

NNPDF studies with Heavy Quarks

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INFN, Sezione di Milano

PDF4LHC, QCD at the LHC workshop
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Outline

What will not be discussed here

- ▶ Basics of the NNPDF methodology → M. U.'s talks
- ▶ Heavy quarks in DIS Theory and the FONLL GM scheme → P. Nason's talk

What we will talk about

- ▶ Impact of heavy quarks on PDFs and LHC observables
- ▶ Impact of the values of m_c and m_b
- ▶ PDFs in fixed-flavour number schemes

HEAVY QUARK MASS EFFECTS IN THE NNPDF FRAMEWORK

FONLL: treatment of subleading terms

- ▶ The FONLL F_{2c} structure function reads

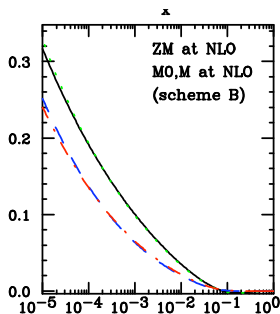
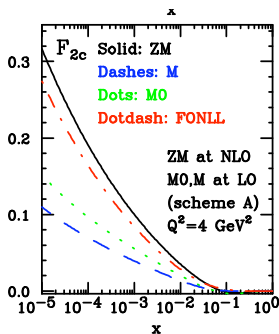
$$F_{2c}^{\text{fonll}}(x, Q^2) = F_{2c}^{(M)}(x, Q^2) + \Theta\left(Q^2 - m_c^2\right) \left(1 - \frac{m_c^2}{Q^2}\right)^2 \left(F_{2c}^{(ZM)}(x, Q^2) - F_{2c}^{(M,0)}(x, Q^2)\right)$$

with $F_{2c}^{(n_f,0)}$ the massless limit of $F_{2c}^{(n_f)}$

- ▶ The *difference term* $\left(F_{2c}^{(ZM)} - F_{2c}^{(M,0)}\right)$ is $\mathcal{O}(\alpha_s^2)$ for $Q^2 \gtrsim m_c^2$, but numerically it turns out to be non-negligible
→ can be suppressed by terms that go to 1 when $Q^2 \gg m_c^2$
- ▶ Possible choices are a **threshold damping factor**, or different forms of the **χ -prescription**
- ▶ This threshold ambiguity is an inherent theoretical uncertainty to any General-Mass VFN scheme.

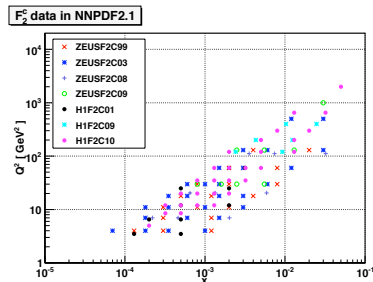
FONLL: treatment of subleading terms

- ▶ FONLL allows to **combine different perturbative orders** in ZM and FFNS terms
- ▶ FONLL-A combines the ZM scheme at $\mathcal{O}(\alpha_s)$ with the FFNS scheme at $\mathcal{O}(\alpha_s)$
→ Identical to S-ACOT
- ▶ FONLL-B combines the ZM scheme at $\mathcal{O}(\alpha_s)$ with the FFNS scheme at $\mathcal{O}(\alpha_s^2)$
- ▶ FONLL-B takes into account consistently $\mathcal{O}(\alpha_s^2)$ massive contributions, phenomenologically important at small x and Q^2 .



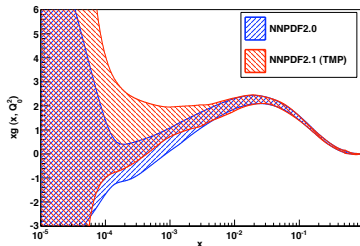
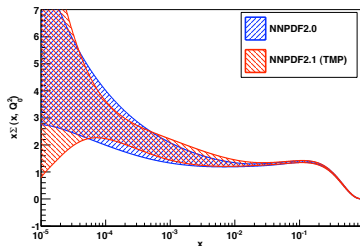
The NNPDF2.1 analysis

- ▶ **FONLL-A-Damp** as a General Mass scheme for NC and CC DIS observables
- ▶ Same dataset as NNPDF2.0 ([arXiv:1002.4407](https://arxiv.org/abs/1002.4407)), supplemented with **HERA F_2^c data**
- ▶ All results shown still **preliminary**



