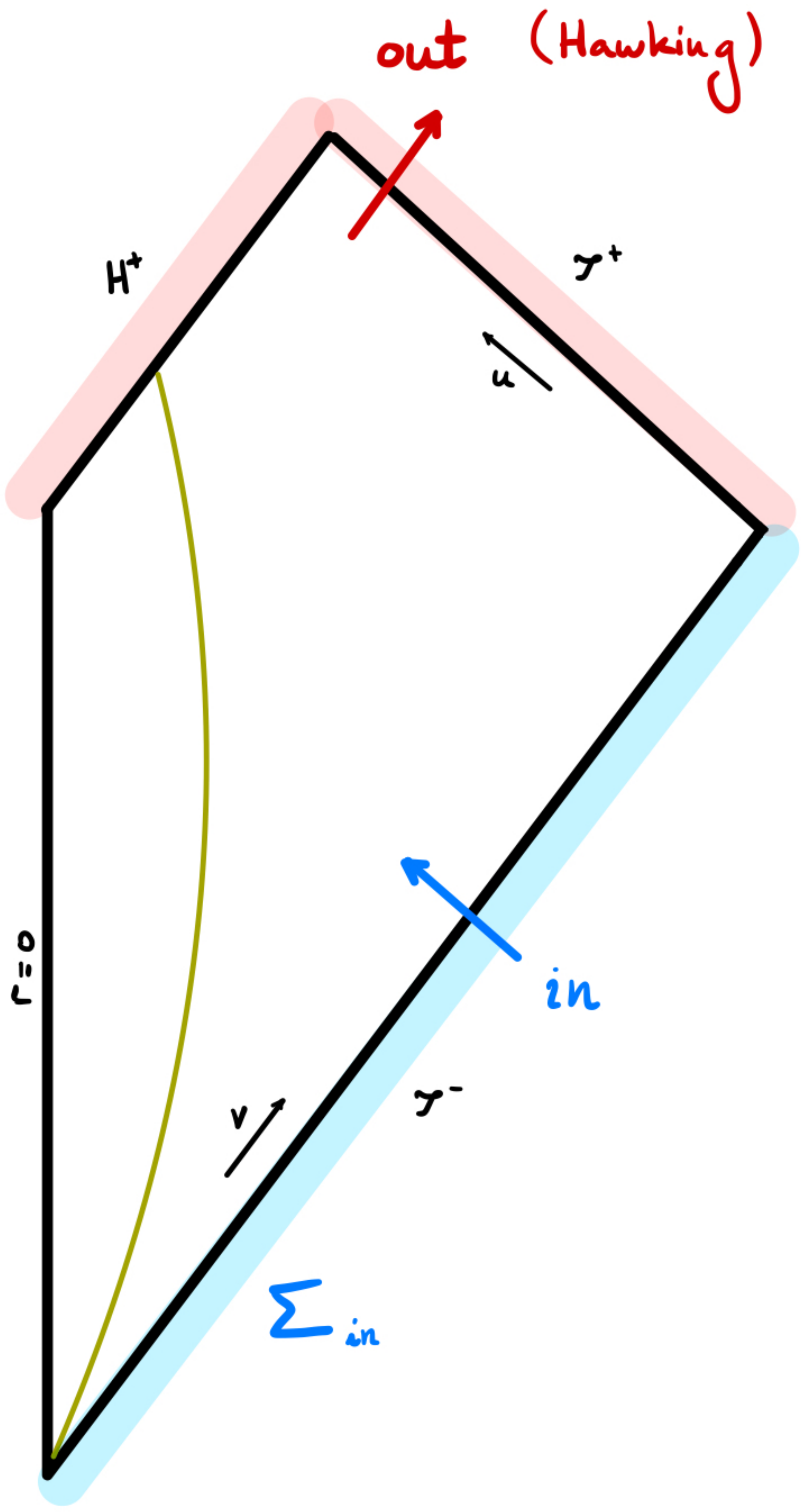
Evolution: 1 Hawking mode = mixture of infinitely many in modes

$$e^{-i\omega u} \xrightarrow{\text{Time evolution}} \int_0^\infty d\tilde{\omega}\left(\alpha_{\omega\tilde{\omega}}\ e^{-i\tilde{\omega}v} + \beta_{\omega\tilde{\omega}}\ e^{i\tilde{\omega}v}\right)$$

$$e^{-i\omega u} \downarrow a_\omega^{out} \qquad e^{-i\tilde{\omega}v} \downarrow a_\omega^{in} \qquad e^{i\tilde{\omega}v} \downarrow a_\omega^{in}$$

Problem: infinite number of degrees of freedom.

Gaussian state formalism and entanglement quantifiers need finite number of degrees of freedom.



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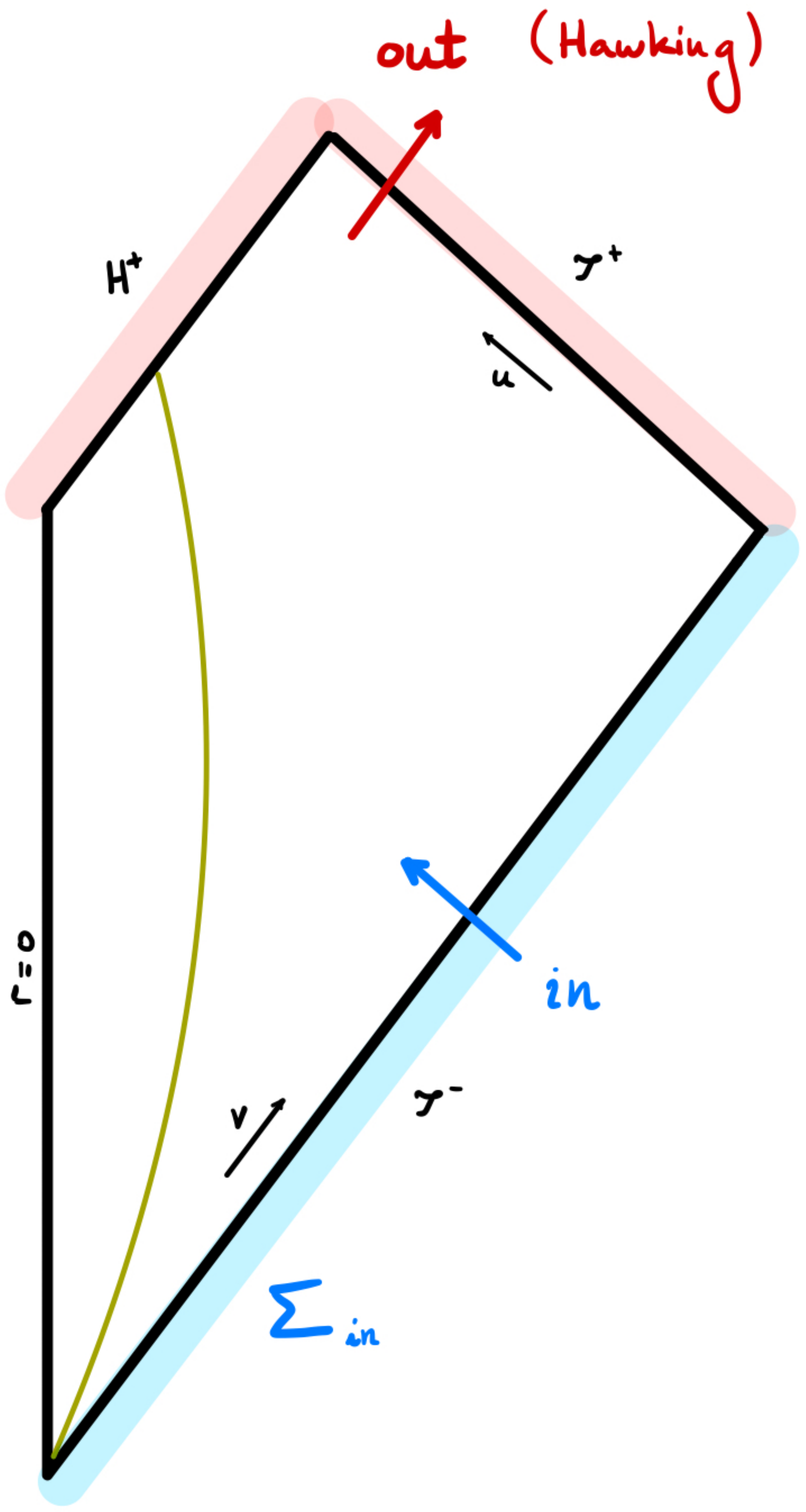
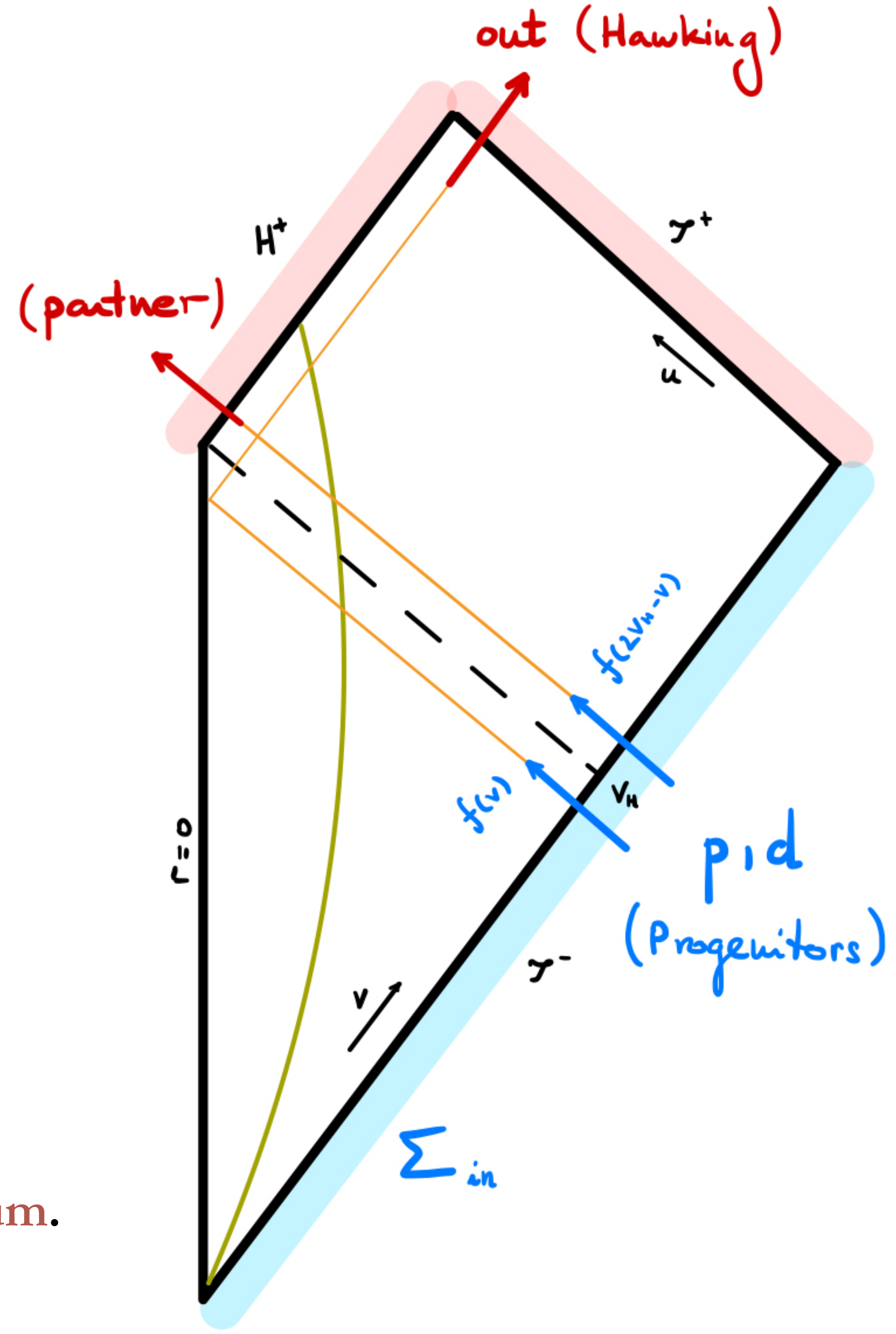
$$e^{-i\omega u} \downarrow a_\omega^{out} \qquad e^{-i\tilde{\omega}v} \downarrow a_\omega^{in} \qquad e^{i\tilde{\omega}v} \downarrow a_\omega^{in}$$

Problem: infinite number of degrees of freedom.



Solution: Use **Wald's Basis** to simplify Hawking pair creation to a $2 \longrightarrow 2$ process

Purifyer of out!



