Jets at NNLO at the LHC

Zoltán Trócsányi

University of Debrecen
and
Institute of Nuclear Research

in collaboration with
G. Somogyi
and
U. Aglietti, P. Bolzoni, V. Del Duca, C. Duhr, S. Moch

Outline

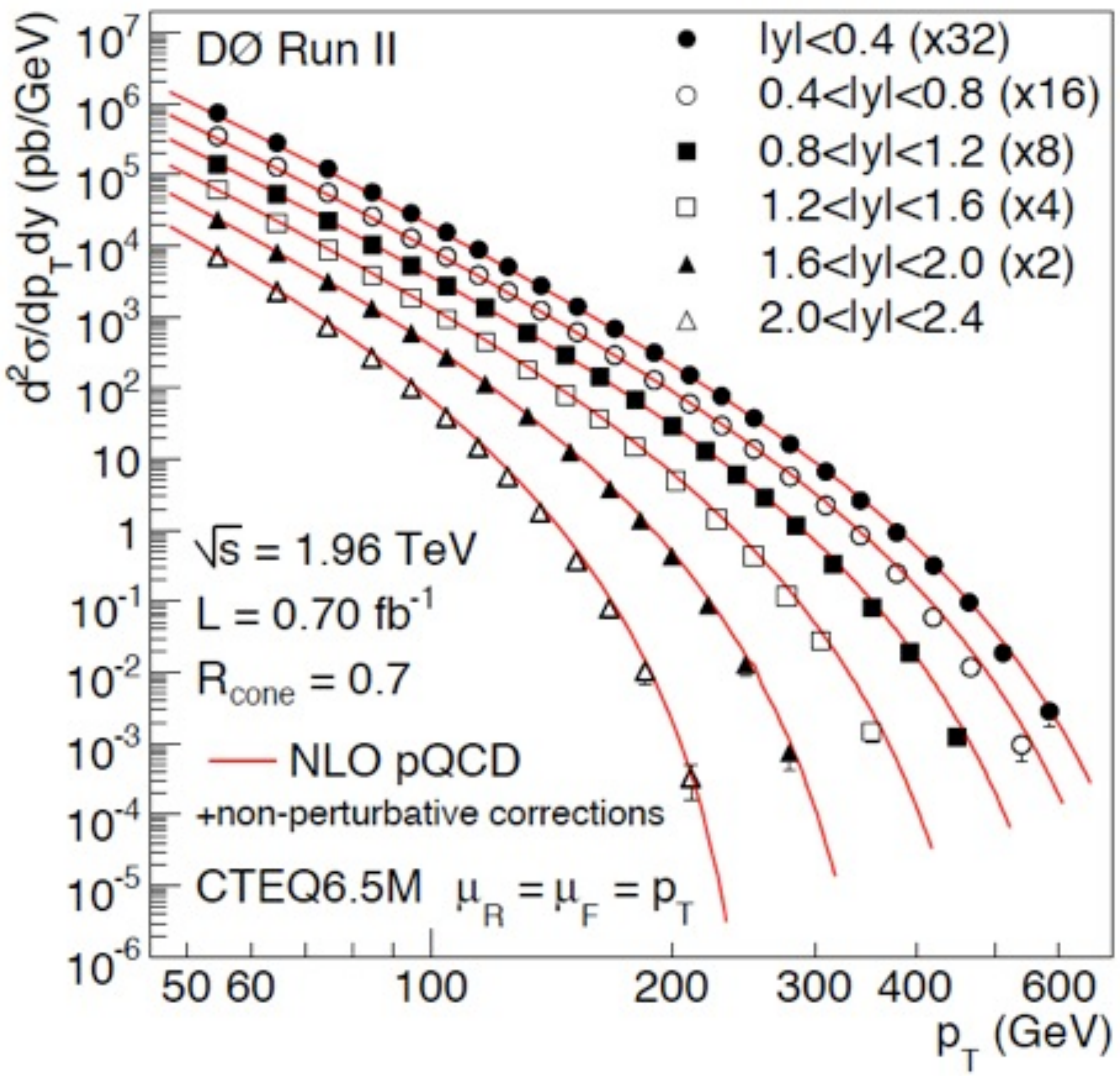
- Motivation
- Recipe for a general subtraction scheme at NNLO
- Integrating the counterterms
- Results
- Conclusions

Motivation

Why jets at NNLO?

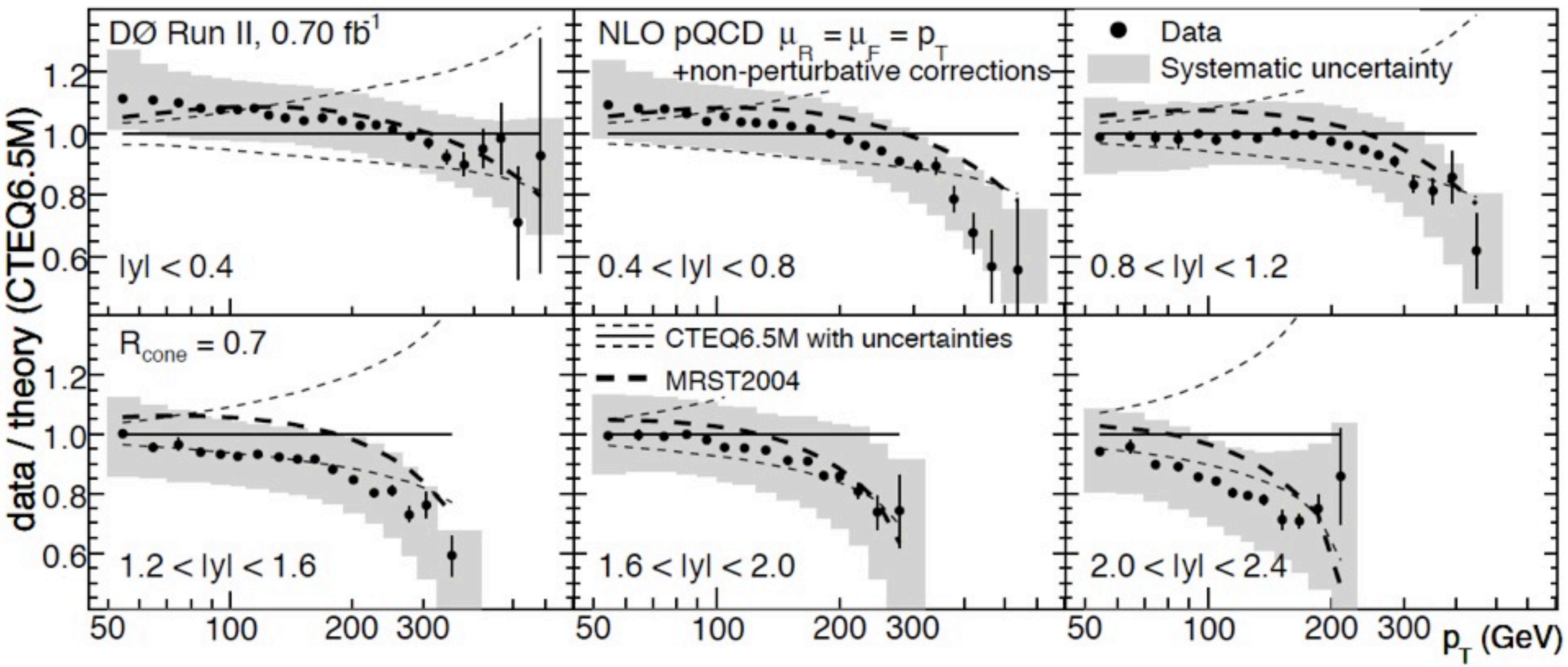
- Jets are essential analysis tools at LHC:
  good understanding is needed
  Status at TeVatron (with midpoint cone):

looks nice,
but have a
closer look



Why jets at NNLO?