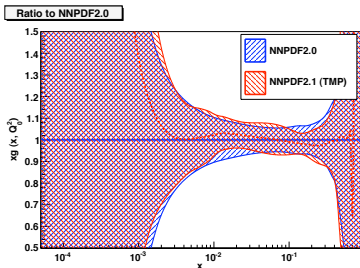
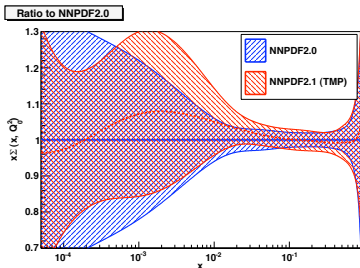
- ▶ For details on the FONLL GM scheme and its implementation in the NNPDF **FastKernel** framework, see [J. Rojo's talks at PDF4LHC 01/10](#) and [07/10](#)

The NNPDF2.1 analysis - PDFs at $Q_0^2 = 2 \text{ GeV}^2$



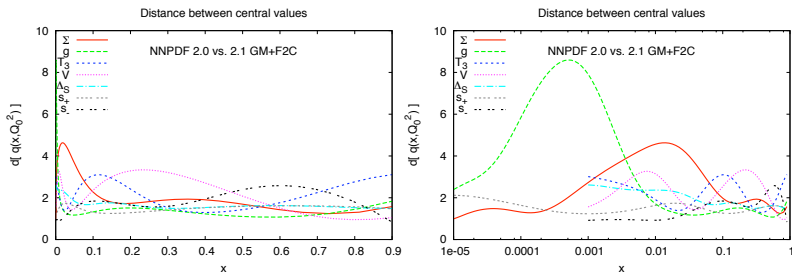
- ▶ HQ mass effects and F_2^c data enhance the singlet and the gluon PDFs at moderate and small- x
- ▶ NNPDF2.1 always within 1σ of NNPDF2.0
→ HQ effects important though not dramatic
- ▶ Harder small- x gluon partly from constraints of $F_2^c(x, Q^2)$ data

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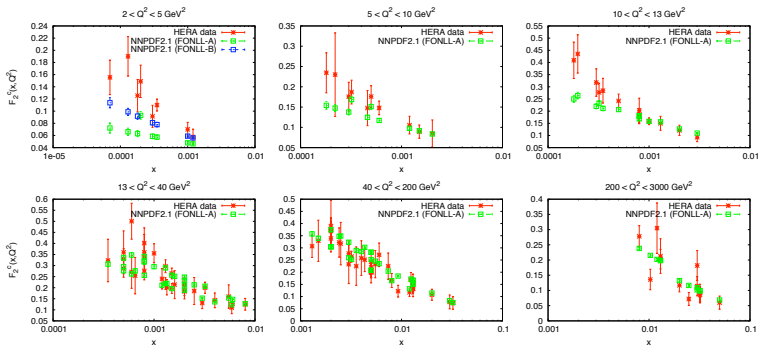
- ▶ Compute **distances** between PDF sets to quantify HQ impact

$$d^2 \left(\langle q^{(1)} \rangle, \langle q^{(2)} \rangle \right) = \frac{(\langle q^{(1)} \rangle_{(1)} - \langle q^{(2)} \rangle_{(2)})^2}{\sigma_{(1)}^2[\langle q^{(1)} \rangle] + \sigma_{(2)}^2[\langle q^{(2)} \rangle]}, \quad \sigma_{(i)}^2[\langle q^{(i)} \rangle] = \frac{1}{N_{\text{rep}}^{(i)}} \sigma_{(i)}^2[q^{(i)}] \quad (1)$$

- ▶ $d \sim 5$ for the singlet at $x \sim 10^{-2}$ at $Q_0^2 = 2 \text{ GeV}^2$
- ▶ $d \sim 8$ for the gluon at $x \sim 10^{-3}$ at $Q_0^2 = 2 \text{ GeV}^2$

Impact of F_2^c data in NNPDF2.1

Good description of F_2^c data except at the smallest x and Q^2 bins
FONLL-A does not account for large $\mathcal{O}(\alpha_s^2)$ corrections to F_2^c in the FFNS