Wald '75

Wald's Basis:

Progenitors of the out modes: $F_p(\omega)$, $F_d(\omega)$

$$F_p(\omega) = N_{\omega\kappa}\left[f(v) + e^{-\frac{\pi\omega}{\kappa}} f(2v_H - v)\right]$$

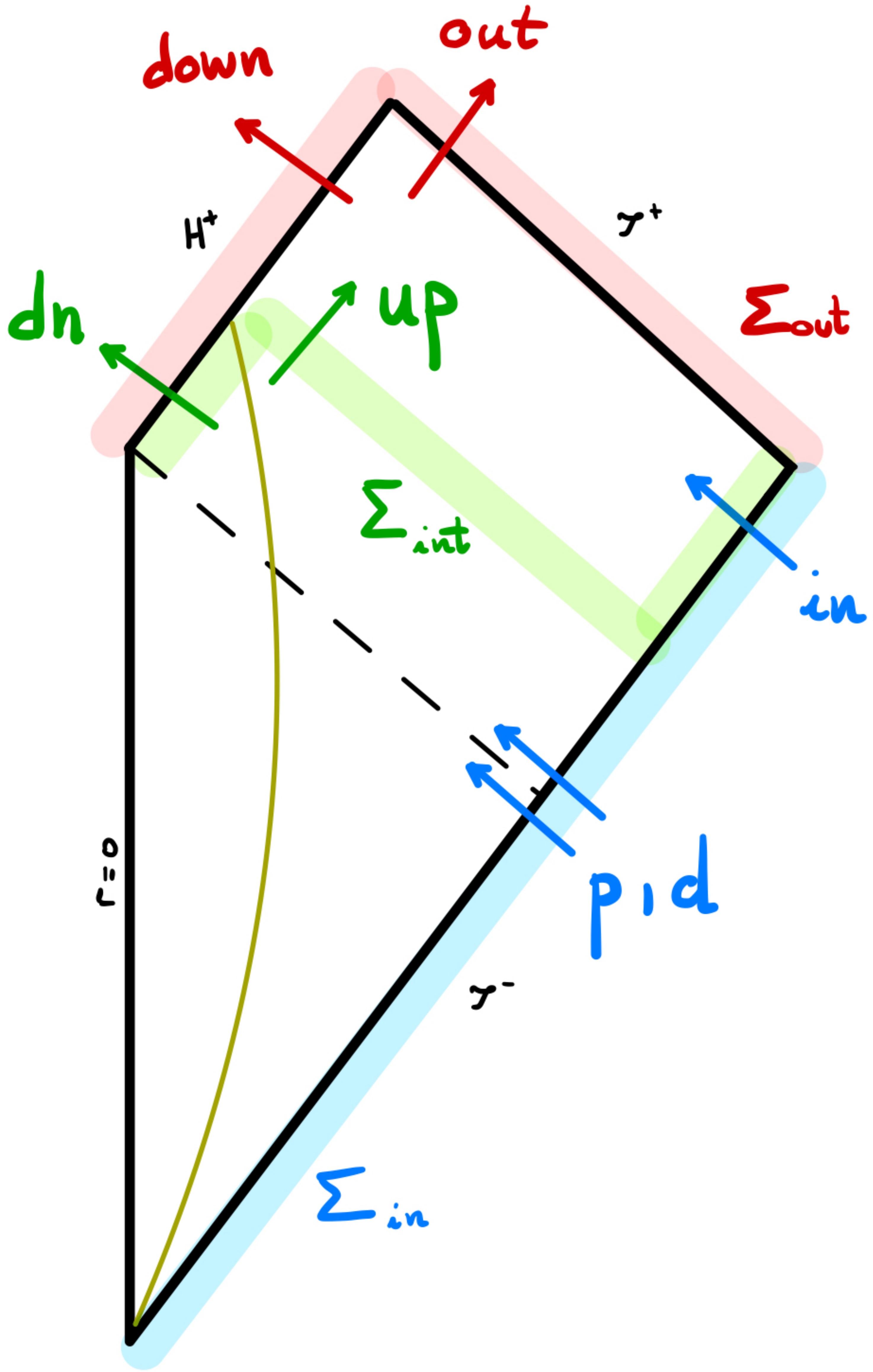
$$F_d(\omega) = N_{\omega\kappa}\left[f^*(v) + e^{-\frac{\pi\omega}{\kappa}} f^*(2v_H - v)\right]$$

Linear combination of positive-frequency in modes hence define the same in vacuum.

Hawking process in two steps

1- Particle creation near horizon, early times: p + d ⟶ up + dn

2-Scattering at potential barrier, late times: up + in ⟶ out + down



Hawking process in two steps

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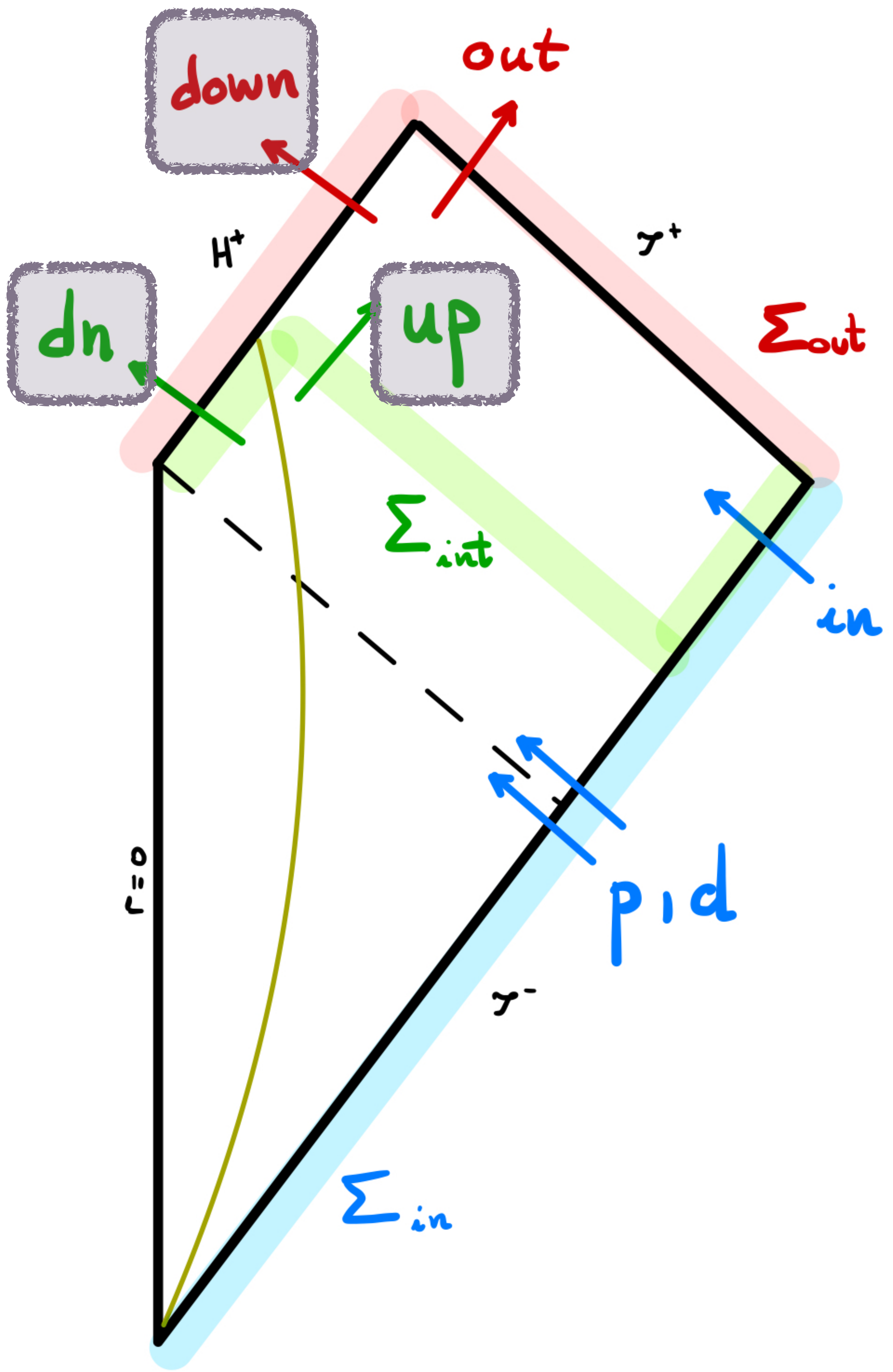
NORM (near the horizon)

Schwarzschild: $sign(\omega)$

Kerr: $sign(\omega - m\Omega_h)$ Superradiant conditon



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Particle creation at the horizon (Scwarzschild and Kerr)

$$\hat{a}^p_\omega \longrightarrow \hat{a}^{up}_\omega = \cosh r_H \, \hat{a}^p_\omega - \sinh r_H \, \hat{a}^{d\,\dagger}_\omega$$

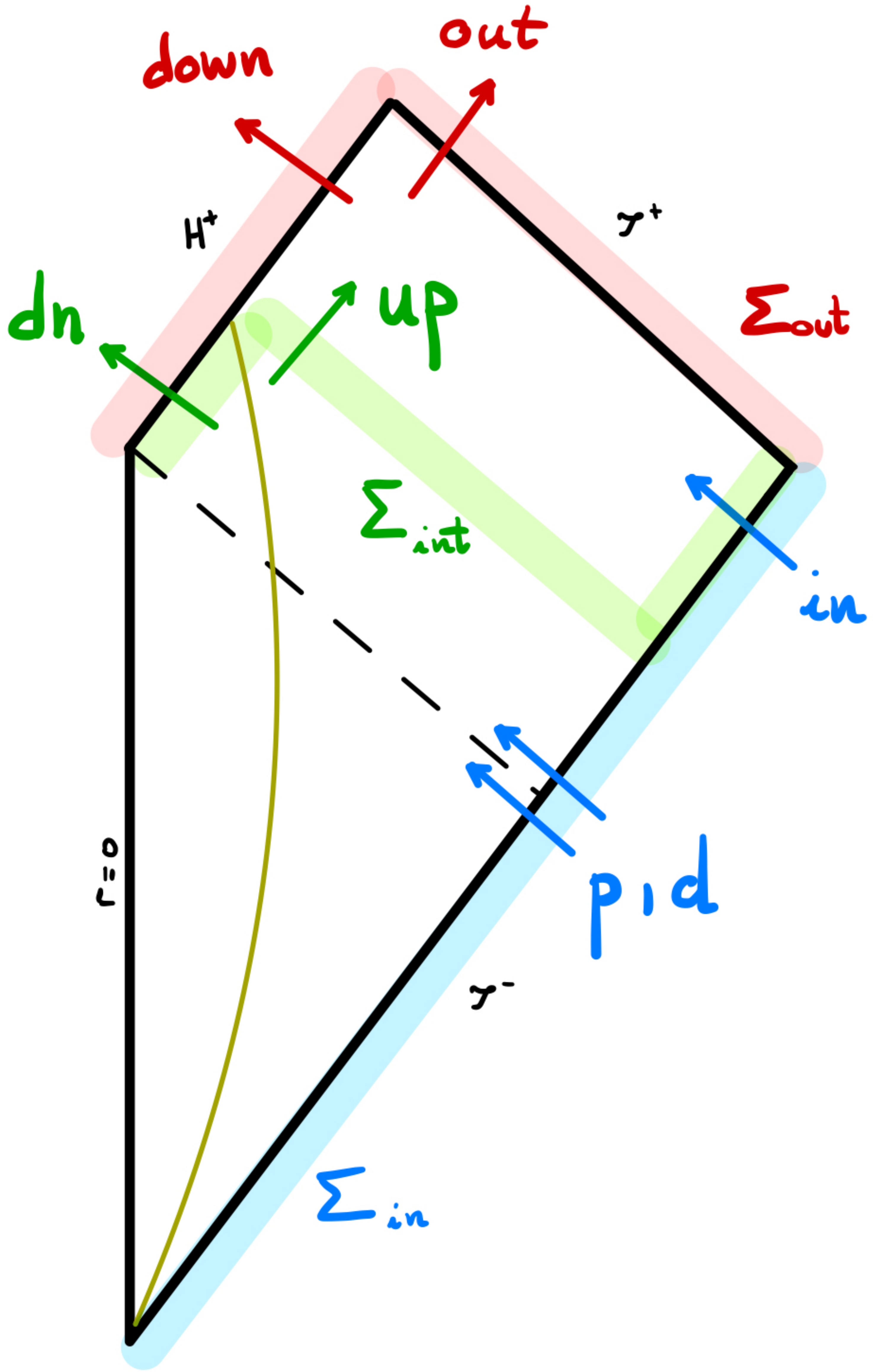
$$\hat{a}^d_\omega \longrightarrow \hat{a}^{dn}_\omega = -\sinh r_H \, \hat{a}^{p\,\dagger}_\omega + \cosh r_H \, \hat{a}^d_\omega$$

TWO-MODE SQUEEZER!

where

$$r_H(\omega, m) = \tanh^{-1} e^{-\frac{\omega - m\Omega_h}{3T_H}}$$

Hawking squeezing intensity



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TWO-MODE SQUEEZER!

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$$r_H(\omega, m) = \tanh^{-1} e^{-\frac{\omega - m\Omega_h}{2T_H}}$$

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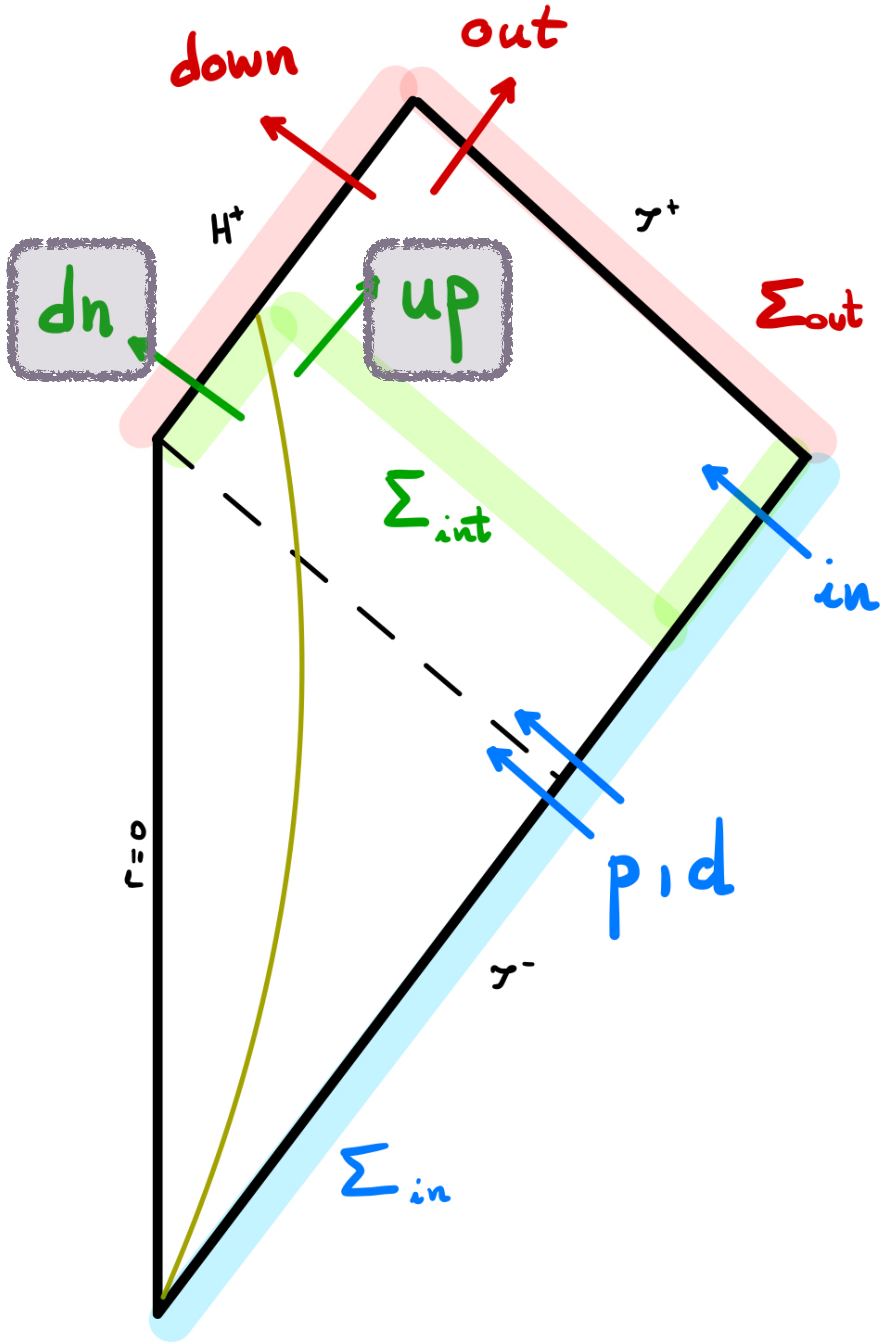