- Jets are essential analysis tools at LHC:
  good understanding is needed
  Status at TeVatron (with midpoint cone):



Why jets at NNLO?

- Jets are essential analysis tools at LHC:
  10% energy-scale uncertainty (G.D.) warrants precision physics
- Precise predictions for 'standard candles':
  inclusive jet, V (+ jet)
- Missing piece for precise determination of pdf's (W.J.S.)
- NLO is effectively LO:
  energy distribution inside jets, jet pT asymmetry (G.D.)

Less sophisticated answer:

Matrix elements are known,but not yet used

Problem

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m}$$
$$\equiv \int_{m+2} {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} + \int_{m+1} {\rm d}\sigma^{\rm RV}_{m+1} J_{m+1} + \int_{m} {\rm d}\sigma^{\rm VV}_{m} J_{m}$$

- matrix elements are known for 0→4 parton (for jet production), V+3 parton (for V+jet production) processes
- the three contributions are separately divergent in d = 4 dimensions:
  - in σ^{RR} kinematical singularities as one or two partons become unresolved yielding ϵ-poles at $O(\epsilon^{-j})$, j = 1-4 after integration over phase space, no explicit poles
  - in σ^{RV} kinematical singularities as one parton becomes unresolved yielding ϵ-poles at $O(\epsilon^{-j})$ after integration over phase space + explicit ϵ-poles at $O(\epsilon^{-j})$, j =1,2
  - in σ^{VV} explicit ϵ-poles at $O(\epsilon^{-j})$, j=1-4

general solution is not yet available

Approaches

Several options available - why a new one?

Sector Decomposition (residuum subtraction)

Binoth, Heinrich, Anastasiou, Melnikov, Petriello

- ✓ First method to yield physical cross sections
- ✓ Calculation is fully numerical
- – Cancellation of poles also, and depends on the jet function
- – Can it handle final states with many coloured partons?

M. Czakon 2010: yes

q_T subtraction

Catani, Grazzini, Cieri, Ferrara, De Florian ...

✓Simple concept, explicit documentation

✓Efficient and fully exclusive calculation

–Limited scope: applicable to production of colorless final states

Antennae subtraction

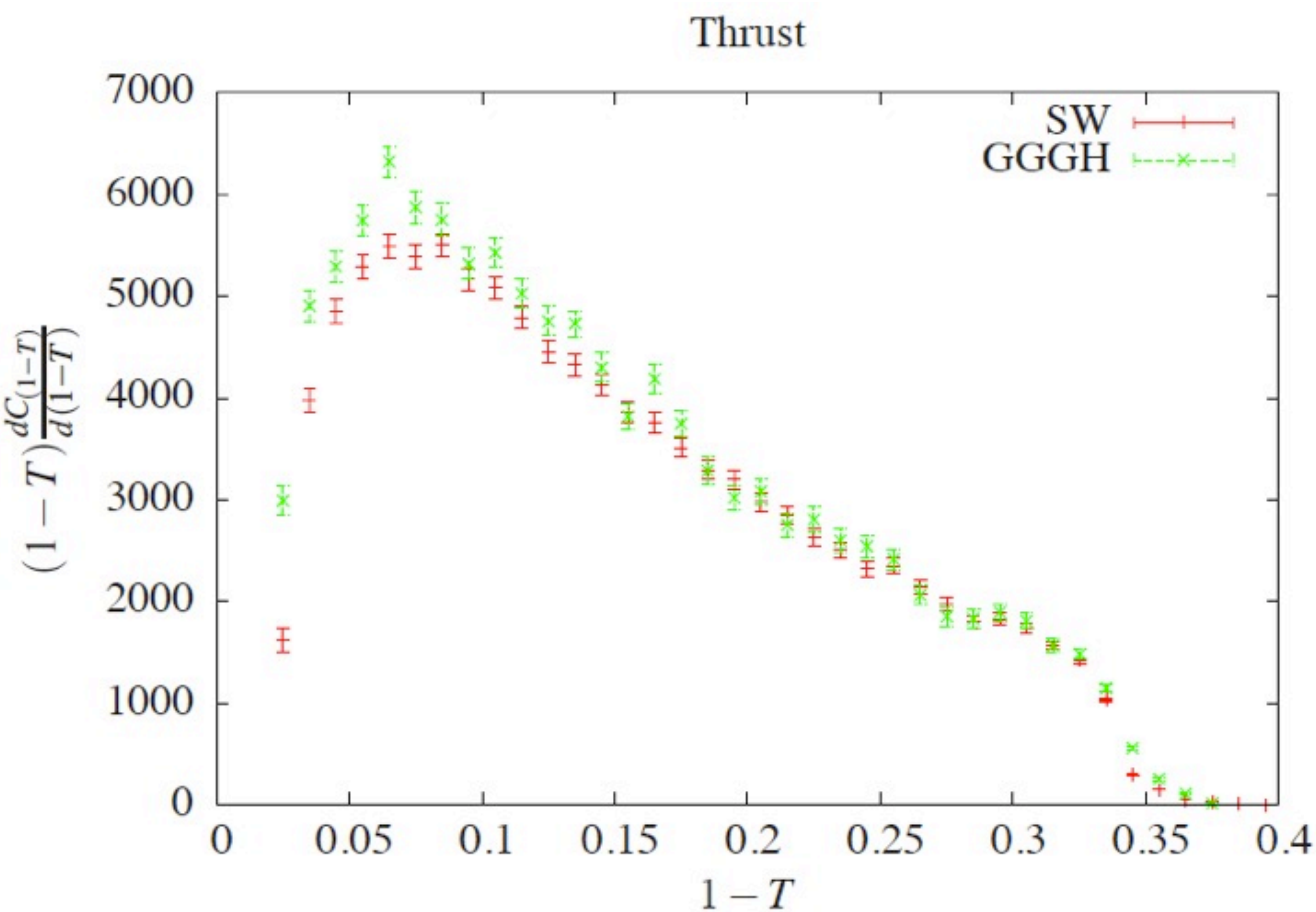
Gehrmann, Gehrmann-De Ridder, Glover, Weinzierl ...

- ✓ Successfully applied to $e^+e^- \to 2, 3$ jets
- ✓ Analytic integration of the antennae over unresolved phase space is understood
- ✳ Extension to hadron collisions is well advanced (more later)
- – Nonlocal counterterms
- – Colour implicit
- – Cannot cut on factorized phase space

Approaches

Is agreement between antennae implementations satisfactory?

