

QCD at the LHC

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- introduction and overview
- HO corrections
- PDFs
- LHC benchmarks
- (double parton scattering)
- summary

apologies for
omitting many
topics of interest!



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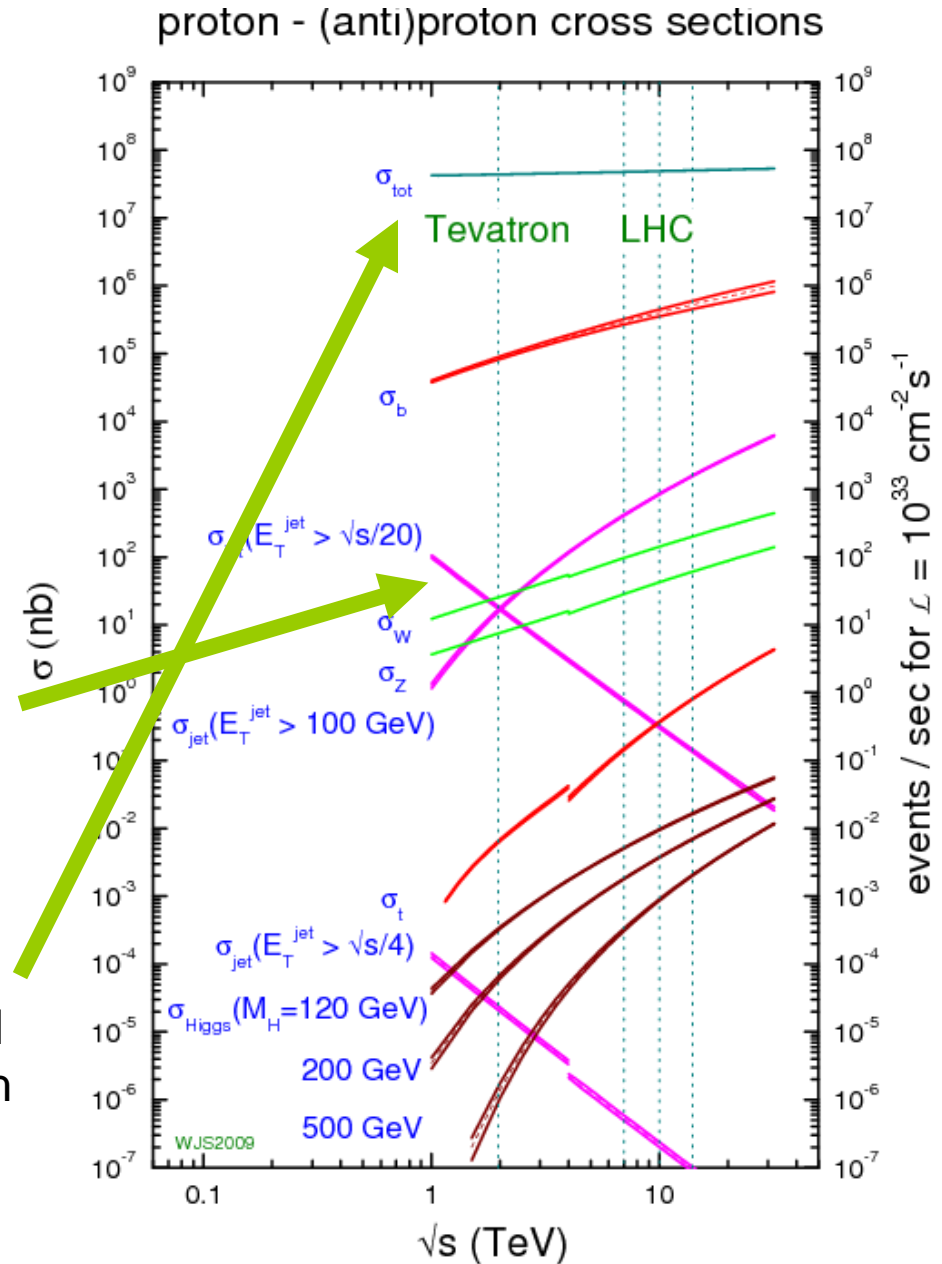
introduction and overview

Scattering processes at high energy hadron colliders can be classified as either **HARD** or **SOFT**

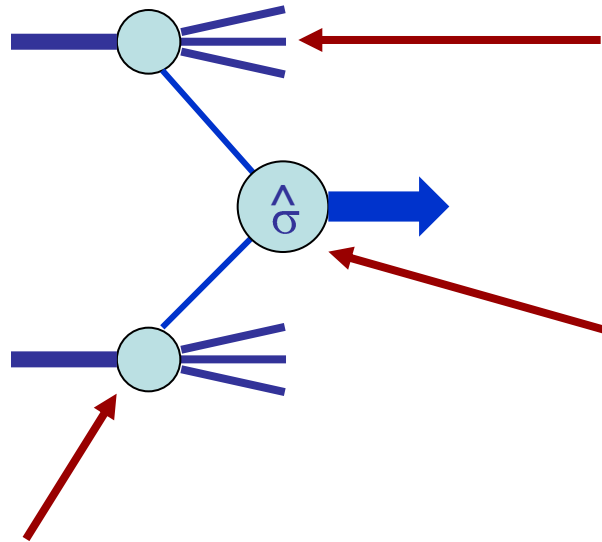
Quantum Chromodynamics (QCD) is the underlying theory for **all** such processes, but the approach (and the level of understanding) is very different for the two cases

For **HARD** processes, e.g. **W** or **high- E_T jet** production, the rates and event properties can be predicted with some precision using **perturbation theory**

For **SOFT** processes, e.g. the **total cross section** or **diffractive** processes, the rates and properties are dominated by **non-perturbative** QCD effects, which are much less well understood



precision pQCD in the LHC era



tuned event simulation
(parton showers + UE) MCs,
interfaced with LO or NLO
hard scattering MEs

LO, NLO, NNLO, ... supplemented
by resummed NⁿLL improvements,
EW corrections, ...

parton distribution functions

the QCD **factorization theorem** for hard-scattering (short-distance) inclusive processes



$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \rightarrow X} \left(\mathbf{x}_1, \mathbf{x}_2, \{\mathbf{p}_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2} \right)$$

precision phenomenology

- *Benchmarking*

- inclusive SM quantities (V , jets, top, ...), calculated to the highest precision available (e.g. NNLO)
- tools needed: robust jet algorithms, decays included, PDFs, ...



e.g. anti- k_T (Cacciari, Salam, Soyez)

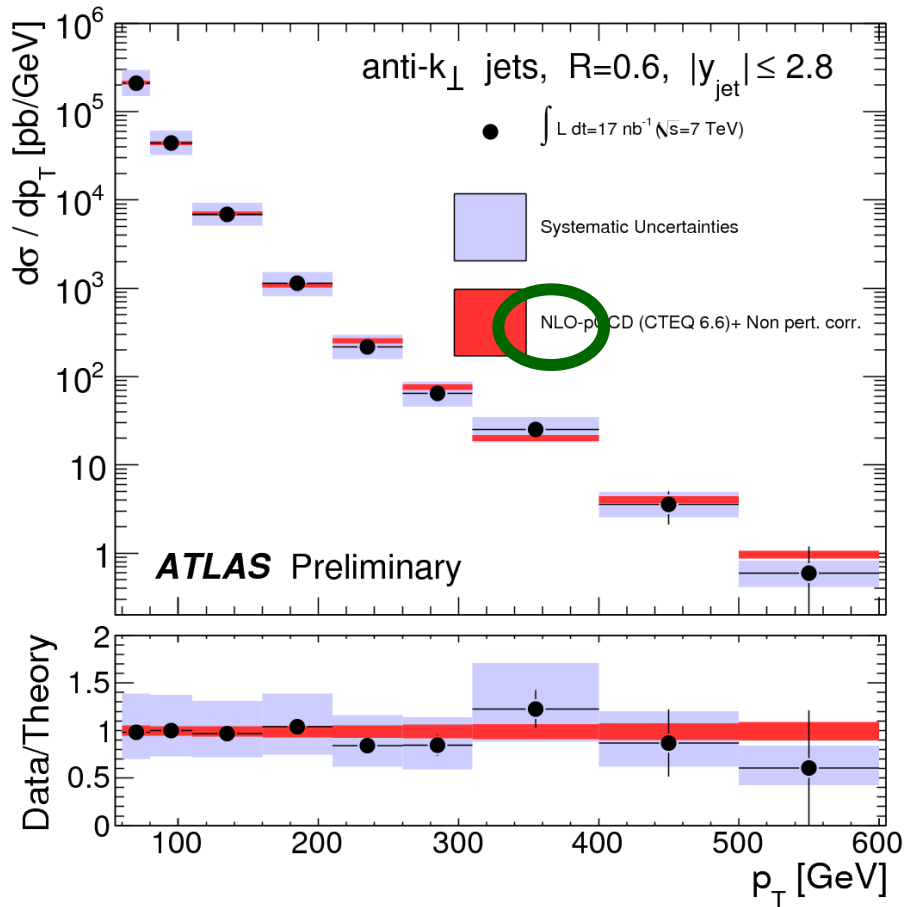
- *Backgrounds*

- new physics generally results in some combination of multijets, multileptons, missing E_T
- therefore, we need to know SM cross sections $\{V, VV, bb, tt, H, \dots\} + \text{jets}$ to high precision \rightarrow 'wish lists'
- ratios can be useful

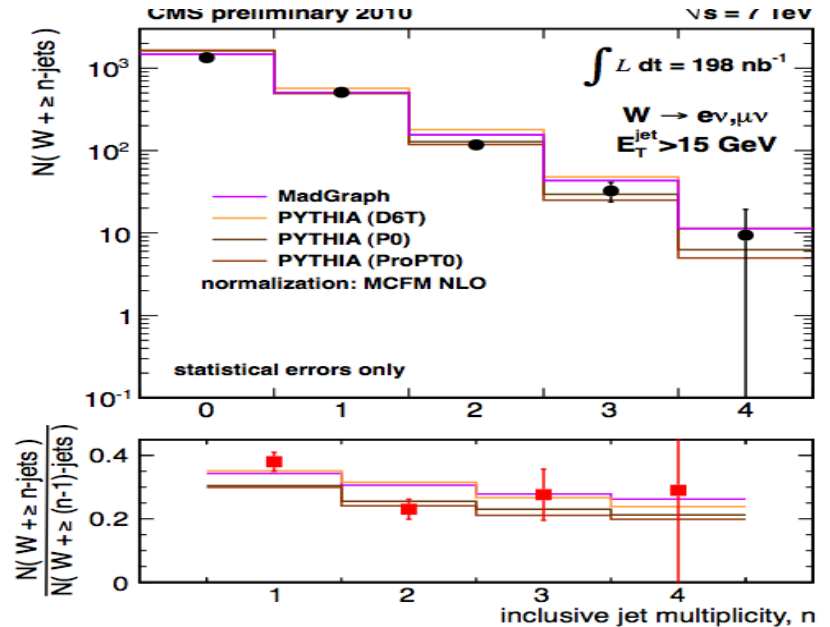
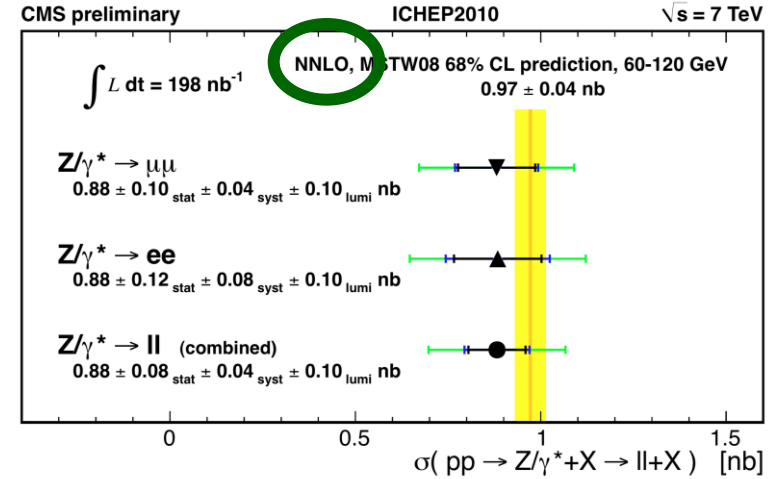
Note: $V = \gamma^*, Z, W^\pm$

emerging 'precision' phenomenology at LHC

jet production

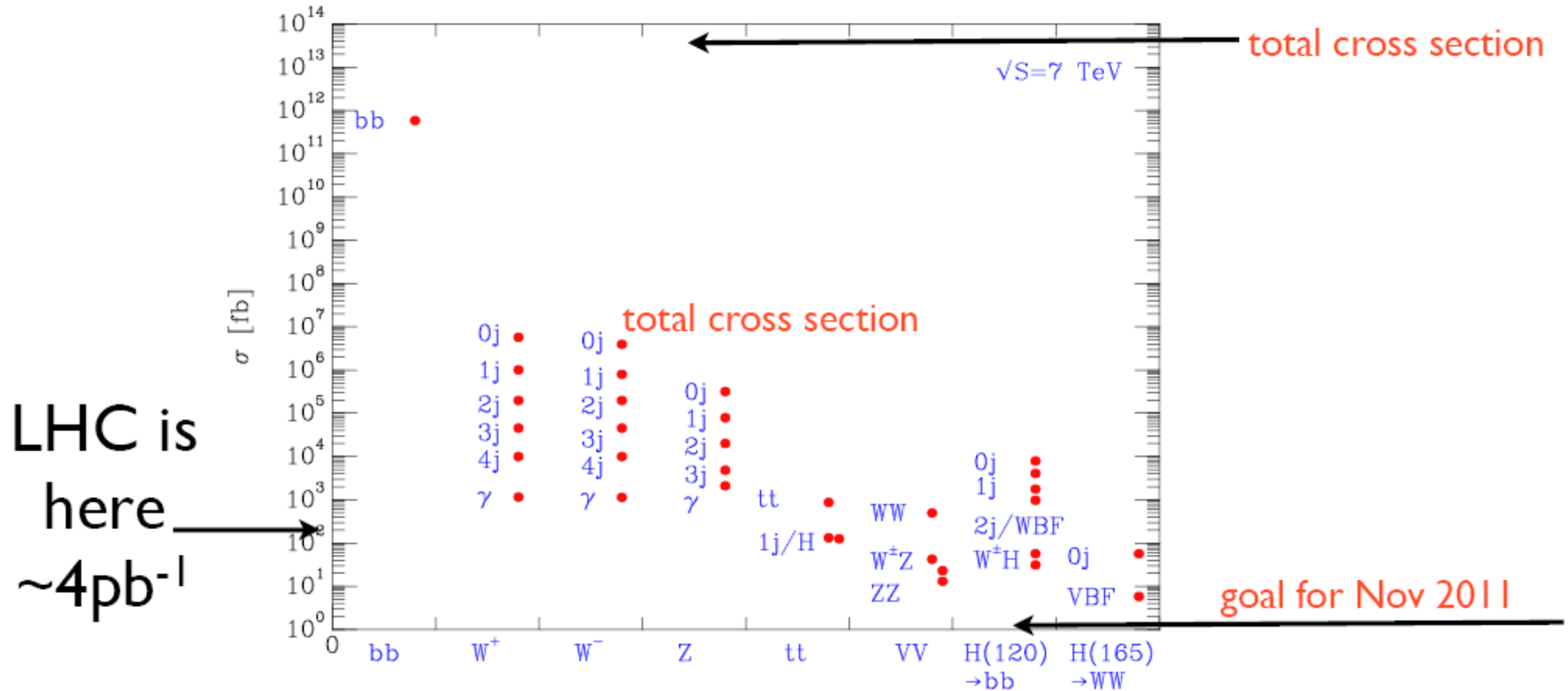


W, Z production



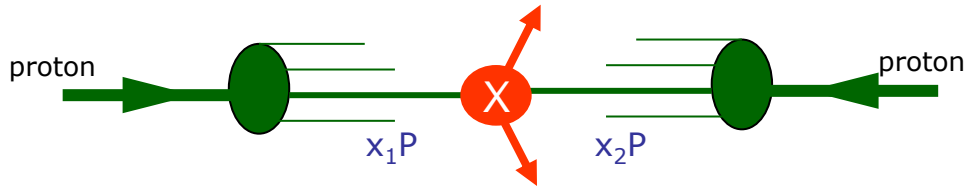
the SM Ladder at 7 TeV

Keith Ellis, MCFM



Includes decay of W/Z to one species of charged lepton and semi-leptonic decay of top ($t \rightarrow b l \nu$) (where applicable) and jets, $E_t > 25$ GeV.