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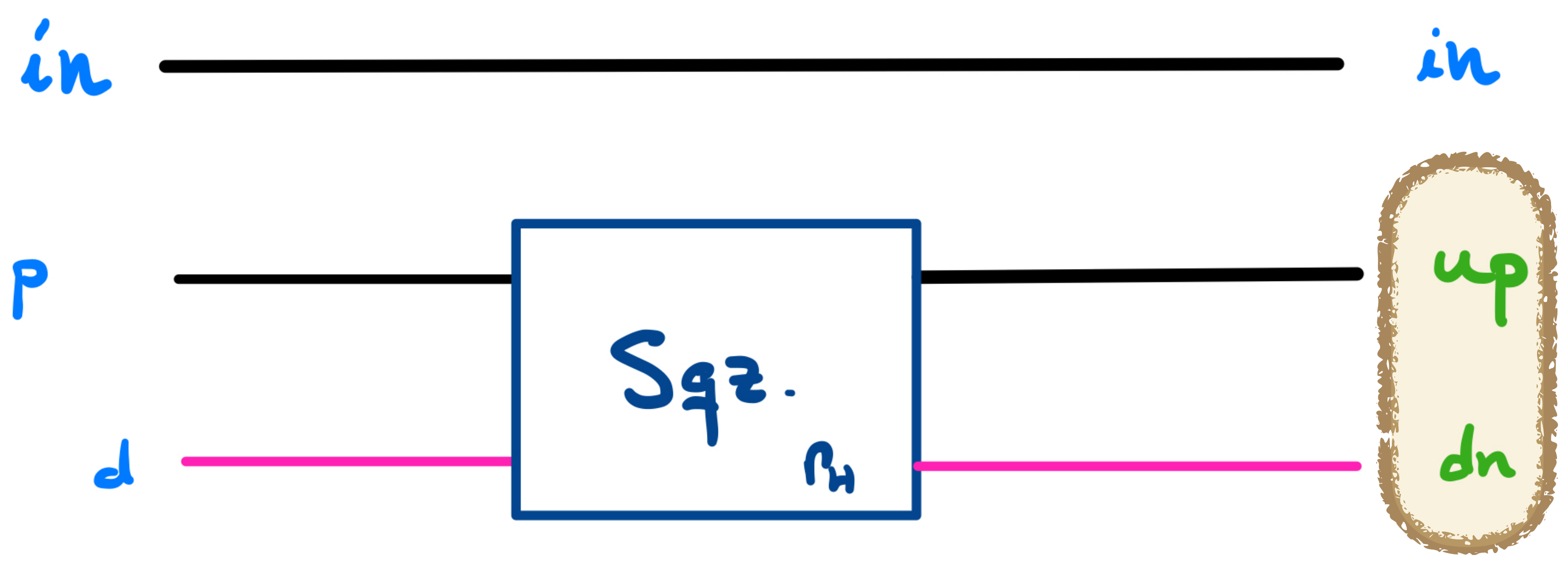
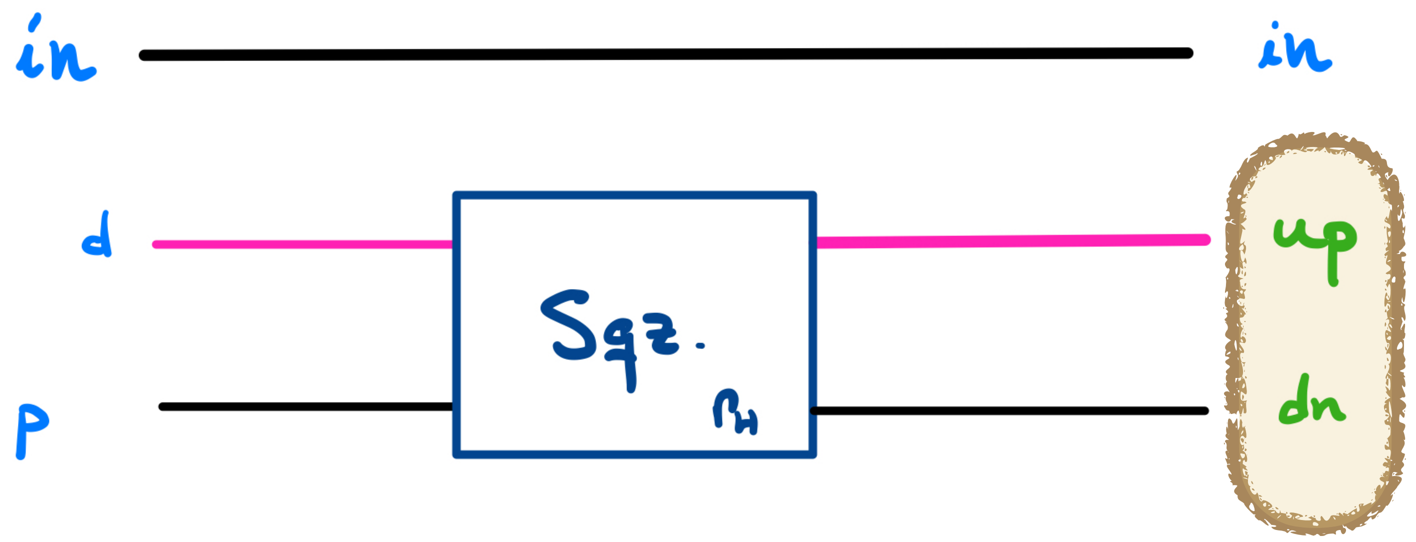
Non-superradiant



Entangled

Superradiant



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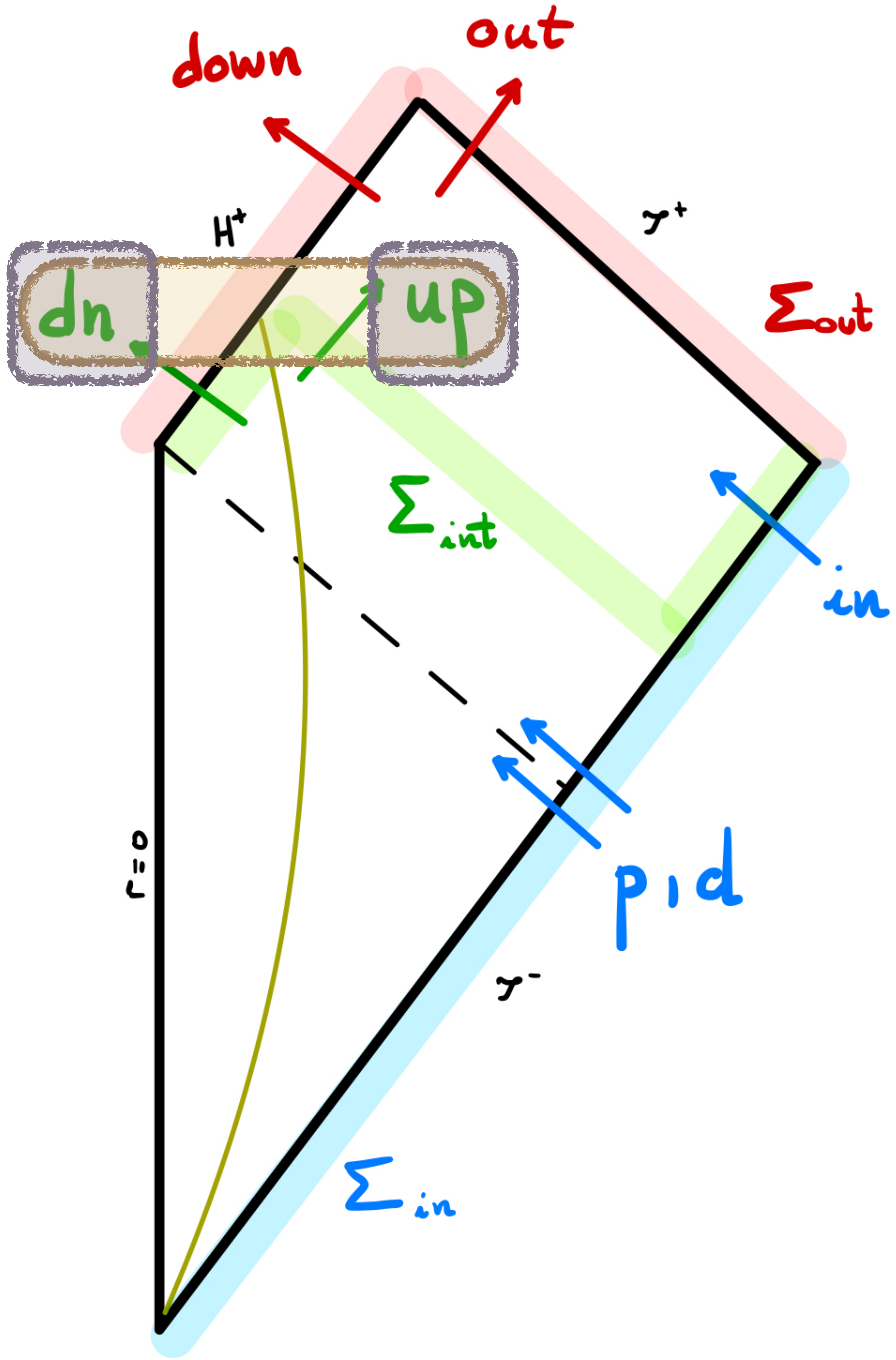

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Scattering with gravitational potential

$$\hat{a}^{up}_{\omega} \longrightarrow \hat{a}^{out}_{\omega} = \cos\theta_\Gamma\, \hat{a}^{up}_{\omega} + \sin\theta_\Gamma\, \hat{a}^{in}_{\omega}$$

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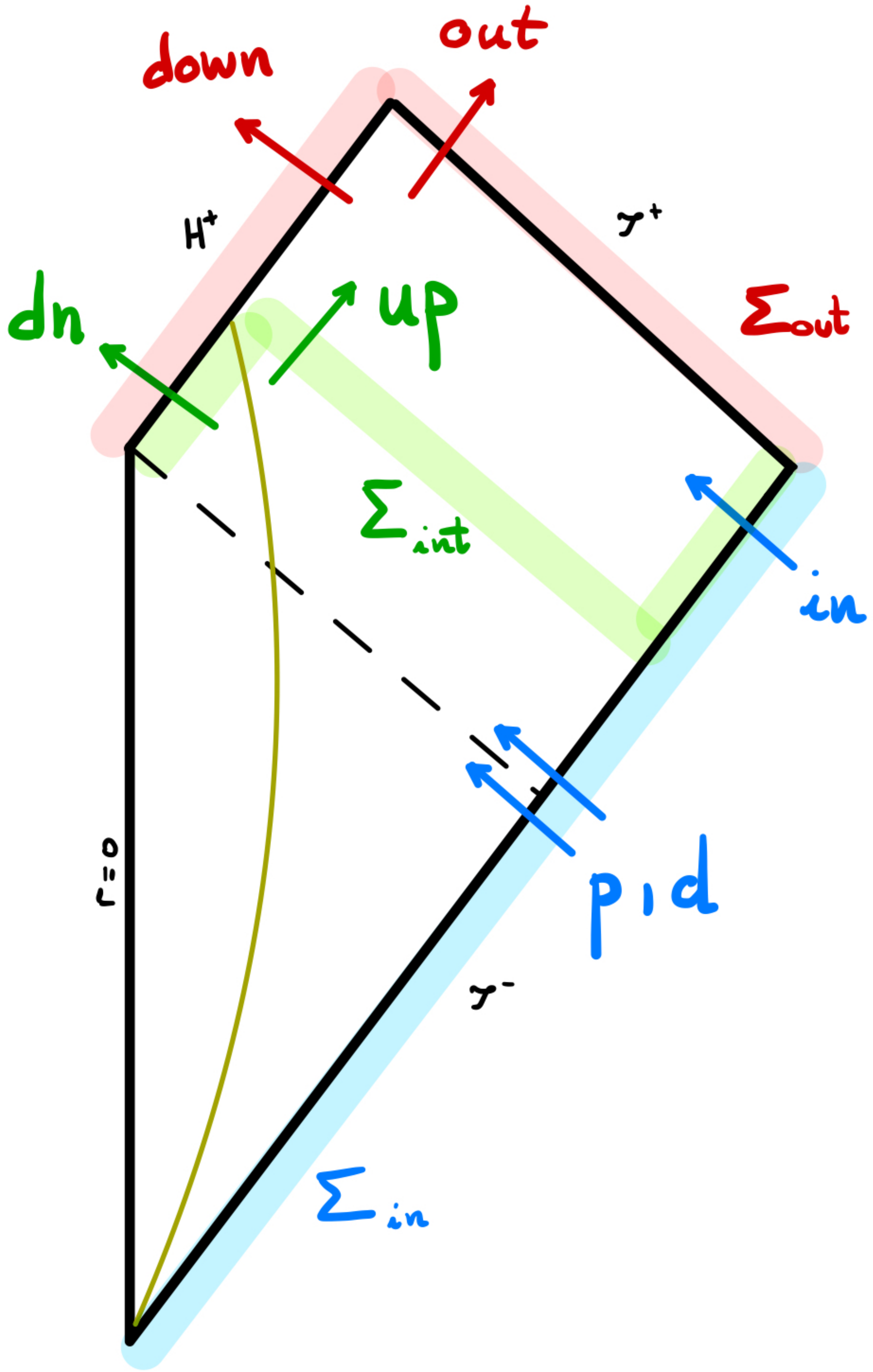
BEAM SPLITTER

Where: $\Gamma_{\omega\ell} = \sin^2\theta_\Gamma$

(greybody factors from Teukolsy equation)