(S. Weinzierl)



Our goal

to devise a subtraction scheme with

- ✓ fully local counterterms
  (efficiency and mathematical rigour)
- ✓ explicit expressions including colour
  (colour space natation is used)
- ✓ completely algorithmic construction
  (valid in any order of perturbation theory)
- ✓ option to constrain subtraction near singular regions (important check)

Our goal

to devise a subtraction scheme with

✓ fully local counterterms
(efficiency and mathematical rigour)

✓ explicit expressions including colour
(colour space natation is used)

✓ completely algorithmic construction
(valid in any order of perturbation theory)

✓ option to constrain subtraction near singular regions (important check)

Useful for automatization

Recipe for a general subtraction scheme at NNLO

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

Structure

of subtractions is governed by jet functions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

Structure

of subtractions is governed by jet functions

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \Big\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \Big({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \Big) \Big\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \Big\{ \Big({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big) J_{m+1} - \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] J_m \Big\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \Big\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \Big({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \Big) + \int_1 \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] \Big\} J_m$$

${\rm d}\sigma^{\rm RR,A_2}_{m+2}$ regularizes the doubly-unresolved limits of ${\rm d}\sigma^{\rm RR}_{m+2}$

Structure

of subtractions is governed by jet functions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

$\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2}$ regularizes the singly-unresolved limits of $\mathrm{d}\sigma^{\mathrm{RR}}_{m+2}$

Structure

of subtractions is governed by jet functions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

$\mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2}$ compensates for the double subtraction in $\mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2}$ and $\mathrm{d}\sigma^{\mathrm{RR,A}}_{m+2}$

Structure

of subtractions is governed by jet functions

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

${\rm d}\sigma^{\rm RV,A_1}_{m+1}$ regularizes the singly-unresolved limits of ${\rm d}\sigma^{\rm RV}_{m+1}$

$\left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1}$ regularizes the singly-unresolved limits of $\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2}$

Definition of subtractions

requires three steps

(1) Matching of limits: to avoid multiple subtraction in overlapping singular regions of PS; easy to find at NLO:

- collinear limit + soft limit - collinear limit of soft limit
- more cumbersome at NNLO or higher order: requries matching of various doubly- and singly-unresolved limits

(2) extension of limit formulae over the whole phase space using momentum mappings that respect factorization and delicate cancellation of IR singularities

(3) integration of subtractions over the phase space measure of the unresolved parton(s)

Antennae subtractions

Consider step (3) the most important

For analytic results

✓ use colour-stripped amplitudes

✓ subtraction derived from physical matrix elements normalized to two-parton matrix elements

⇒ can use integration techniques developed for loop-amplitudes

⇒ part of step (1) comes free

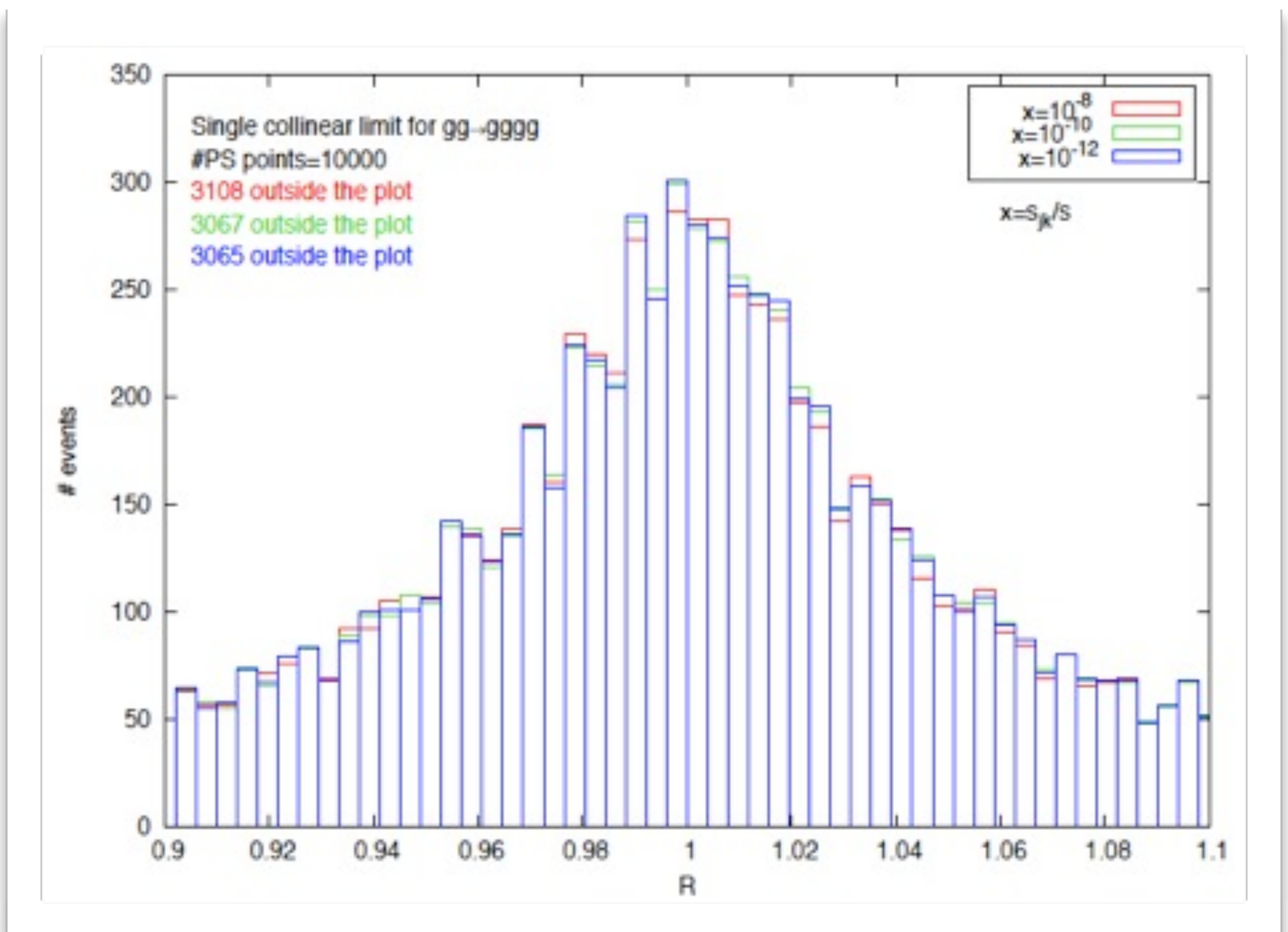
- Price: less numerical control (non-local subtractions, adventage is lost if phase space for subtractions are constrained)