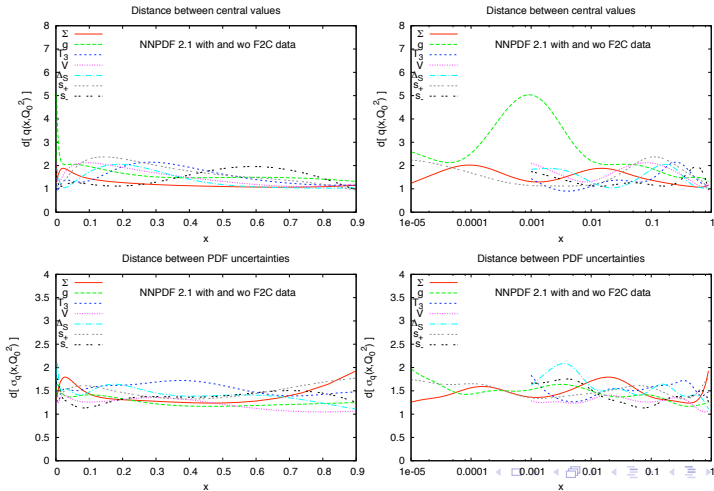


Will update the analysis with Combined HERA F_2^c dataset and with the FONLL-B

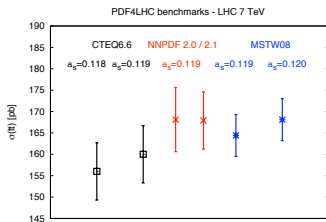
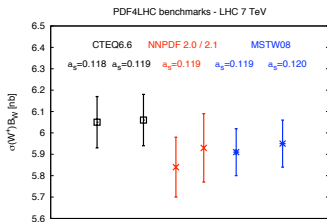
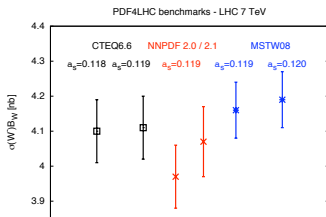
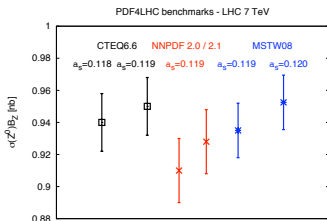
Impact of F_2^c data in NNPDF2.1

F_2^c data lead to an important constraint on the **small- x gluon**

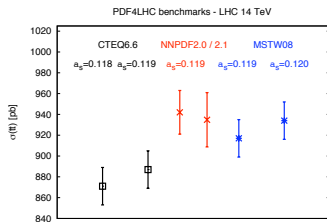
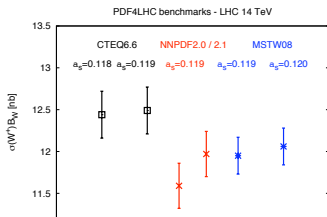
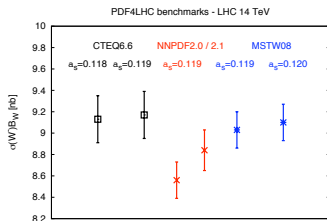
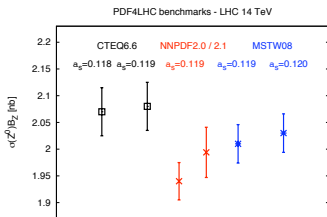
→ **$\sim 1/2$ -sigma** shift at $x \sim 10^{-3}$



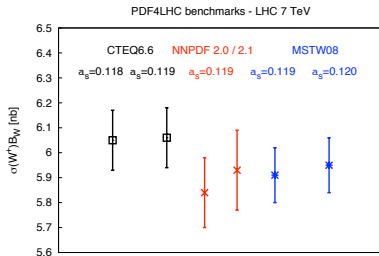
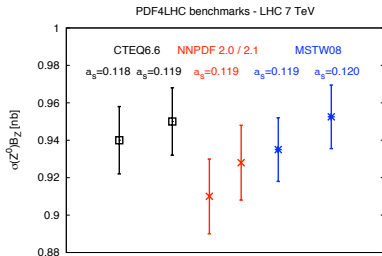
Impact on LHC observables - 7 and 14 TeV



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Impact on LHC observables - 7 and 14 TeV



- ▶ **HQ mass effects** and F_2^c data amount to an about ~ 1 -sigma shift in LHC observables at 7 TeV and at 14 TeV
- ▶ **NNPDF2.1** predictions in excellent agreement with **MSTW08** for all observables
- ▶ Only **marginal agreement** with **CTEQ6.6** for most observables (also Higgs)
- ▶ Using **common** α_s increases the agreement

HEAVY QUARK PDFs

Heavy quark PDFs

- ▶ Heavy quark PDFs, $c(x, Q^2)$ and $b(x, Q^2)$, are much more dependent on the heavy quark mass chosen than on the VFN scheme adopted
- ▶ Crucial problem → Agree on the best possible estimate of m_c and m_b and their associated uncertainties (analogously to the $\alpha_s(M_Z)$ case)
- ▶ The issue of the possibility of extracting m_c , m_b from the global fit should be separated from the choice of best m_h and δm_h , determined from many other external measurements