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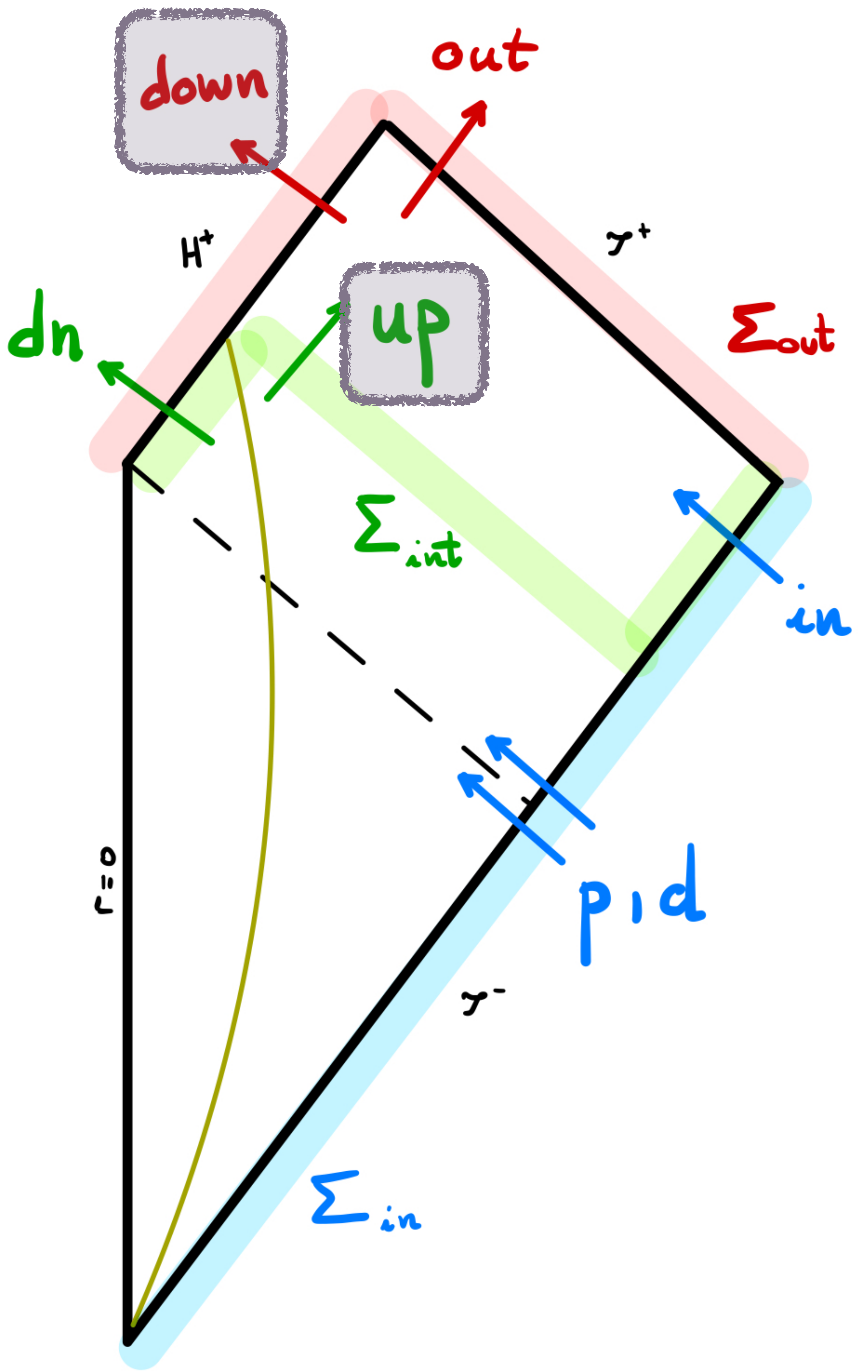

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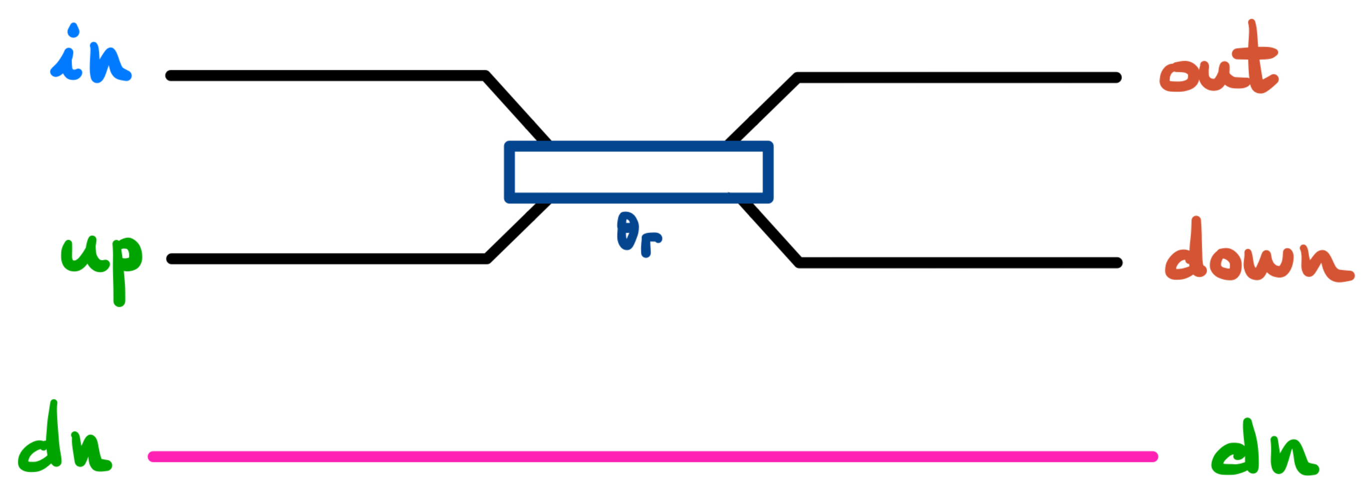

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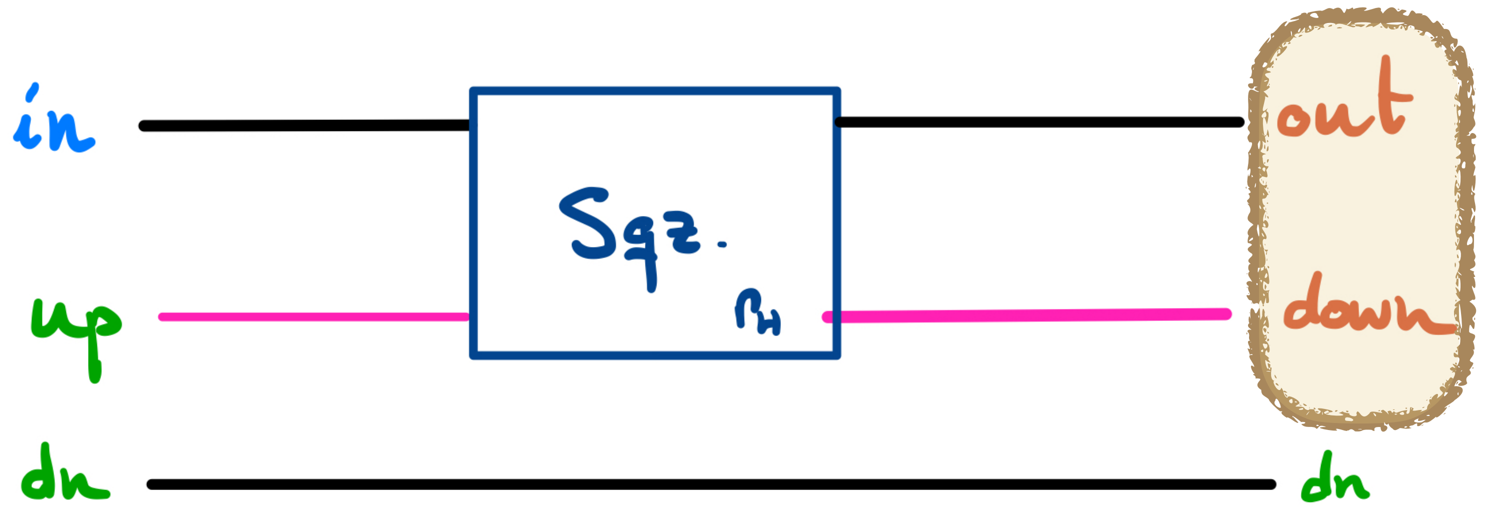
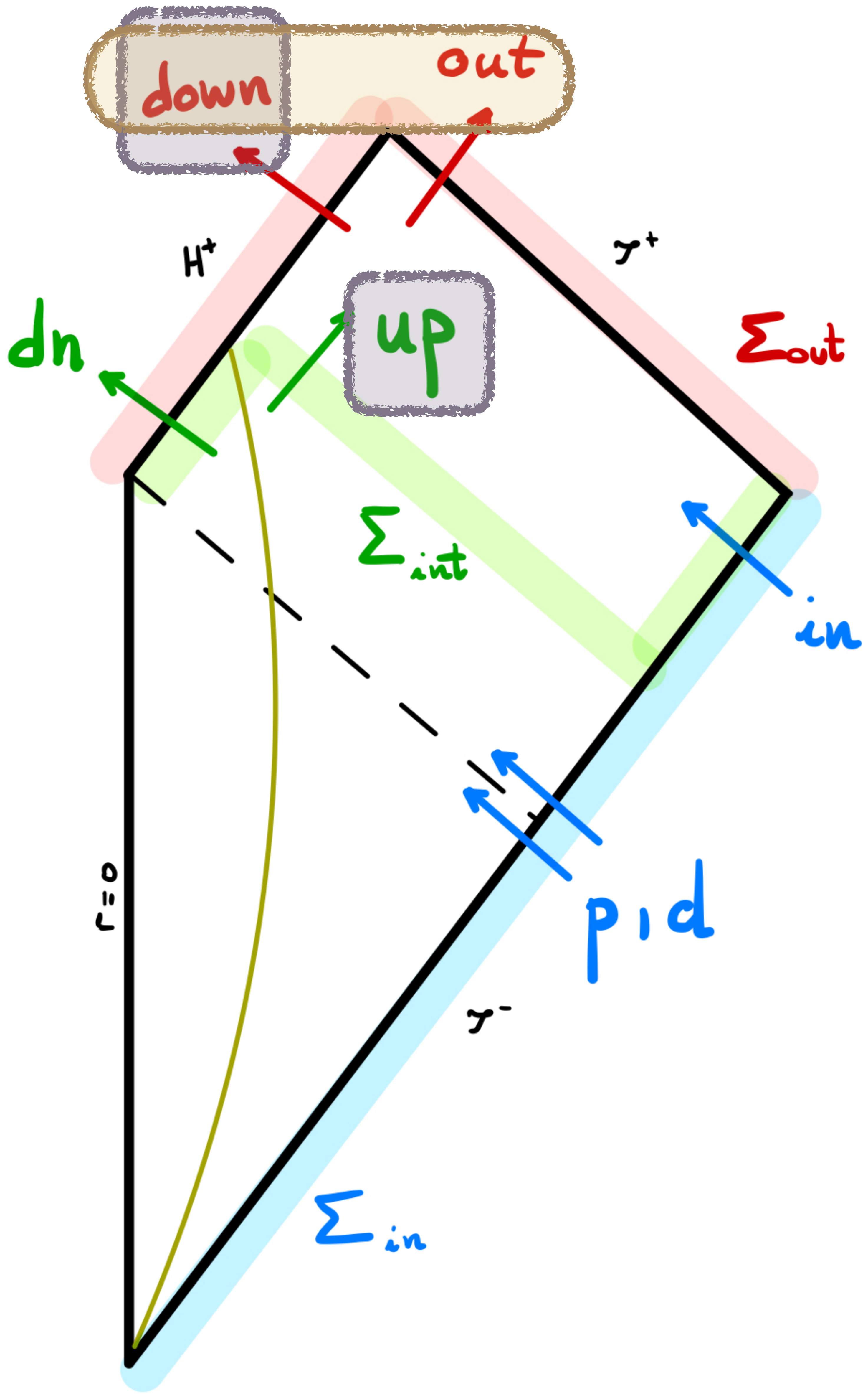
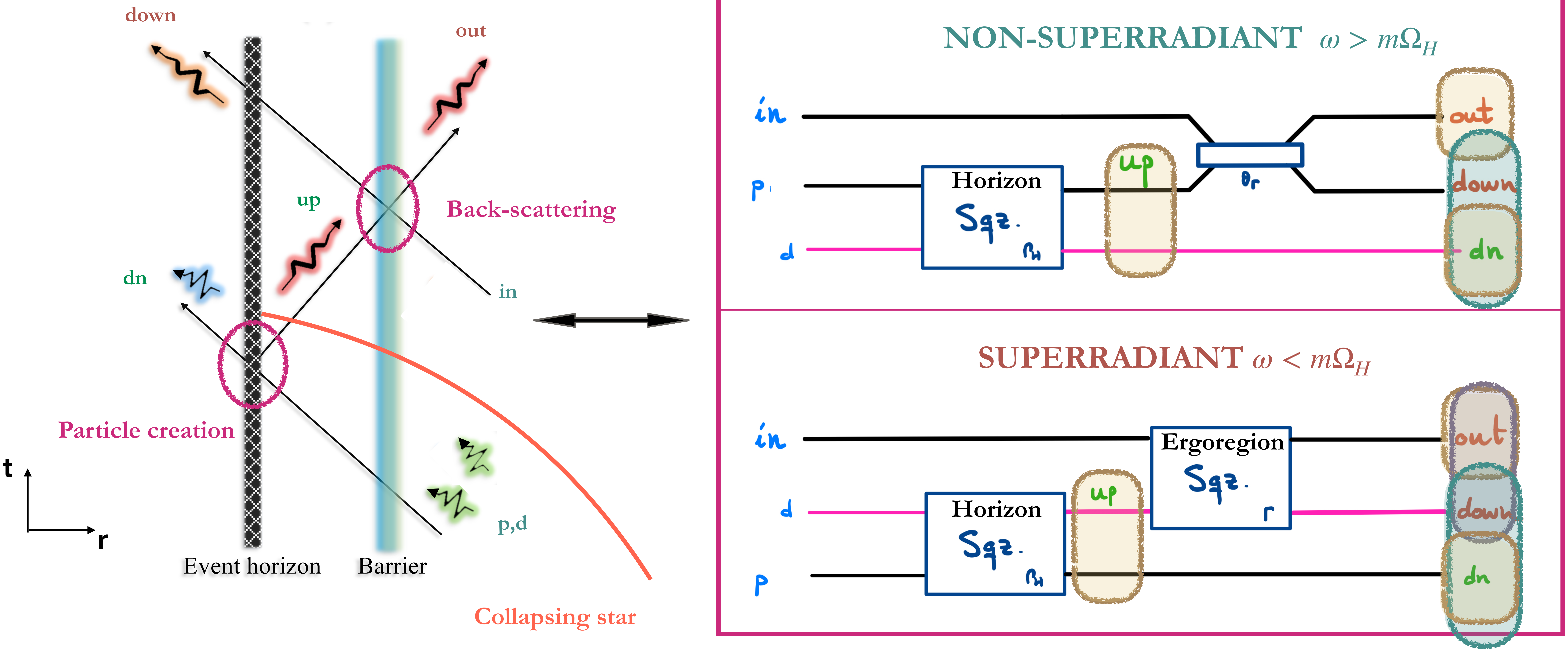




arXiv:2307.06215

Summary of Hawking process in rotating BHs



Hawking process in two steps

Ergoregion is a source of entanglement, as much as the horizon is. However:

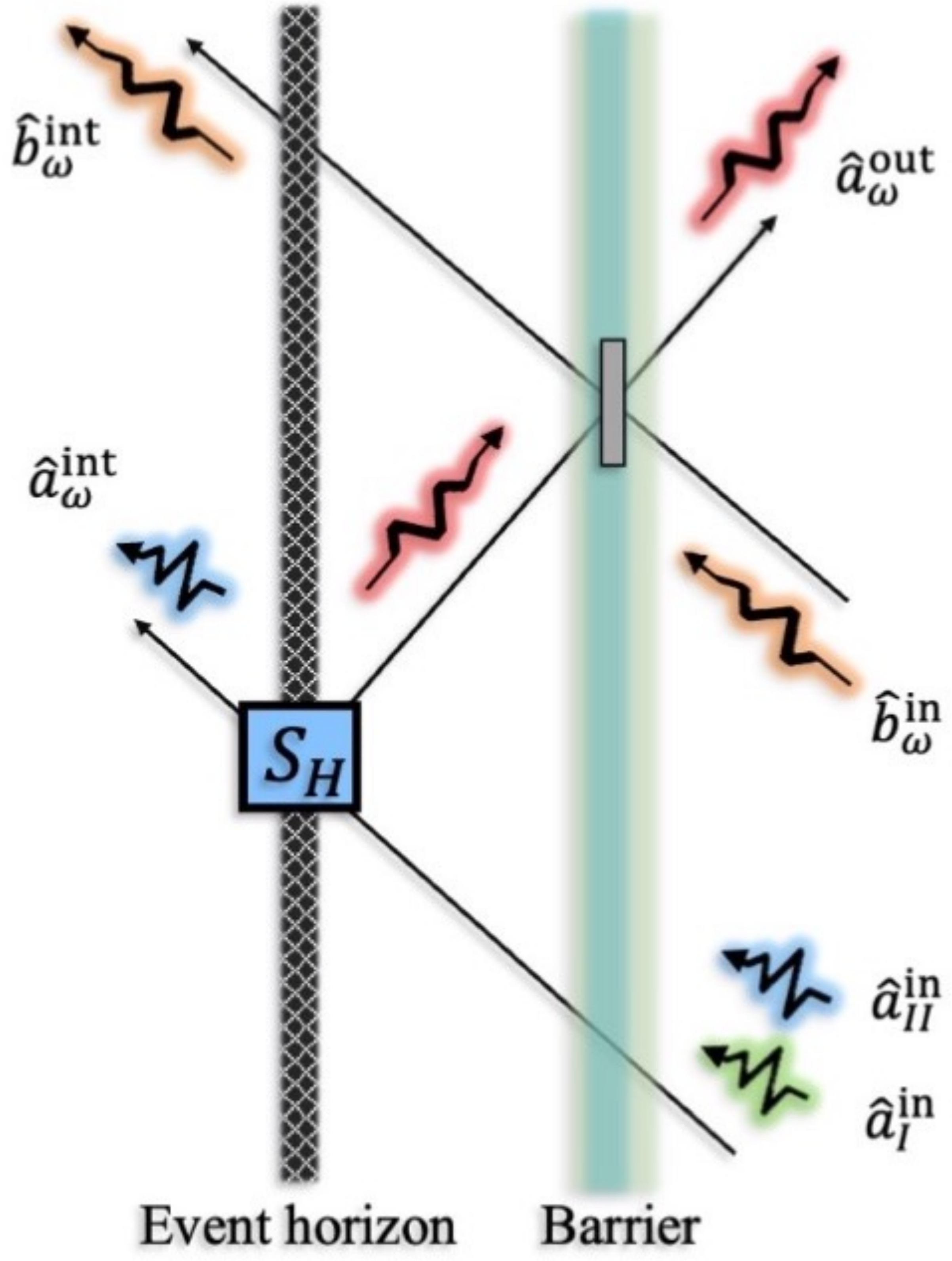
Ergoregion squeezer does not have a temperature: not black-body radiation

Ergoregion squeezer is stimulated by thermal radiation coming our from the horizon

The Hawking process involves an interplay between two squeezing process. One associated with Hawking pair creation and another associated with Superradiance

Entanglement degradation from CMB temperature

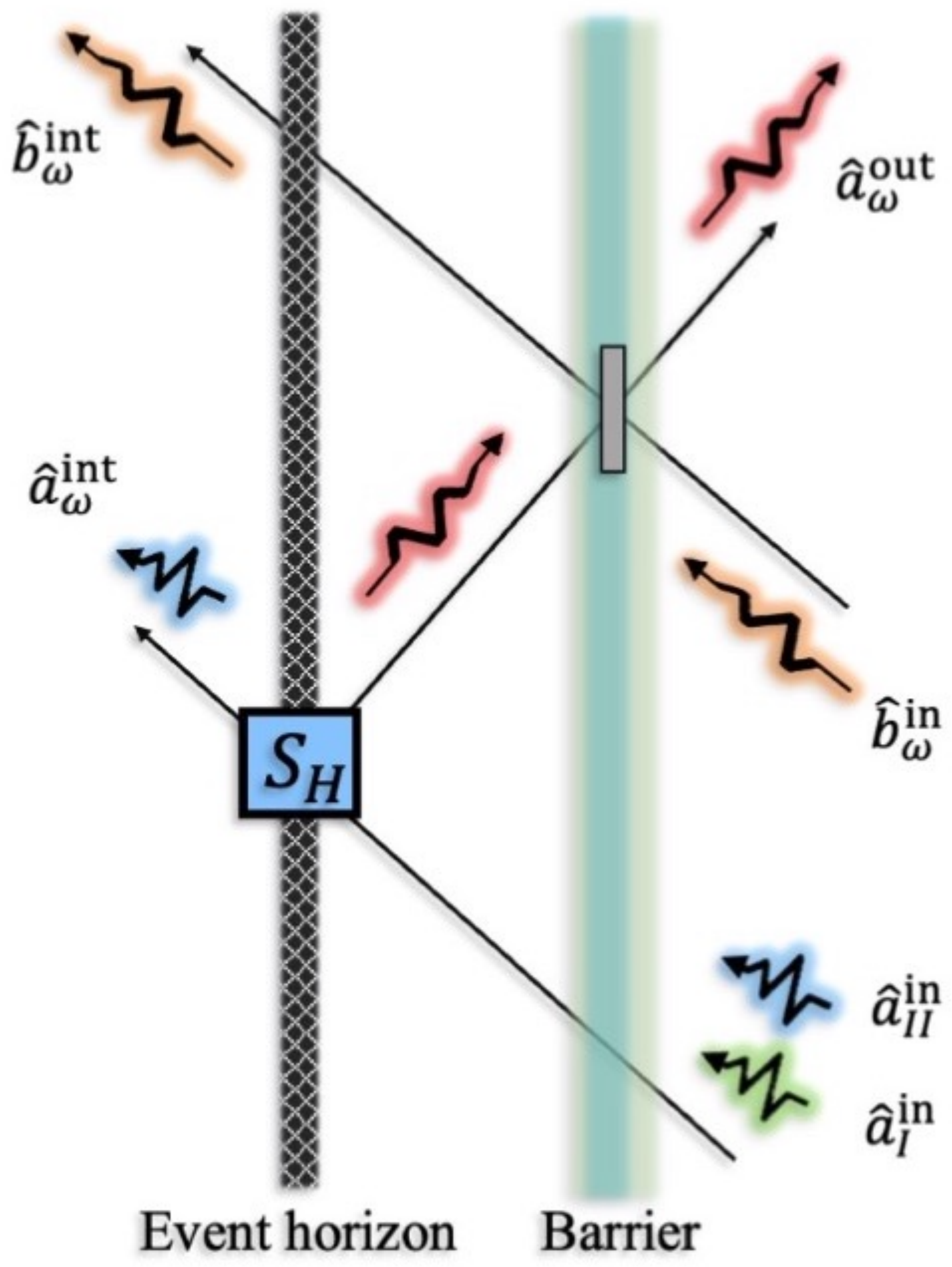
Entanglement degradation from CMB temperature



← **Thermal input** **(CMB)**

← **Vacuum input**
(very UV, not seeded by CMB)

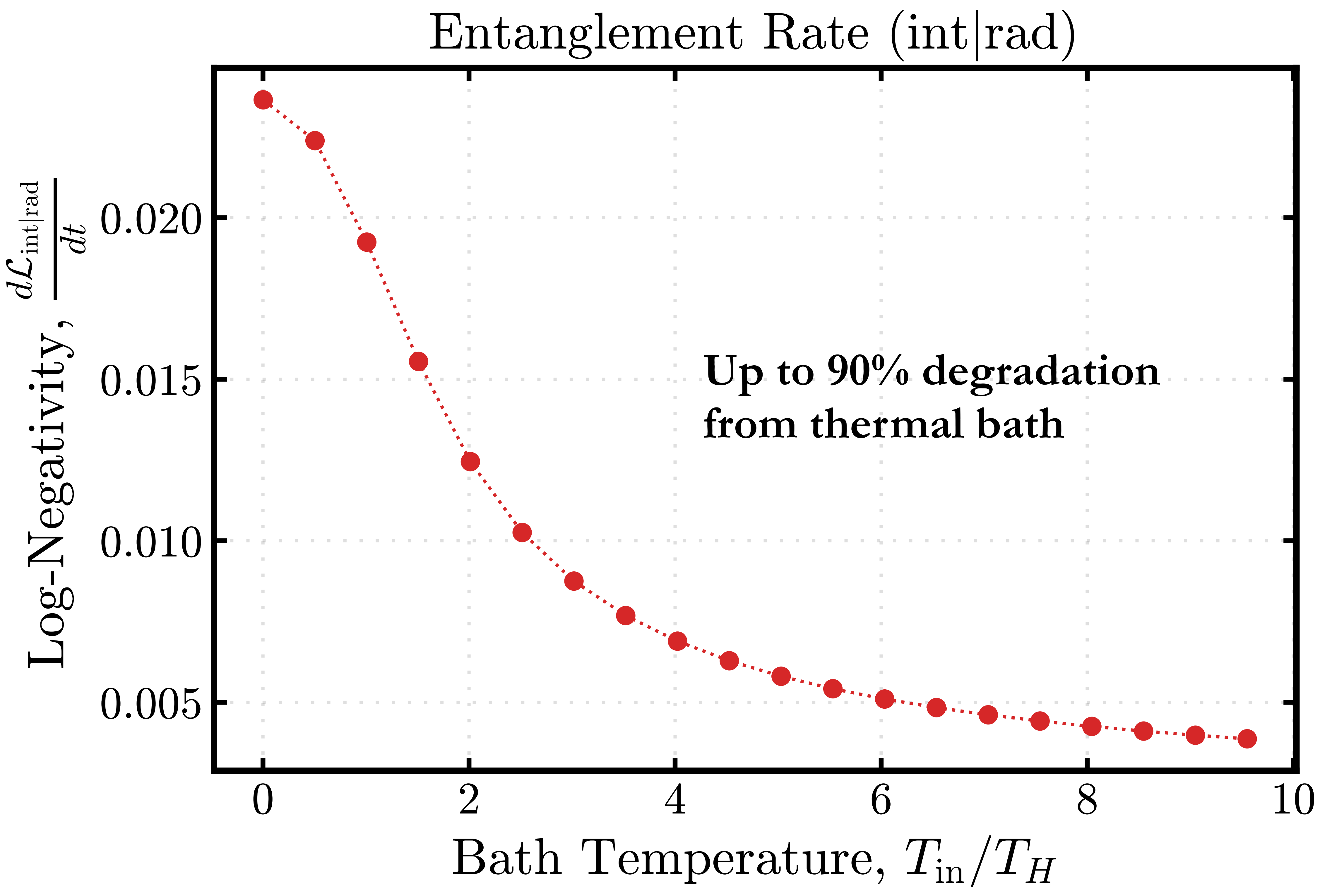
Entanglement degradation from CMB temperature



← Thermal input (CMB)

← Vacuum input (very UV, not seeded by CMB)