Parton Distribution Functions



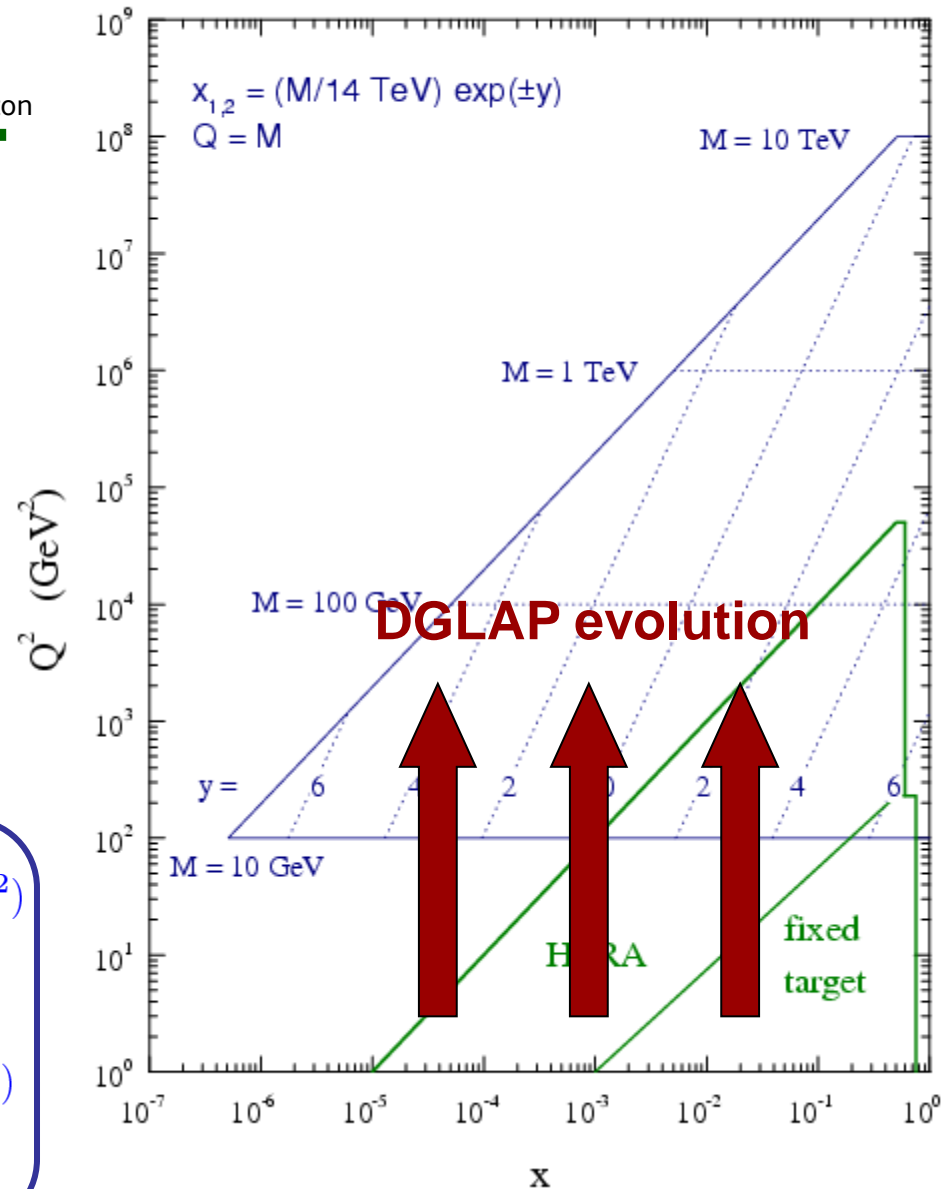
momentum fractions x_1 and x_2
determined by mass and rapidity of **X**

x dependence of $f_i(x, Q^2)$ determined
by 'global fit' to deep inelastic
scattering and other data, Q^2
dependence determined by **DGLAP**
equations:

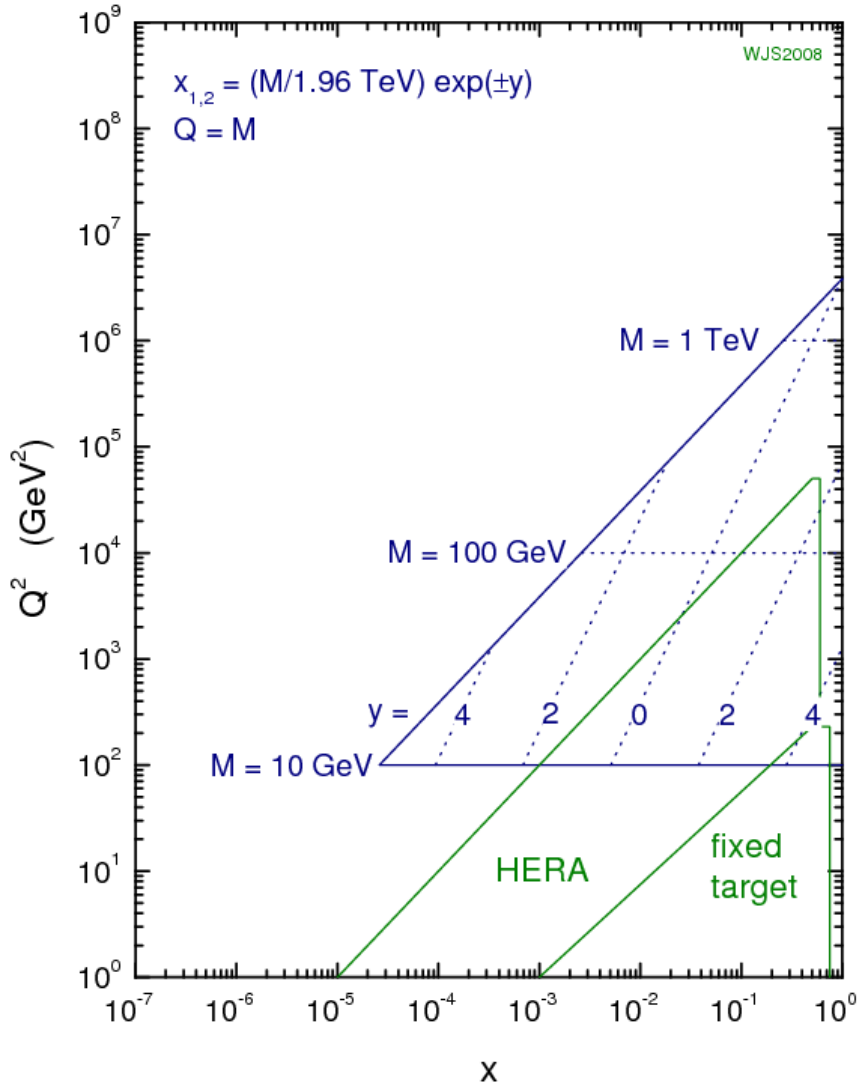
$$\frac{\partial q_i(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{dy}{y} \left\{ P_{q_i q_j}(y, \alpha_S) q_j\left(\frac{x}{y}, Q^2\right) + P_{q_i g}(y, \alpha_S) g\left(\frac{x}{y}, Q^2\right) \right\}$$

$$\frac{\partial g(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{dy}{y} \left\{ P_{g q_j}(y, \alpha_S) q_j\left(\frac{x}{y}, Q^2\right) + P_{g g}(y, \alpha_S) g\left(\frac{x}{y}, Q^2\right) \right\}$$

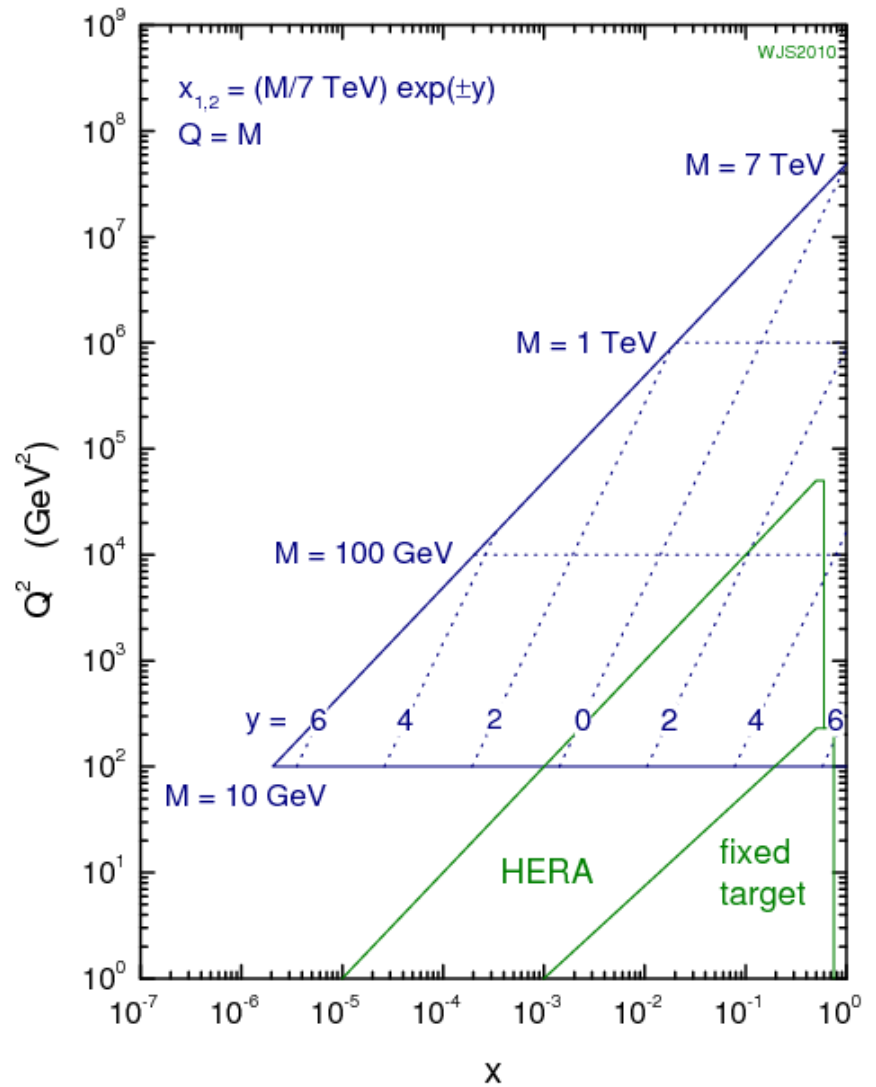
LHC parton kinematics



Tevatron parton kinematics

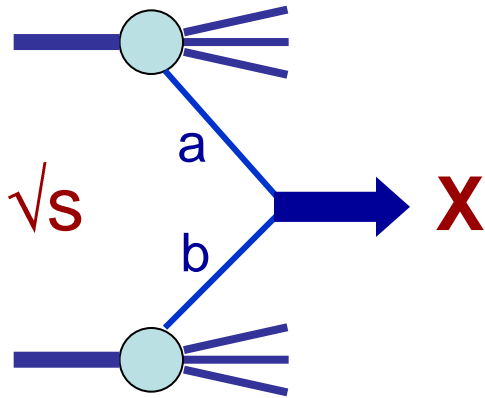


7 TeV LHC parton kinematics



parton luminosity functions

- a quick and easy way to assess the mass, collider energy and pdf dependence of production cross sections



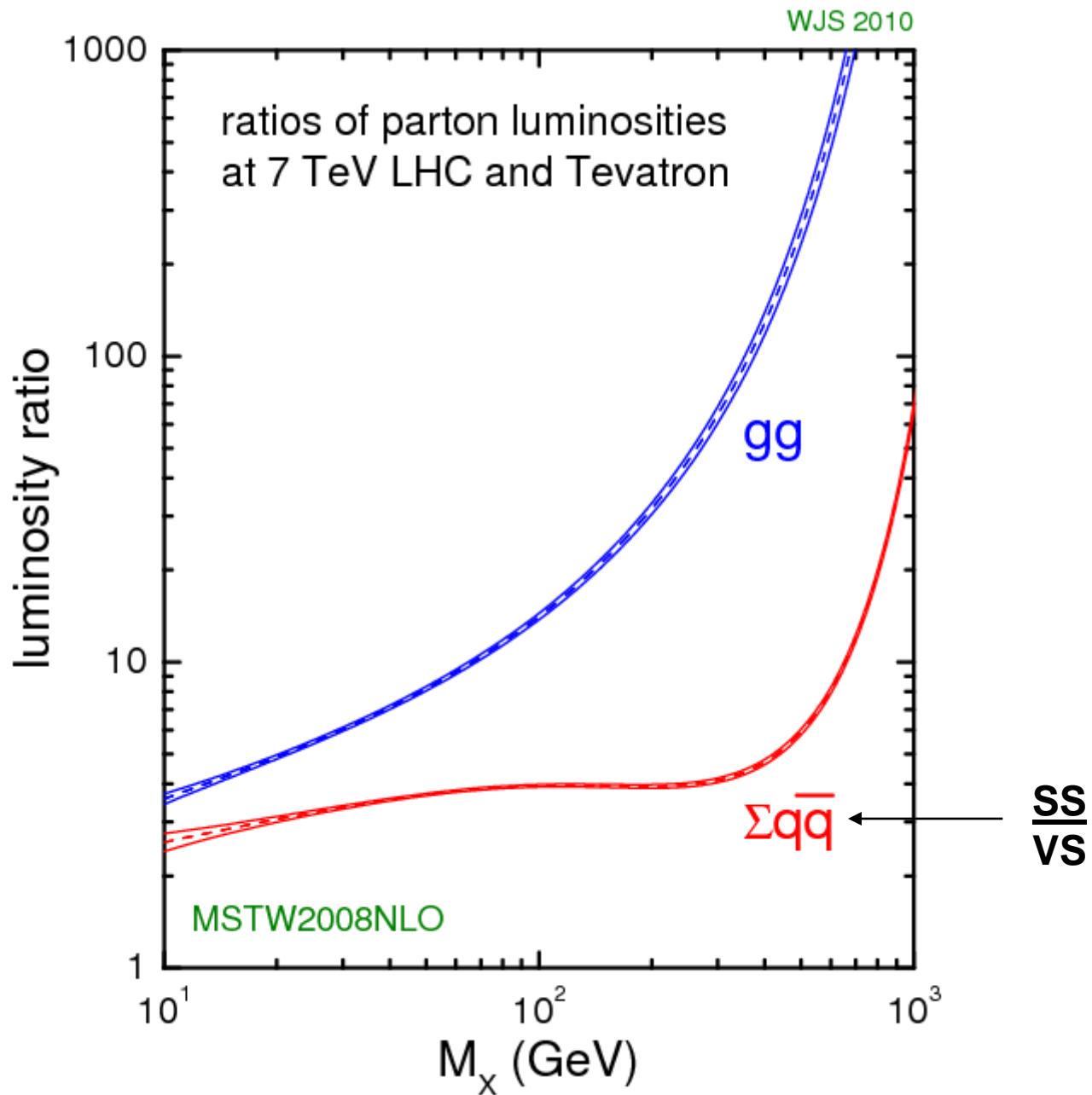
$$\hat{\sigma}_{ab \rightarrow X} = C_X \delta(\hat{s} - M_X^2)$$

$$\sigma_X = \int_0^1 dx_a dx_b f_a(x_a, M_X^2) f_b(x_b, M_X^2) C_X \delta(x_a x_b - \tau)$$