Impact of δm_c on LHC observables - 7 TeV

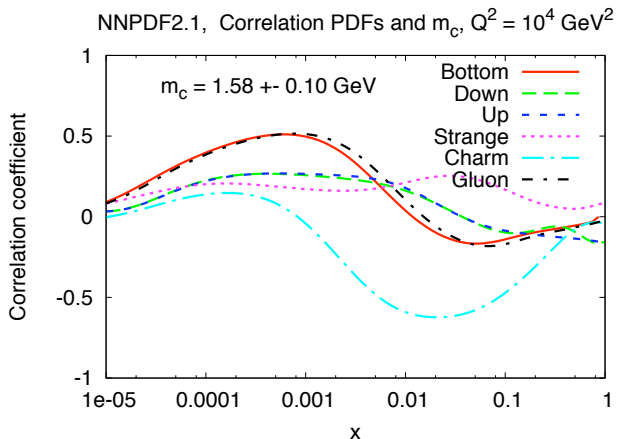
NNPDF2.1 analysis repeated for different m_c values

	$W^+ B_{l\nu}$	$W^- B_{l\nu}$	$Z^0 B_{\bar{l}l}$	$t\bar{t}$
$m_c^2 = 2$	5.93 ± 0.16	4.07 ± 0.10	0.930 ± 0.02	167 ± 7
$m_c^2 = 2.5$	6.04 ± 0.10	4.11 ± 0.07	0.945 ± 0.013	164 ± 5
$m_c^2 = 2.9$	6.10 ± 0.15	4.16 ± 0.10	0.956 ± 0.02	163 ± 7

- ▶ Non-negligible impact of m_c variations, but not dramatic
- ▶ Uncertainties $\delta m_c \sim 0.10$ (PDG uncertainty) induce variations in $\sigma(W^\pm)$ and $\sigma(Z)$ below the 1-sigma PDF uncertainty
- ▶ Similar studies performed by MSTW and HERAPDF
- ▶ Crucial problem \rightarrow Agree on the best estimate for δm_c .

Impact of δm_c on LHC observables - 7 TeV

- ▶ The correlation between cross-sections and m_c can be easily computed in the NNPDF approach

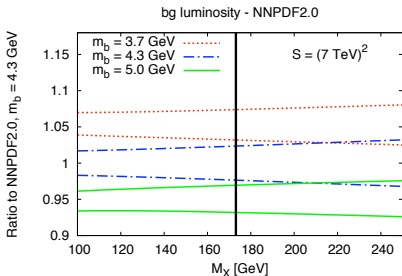


Impact of δm_b on LHC observables - 7 TeV

Taking into account uncertainty induced by m_b (correlated with the b -PDF) crucial for important LHC processes: **single-top, MSSM Higgs, ...**

Example: single top t-channel production: m_b -uncertainty \gg PDF uncertainty

Differences both from PDF luminosity and from matrix element



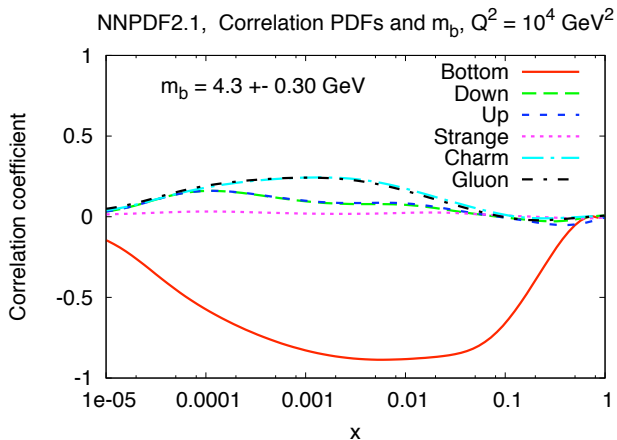
NNPDF2.0	$\sigma(t)_{t\text{-channel}}$
$m_b = 3.7$ GeV	46.77 ± 0.36 pb
$m_b = 4.3$ GeV	44.33 ± 0.32 pb
$m_b = 5.0$ GeV	41.04 ± 0.32 pb

The **uncertainty in m_b** and its correlation with the b PDF are crucial for **b -initiated processes** at the LHC

Crucial to determine best estimates for m_b , δm_b

Impact of δm_b on LHC observables - 7 TeV

- ▶ The correlation between cross-sections and m_b can be easily computed in the NNPDF approach