LogNeg rate of emission

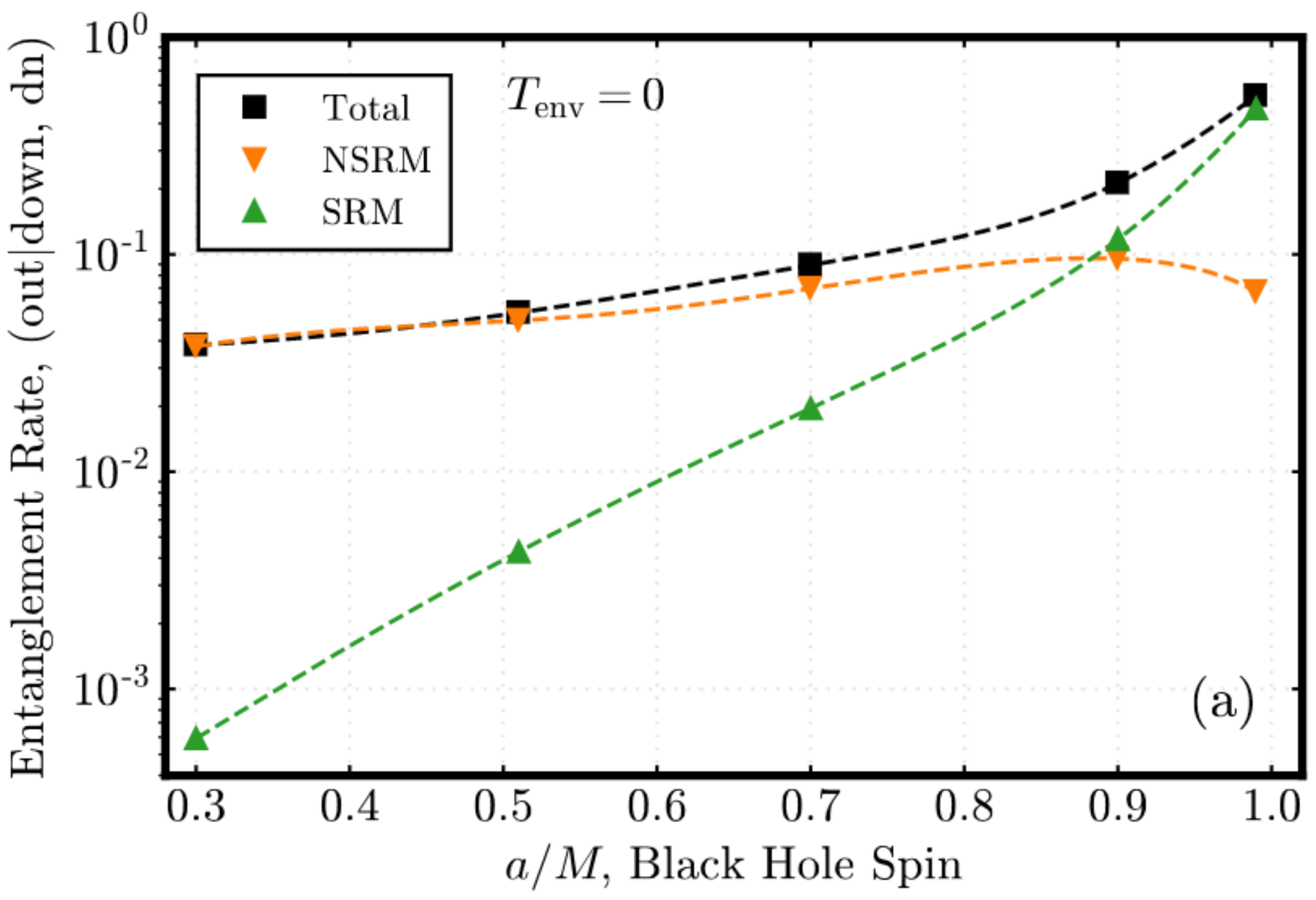


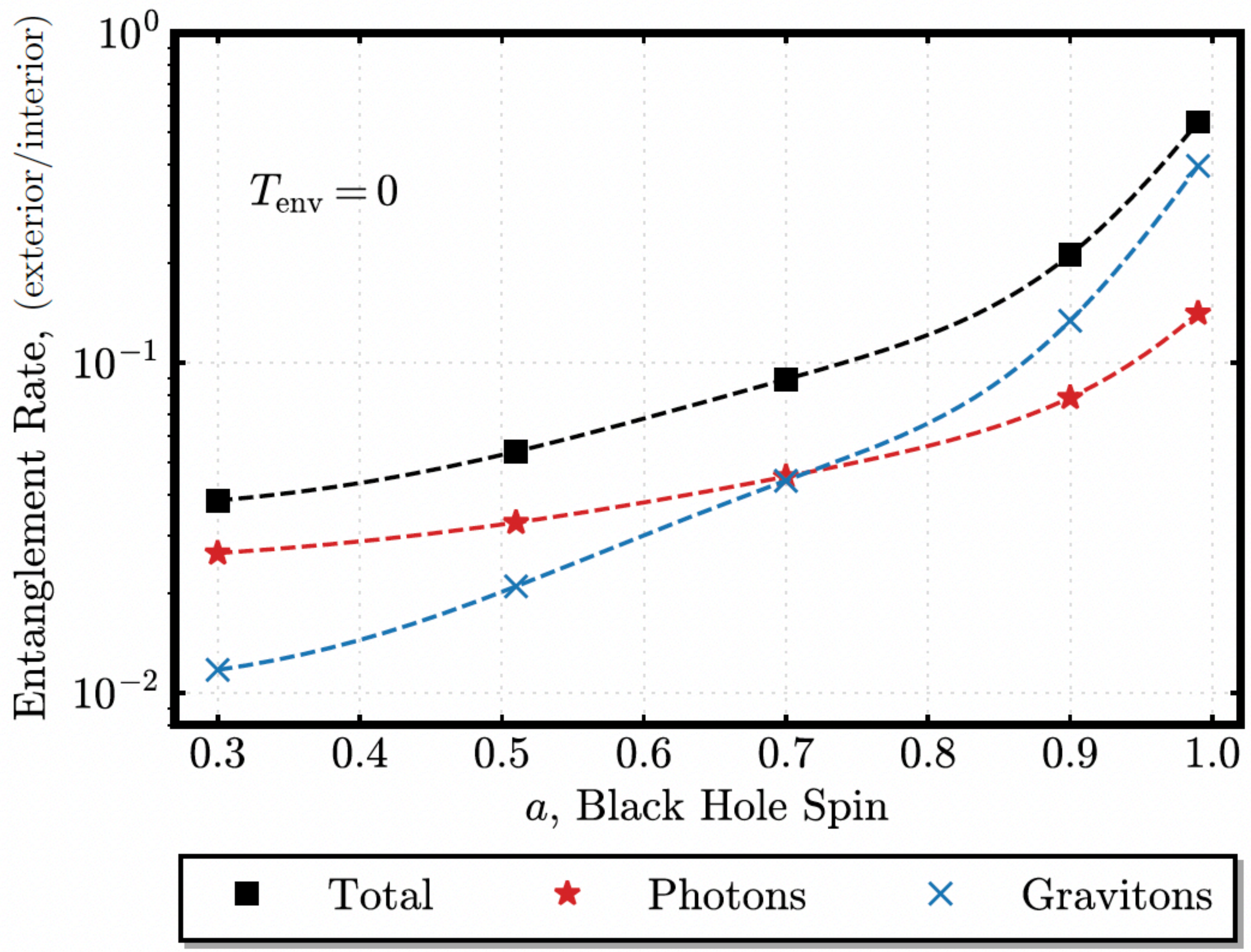
Sum over ℓ, m and w

Entanglement from rotating BHs

arXiv:2307.06215

LogNeg rate of emission





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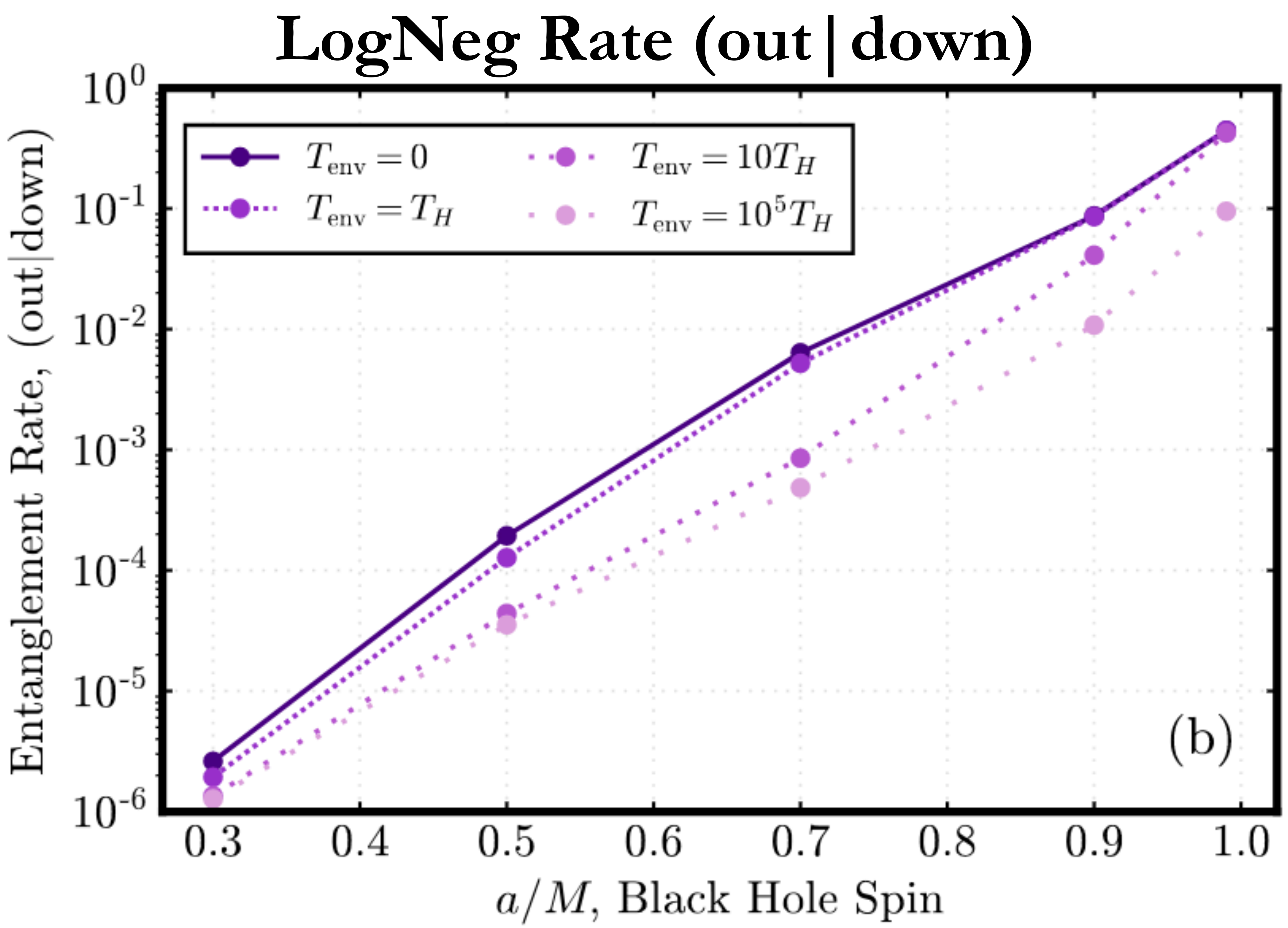


Signature of Superradiance!

Sum over ω, ℓ, m

Towards experimental detection with polariton fluids

Rotational superradiance

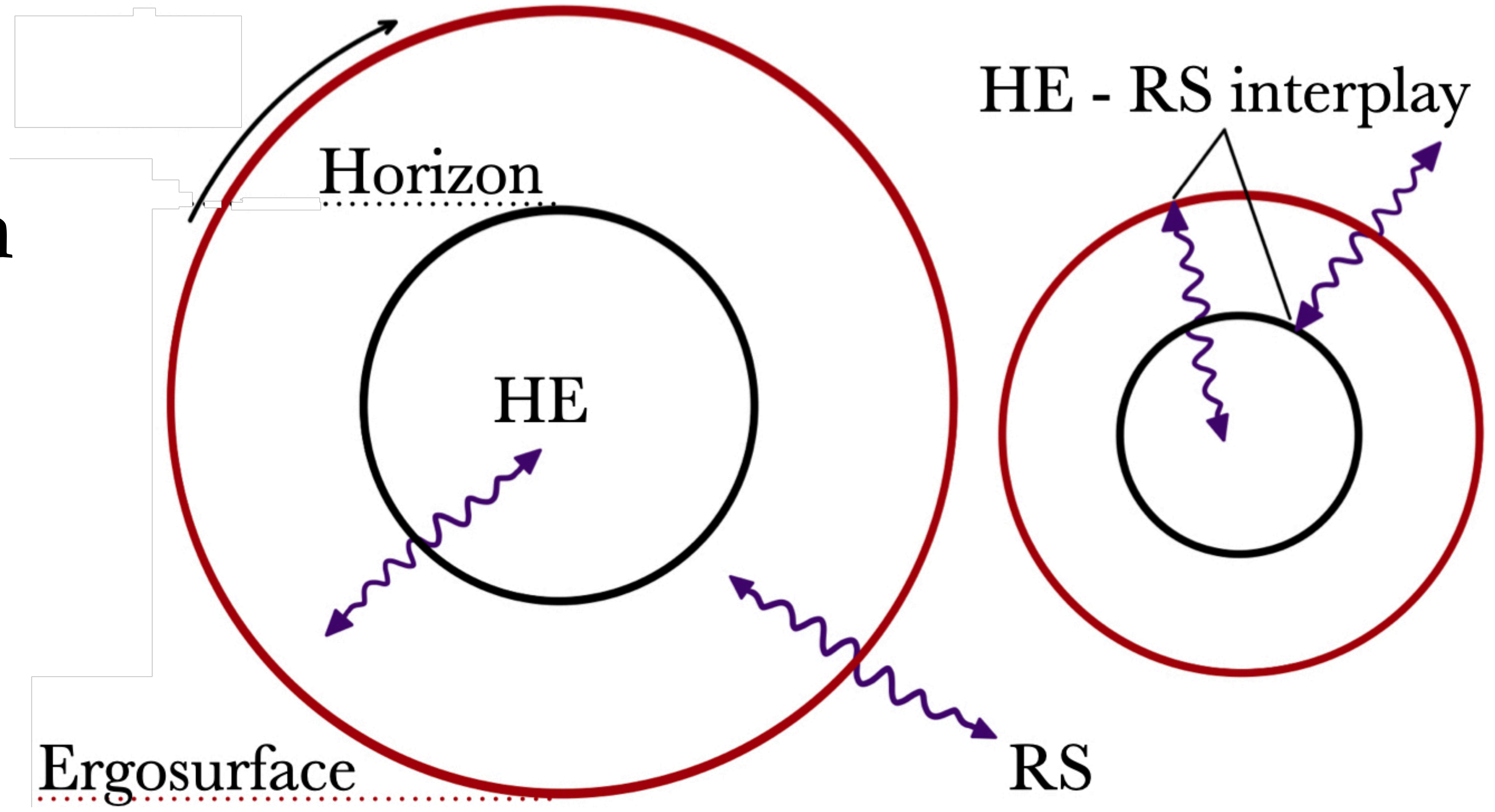
Rotational superradiance tied to **ergoregions**: co-rotating waves can be amplified

Ergoregion instability forbids isolated ergoregions

Friedman 78'

Kerr BHs are stable because horizon acts as one-way membrane for the instability.