Antennae subtractions

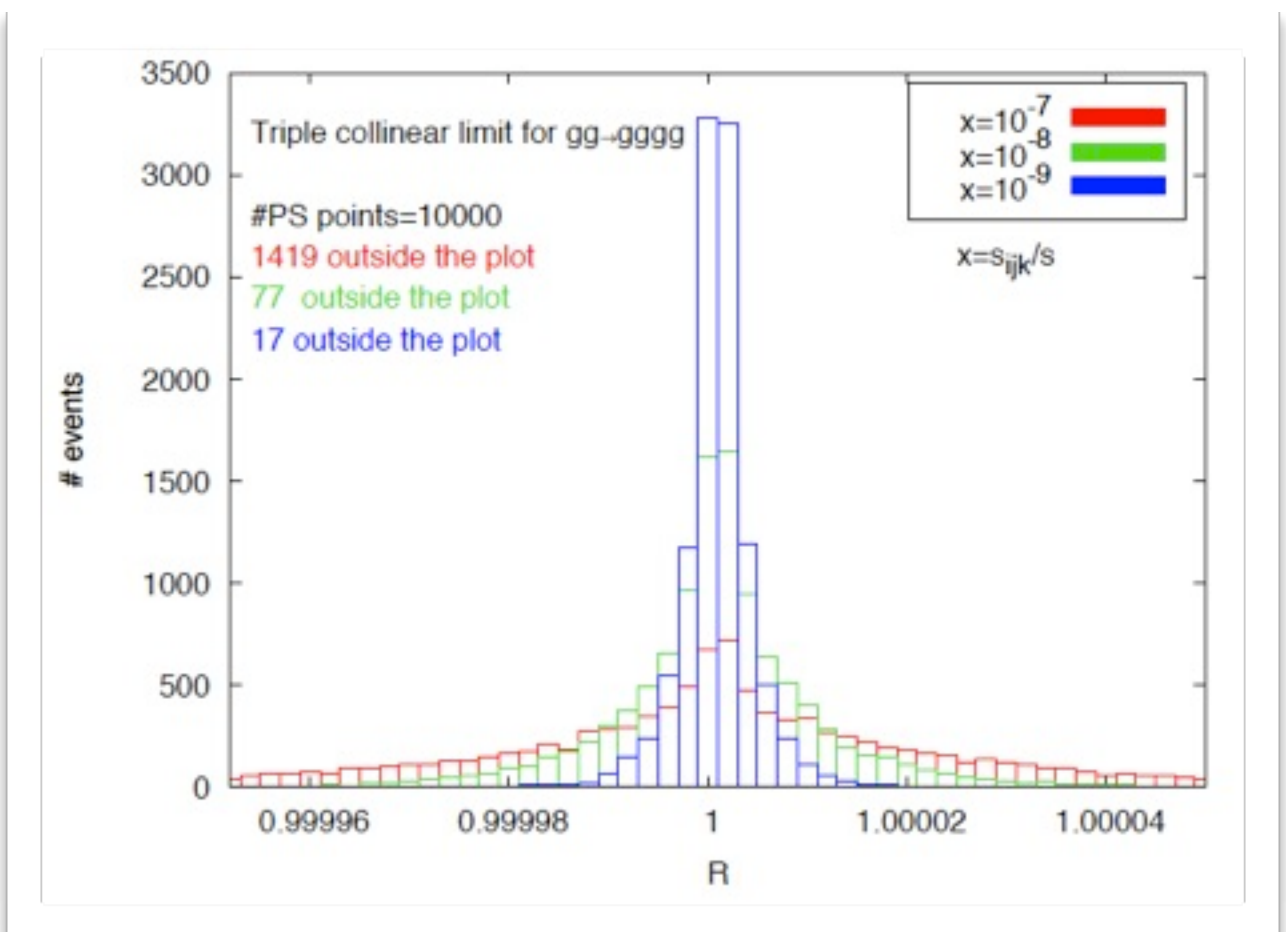
Puzzle in testing NNLO antennae for gluon scattering: azimuthal correlations in gluon splitting

Pires, Glover arXiv:1003.2824

single collinear



triple collinear



Our subtractions

are based on

✓ universal IR structure of QCD squared matrix elements

- ϵ-poles of one- and two-loop amplitudes
- soft and collinear factorization of QCD matrix elements

✓ simple and general procedure for separating overlapping singularities (using a physical gauge)

▸ extension over phase space using momentum mappings that

- implement exact momentum conservation
- lead to exact phase-space factorization
- use different mappings for collinear and soft-type subtractions
- distribute recoil democratically

⇒ can be generalized to any number of unresolved partons

Our subtractions

are

- ✓ given explicitly for processes with colorless particles in the initial state (extension to hadronic processes is known at NLO)

- ✓ fully local in color and spin space
  - ➡ no need to consider color subamplitudes of real emission matrix elements
  - ➡ azimuthal correlations in gluon splitting treated exactly
  - ➡ ratio of the sum of counterterms to matrix elements of real emission tend to one in kinematically degenerate phase-space points

- ✓ can be constrained to near singular regions
  - ➡ leads to gain in efficiency
  - ➡ independence of phase space cut provides strong check

Subtractions

that need momentum mappings only

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

The tedious part:
Integrating the subtraction terms

G. Somogyi, ZT arXiv:0807.0509

U. Aglietti, V. Del Duca, C. Duhr, G. Somogyi, ZT arXiv:0807.0514

P. Bolzoni, S. Moch, G. Somogyi, ZT arXiv:0905.4390

Integrated counterterms

counterterm	types of integrals	done
$d\sigma^{RR,A_1}_{m+2}$	tree-level singly-unresolved	✓
$d\sigma^{RV,A_1}_{m+1}$	one-loop singly-unresolved	✓
$\left(\int_1 d\sigma^{RR,A_1}_{m+2}\right)^{A_1}$	tree-level iterated singly-unresolved (1)	✓
$d\sigma^{RR,A_{12}}_{m+2}$	tree-level iterated singly-unresolved (2)	✓
$d\sigma^{RR,A_2}_{m+2}$	tree-level doubly-unresolved	✗

Integrated counterterms

In this talk

counterterm	types of integrals	done
$\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1}$	tree-level singly-unresolved	✓
$\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1}$	one-loop singly-unresolved	✓
$\left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1}\right)^{\mathrm{A}_1}$	tree-level iterated singly-unresolved (1)	✓
$\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}}$	tree-level iterated singly-unresolved (2)	✓
$\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2}$	tree-level doubly-unresolved	✗

Integrating iterated counterterms

One of 25 subtraction terms: collinear-double collinear subtraction

$$\mathcal{C}_{kt}\mathcal{C}^{(0)}_{ir;kt} = (8\pi\alpha_{\rm s}\mu^{2\epsilon})^2 \frac{1}{s_{kt}}\frac{1}{\widehat{s}_{ir}} \langle \mathcal{M}^{(0)}_m(\{\tilde{p}\})|P^{(0)}_{f_k f_t}(z_{t,k};\epsilon) P^{(0)}_{f_i f_r}(\widehat{z}_{r,i};\epsilon)|\mathcal{M}^{(0)}_m(\{\tilde{p}\})\rangle$$
$$\times\,(1-\alpha_{kt})^{2d_0-2m(1-\epsilon)}(1-\widehat{\alpha}_{kt})^{2d_0-2m(1-\epsilon)}\Theta(\alpha_0-\alpha_{kt})\Theta(\alpha_0-\widehat{\alpha}_{ir})$$

obtained by an iterated mapping

$$\{p\}_{m+2} \xrightarrow{\mathsf{C}_{kt}} \{\hat{p}\}_{m+1} \xrightarrow{\mathsf{C}_{\hat{i}\hat{r}}} \{\tilde{p}\}: \quad {\rm d}\phi_{m+2}(\{p\};Q) = {\rm d}\phi_m(\{\tilde{p}\};Q)[{\rm d}\widehat{p}_{1,m}][{\rm d}p_{1,m+1}]$$

Then we define the function $\mathrm{C}_{kt}\mathrm{C}^{(0)}_{ir;kt}(\widetilde{x}_{kt},\widetilde{x}_{ir},\epsilon,\alpha_0,d_0)$ by

$$\int [{\rm d}\widehat{p}_{1,m}][{\rm d}p_{1,m+1}]\mathcal{C}_{kt}\mathcal{C}^{(0)}_{ir;kt} \equiv \left[\frac{\alpha_{\rm s}}{2\pi}S_\epsilon\left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \mathrm{C}_{kt}\mathrm{C}^{(0)}_{ir;kt}\boldsymbol{T}^2_{kt}\boldsymbol{T}^2_{ir}|\mathcal{M}^{(0)}_m(\{\tilde{p}\})|^2$$

Integrating iterated counterterms

Use explicit parametrization of $[\mathrm{d}\widehat{p}_{1,m}]$ and $[\mathrm{d}p_{1,m+1}]$ to write