PDF and heavy quark mass uncertainties

NNPDF2.1 sets for a range of different values of m_c and m_b will be provided

→ Combined PDF+ m_h uncertainties, exact error propagation to physical observables

$$\langle \mathcal{F} \rangle_{\text{rep}} = \frac{1}{N_{\text{rep}}} \sum_{i=1}^{N_{m_c}} \sum_{j=1}^{N_{m_b}} \sum_{k_{ij}=1}^{N_{\text{rep}}^{(i,j)}} \mathcal{F} \left(\text{PDF}^{(k_{ij}, i, j)}, m_c^{(i)}, m_b^{(j)} \right),$$

PDF $^{(k_{ij}, i, j)}$ stands for the replica k_{ij} of the PDF fit obtained using $m_c^{(i)}$ and $m_b^{(j)}$

$$N_{\text{rep}} = \sum_i^{N_{m_c}} \sum_j^{N_{m_b}} N_{\text{rep}}^{(i,j)},$$

$N_{\text{rep}}^{(i,j)}$ number of PDF replicas randomly selected from the fit obtained with $m_c^{(i)}, m_b^{(j)}$

$$N_{\text{rep}}^{(i,j)} \propto \exp \left(- \frac{(m_c^{(i)} - m_c^{(0)})^2}{2\delta_{m_c}^2} - \frac{(m_b^{(j)} - m_b^{(0)})^2}{2\delta_{m_b}^2} \right).$$

Important advantage: No extra CPU time required! (Set $N_{\text{rep}} = 100$)

Another advantage: both $m_c^{(0)}, m_b^{(0)}$ and $\delta_{m_c}, \delta_{m_b}$ can be decided by the PDF user

PDFs WITH FIXED-FLAVOR NUMBER

PDFs with Fixed Flavor Number

- ▶ PDF in the Fixed Flavour $N_f = 3$ and $N_f = 4$ schemes important for **LHC phenomenology**
- ▶ FFN sets can easily be obtained from $N_f = 5$ GM PDF sets by **matching PDFs and α_s** at the HQ mass threshold

$$\text{PDFs}^{(N_f)}(Q^2 = m_h^2) = \text{PDFs}^{(N_f+1)}(Q^2 = m_h^2)$$

$$\alpha_s^{(N_f)}(Q^2 = m_h^2) = \alpha_s^{(N_f+1)}(Q^2 = m_h^2) ,$$

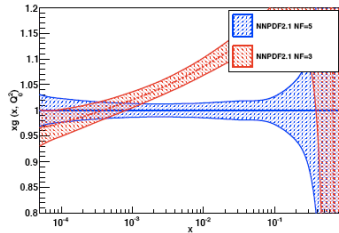
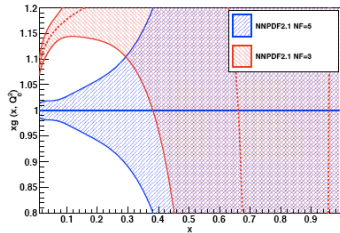
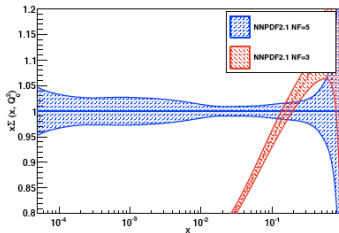
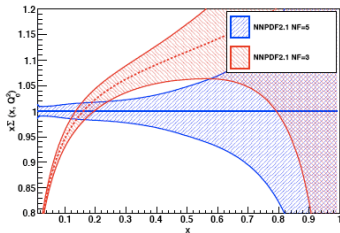
and then evolving upwards with fixed N_f

- ▶ Bypass problems related to **unknown massive FFN** coefficient functions for jets and DY
- ▶ Same approach adopted by **CT** and **MSTW**

NNPDF2.1 $N_f = 3$ PDFs

$N_f = 3$ and $N_f = 4$ sets of NNPDF2.1 will be provided

Compare $N_f = 3$ with $N_f = 5$ PDFs at LHC scale $Q^2 = 10^4 \text{ GeV}^2$



Summary

- ▶ The NNPDF2.1 analysis is based in the FONLL General Mass scheme for heavy quark effects. Will be released in the coming weeks.
- ▶ NNPDF2.1 sets with different values of m_c and m_b will be provided
- ▶ $N_f = 3$ and $N_f = 4$ PDF sets will also be provided
- ▶ The impact of m_c variations on LHC observables is comparable in size to GM/ZM differences
- ▶ The b -PDF depends crucial on the value of m_b → Important phenomenological impact in b -initiated LHC processes
- ▶ Within NNPDF, easy to compute and propagate the correlation between PDFs and heavy quark masses
- ▶ The choice of the heavy quark mass m_h can be as important as the ZM/GM difference → Crucial problem to converge on a common choice of *best estimates* for m_h and δm_h