But… horizon sources ergoregion with radiation



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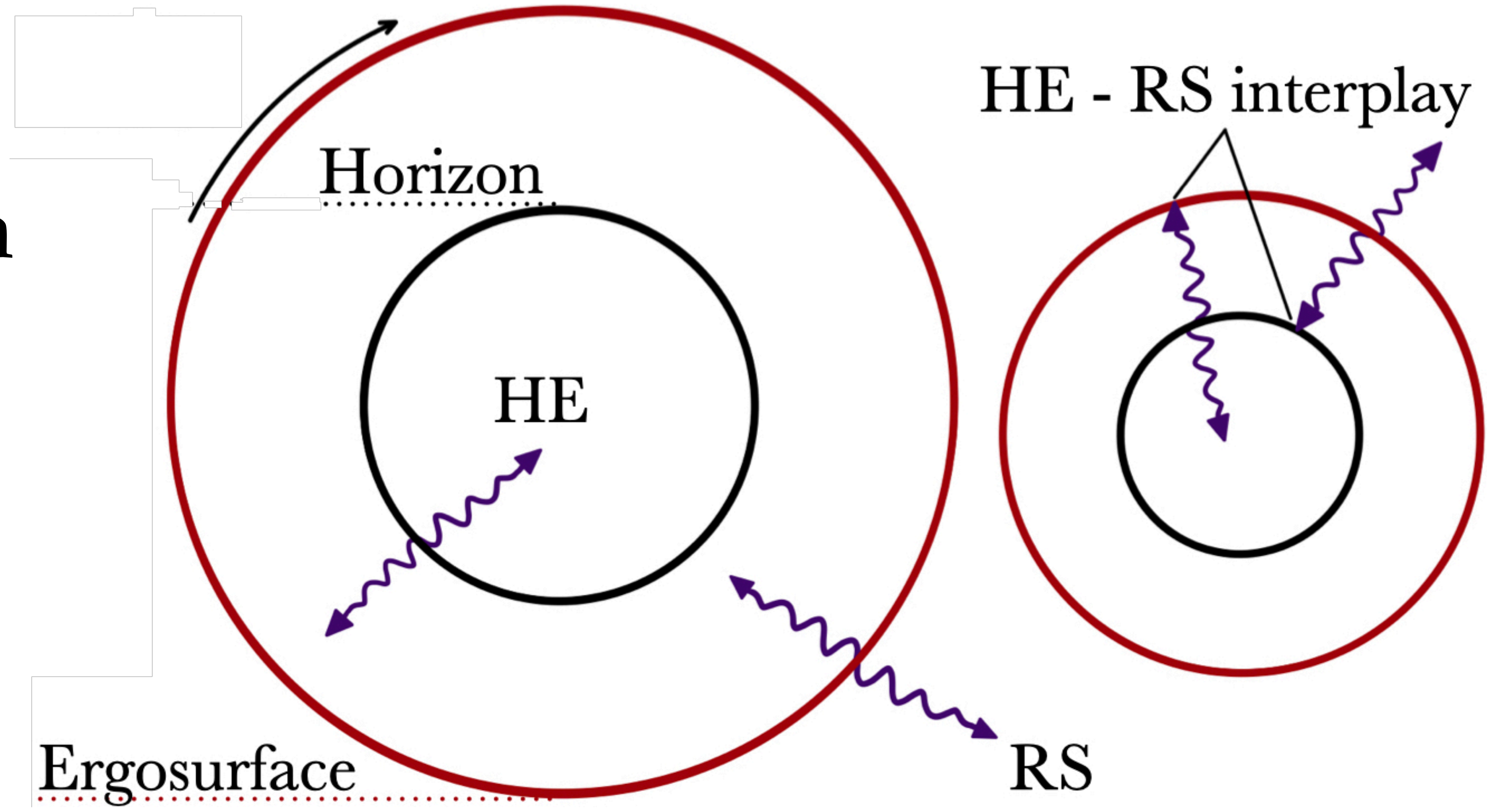
Goal: Quantify and measure HE-SR interplay



Need to characterize rotational SR in isolation from horizons!

New proposal: horizonless ergoregions thanks to dissipative dynamics

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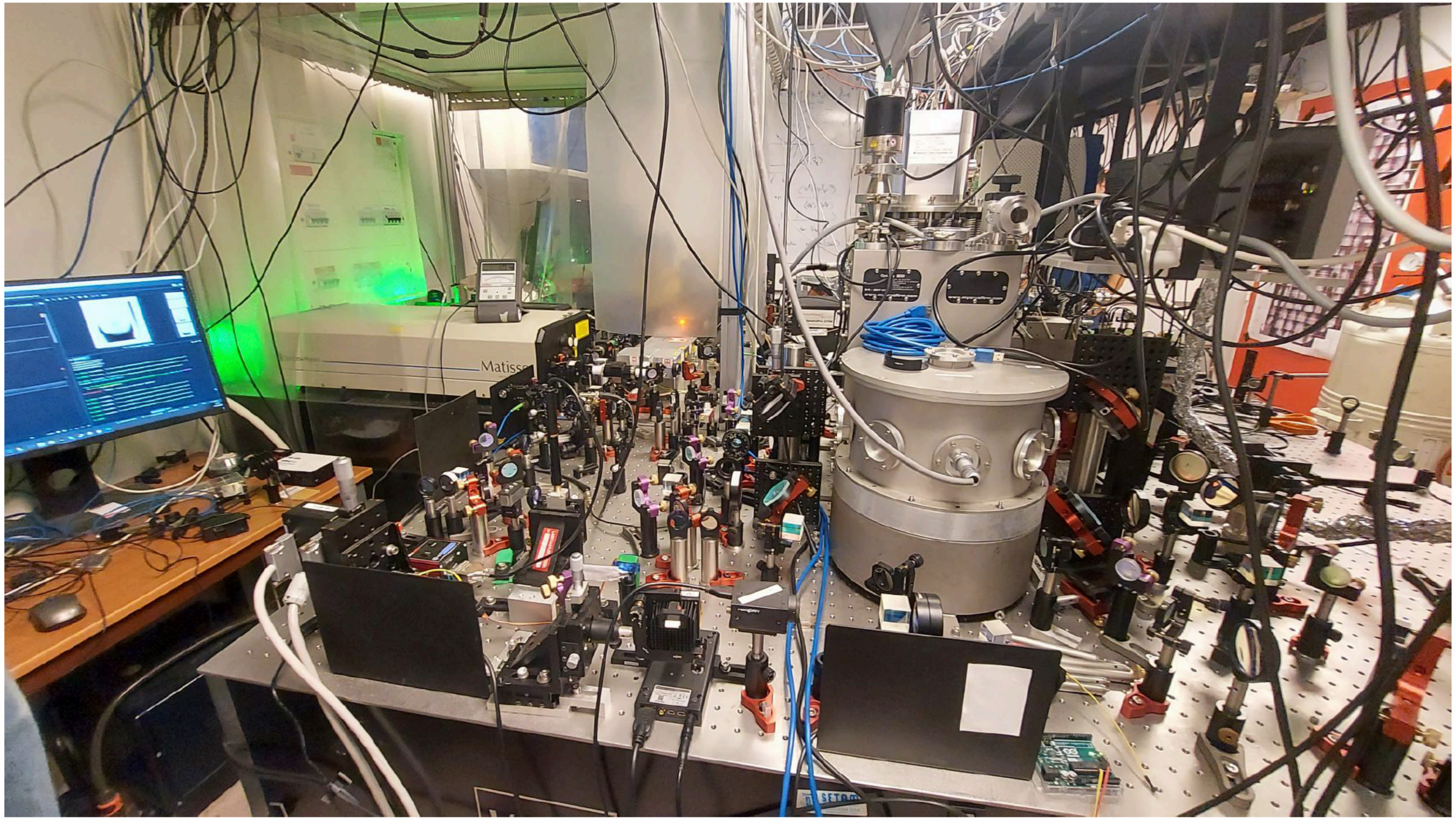
Polariton fluids: a bosonic 2D fluid of light in semiconductor cavity (made of electrons, holes, and photons)

Intrinsically driven-dissipative

Fluid profile controlled by laser drive. Highly tunable.

Rotational superradiance in the lab

New proposal: horizonless ergoregions thanks to dissipative dynamics



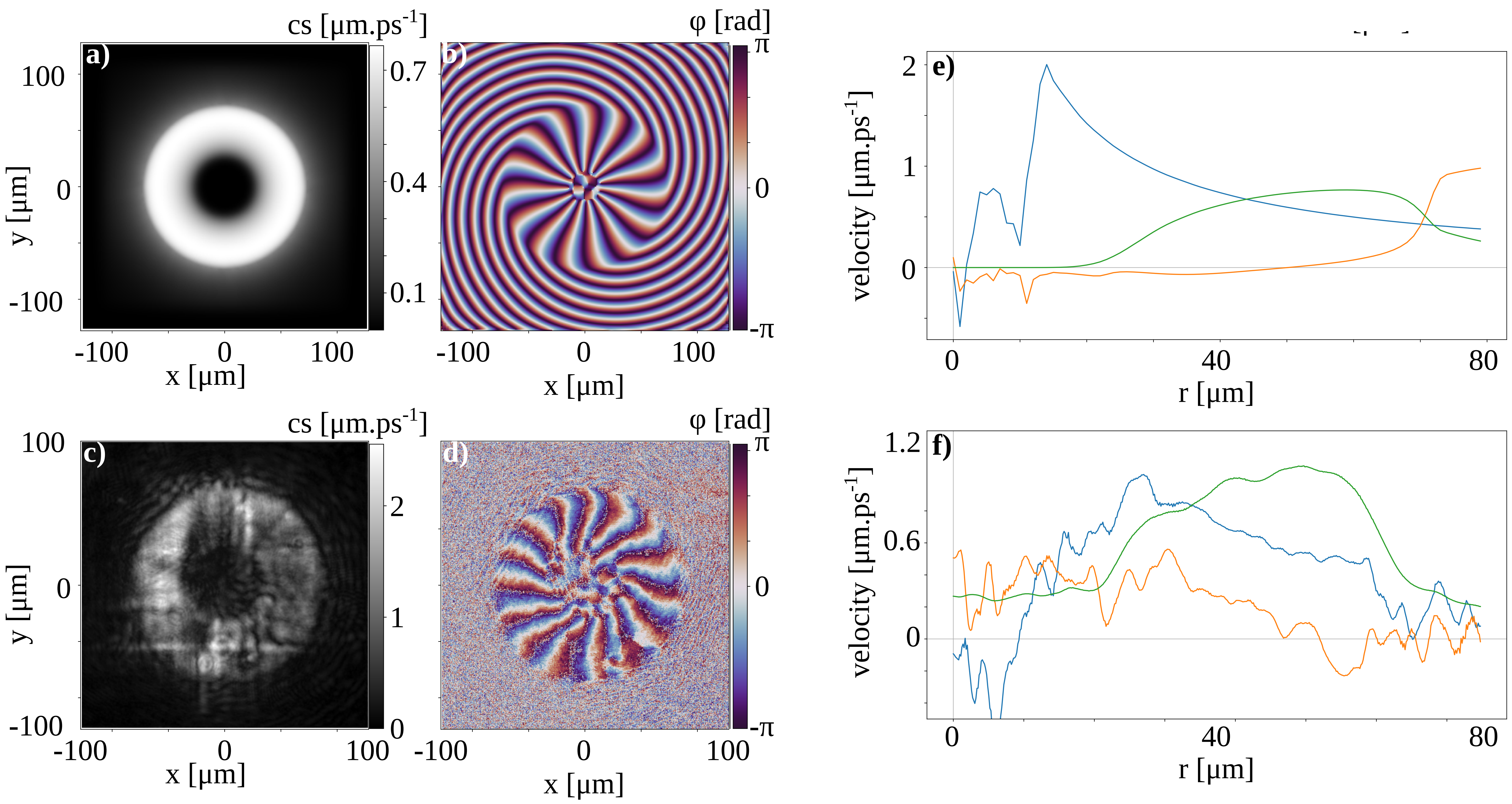
LKB, Sorbonne Université

New proposal: horizonless ergoregions thanks to dissipative dynamics

Preliminar evidence of stable horizonless ergoregions in dissipative quantum fluids of light

Numerical Simulations

Experimental Data (Preliminary)