$\mathrm{C}_{kt}\mathrm{C}^{(0)}_{ir;kt}(\widetilde{x}_{kt}, \widetilde{x}_{ir}, \epsilon, \alpha_0, d_0)$ as a linear combination of basic integrals

$$\mathcal{I}_{\mathcal{C}}^{(4)}(x_k, x_i; \epsilon, \alpha_0, d_0, k, l) = x_k x_i$$

$$\times \int_0^{\alpha_0} \mathrm{d}\beta\,(1-\beta)^{2d_0-2+2} \; s_{ir}(\beta, x_i)^{-1-\epsilon}$$

$$\times \int_0^{\alpha_0} \mathrm{d}\alpha\,(1-\alpha)^{2d_0-1} \; s_{kt}(\alpha, \beta, x_k)^{-1-\epsilon}$$

$$\times \int_0^{1} \mathrm{d}u\, u^{-\epsilon}(1-u)^{-\epsilon} \left(Z_{r;i}(\beta, x_i, u) \right)^{l}$$

$$\times \int_0^{1} \mathrm{d}v\, v^{-\epsilon}(1-v)^{-\epsilon} \left(Z_{k;t}(\alpha, \beta, x_i, v) \right)^{k}, \qquad k, l = -1, 0, 1, 2$$

Integrating iterated counterterms

Another example: abelian soft-double soft subtraction

$$\left[\mathcal{S}_t\mathcal{S}_{rt}^{(0,0)}\right]^{\mathrm{ab}} = (8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon})^2\frac{1}{2}\sum_{i,j,k,l}\frac{s_{\hat{i}\hat{k}}}{s_{\hat{i}\hat{r}}s_{\hat{k}\hat{r}}}\frac{s_{jl}}{s_{jt}s_{lt}}|\mathcal{M}_{m,(i,k)(j,l)}^{(0)}(\{\tilde{p}\}_m^{(\hat{r},t)})|^2$$

$$\times(1-y_{tQ})^{d_0'-m(1-\epsilon)}(1-y_{\hat{r}Q})^{d_0'-m(1-\epsilon)}\Theta(y_0-y_{tQ})\Theta(y_0-y_{\hat{r}Q})$$

obtained by an iterated mapping

$$\{p\}\xrightarrow{\mathsf{S}_t}\{\hat{p}\}_{m+1}^{(t)}\xrightarrow{\mathsf{S}_{\hat{r}}}\{\tilde{p}\}_m^{(\widehat{r},t)}\quad:\quad \mathrm{d}\phi_{m+2}(\{p\})=\mathrm{d}\phi_m(\{\tilde{p}\}_m)[\mathrm{d}\widehat{p}_{1,m}][\mathrm{d}p_{1,m+1}]$$

Then we define the function $[S_tS_{rt}^{(0)}]_{ikjl}(p_i,p_j,p_k,p_l,\epsilon,y_0,d_0')$ by

$$\int[\mathrm{d}\widehat{p}_{1,m}][\mathrm{d}p_{1,m+1}]\left[\mathcal{S}_t\mathcal{S}_{rt}^{(0,0)}\right]^{\mathrm{ab}}\equiv\left[\frac{\alpha_{\mathrm{s}}}{2\pi}S_\epsilon\left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2\sum_{i,j,k,l}[S_tS_{rt}^{(0)}]_{ikjl}|\mathcal{M}_{m,(i,k)(j,l)}^{(0)}(\{\tilde{p}\}_m^{(\hat{r},t)})|^2$$

Integrating iterated counterterms

For simplicity, consider terms with j = i and k = l: $[S_t S_{rt}^{(0)}]_{ikik}$

kinematical dependence through $\chi_{ik} = \angle(p_i, p_k)\,,\ \cos\chi_{ik} \equiv 1 - 2Y_{ik,Q}$

The integrated counterterm is proportional to $\qquad Y_{ik,Q} = \dfrac{y_{ik}}{y_{iQ}y_{kQ}}$

$$\mathcal{I}_{\mathcal{S}}^{(11)}(Y_{ik,Q};\epsilon,y_0,d'_0) = -\frac{4\Gamma^4(1-\epsilon)}{\pi\Gamma^2(1-\epsilon)}\frac{B_{y_0}(-2\epsilon,d'_0+1)}{\epsilon}Y_{ik,Q}$$
$$\times\int_0^{y_0}\mathrm{d}y\,y^{-1-2\epsilon}(1-y)^{d'_0-1+\epsilon}\int_{-1}^{1}\mathrm{d}(\cos\vartheta)\,(\sin\vartheta)^{-2\epsilon}$$
$$\times\int_{-1}^{1}\mathrm{d}(\cos\varphi)\,(\sin\varphi)^{-1-2\epsilon}\left[f(\vartheta,\varphi;0)\right]^{-1}\left[f(\vartheta,\varphi;Y_{ik,Q})\right]^{-1}$$
$$\times\left[Y(y,\vartheta,\varphi;Y_{ik,Q})\right]^{-\epsilon}{}_2F_1\big(-\epsilon,-\epsilon,1-\epsilon;1-Y(y,\vartheta,\varphi;Y_{ik,Q})\big)$$

$$f(\vartheta,\varphi;Y_{ik,Q}) = 1-2\sqrt{Y_{ik,Q}(1-Y_{ik,Q})}\sin\vartheta\cos\varphi-(1-2Y_{ik,Q})\chi\cos\vartheta$$

$$Y(y,\vartheta,\varphi;\chi) = \frac{4(1-y)Y_{ik,Q}}{[2(1-y)+y\,f(\vartheta,\varphi;0)][2(1-y)+y\,f(\vartheta,\varphi;Y_{ik,Q})]}$$