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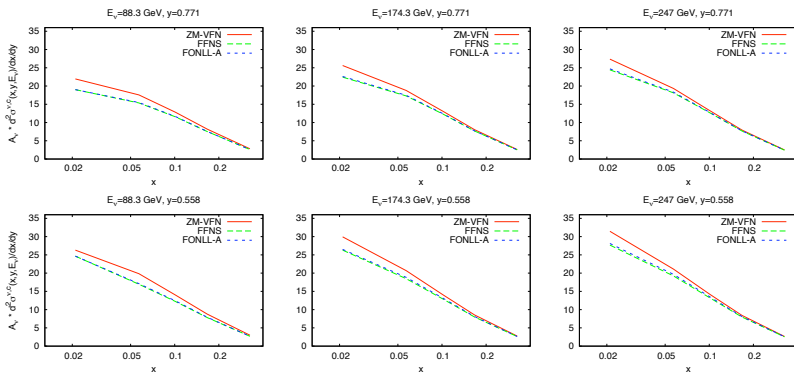
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EXTRA MATERIAL

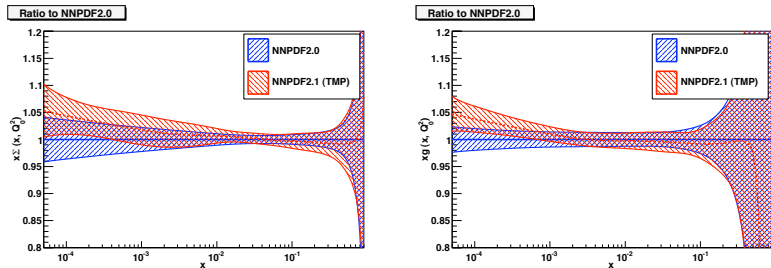
FONLL-A for Charged Current

The FONLL-A GM scheme also applies to **CC structure functions**
In the NuTeV kinematical region \rightarrow **FONLL-A very close to FFNS**
(Les Houches heavy quark benchmark settings)



The NNPDF2.1 analysis - PDFs at $Q_0^2 = 10^4 \text{ GeV}^2$

Compare PDFs at the LHC scale \rightarrow Assess effects of **quark-gluon mixing in DGLAP evolution**

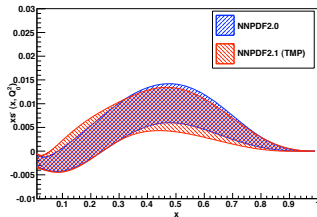
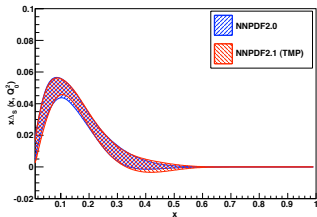
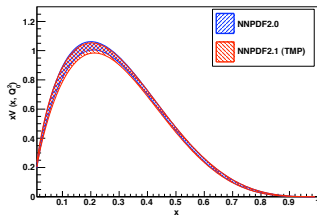
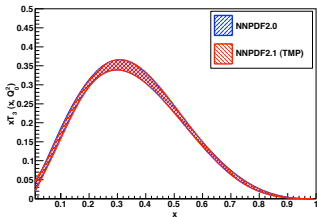


Note greatly **reduced small- x PDF uncertainties**

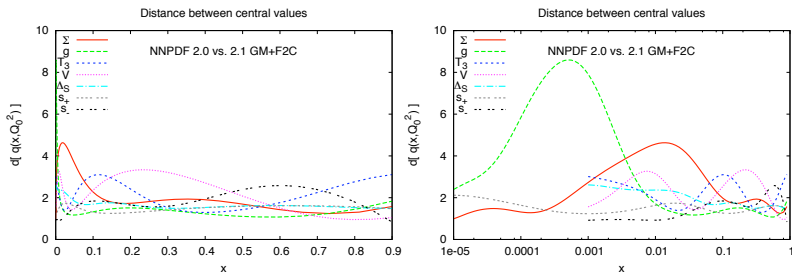
NNPDF2.0 and 2.1 always consistent within uncertainties