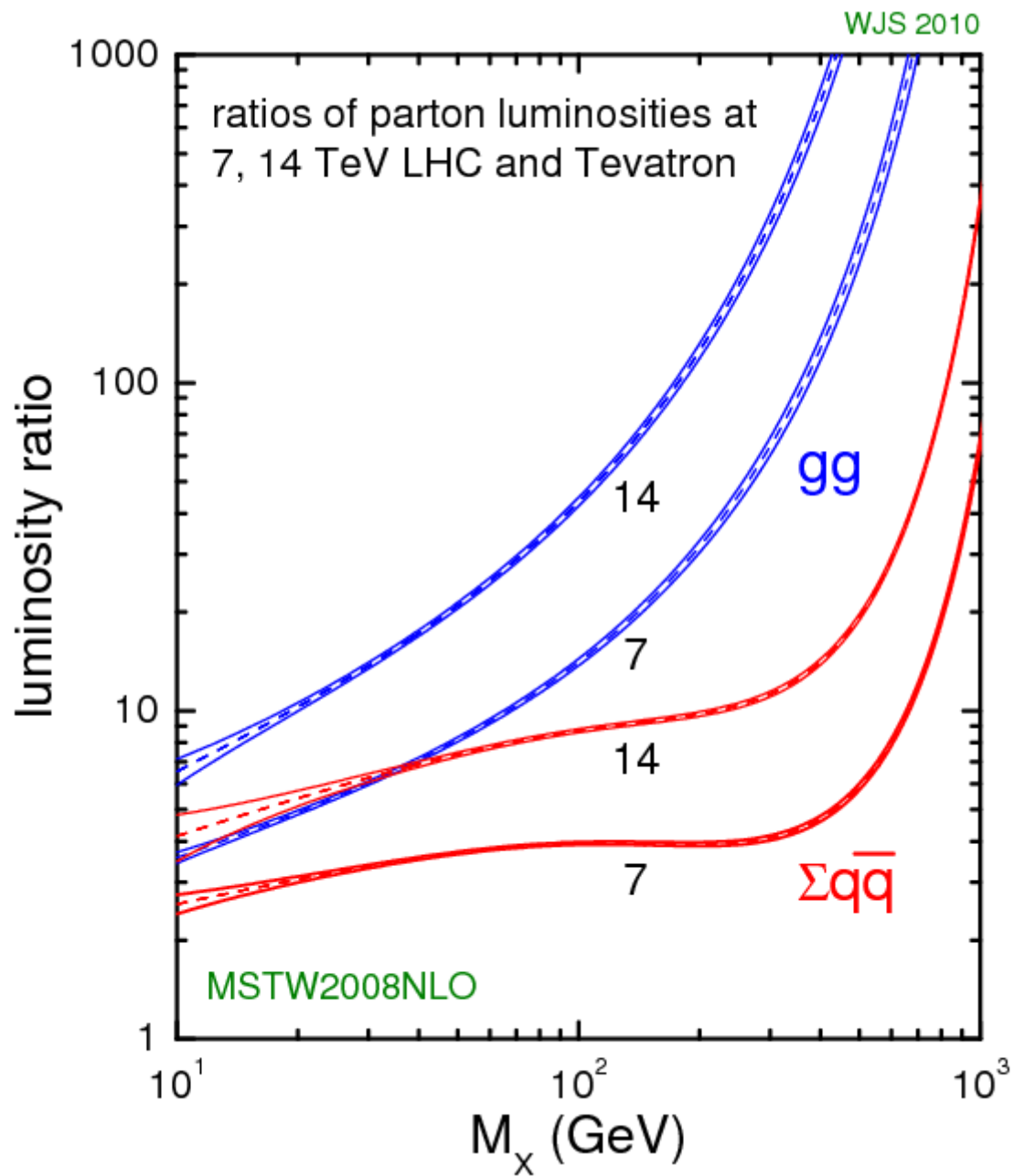
$$\equiv C_X \left[\frac{1}{s} \frac{\partial \mathcal{L}_{ab}}{\partial \tau} \right] \quad (\tau = M_X^2/s)$$

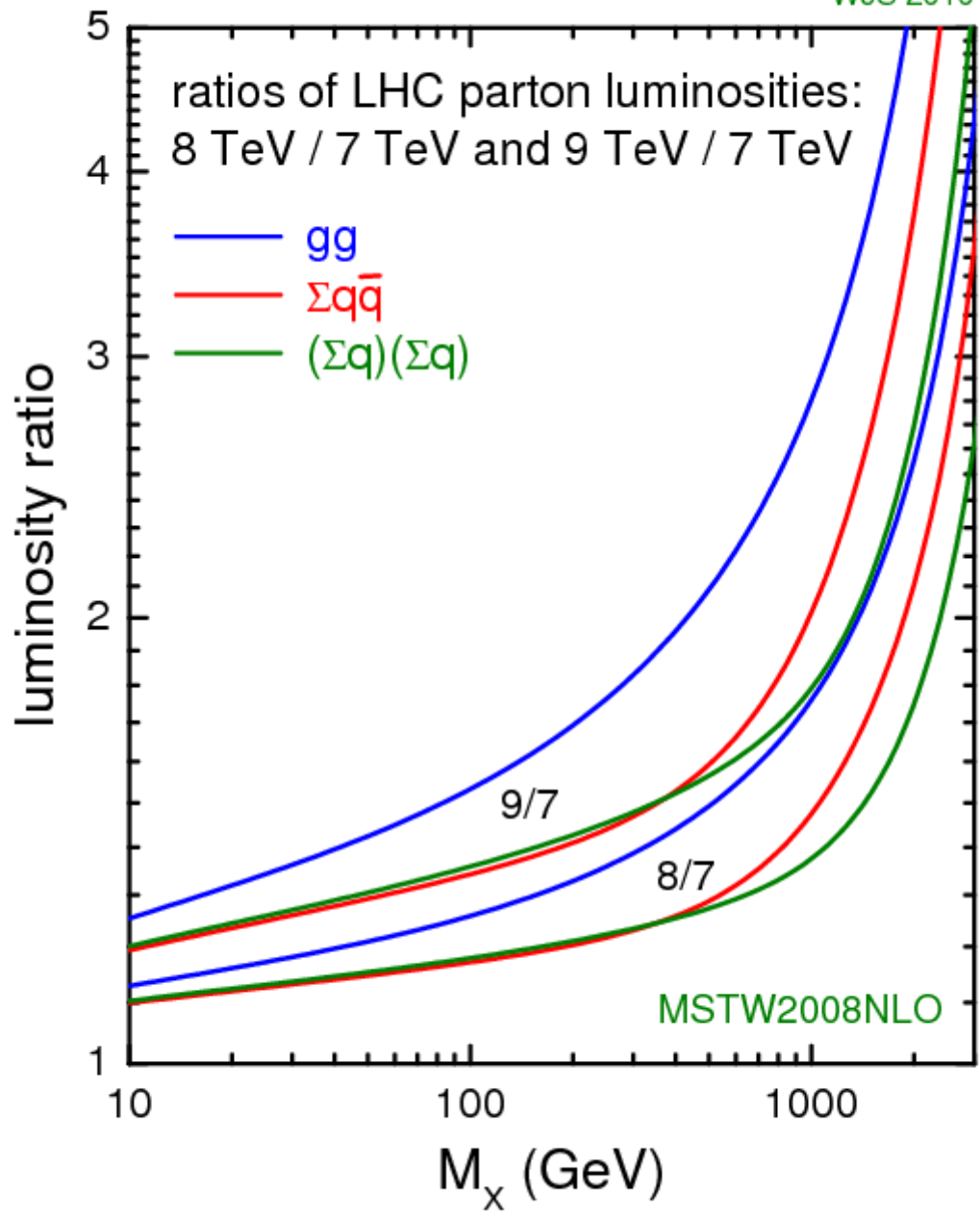
$$\frac{\partial \mathcal{L}_{ab}}{\partial \tau} = \int_0^1 dx_a dx_b f_a(x_a, M_X^2) f_b(x_b, M_X^2) \delta(x_a x_b - \tau)$$

- i.e. all the mass and energy dependence is contained in the **X**-independent parton luminosity function in []
- useful combinations are $ab = gg, \sum_q q\bar{q}, \dots$
- and also useful for assessing the uncertainty on cross sections due to uncertainties in the pdfs (see later)



more such luminosity plots available at www.hep.phy.cam.ac.uk/~wjs/plots/plots.html





e.g.
 $gg \rightarrow H$
 $q\bar{q} \rightarrow VH$
 $qq \rightarrow qqH$

2

higher-order perturbative QCD corrections

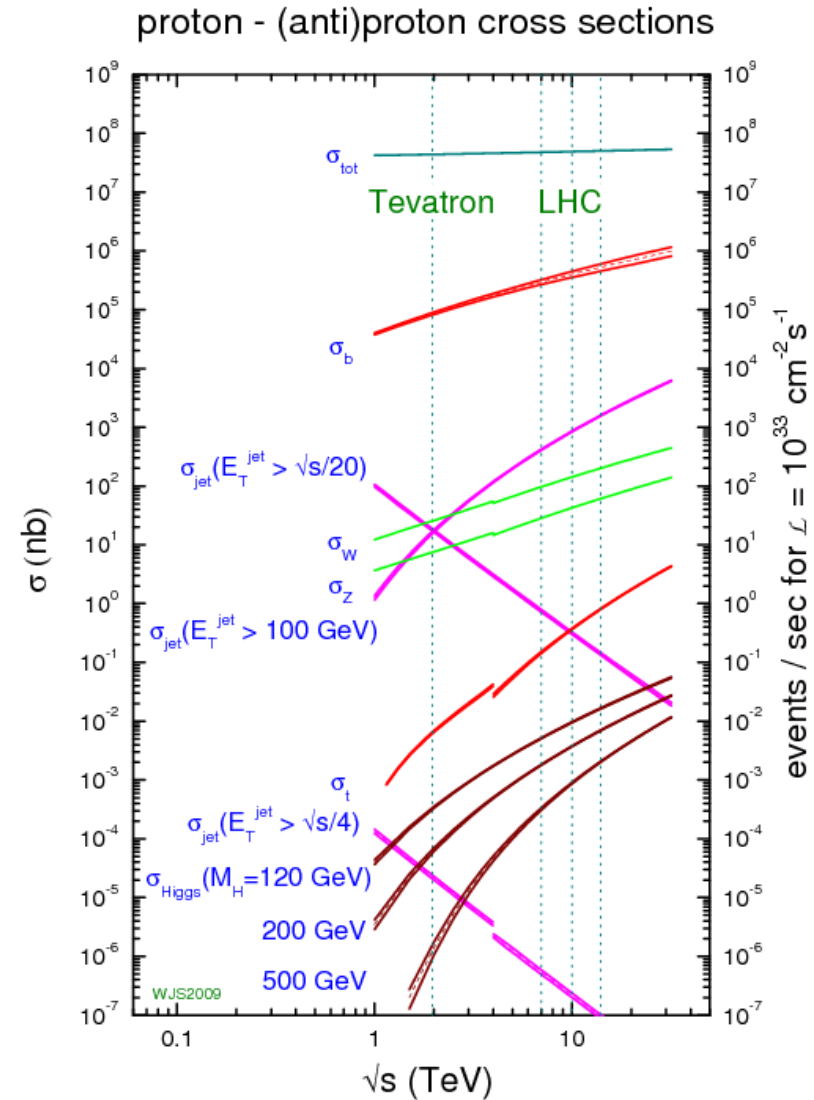
how precise?

- **LO** for generic PS Monte Carlos, tree-level MEs
 - **NLO** for NLO-MCs and many parton-level signal and background processes
 - in principle, less sensitivity to unphysical renormalisation and factorisation scales, μ_R and μ_F
 - parton merging to give structure in jets
 - more types of incoming partons
 - more reliable pdfs
 - better description of final state kinematics
 - **NNLO** for a limited number of 'precision observables' (W, Z, DY, H, ...)
- + E/W corrections, resummed HO terms etc...

$$\delta\sigma_{\text{th}} = \delta\sigma_{\text{UHO}} \oplus \delta\sigma_{\text{pdf}} \oplus \delta\sigma_{\text{param}} \oplus \dots$$

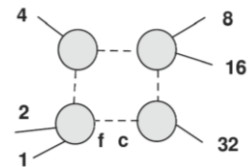


what does this mean?!



recent developments at NLO

- traditional methods based on Feynman diagrams, then reduction to known (scalar box, triangle, bubble and tadpole) integrals
- ... and new methods based on unitarity and on-shell recursion: assemble loop-diagrams from individual tree-level diagrams
 - basic idea: [Bern, Dixon, Kosower 1993](#)
 - cuts with respect to on-shell complex loop momenta: [Cachazo, Britto, Feng 2004](#)
 - tensor reduction scheme: [Ossola, Pittau, Papadopoulos 2006](#)
 - integrating the OPP procedure with unitarity: [Ellis, Giele, Kunszt 2008](#)
 - D-dimensional unitarity: [Giele, Kunszt, Melnikov 2008](#)
 - ...
- ... and the appearance of automated programmes for one-loop, multi-leg amplitudes, either based on
 - traditional or numerical Feynman approaches ([Golem, ...](#))
 - unitarity/recursion ([BlackHat, CutTools, Rocket, ...](#))



➔ see talk by Lance Dixon

recent NLO results...*

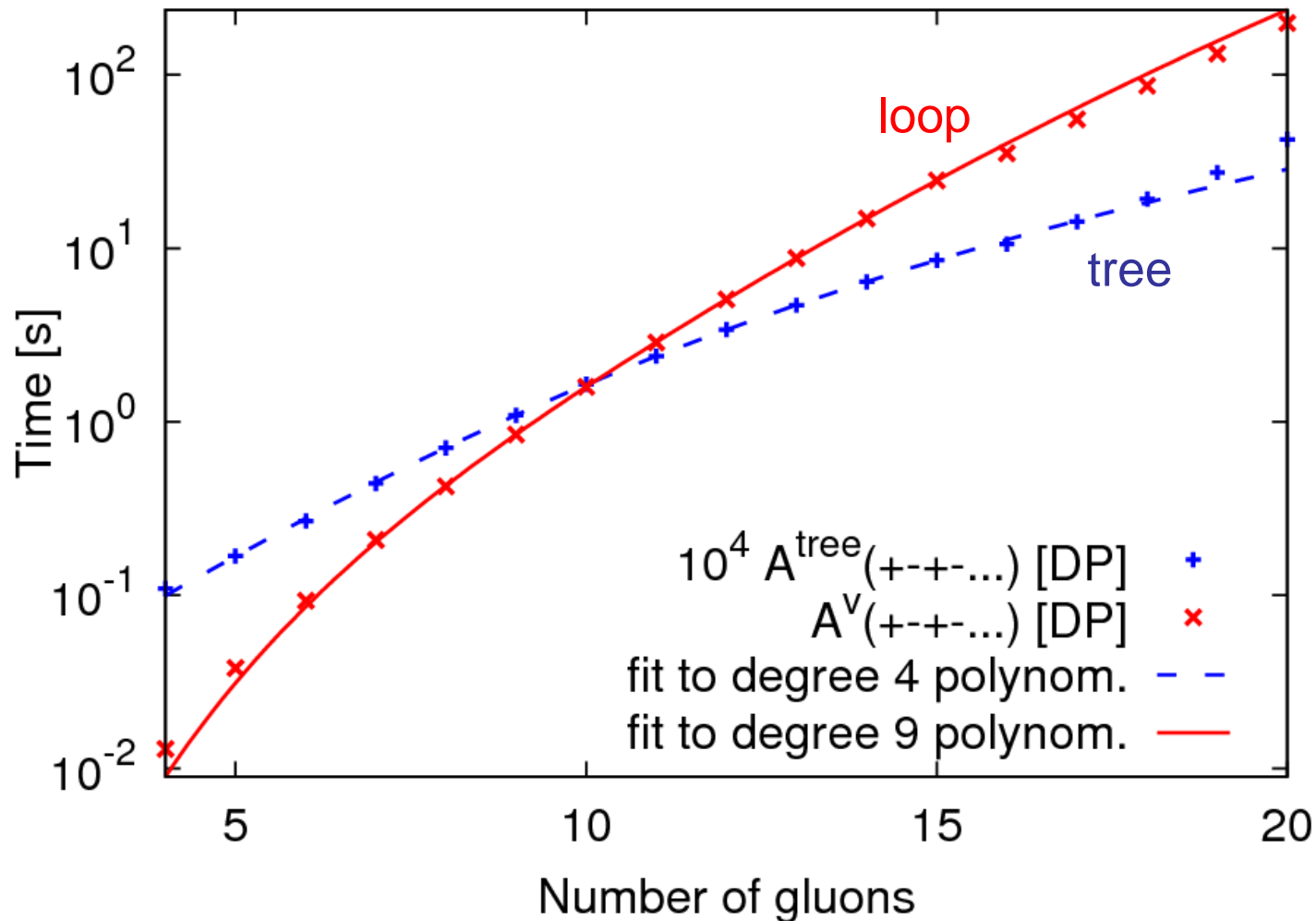
- $pp \rightarrow W+3j$ [Rocket: Ellis, Melnikov & Zanderighi] [unitarity]
- $pp \rightarrow W+3j$ [BlackHat: Berger et al] [unitarity]
- $pp \rightarrow tt\ bb$ [Bredenstein et al] [traditional]
- $pp \rightarrow tt\ bb$ [HELAC-NLO: Bevilacqua et al] [unitarity]
- $pp \rightarrow qq \rightarrow 4b$ [Golem: Binoth et al] [traditional]
- $pp \rightarrow tt+2j$ [HELAC-NLO: Bevilacqua et al] [unitarity]
- $pp \rightarrow Z+3j$ [BlackHat: Berger et al] [unitarity]
- $pp \rightarrow W+4j$ [BlackHat: Berger et al, partial] [unitarity]
- ...

with earlier results on $V,H + 2\text{ jets}$, $VV,tt + 1\text{ jet}$, VVV , ttH , ttZ , ...