Integrating iterated counterterms

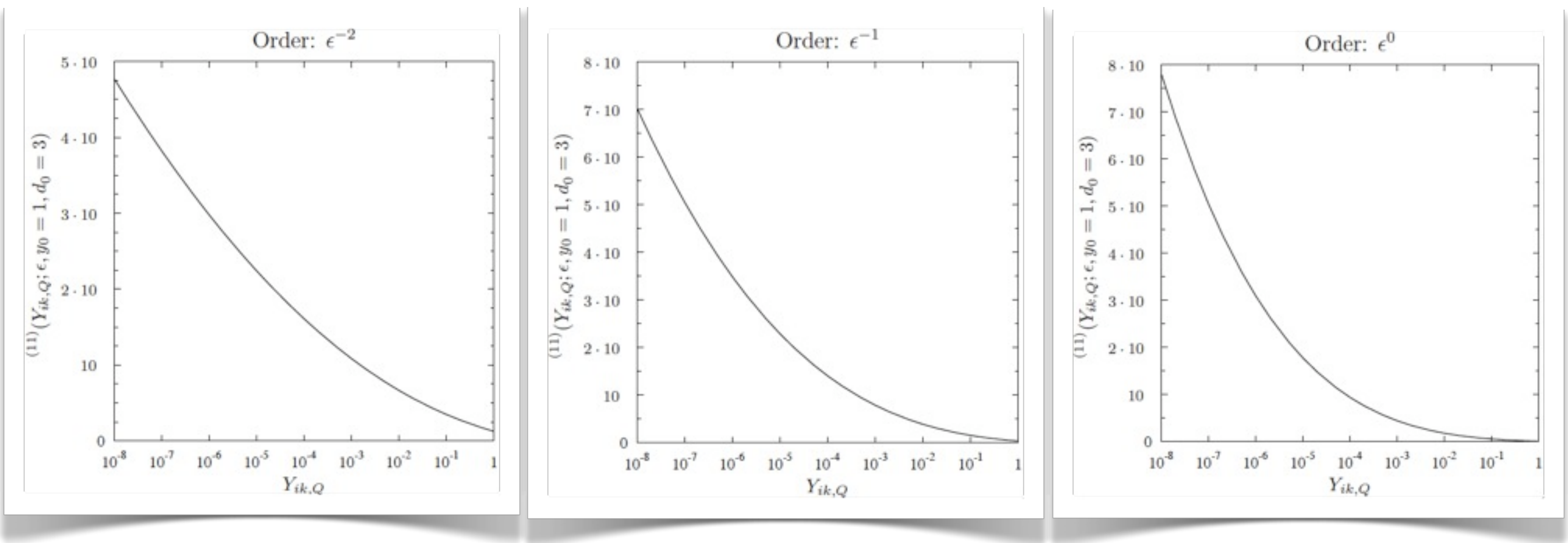
This integral is equal to

$$\mathcal{I}_{\mathcal{S}}^{(11)}(Y_{ik,Q};\epsilon,y_0,d'_0) = \frac{1}{\epsilon^4} - 2\left[\ln(Y_{ik,Q}) + \Sigma(y_0,D'_0) + \Sigma(y_0,D'_0-1)\right]\frac{1}{\epsilon^3} + \mathrm{O}(\varepsilon^{-2})$$

where $D'_0 = d'_0|_{\epsilon=0}$ and

$$\Sigma(z,N) = \ln z - \sum_{k=1}^{N} \frac{1-(1-z)^k}{k}$$

We compute the higher order coefficients numerically $(y_0 = 1\,,\; D'_0 = 3)$



Three methods

to compute the integrals:

- IBP's to reduce to master integrals + solution of MI's by differential equations
- Mellin-Barnes representations to extract poles structure + summation of nested series
- Sector decomposition

Three methods

Method	Analytical	Numerical
IBP	✓ Singly-unresolved integrals - Bottleneck is the proliferation of denominators	✓ Evaluating analytical expressions - No numbers without full analytical results
MB	✓ Iterated singly unresolved integrals - Bottleneck is the evaluation of sums	✓ Direct numerical evalution of MB integrals possible ✓ Fast and accurate
SD	✓ Easy to automate - Only in principle, except for leading pole	✓ Straightforward - In general slower & less accurate than MB

Three methods

Method	Analytical	Numerical
IBP	✓ Singly-unresolved integrals - Bottleneck is the proliferation of denominators	✓ Evaluating analytical expressions - No numbers without full analytical results
MB	✓ Iterated singly unresolved integrals - Bottleneck is the evaluation of sums	✓ Direct numerical evalution of MB integrals possible ✓ Fast and accurate
SD	✓ Easy to automate - Only in principle, except for leading pole	✓ Straightforward - In general slower & less accurate than MB

Three methods

Method	Analytical	Numerical
IBP	✓ Singly-unresolved integrals - Bottleneck is the proliferation of denominators	✓ Evaluating analytical expressions - No numbers without full analytical results
MB	✓ Iterated singly unresolved integrals - Bottleneck is the evaluation of sums	✓ Direct numerical evalution of MB integrals possible ✓ Fast and accurate
SD	✓ Easy to automate - Only in principle, except for leading pole	✓ Straightforward - In general slower & less accurate than MB

Analytical vs. numerical

Matter of principle:

- Cancellation of poles requires the coefficients of poles in integrated counterterms in analytical form
- Analytical forms are fast and accurate compared to numerical ones

However:

- Analytical results show that the integrated counterterms are smooth functions of the kinematic variables

Hence:

- Finite terms of integrated counterterms can be given in form of interpolating tables or approximating functions. Thus numerical form – computed once with required precision – is sufficient.

Results

Integrated iterated counterterms

After summation over unresolved flavours:

$$\int_2 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A}_{12}} = \mathrm{d}\sigma_m^{\mathrm{B}} \otimes \boldsymbol{I}_{12}^{(0)}(\{p\}_m;\epsilon)$$

$$\begin{aligned}\boldsymbol{I}_{12}^{(0)}(\{p\}_m;\epsilon) \propto \Bigg\{ &\sum_i \Big[\mathrm{C}_{12,f_i}^{(0)} \boldsymbol{T}_i^2 + \sum_k \mathrm{C}_{12,f_i f_k}^{(0)} \boldsymbol{T}_k^2\Big] \boldsymbol{T}_i^2 \\ &+ \sum_{j,l} \Big[\mathrm{S}_{12}^{(0),(j,l)} C_{\mathrm{A}} + \sum_i \mathrm{CS}_{12,f_i}^{(0),(j,l)} \boldsymbol{T}_i^2\Big] \boldsymbol{T}_j \boldsymbol{T}_l \\ &+ \sum_{i,k,j,l} \mathrm{S}_{12}^{(0),(i,k)(j,l)} \{\boldsymbol{T}_i \boldsymbol{T}_k, \boldsymbol{T}_j \boldsymbol{T}_l\} \Bigg\}\end{aligned}$$

The coefficients depend on ϵ (poles starting at $O(\epsilon^{-4})$), kinematics and PS cut parameters

[Same structure, but different coeff's for I_2]

Insertion operator I_{12}

Illustration: $e^+e^- \to 2$ jets

Born squared matrix element: $|\mathcal{M}_2^{(0)}(1_q, 2_{\bar{q}})|^2$

Colour and kinematics are trivial:

$$\boldsymbol{T}_1^2 = \boldsymbol{T}_2^2 = -\boldsymbol{T}_1 \boldsymbol{T}_2 = C_{\mathrm{F}}\,, \qquad y_{12} = \frac{2p_1 \cdot p_2}{Q^2} = 1$$

Insertion operator from iterated subtraction:

$$\boldsymbol{I}_{12}^{(0)}(p_1, p_2; \epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 C_{\mathrm{F}}^2 \Bigg\{ (3-x)\frac{2}{\epsilon^4} + \frac{1}{6}\Big[20x + 81 - 4y_f + (36x-24)\Sigma(y_0, D_0') + (24x-12)\Sigma(y_0, D_0'-1)\Big]\frac{1}{\epsilon^3} + \mathrm{O}(\varepsilon^{-2}) \Bigg\}$$

$$x = \frac{C_{\mathrm{A}}}{C_{\mathrm{F}}}\,, \qquad y_f = \frac{T_{\mathrm{R}}}{C_{\mathrm{F}}} n_{\mathrm{f}}$$

Insertion operator I_{12}

Illustration: $e^+e^- \to 2$ jets

We compute higher order expansion coefficients numerically

$$\boldsymbol{I}_{12}^{(0)}(p_1,p_2;\epsilon) = \left[\frac{\alpha_s}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 C_F^2 \sum_{i=-4}^{0} \frac{1}{\epsilon^i} \sum_{C_x} C_x \mathcal{I}_{12,2j}^{(C_x,i)} + O(\varepsilon^1)$$

C_x	$O(\epsilon^{-4})$	$O(\epsilon^{-3})$	$O(\epsilon^{-2})$	$O(\epsilon^{-1})$	$O(\epsilon^{0})$
1	6	76/3	32.09	-87.9	-554.5
x	-2	−27/2	-52.4	-150.7	-339.5
y_f	0	-1	-6.332	-17.65	1.013