The NNPDF2.1 analysis - PDFs at $Q_0^2 = 2 \text{ GeV}^2$

Large- x valence PDFs **consistently unaffected** by HQ effects



The NNPDF2.1 analysis - PDFs at $Q_0^2 = 2 \text{ GeV}^2$



- Compute **distances** between PDF sets to quantify HQ impact

$$d^2(\langle q^{(1)} \rangle, \langle q^{(2)} \rangle) = \frac{(\langle q^{(1)} \rangle_{(1)} - \langle q^{(2)} \rangle_{(2)})^2}{\sigma_{(1)}^2[\langle q^{(1)} \rangle] + \sigma_{(2)}^2[\langle q^{(2)} \rangle]}, \quad \sigma_{(i)}^2[\langle q^{(i)} \rangle] = \frac{1}{N_{\text{rep}}^{(i)}} \sigma_{(i)}^2[q^{(i)}] \quad (2)$$

- $d \sim 5$ for the singlet at $x \sim 10^{-2}$ at $Q_0^2 = 2 \text{ GeV}^2$
- $d \sim 8$ for the gluon at $x \sim 10^{-3}$ at $Q_0^2 = 2 \text{ GeV}^2$

FONLL in a nutshell

- Express the massive result $F^{(n_l)}$ in terms of the massless PDFs and α_s (non trivial from $\mathcal{O}(\alpha_s^2)$)

$$F^{(n_l)}(x, Q^2) = x \int_x^1 \frac{dy}{y} \sum_{i=q, \bar{q}, g} B_i \left(\frac{x}{y}, \frac{Q^2}{m^2}, \alpha_s^{(n_l+1)}(Q^2) \right) f_i^{(n_l+1)}(y, Q^2),$$

- Define **massless limit of the massive computation** as

$$F^{(n_l, 0)}(x, Q^2) \equiv x \int_x^1 \frac{dy}{y} \sum_{i=q, \bar{q}, g} B_i^{(0)} \left(\frac{x}{y}, \frac{Q^2}{m^2}, \alpha_s^{(n_l+1)}(Q^2) \right) f_i^{(n_l+1)}(y, Q^2),$$

$$\lim_{m \rightarrow 0} \left[B_i \left(x, \frac{Q^2}{m^2} \right) - B_i^{(0)} \left(x, \frac{Q^2}{m^2} \right) \right] = 0$$

- The FONLL approximation is then

$$F^{\text{FONLL}}(x, Q^2) \equiv F^{(d)}(x, Q^2) + F^{(n_l)}(x, Q^2),$$

$$F^{(d)}(x, Q^2) \equiv \left[F^{(n_l+1)}(x, Q^2) - F^{(n_l, 0)}(x, Q^2) \right]$$

Important technical advantage: PDFs and α_s expressed always in the $(n_l + 1)$ scheme

FONLL in a nutshell

- ▶ Far from threshold, $Q^2 \gg m^2$ $F^{(n_l, 0)}(x, Q^2) \sim F^{(n_l)}(x, Q^2) \rightarrow$ the massless computation recovered

$$F^{\text{FONLL}}(x, Q^2) \sim F^{(n_l+1)}(x, Q^2)$$

- ▶ Near threshold the “difference term” is formally higher order but unreliable, so one can correct it by mass suppressed terms, using for example a **damping factor** (FONLL default)

$$F^{(d, th)}(x, Q^2) \equiv f_{\text{thr}}(x, Q^2) F^{(d)}(x, Q^2), \quad f_{\text{thr}}(x, Q^2) = \Theta(Q^2 - m^2) \left(1 - \frac{Q^2}{m^2}\right)^2,$$

or some form of **χ -scaling**,

$$F^{(d, \chi)}(x, Q^2) \equiv F^{(d)}(x, Q^2) = x \int_{\chi(x, Q^2)} \frac{dy}{y} C\left(\frac{\chi(x, Q^2)}{y}, \alpha(Q^2)\right) f(y, Q^2),$$

$$F^{(d, \chi, v^2)}(x, Q^2) \equiv F^{(d)}(\chi(x, Q^2), Q^2), \quad \chi = x \left(1 + \frac{4m^2}{Q^2}\right).$$

The choice of **threshold prescription** represent an **intrinsic ambiguity** of the matching procedure. Can this **ambiguity** be minimized?