In contrast, for **NNLO** we still only have inclusive γ^*,W,Z,H with rapidity distributions and decays (although much progress on *top*, *single jet*, ...)

calculation time: one-loop pure gluon amplitudes

Giele and Zanderighi, 2008



general structure of a QCD perturbation series

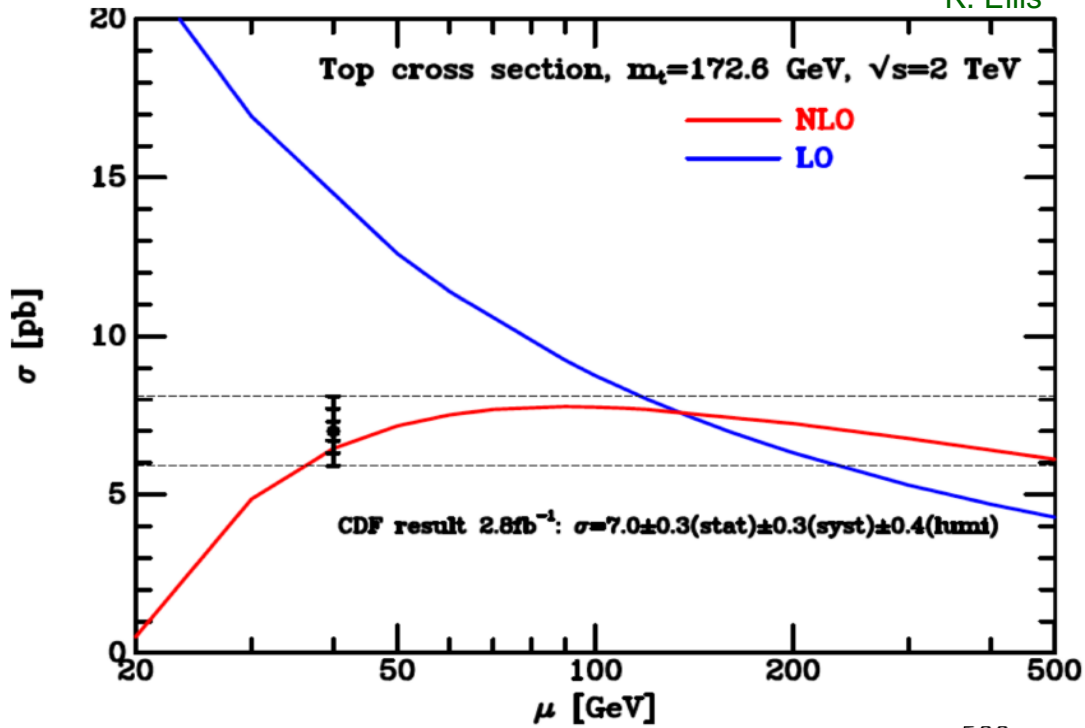
- choose a renormalisation scheme (e.g. MSbar)
- calculate cross section to some order (e.g. NLO)

$$\sigma(P) = A \alpha_S^N(\mu) + \alpha_S^{N+1}(\mu) \left[B + \frac{NAb}{2\pi} \ln \frac{\mu}{P} \right] + \dots$$

The diagram illustrates the structure of a QCD perturbation series. The equation is $\sigma(P) = A \alpha_S^N(\mu) + \alpha_S^{N+1}(\mu) \left[B + \frac{NAb}{2\pi} \ln \frac{\mu}{P} \right] + \dots$. Three green boxes with arrows point to specific parts of the equation: 'physical variable(s)' points to P ; 'process dependent coefficients depending on P ' points to A and B ; and 'renormalisation scale' points to μ .

- note $d\sigma/d\mu=0$ “to all orders”, but in practice $d\sigma^{(N+n)}/d\mu = O((N+n)\alpha_S^{N+n+1})$
- can try to help convergence by using a “physical scale choice”, $\mu \sim P$, e.g. $\mu = M_Z$ or $\mu = E_T^{\text{jet}}$
- what if there is a wide range of P 's in the process, e.g. $W + n$ jets?

K. Ellis

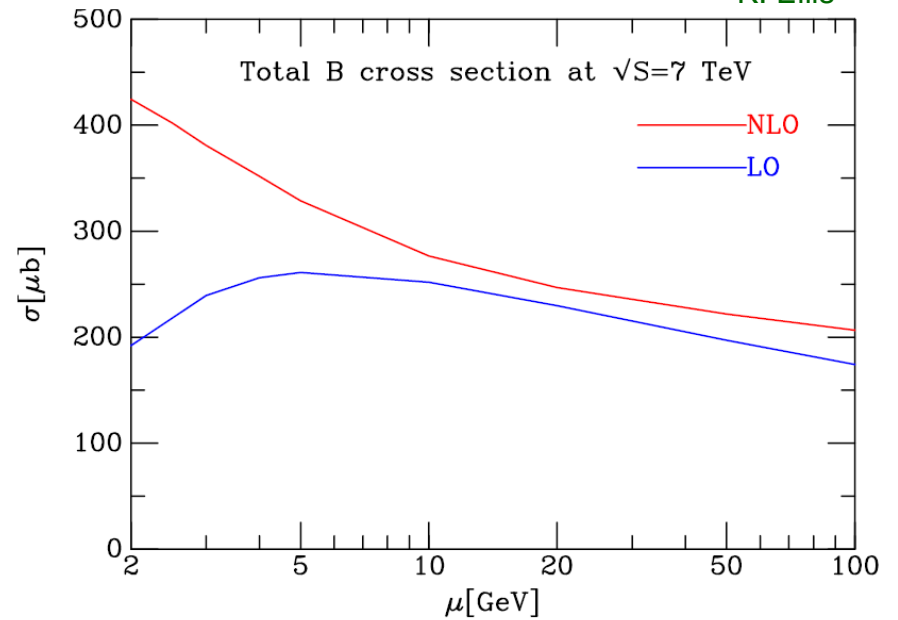


Top at Tevatron

Bottom at LHC

reason: new processes open up at NLO!

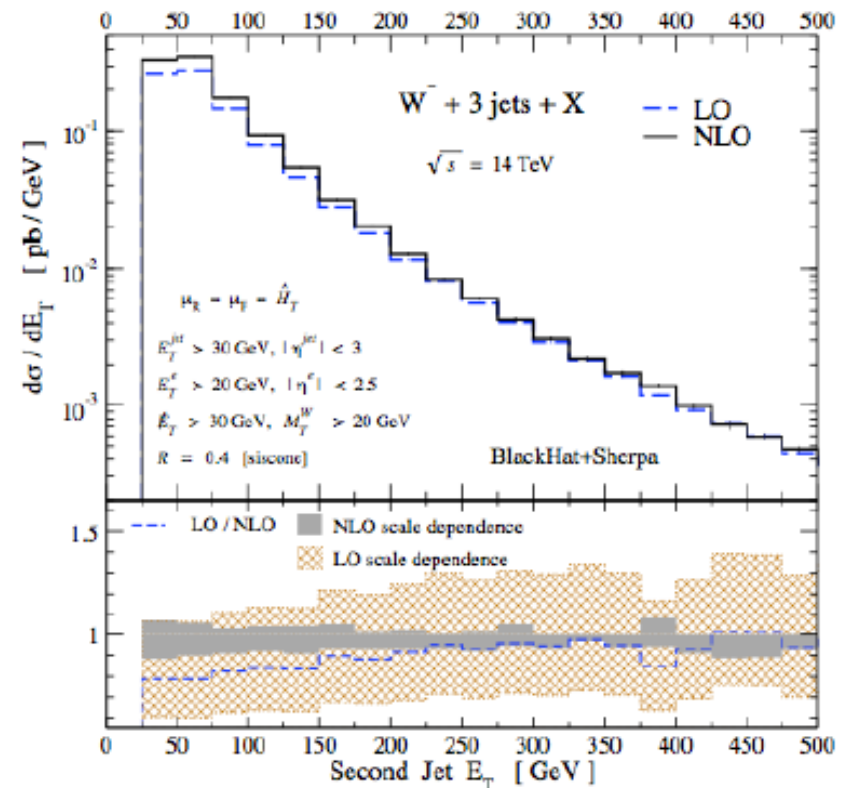
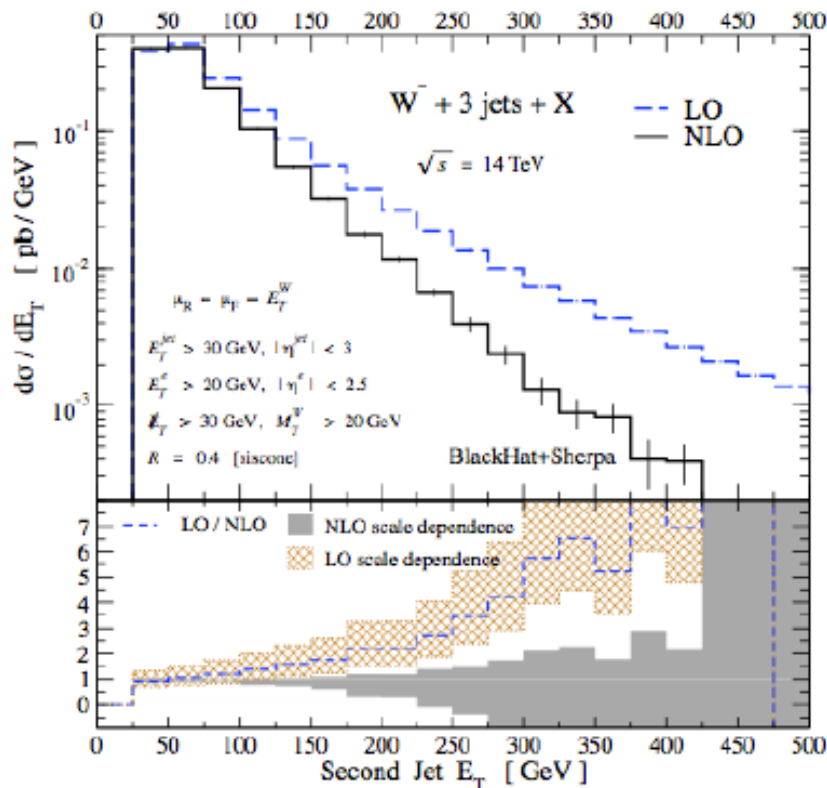
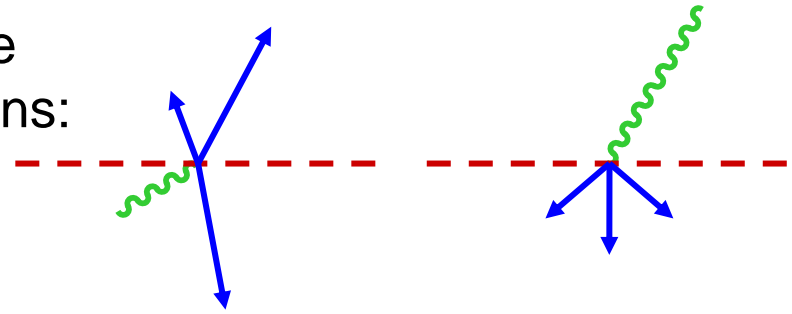
K. Ellis



in complicated processes like $W + n$ jets, there are often many 'reasonable' choices of scales: $\mu = M_W, E_{TW}, \langle E_{T\text{jet}} \rangle, H_T, \dots$

'blended' scales like H_T can seamlessly take account of different kinematical configurations:

$$H_T = \sum_{i=\text{partons}} E_{Ti} + E_{Te} + E_{T\text{miss}}$$



using the W^{\pm} charge asymmetry at the LHC

- at the Tevatron $\sigma(W^+) = \sigma(W^-)$, whereas at LHC $\sigma(W^+) \sim (1.4 - 1.3) \sigma(W^-)$
- can use this asymmetry to calibrate backgrounds to new physics, since typically $\sigma_{\text{NP}}(X \rightarrow W^+ + \dots) = \sigma_{\text{NP}}(X \rightarrow W^- + \dots)$

- **example:**

$$gg \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b} \rightarrow W^{\pm}(\rightarrow l^{\pm} + \nu) + 4\text{jets}$$

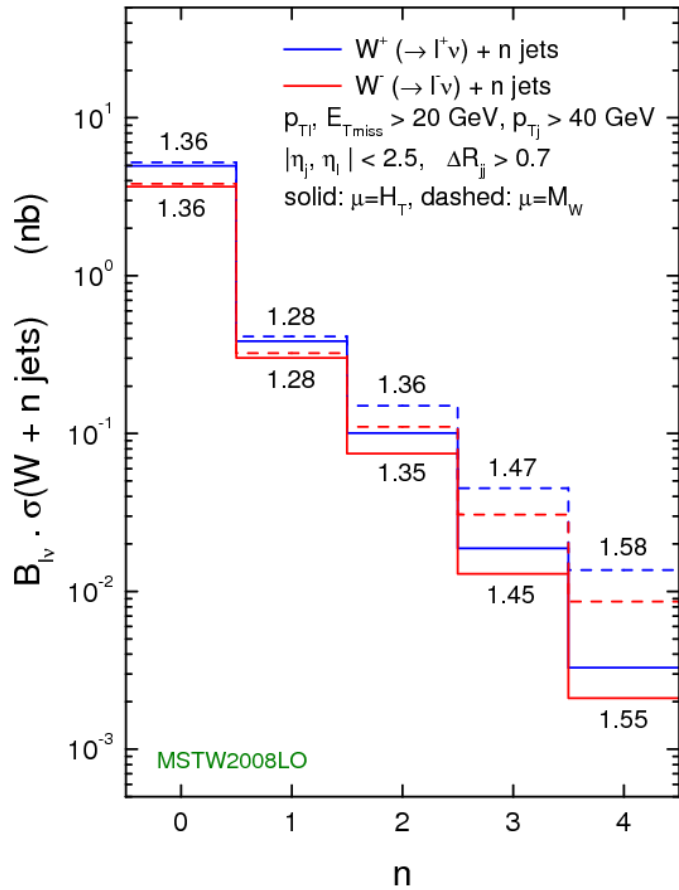
in this case

$$\sigma_{\text{signal}}(W^+ + 4\text{jets}) = \sigma_{\text{signal}}(W^- + 4\text{jets})$$