Preliminary

$$x = \frac{C_A}{C_F}, \qquad y_f = \frac{T_R}{C_F} n_f$$

Insertion operator I_{12}

Illustration: $e^+e^- \to 3$ jets

Born squared matrix element: $|\mathcal{M}_3^{(0)}(1_q, 2_{\bar{q}}, 3_g)|^2$

Colour is still trivial:

$$\boldsymbol{T}_1^2 = \boldsymbol{T}_2^2 = C_\mathrm{F}\,, \quad \boldsymbol{T}_3^2 = C_\mathrm{A}\,, \quad \boldsymbol{T}_1\boldsymbol{T}_2 = \frac{C_\mathrm{A} - 2C_\mathrm{F}}{2}\,, \quad \boldsymbol{T}_1\boldsymbol{T}_3 = \boldsymbol{T}_2\boldsymbol{T}_3 = -\frac{C_\mathrm{A}}{2}$$

Insertion operator from iterated subtraction:

$$\boldsymbol{I}_{12}^{(0)}(p_1, p_2, p_3; \epsilon) = \left[\frac{\alpha_\mathrm{s}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 C_\mathrm{F}^2 \left\{\frac{x^2 + 2x + 6}{\epsilon^4}\right.$$

$$+ \left[\frac{11x^2}{2} + \frac{50x}{3} + 12 - \frac{1}{3}xy_f - x^2 y_f - 4y_f\right.$$

$$+ \left(\frac{5x^2}{2} - x - 8\right)\ln y_{12} - \left(\frac{5x^2}{2} + 4x\right)(\ln y_{13} + \ln y_{23})$$

$$\left.\left. + (x^2 + 12x - 4)\Sigma(y_0, D_0') + 4(x-1)\Sigma(y_0, D_0' - 1)\right]\frac{1}{\epsilon^3} + \mathrm{O}(\varepsilon^{-2})\right\}$$

Insertion operator I_{12}

Illustration: $e^+e^- \to 3$ jets

We compute higher order expansion coefficients numerically

$$\boldsymbol{I}_{12}^{(0)}(p_1,p_2,p_3;\epsilon) = \left[\frac{\alpha_s}{2\pi}S_\epsilon\left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 C_F^2 \sum_{i=-4}^{0}\frac{1}{\epsilon^i}\sum_{C_x} C_x \mathcal{I}_{12,3j}^{(C_x,i)} + O(\varepsilon^1)$$

$y_{12} = 0.333333$, $y_{13} = 0.333333$, $y_{23} = 0.333333$

C_x	$O(\epsilon^{-4})$	$O(\epsilon^{-3})$	$O(\epsilon^{-2})$	$O(\epsilon^{-1})$	$O(\epsilon^{0})$
1	6	34.12	82.98	34.59	-543.8
x	2	9.721	1.209	-142.2	-696.6
x^2	1	6.497	12.80	15.87	-47.92
y_f	0	-13/3	-32.40	-127.9	-355.2
$x\, y_f$	0	-3/2	-12.01	-46.90	-104.1

Preliminary

Insertion operator I_{12}

Illustration: $e^+e^- \to 3$ jets

We compute higher order expansion coefficients numerically

$$\boldsymbol{I}_{12}^{(0)}(p_1,p_2,p_3;\epsilon) = \left[\frac{\alpha_s}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 C_F^2 \sum_{i=-4}^{0} \frac{1}{\epsilon^i} \sum_{C_x} C_x \mathcal{I}_{12,3j}^{(C_x,i)} + O(\varepsilon^1)$$

$y_{12} = 0.238667$, $y_{13} = 0.758153$, $y_{23} = 0.003180$

C_x	$O(\epsilon^{-4})$	$O(\epsilon^{-3})$	$O(\epsilon^{-2})$	$O(\epsilon^{-1})$	$O(\epsilon^{0})$
1	6	36.79	106.0	120.6	-431.0
x	2	25.38	143.6	537.3	1505
x^2	1	15.24	119.5	660.5	2902
y_f	0	-13/3	-31.30	-121.7	-346.0
$x\,y_f$	0	-3/2	-17.72	-109.1	-470.9

Preliminary

Insertion operator I_{12}

Illustration: $e^+e^- \to 3$ jets

We compute higher order expansion coefficients numerically

$$\boldsymbol{I}_{12}^{(0)}(p_1,p_2,p_3;\epsilon) = \left[\frac{\alpha_s}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 C_F^2 \sum_{i=-4}^{0} \frac{1}{\epsilon^i} \sum_{C_x} C_x \mathcal{I}_{12,3j}^{(C_x,i)} + O(\varepsilon^1)$$

$y_{12} = 0.937004$, $y_{13} = 0.024207$, $y_{23} = 0.038749$

C_x	$O(\epsilon^{-4})$	$O(\epsilon^{-3})$	$O(\epsilon^{-2})$	$O(\epsilon^{-1})$	$O(\epsilon^0)$
1	6	25.85	34.59	-84.25	-566.8
x	2	27.79	136.8	330.6	46.20
x^2	1	21.02	195.4	1174	5355
y_f	0	-13/3	-57.59	-405.2	-2120
$x\, y_f$	0	-3/2	-24.07	-194.7	-1083

Preliminary

Present status

Integration of the doubly-unresolved counterterms in progress (most difficult)

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

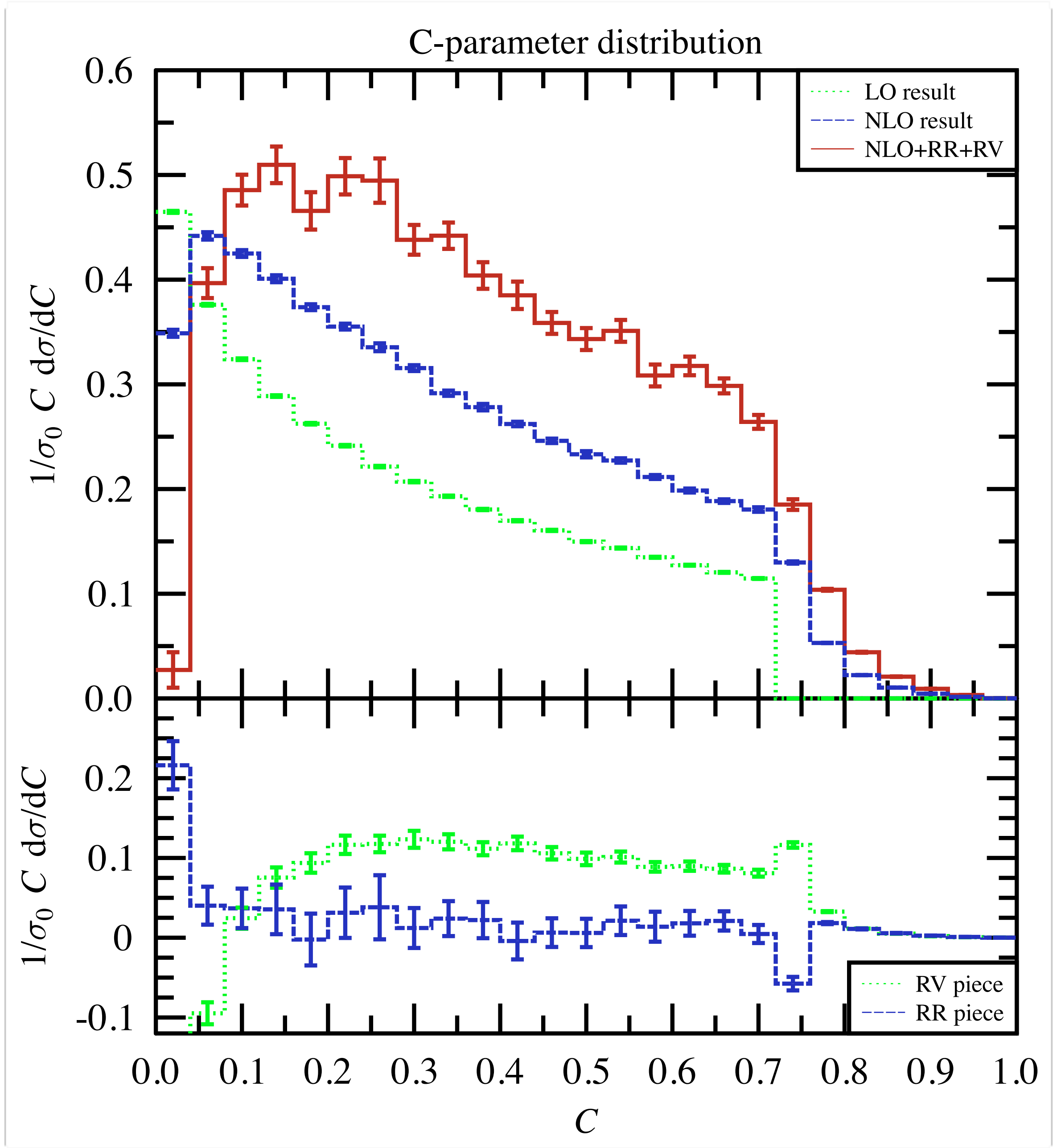
$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