Perturbative ordering in FONLL

Three FONLL schemes for different **ordering of the perturbative expansion** can be defined:

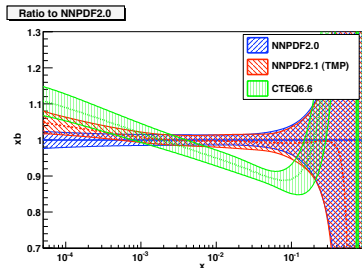
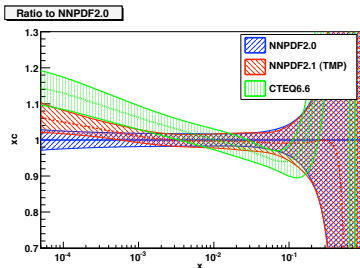
1. Scheme A $\rightarrow \mathcal{O}(\alpha_s)$ in massless and in massive
2. Scheme B $\rightarrow \mathcal{O}(\alpha_s)$ in massless and $\mathcal{O}(\alpha_s^2)$ in massive
3. Scheme C $\rightarrow \mathcal{O}(\alpha_s^2)$ in massless and in massive

In any of the three schemes, **any threshold prescription** can be implemented
These schemes can be related to **existing approaches**

1. Scheme A is identical to S-ACOT
2. Scheme B was formulated with similar scope as TR (use the information from the $\mathcal{O}(\alpha_s^2)$ massive computation in a NLO GM-VFN scheme), but they turn to be **different**
3. Scheme C should be S-ACOT at NNLO?

Heavy quark PDFs

Ratio to NNPDF2.0 at $Q^2 = 10^4 \text{ GeV}^2$

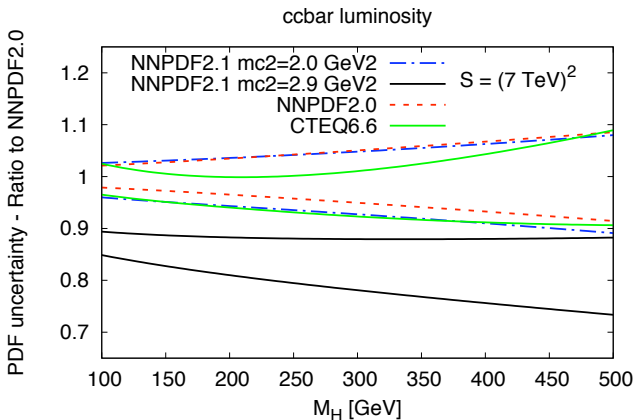


- ▶ Same pattern for $c(x, Q^2)$ and $b(x, Q^2)$ (Common evolution from singlet and gluon)
- ▶ Systematic discrepancy in b PDF for $x \in [0.01, 0.1]$ unrelated to ZM/GM differences, rather to different choices for m_b

Heavy quark PDFs

Luminosity $c\bar{c}$ at 7 TeV, Dependence on the charm quark mass and the GM scheme

The value of m_c more important than ZM/GM difference

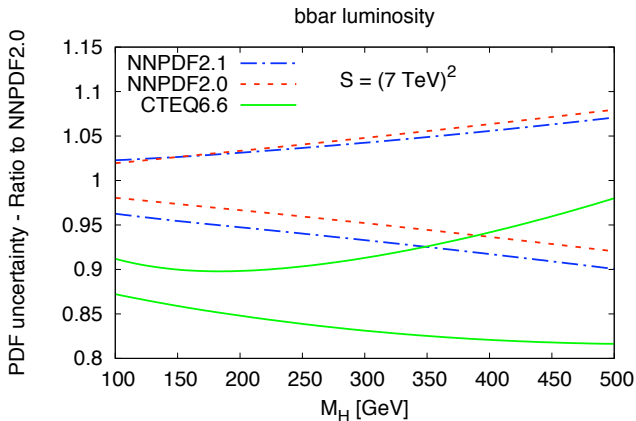


The b PDF

The $b(x, Q^2)$ PDF is **anticorrelated** with m_b

Different values of m_b lead to very different $b\bar{b}$ luminosities

The differences in m_b much larger than the GM-ZM differences

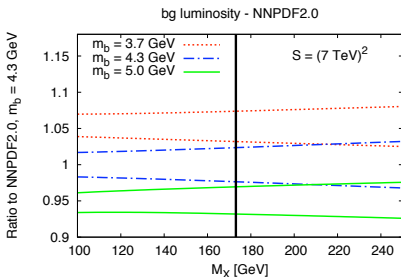


The b PDF

Taking into account uncertainty induced by m_b (correlated with the b -PDF) crucial for important LHC processes: **single-top, MSSM Higgs, ...**

Example: single top t-channel production: m_b -uncertainty \gg PDF uncertainty

Differences both from PDF luminosity and from matrix element



NNPDF2.0	$\sigma(t)_{t\text{-channel}}$
$m_b = 3.7$ GeV	46.77 ± 0.36 pb
$m_b = 4.3$ GeV	44.33 ± 0.32 pb
$m_b = 5.0$ GeV	41.04 ± 0.32 pb

The **uncertainty in m_b** and its correlation with the b PDF are crucial for **b -initiated processes** at the LHC

Crucial to determine best estimates for m_b , δm_b