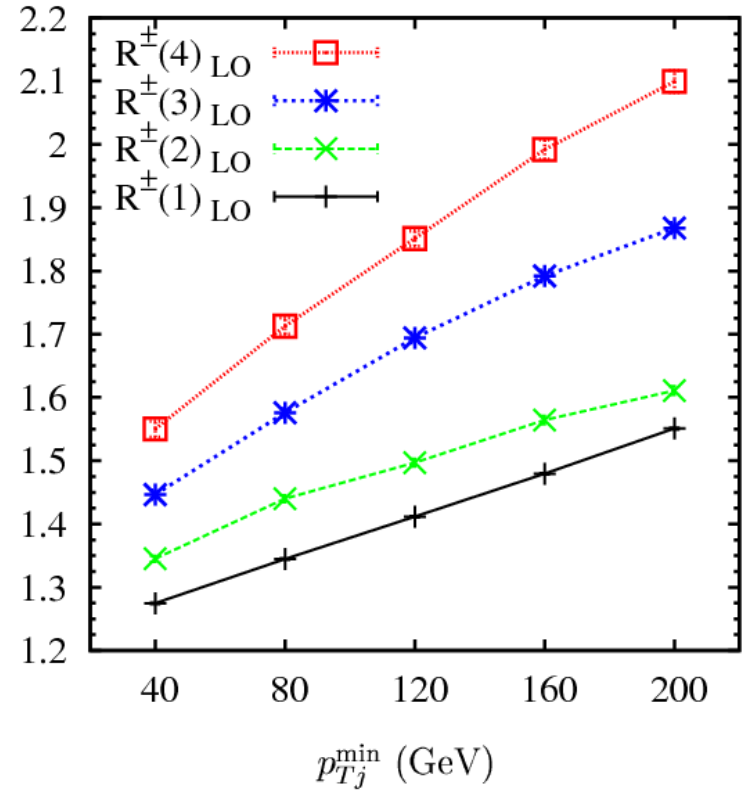
whereas...

$$\sigma_{\text{QCD bkgd}}(W^+ + 4\text{jets}) \neq \sigma_{\text{QCD bkgd}}(W^- + 4\text{jets})$$

which can in principle help distinguish signal and background



R^\pm increases with jet p_T^{\min}



R^\pm larger at 7 TeV LHC

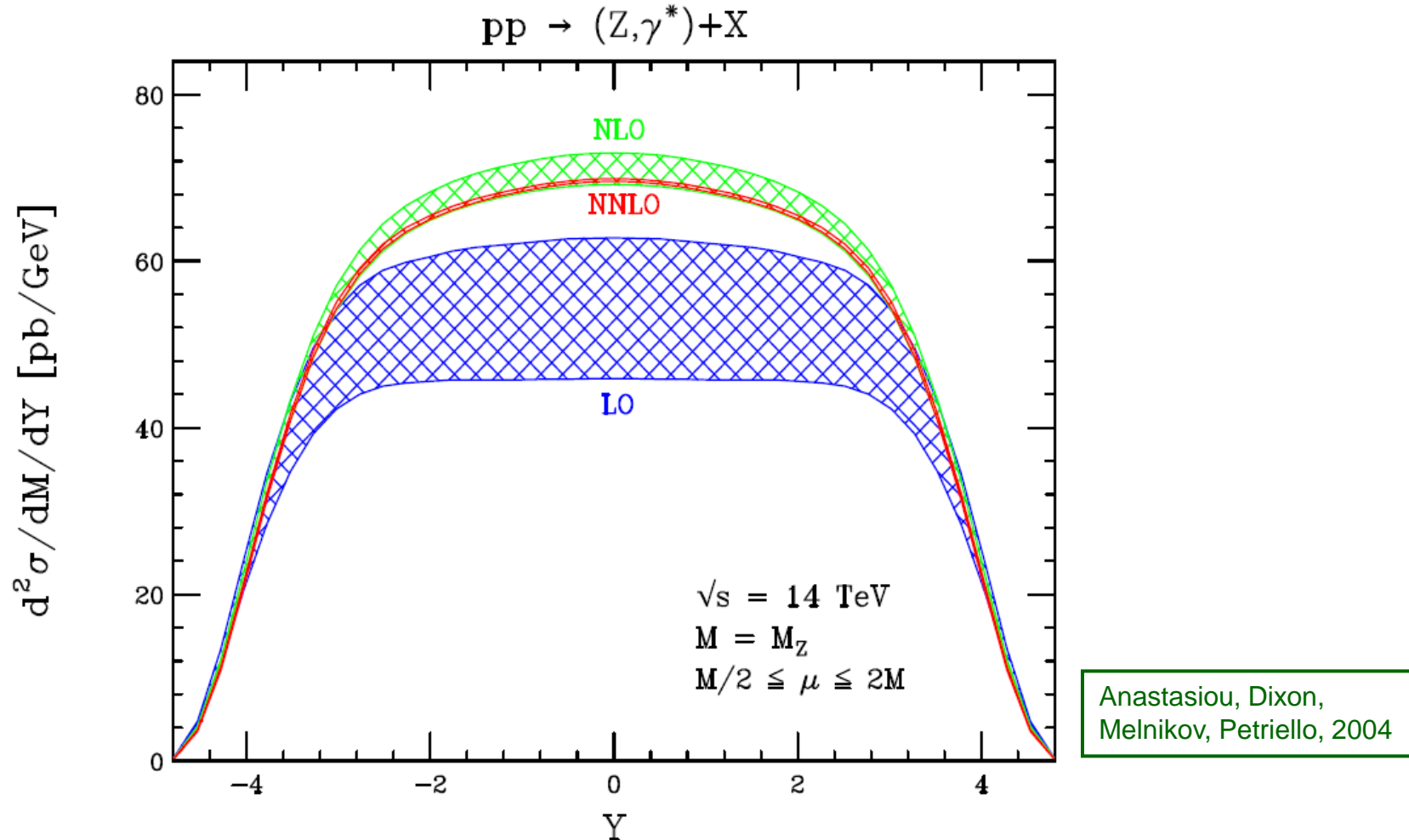


n	$\sqrt{s} = 7 \text{ TeV}$	$\sqrt{s} = 14 \text{ TeV}$
0	$1.52 \pm 0.01 \text{ (scl)} \pm 0.02 \text{ (pdf)}$	$1.31 \pm 0.01 \text{ (scl)} \pm 0.01 \text{ (pdf)}$
1	$1.45 \pm 0.01 \text{ (scl)} \pm 0.01 \text{ (pdf)}$	$1.27 \pm 0.01 \text{ (scl)} \pm 0.01 \text{ (pdf)}$
2	$1.56 \pm 0.02 \text{ (scl)} \pm 0.02 \text{ (pdf)}$	$1.33 \pm 0.02 \text{ (scl)} \pm 0.01 \text{ (pdf)}$
3	$1.72 \pm 0.03 \text{ (scl)} \pm 0.03 \text{ (pdf)}$	$1.45 \pm 0.03 \text{ (scl)} \pm 0.02 \text{ (pdf)}$
4	$1.87 \pm 0.04 \text{ (scl)} \pm 0.03 \text{ (pdf)}$	$1.55 \pm 0.04 \text{ (scl)} \pm 0.02 \text{ (pdf)}$

Berger et al (arXiv:1009.2338)
- 7 TeV, slightly different cuts

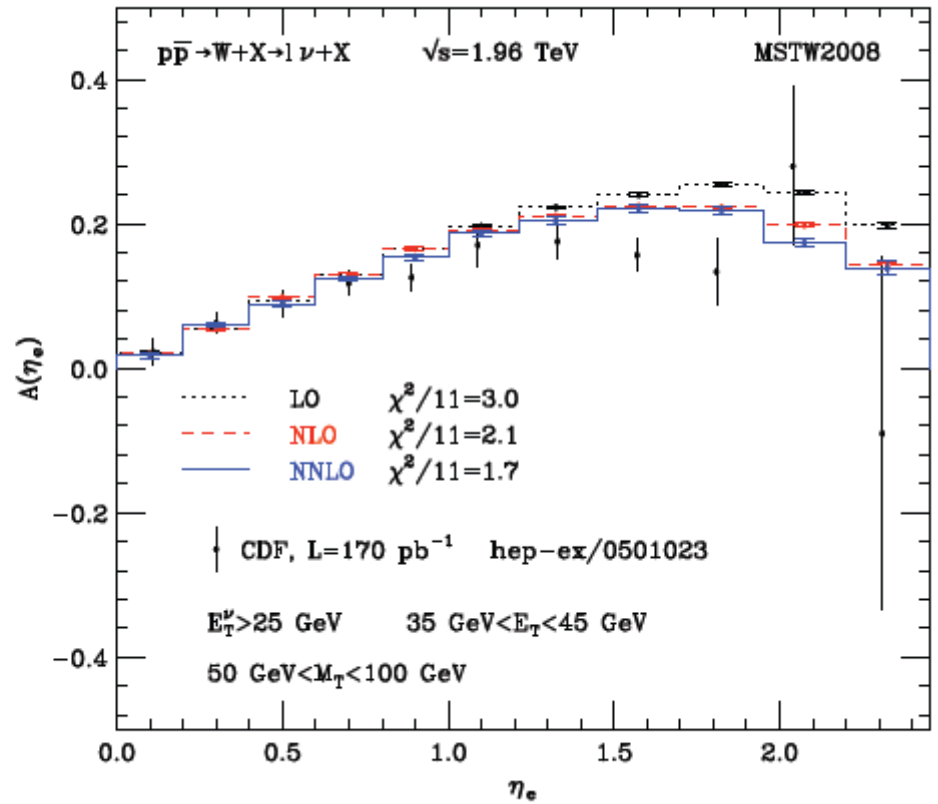
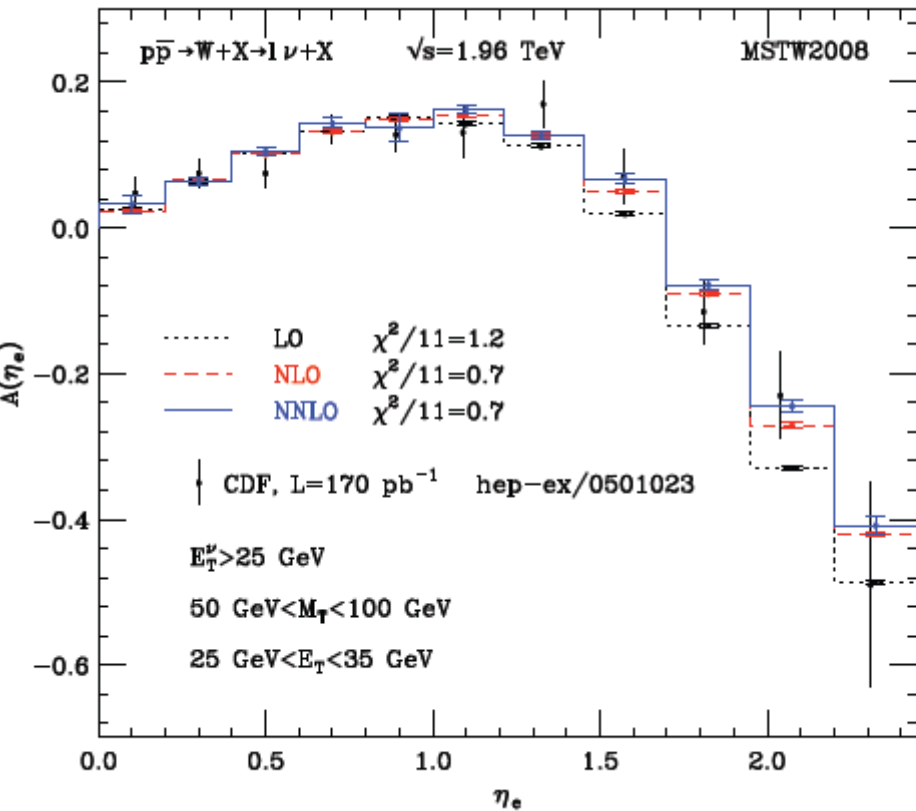
$W^+/W^- \text{ LO}$	$W^+/W^- \text{ NLO}$
1.656(0.001)	1.580(0.004)
1.507(0.002)	1.498(0.009)
1.596(0.003)	1.57(0.02)
1.694(0.005)	1.66(0.02)
1.817(0.003)	—

the impact of NNLO: W,Z



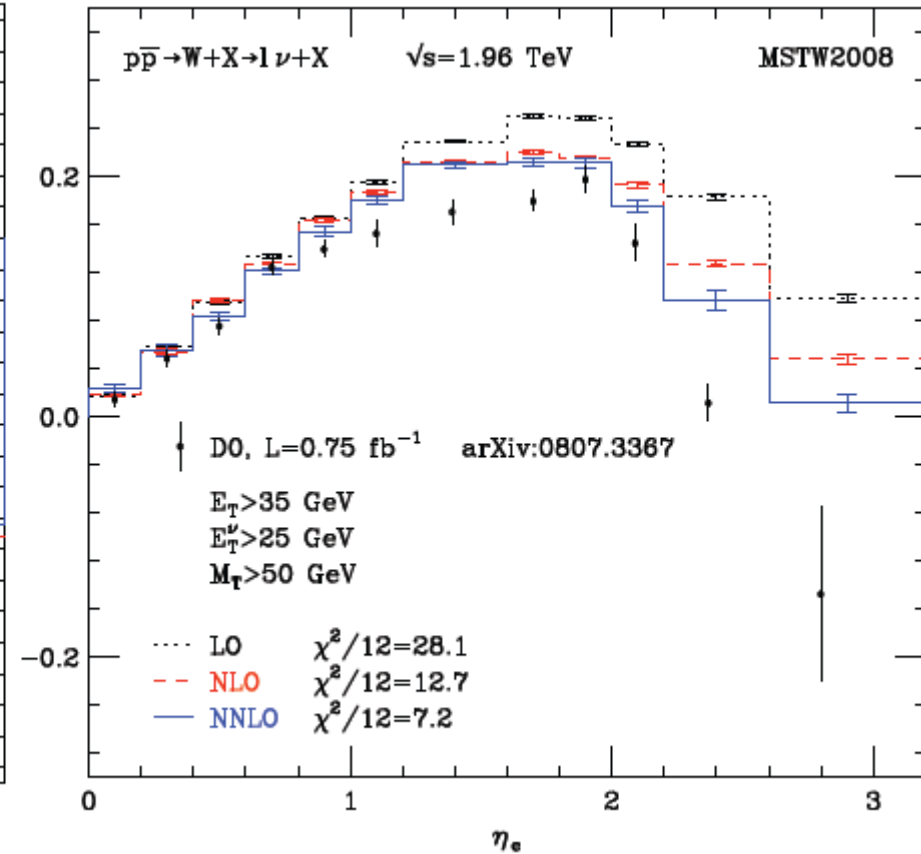
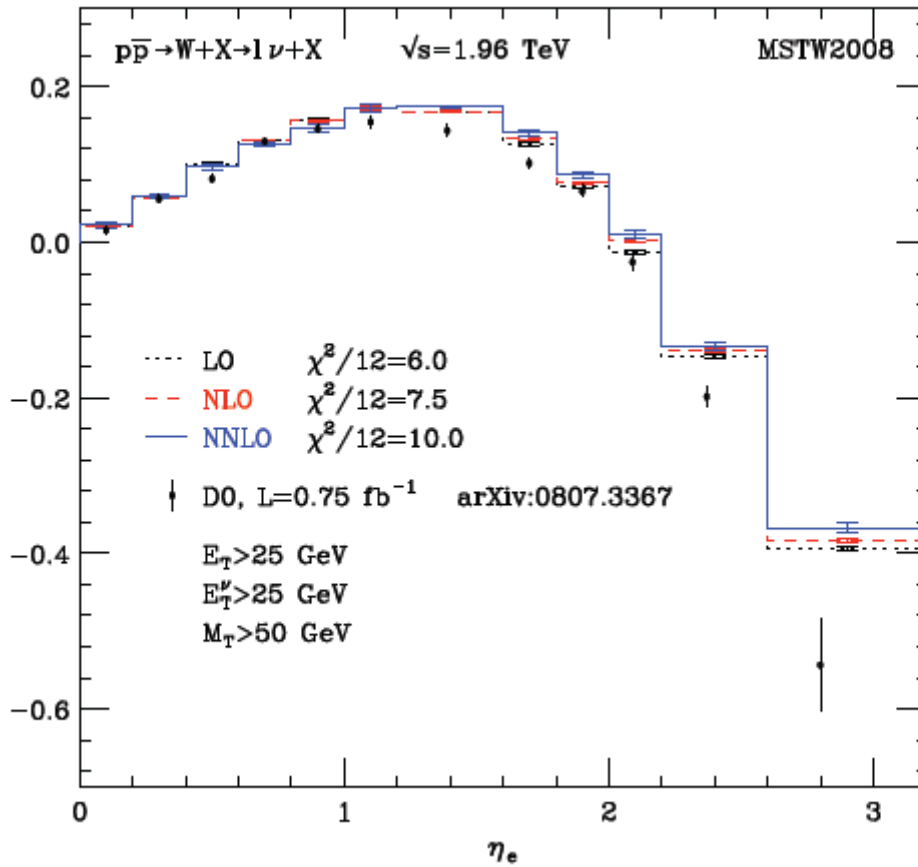
- only scale variation uncertainty shown
- central values calculated for a *fixed* set pdfs with a *fixed* value of $\alpha_s(M_Z^2)$