Rotational superradiance in the lab

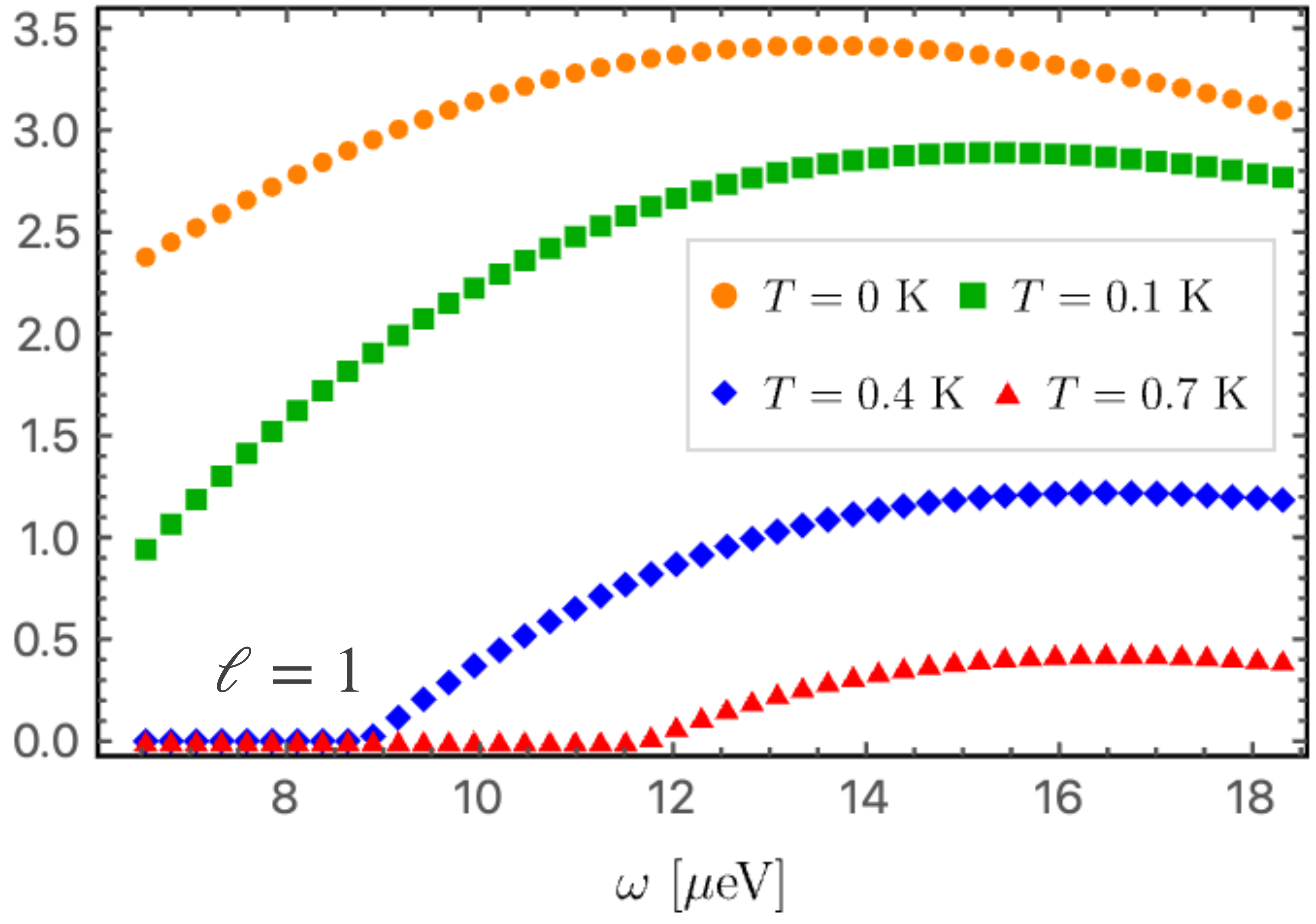
arXiv:2310.16031

Entanglement Entropy vacuum input / Entanglement Entropy squeezed input

Rotational superradiance in the lab

arXiv:2310.16031

LogNeg thermal input



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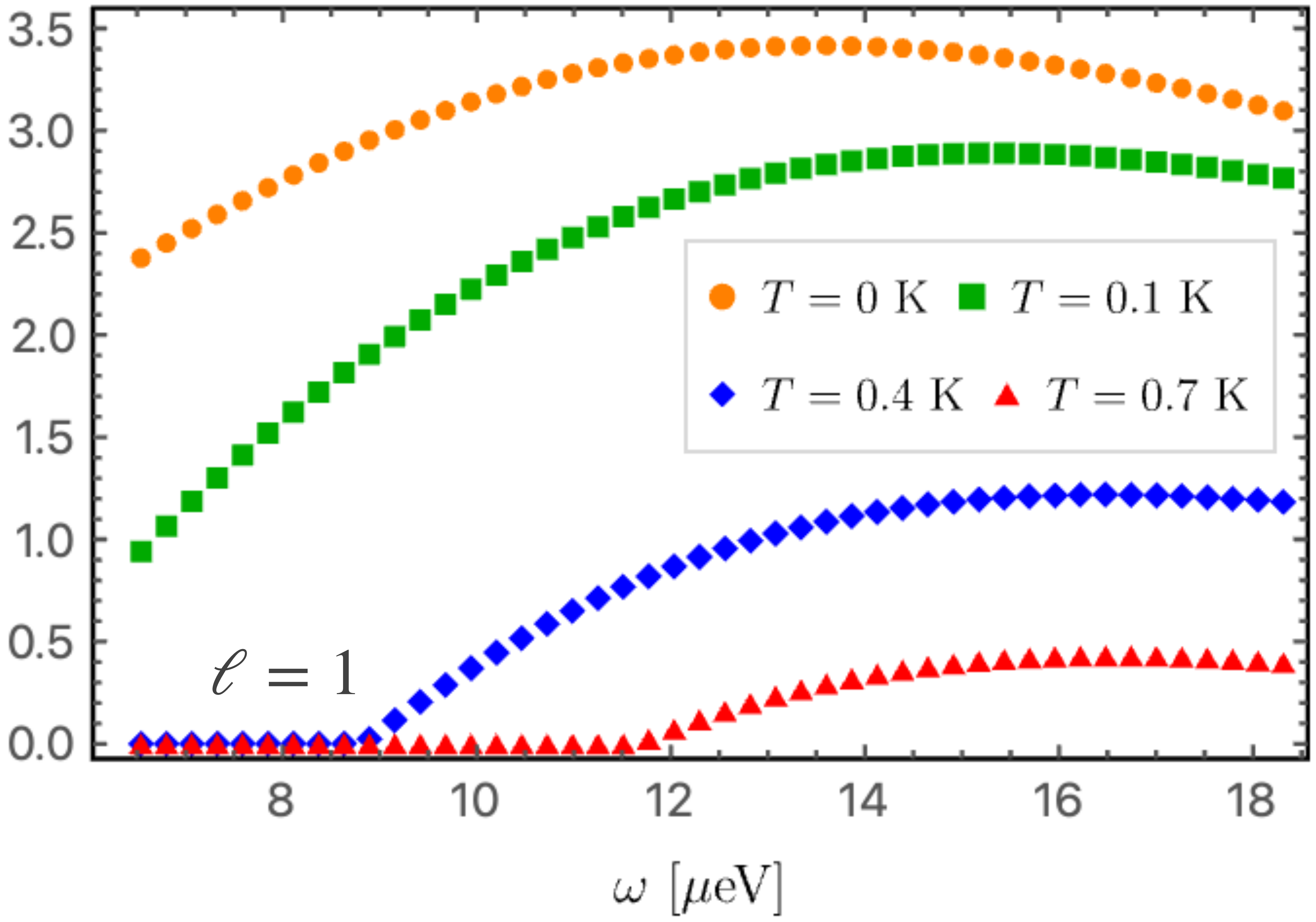
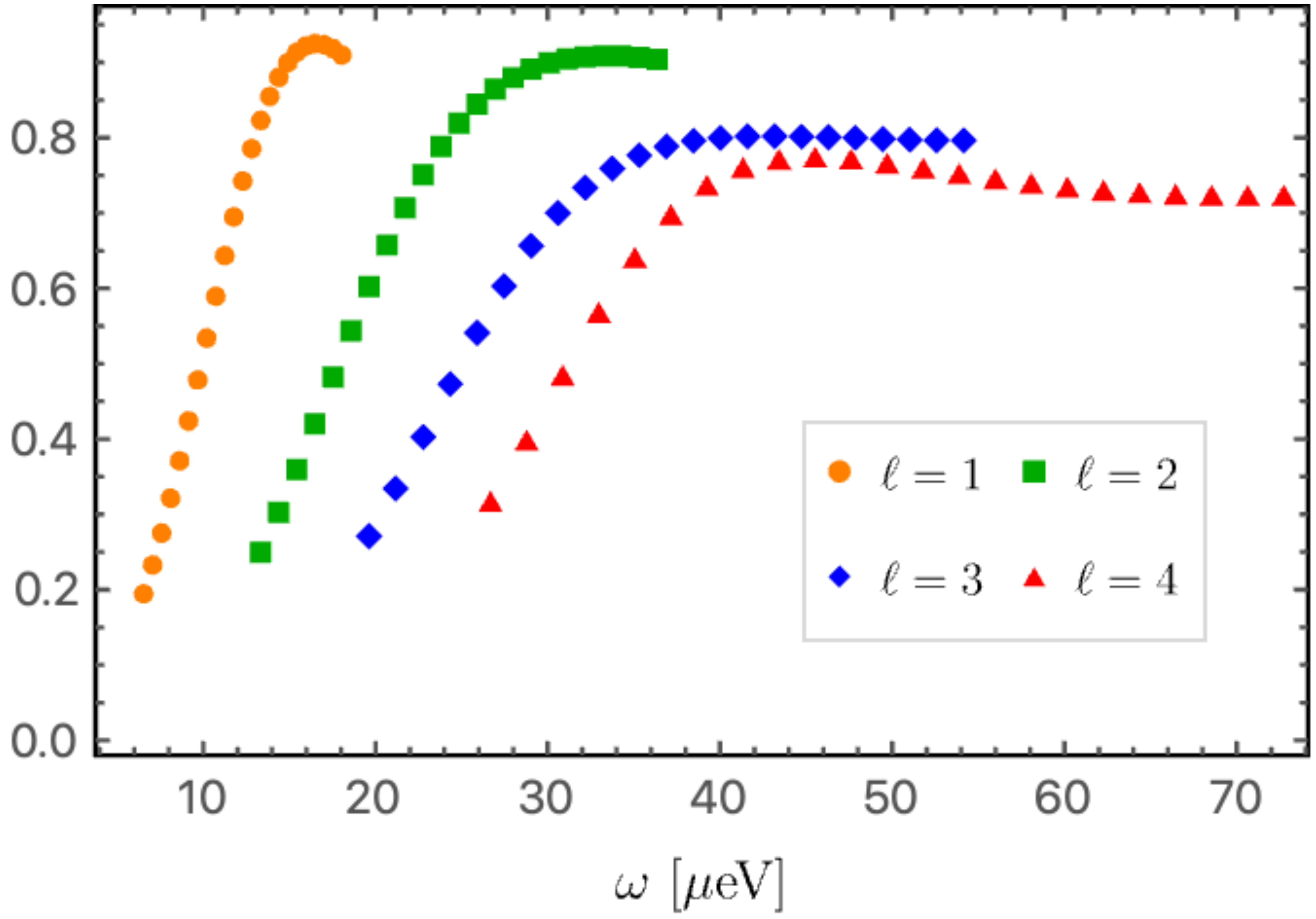


Critical Temperature (LogNeg=0)

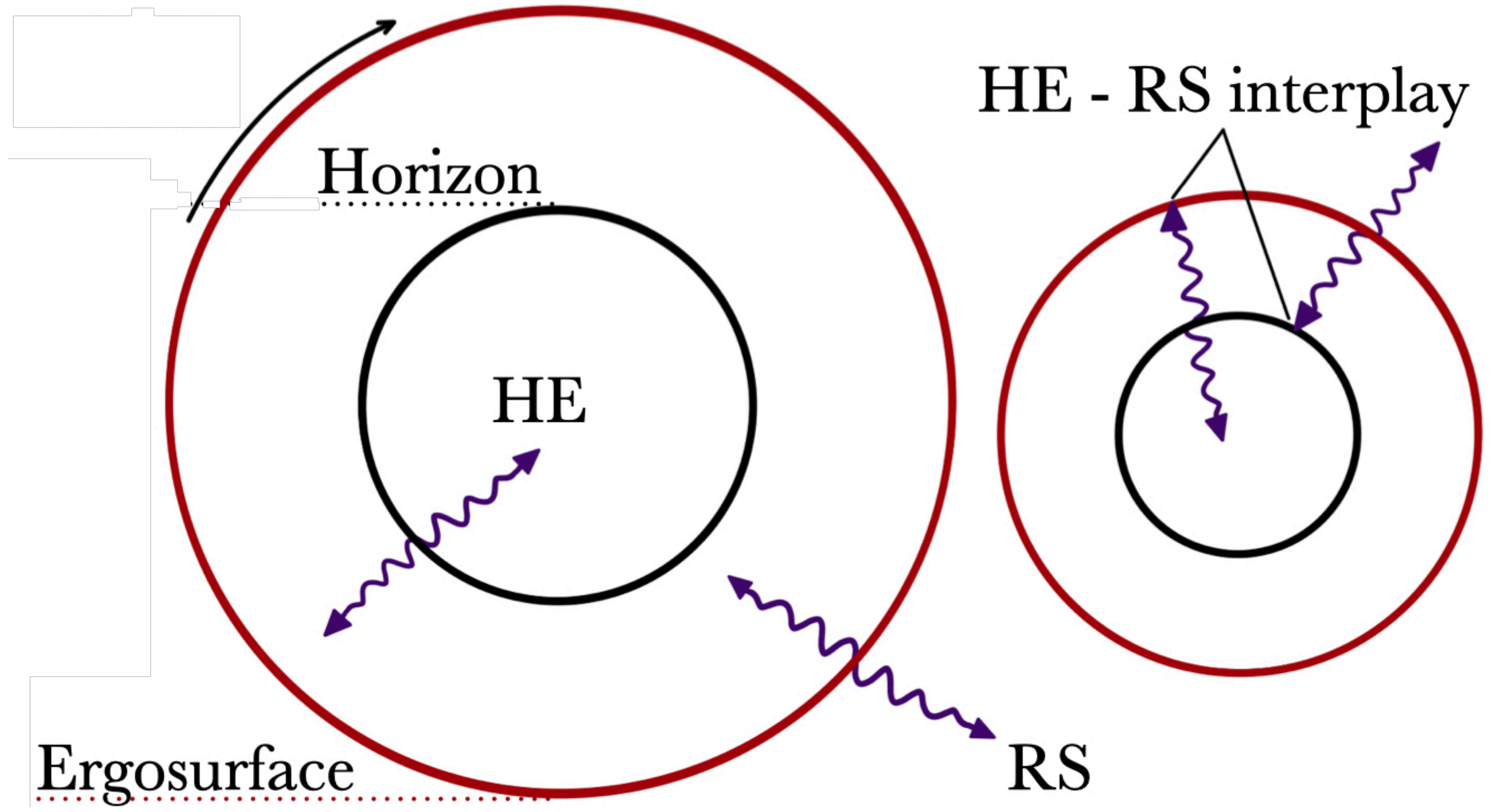


Analogue Rotating BH

Polaritons can in principle simulate rotating BHs by including radial inwards velocity

Goal: Quantify and measure HE-SR interplay

Experimental and theoretical challenges…
but we will make it!



Conclusions

- Superradiance is inherently quantum: it generates entanglement (even for classical inputs).
- CMB radiation degrades entanglement radiated by BHs, but cannot estinguish it.
- SR changes entanglement structure of Hawking radiation with respect to non-rotating BHs.
- Interesting interplay Hawking radiation - Superradiance at the level of entanglement generation
- Ergoregion instability can be quenched with dissipation (no need for horizons).
- Isolated ergoregions generate entangled radiation, observable in dissipative quantum fluids of light.
- Hawking Radiation - Superradiance interplay can potentially be observed in analogues.

THANKS FOR ABIDING!