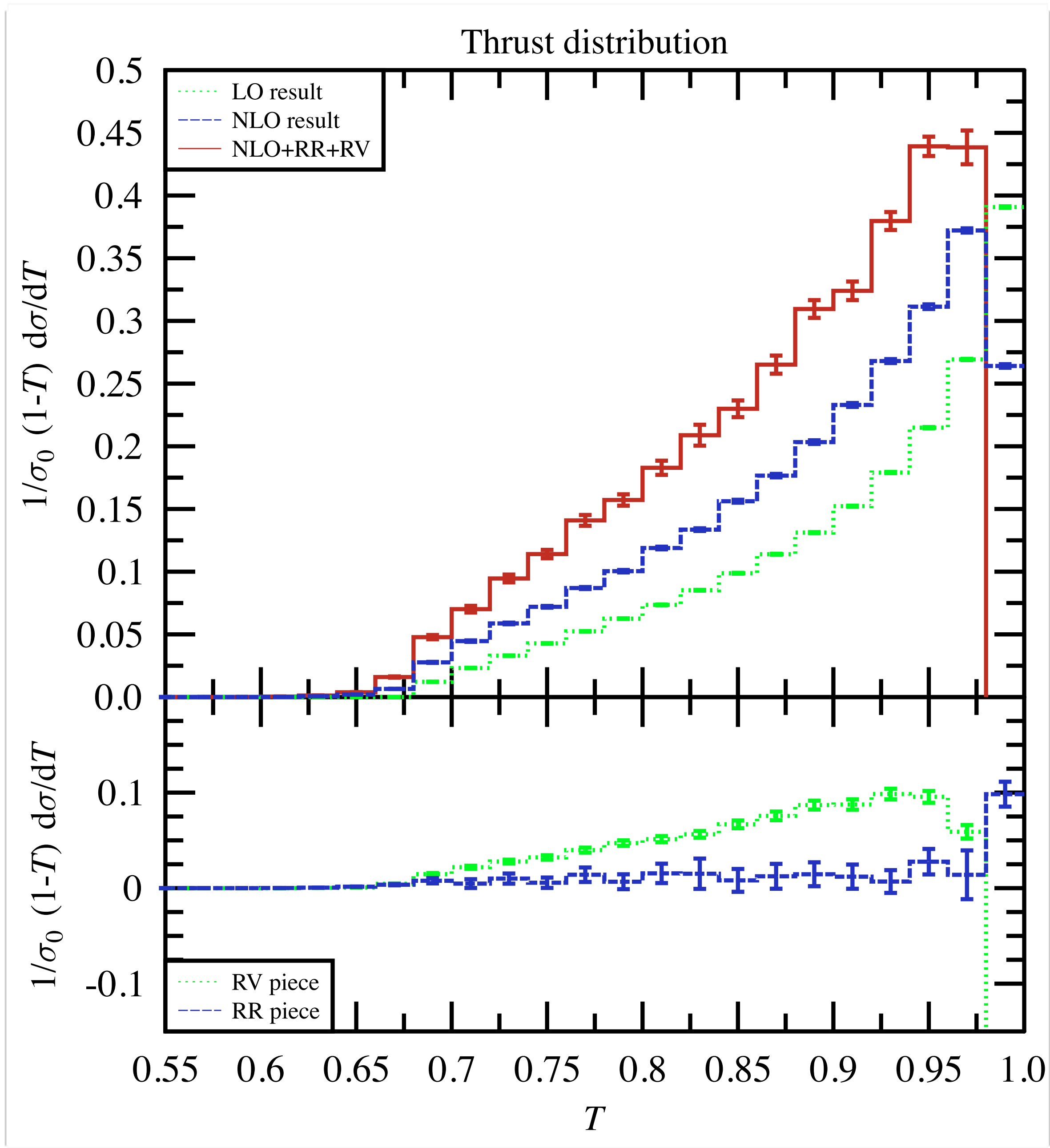
$$\sigma^{\rm NNLO}_{m} = \int_m \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

Present status

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{NNLO}}_{m}$$

by numerical Monte Carlo integrations (on a single CPU in 50 h)





Conclusions

Conclusions

- ✓ Matrix elements are known for jet, V + jet hadroproduction
- Three different subtraction methods are being developed
- Sector decomposition matched with residuum subtraction: a promise
- Antennae subtraction: subtractions and integrated subtractions are
  - ✓ known for final-final and initial-final emitter-spectator configurations
  - ✳ in progress for initial-initial configurations

Conclusions

Status of integrated antennae

	final-final	initial-final	initial-initial
X_3^0	✓	✓	✓
X_4^0	✓	✓	?
$X_3^0 \otimes X_3^0$	✓	✓	?
X_3^1	✓	✓	?

Conclusions

- Our subtraction scheme: completely algorithmic,
  - ✓ set up for processes with no coloured particles in the initial state
  - hadroproduction is obtained by crossing

✓ We have investigated various methods to integrate the counterterms

✓ We used the MB method to perform the integrat-ion of all but doubly-unresolved counterterms. The SD method was used to provide independent check

✳ The integration of the doubly-unresolved counter-term is feasible with our methods, and is in progress

The end

Appendix: some angular integrals

Are these integrals known?

$$\Omega_{j_1 \ldots j_n} = \int \mathrm{d}\Omega_{d-1}(\vec{r}) \frac{1}{(\vec{e}_1 \cdot \vec{r})^{j_1} \ldots (\vec{e}_1 \cdot \vec{r})^{j_n}}$$

for

two denominators and one or two masses:

$$\Omega_{jl}(\cos\chi, \beta_1, \beta_2) = \int_{-1}^{1} \mathrm{d}(\cos\vartheta)\, \mathrm{d}(\cos\varphi)\, (\sin\vartheta)^{-2\epsilon} (\sin\varphi)^{-1-2\epsilon}$$
$$\times\, (1 - \beta_1 \cos\vartheta)^{-j} \Big(1 - \beta_2(\sin\chi \sin\vartheta \cos\varphi + \cos\chi \cos\vartheta)\Big)^{-l}$$

three denominators, massless:

$$\Omega_{j_1 j_2 j_3}(\cos\chi_{12}, \cos\chi_{13}, \cos\chi_{23})$$