Lepton asymmetry at NNLO and CDF data



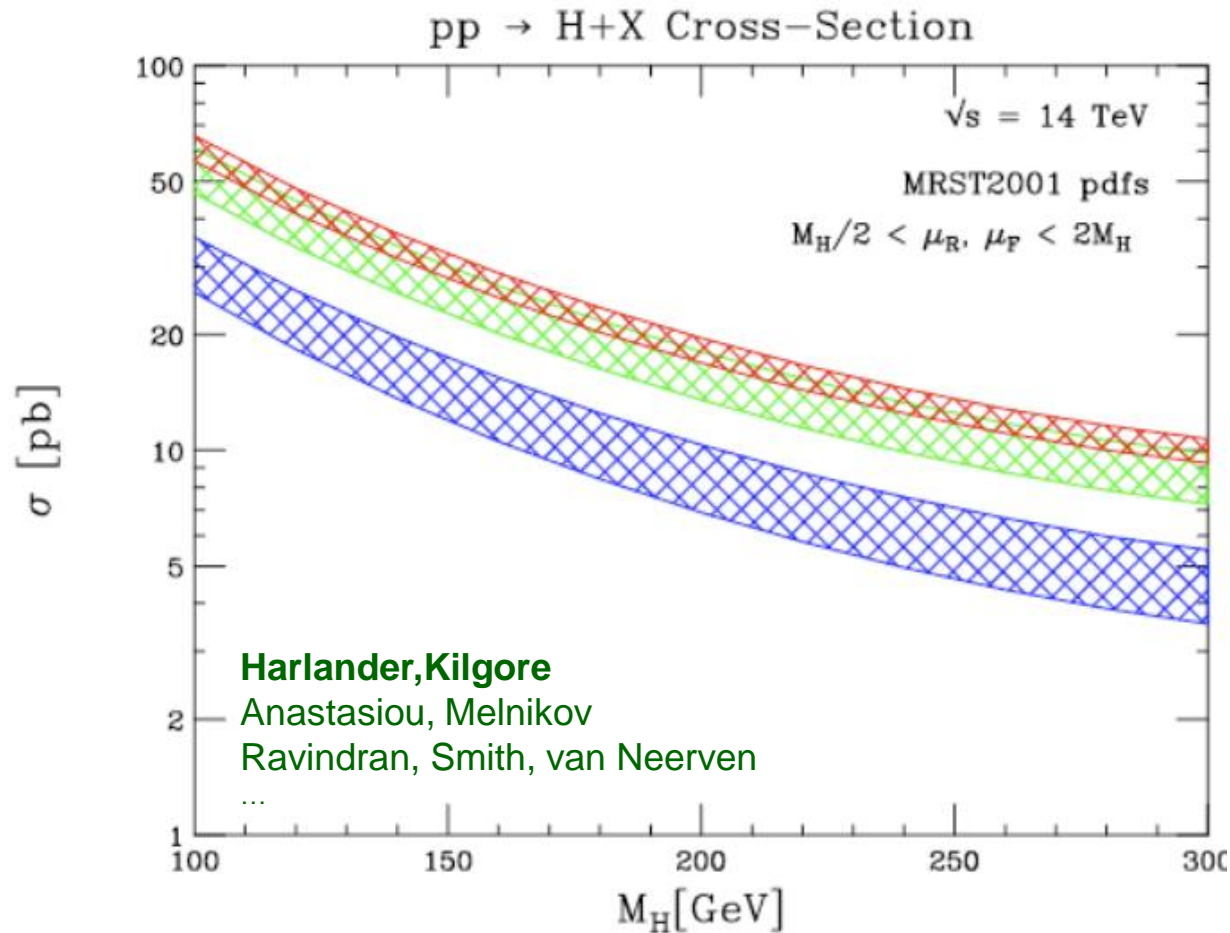
Catani, Ferrara, Grazzini, arXiv:1002.3115

Lepton asymmetry at NNLO and D0 data



Catani, Ferrara, Grazzini, arXiv:1002.3115

the impact of NNLO: H



- the NNLO band is about $\pm 10\%$, or $\pm 15\%$ if μ_R and μ_F varied independently

scale variation in $gg \rightarrow H$?

- ‘conventional’ approach (NNLO):

$$\mu_R = \mu_F = \mu, \quad \frac{\mu_0}{2} \leq \mu \leq 2\mu_0, \quad \mu_0 = M_H \quad \begin{array}{l} +10\% \\ -10\% \end{array}$$

- ‘conservative’ approach (Baglio and Djouadi), NNLO normalised to NNLL

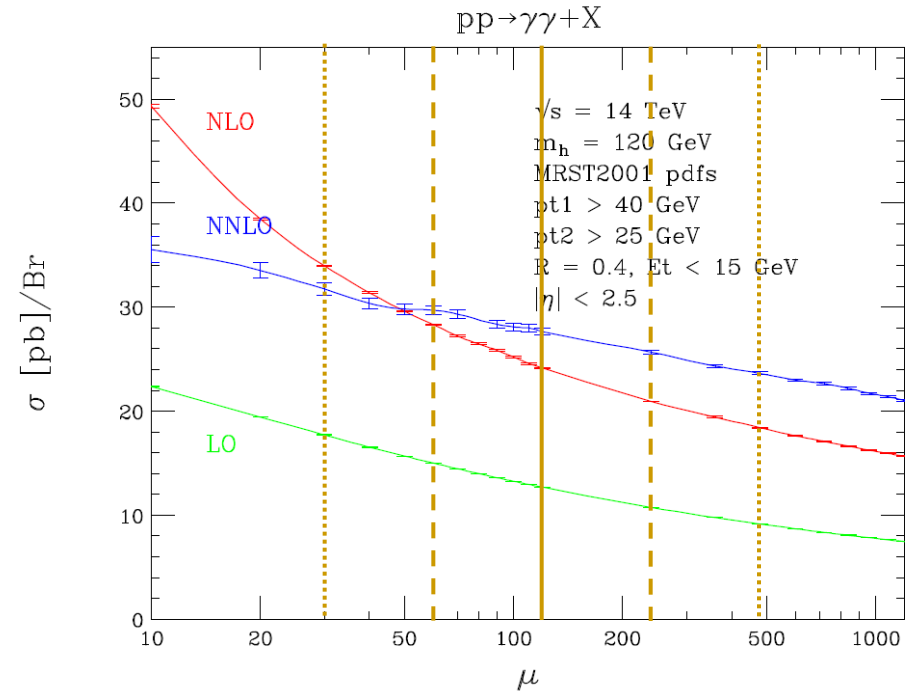
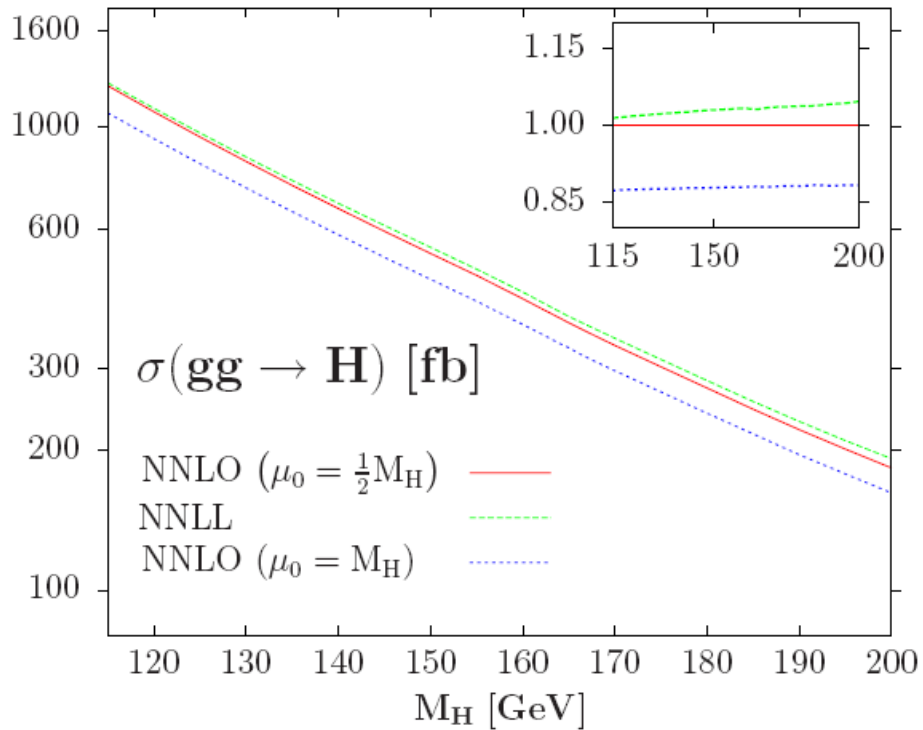
$$\mu_R = \mu_F = \mu, \quad \frac{\mu_0}{3} \leq \mu \leq 3\mu_0, \quad \mu_0 = \frac{1}{2}M_H \quad \begin{array}{l} +15\% \\ -20\% \end{array}$$

- ‘radical approach’: N³LL (Ahrens, Becher, Neubert, Yang, 1008.3162)

$$\mu_R = \mu_F = \mu, \quad \frac{\mu_0}{2} \leq \mu \leq 2\mu_0, \quad \mu_0 = M_H \quad \begin{array}{l} +3\% \\ -3\% \end{array}$$



choice of scale and range – flat prior?



Anastasiou, Melnikov, Petriello (2005)

...with scale variation factor 1/2, 1/4, 2, 4

Baglio and Djouadi, arXiv:1009.1363

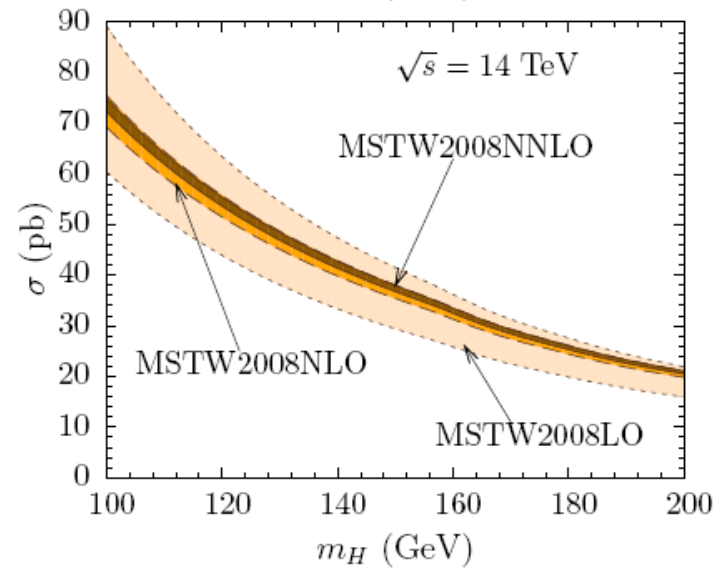
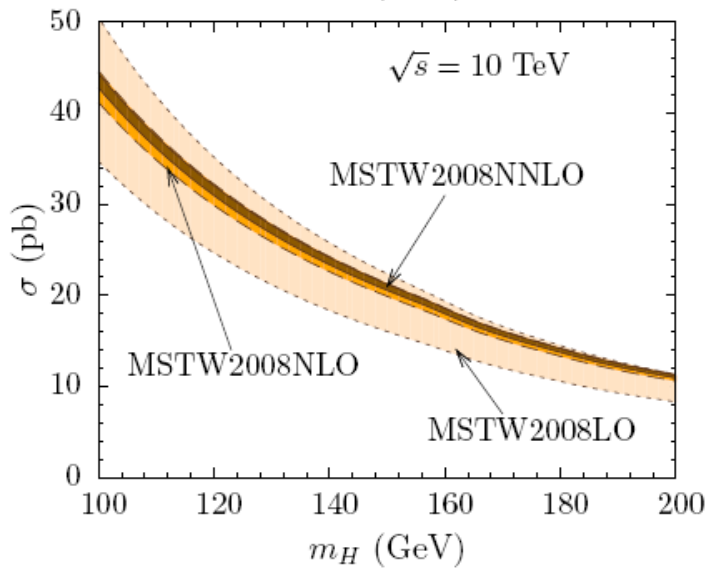
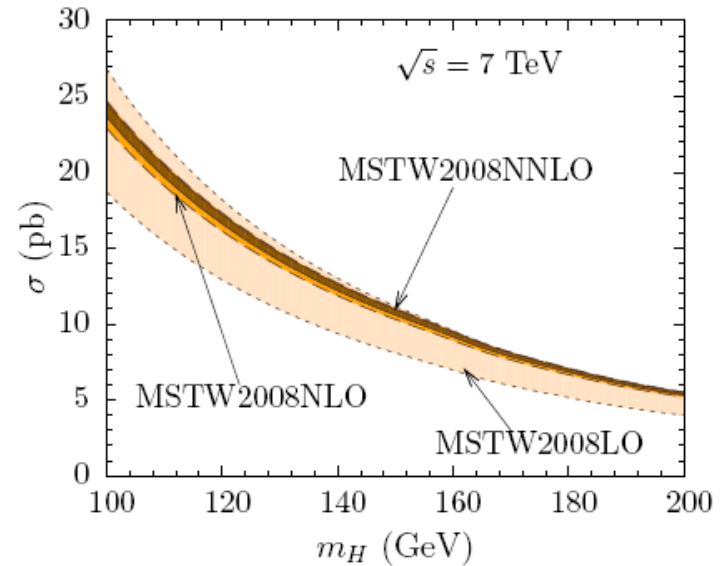
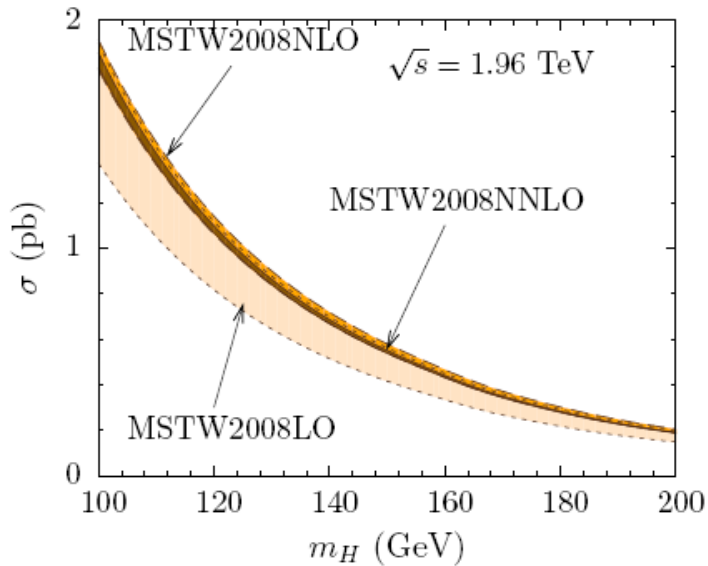
NNLO: Anastasiou, Boughezal, Petriello (2009)

NNLL: de Florian and Grazzini (2009)

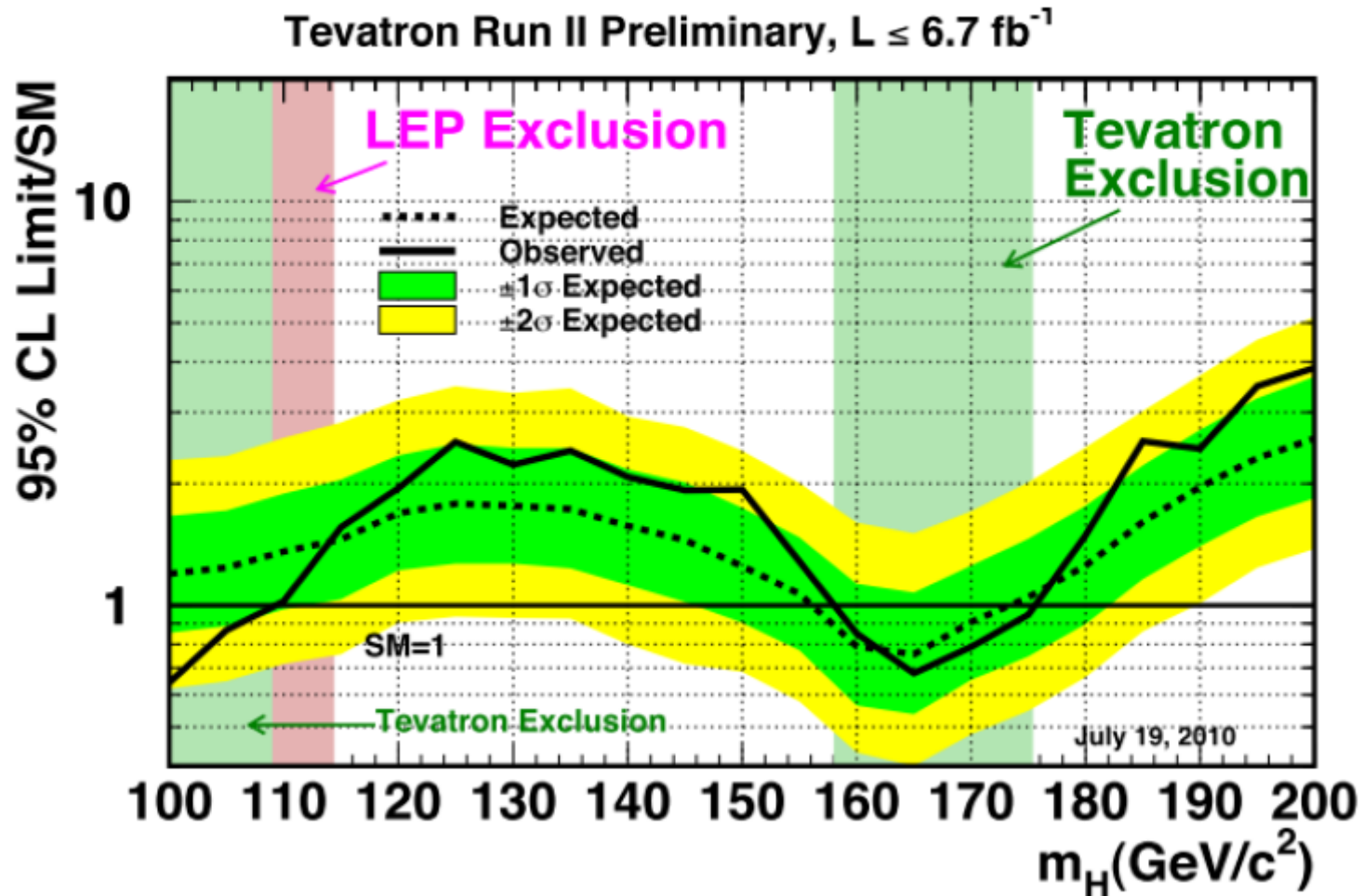
$\delta\sigma_{\text{PDF}}$ (see below) attempts to be a genuine $\pm 1\sigma$ uncertainty related to global fit data

$\delta\sigma_{\text{SCL}}$ is an estimate of the impact of the unknown higher-order pQCD corrections

— there is no unique prescription for combining them!



SM Higgs: Tevatron exclusion limits

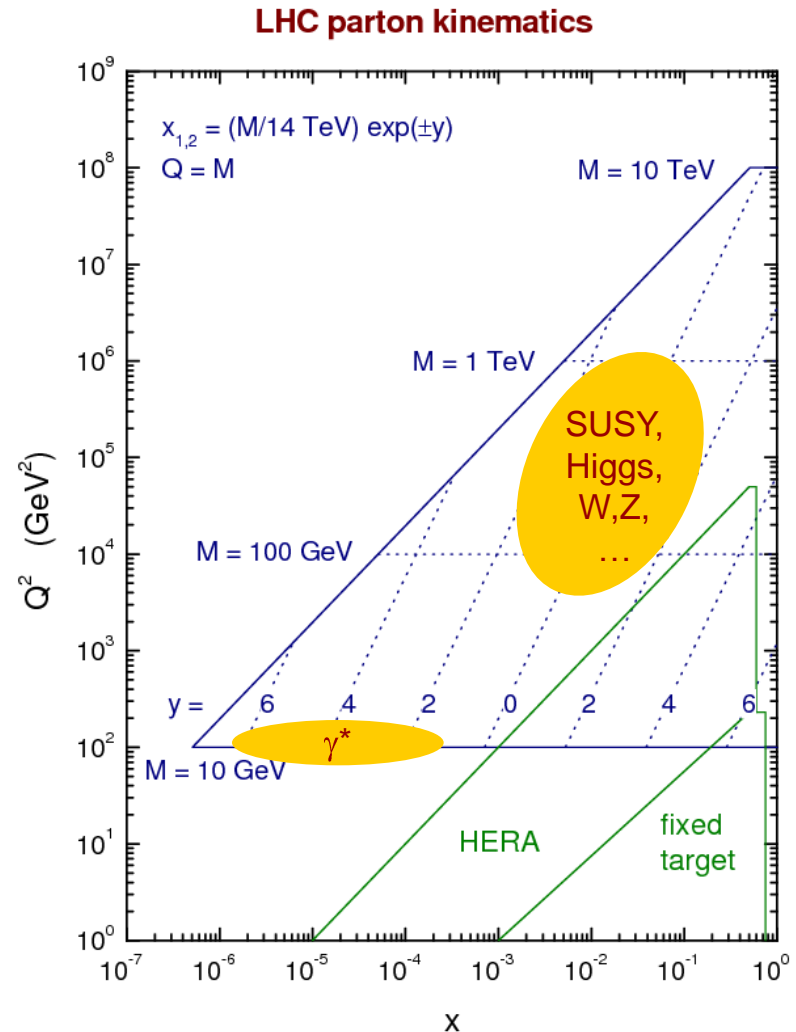


3

parton distribution functions

pdfs @ LHC

- most SM and new physics sample pdfs in a region of x where they are already well known
- current pdf uncertainties provide the benchmark for whether LHC can add new information
- low-mass forward production (e.g. b quarks, Drell-Yan) might provide new information on **small-x** partons



how pdfs are obtained*

- choose a factorisation scheme (e.g. $\overline{\text{MS}}$), an order in perturbation theory (LO, NLO, NNLO) and a ‘starting scale’ Q_0 where pQCD applies (e.g. 1-2 GeV)
- parametrise the quark and gluon distributions at Q_0 , e.g.

$$f_i(x, Q_0^2) = A_i x^{a_i} [1 + b_i \sqrt{x} + c_i x] (1 - x)^{d_i}$$

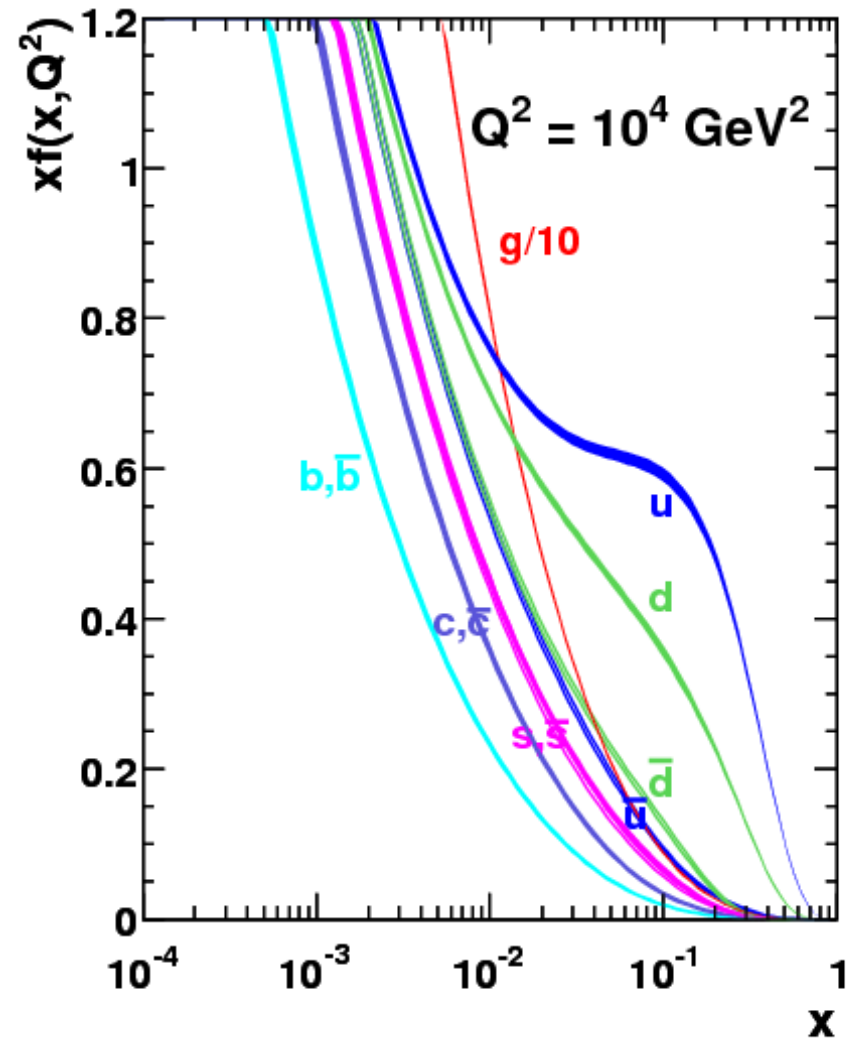
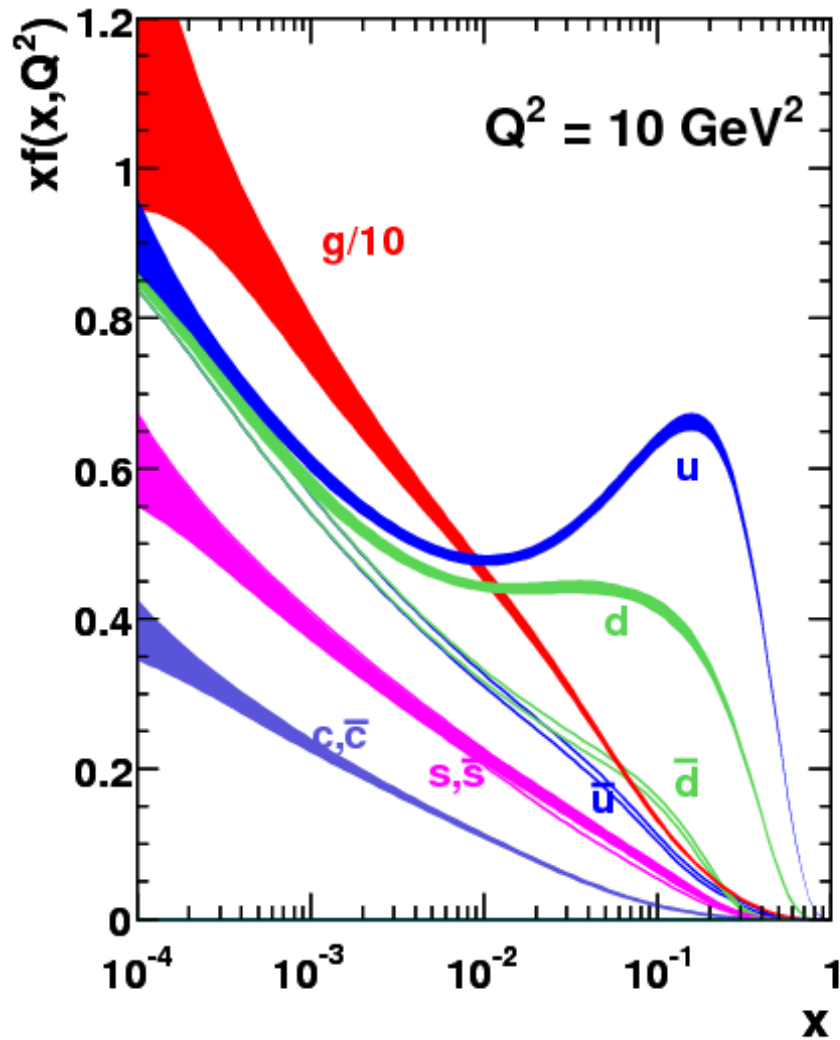
- solve DGLAP equations to obtain the pdfs at any x and scale $Q > Q_0$; fit data for parameters $\{A_i, a_i, \dots, \alpha_S\}$
- approximate the exact solutions (e.g. interpolation grids, expansions in polynomials etc) for ease of use; thus the output ‘global fits’ are available ‘off the shelf’, e.g.

SUBROUTINE PDF (X, Q, U, UBAR, D, DBAR, ..., BBAR, GLU)

input |

output

MSTW 2008 NLO PDFs (68% C.L.)



the pdf industry

- many groups now extracting pdfs from ‘global’ data analyses (MSTW, CTEQ, NNPDF, ...)
- broad agreement, but differences due to
 - choice of data sets (including cuts and corrections)
 - treatment of data errors
 - treatment of heavy quarks (s,c,b)
 - order of perturbation theory
 - parameterisation at Q_0
 - theoretical assumptions (if any) about:
 - flavour symmetries
 - $x \rightarrow 0, 1$ behaviour
 - ...

HERA-DIS
FT-DIS
Drell-Yan
Tevatron jets
Tevatron W,Z
other

recent global or quasi-global pdf fits

pdfs	authors	arXiv
ABKM	S. Alekhin, J. Blümlein, S. Klein, S. Moch, and others	1007.3657, 0908.3128, 0908.2766, ...
CTEQ	H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. Nadolsky, J. Pumplin, C.-P. Yuan, and others	1007.2241, 1004.4624, 0910.4183, 0904.2424, 0802.0007, ...
GJR	M. Glück, P. Jimenez-Delgado, E. Reya, and others	0909.1711, 0810.4274, ...
HERAPDF	H1 and ZEUS collaborations	1006.4471, 0906.1108, ...
MSTW	A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt	1006.2753, 0905.3531, 0901.0002, ...
NNPDF	R. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. Latorre, J. Rojo, M. Ubiali, and others	1005.0397, 1002.4407, 0912.2276, 0906.1958, ...

	MSTW08	CTEQ6.6 ^x	NNPDF2.0	HERAPDF1.0	ABKM09 ^x	GJR08
HERA DIS	✓	✓	✓*	✓*	✓	✓
F-T DIS	✓	✓	✓	✗	✓	✓
F-T DY	✓	✓	✓	✗	✓	✓
TEV W,Z	✓	✓+	✓	✗	✗	✗
TEV jets	✓	✓+	✓	✗	✗	✓
GM-VFNS	✓	✓	✗	✓	✓	✗
NNLO	✓	✗	✗	✗	✓	✓

+ Run 1 only

* includes new combined H1-ZEUS data → few% increase in quarks at low x

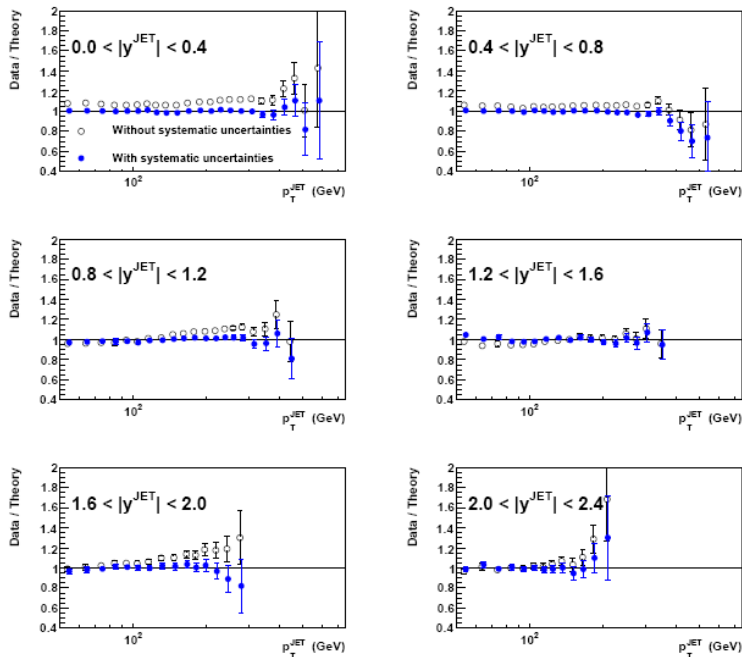
✗ new (July 2010) ABKM and CTEQ updates: ABKM includes new combined H1-ZEUS data + new small-x parametrisation + partial NNLO HQ corrections; CT10 includes new combined H1-ZEUS data + Run 2 jet data + extended gluon parametrisation + ... → more like MSTW08

impact of Tevatron jet data on fits

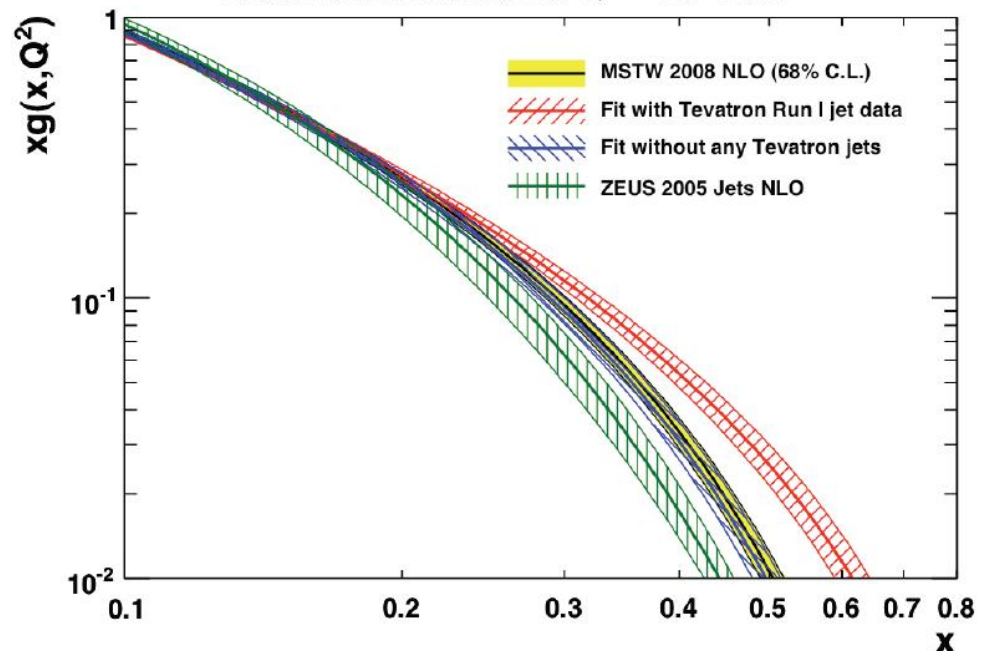
- a distinguishing feature of pdf sets is whether they use (MRST/MSTW, CTEQ, NNPDF, GJR,...) or do not use (HERAPDF, ABKM, ...) Tevatron jet data in the fit: the impact is on the *high-x gluon*
(Note: Run II data requires slightly softer gluon than Run I data)
- the (still) missing ingredient is the full NNLO pQCD correction to the cross section, but not expected to have much impact in practice [Kidonakis, Owens (2001)]

$D\bar{D}$ Run II inclusive jet data (cone, $R = 0.7$)

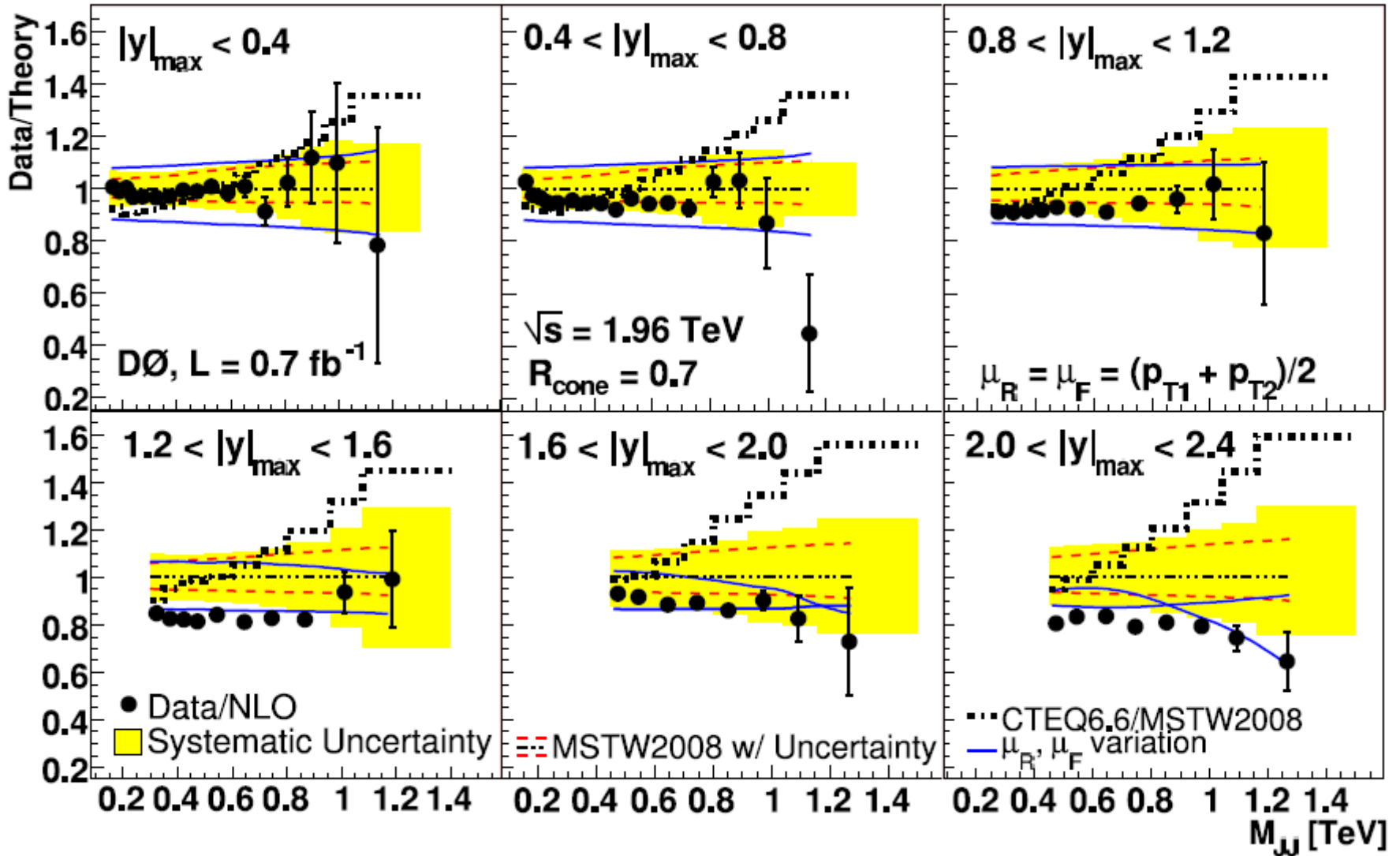
MSTW 2008 NLO PDF fit ($\mu_R = \mu_F = p_T^{\text{JET}}$, $\chi^2 = 114$ for 110 pts.)



Gluon distribution at $Q^2 = 10^4 \text{ GeV}^2$



dijet mass distribution from D0



LO vs NLO vs NNLO?

in the MSTW2008 fit

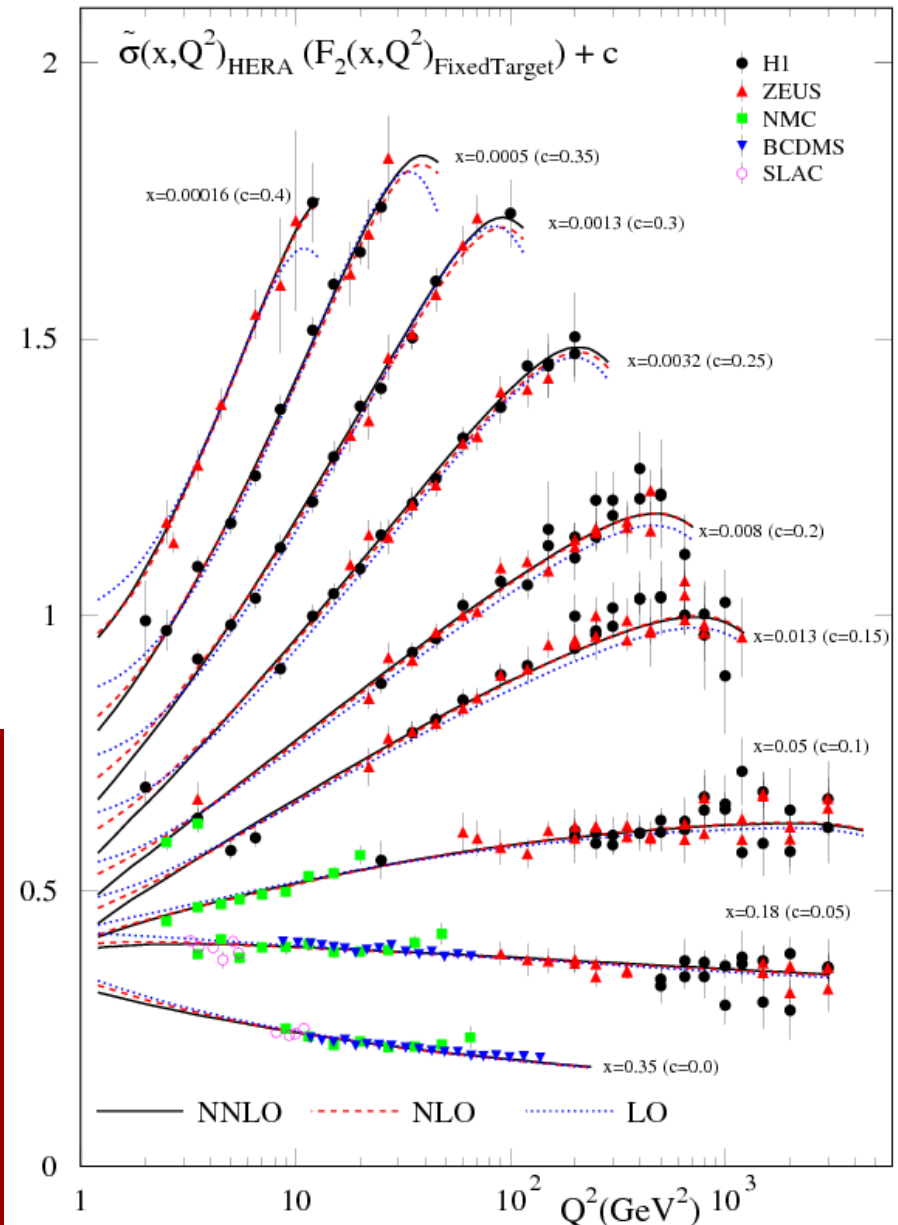
$$\chi^2_{\text{global}} / \text{dof} = \begin{array}{l} 3066/2598 \text{ (LO)} \\ 2543/2699 \text{ (NLO)} \\ 2480/2615 \text{ (NNLO)} \end{array}$$

LO evolution too slow at small x ;
NNLO fit marginally better than NLO

Note:

- an important ingredient missing in the full NNLO global pdf fit is the NNLO correction to the Tevatron high E_T jet cross section
- LO can be improved (e.g. LO*) for MCs by adding K-factors, relaxing momentum conservation, etc.

MSTW 2008



pdf uncertainties

- all groups produce ‘pdfs with errors’
- typically, 20-40 ‘error’ sets based on a ‘best fit’ set to reflect 1σ variation of all the parameters* $\{A_j, a_j, \dots, \alpha_S\}$ inherent in the fit
- these reflect the uncertainties on the **data** used in the global fit (e.g. $\delta F_2 \approx 3\% \rightarrow \delta u \approx 3\%$)
- however, there are also systematic pdf uncertainties reflecting theoretical assumptions/prejudices in the way the global fit is set up and performed (see earlier slide)

* e.g. $f_i(x, Q_0^2) = A_i x^{a_i} [1 + b_i \sqrt{x} + c_i x] (1 - x)^{d_i}$

pdf uncertainties (contd.)

- **NNPDF** create many replicas of data and obtain PDF replicas in each case by fitting to training set and comparing to validation set → uncertainty determined by spread of replicas. Direct relationship to χ^2 in global fit not trivial.
- **NNPDF** and **MSTW** (due to extra parameters) have more complicated shape for gluon at smaller x and bigger small-x uncertainty, ditto for **CTEQ** at large x
- different theory assumptions in strange quark pdf leads to vastly different uncertainties — e.g. **MSTW** small, **NNPDF** large; feeds into other ‘light’ quarks
- perhaps surprisingly, all get rather similar uncertainties for pdfs and predicted cross sections — see later

pdfs and $\alpha_S(M_Z^2)$

- **MSTW08, ABKM09 and GJR08:**
 $\alpha_S(M_Z^2)$ values and uncertainty determined by global fit
- NNLO value about 0.003 – 0.004 lower than NLO value, e.g. for **MSTW08**

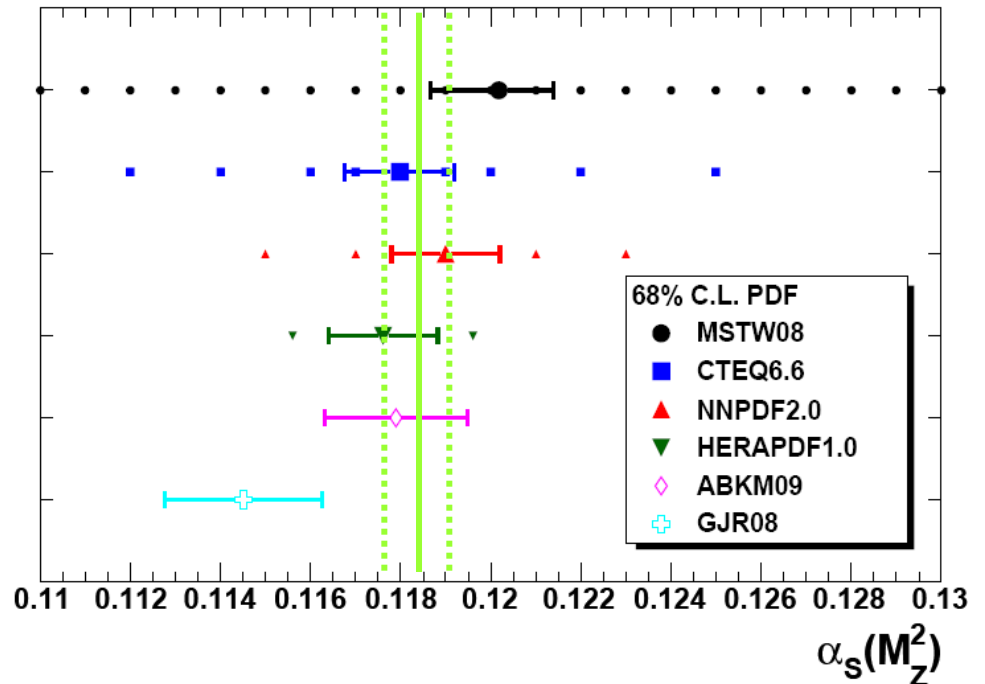
$$\alpha_S^{\overline{MS},NLO}(M_Z^2) = 0.1202^{+0.012}_{-0.015}$$

$$\alpha_S^{\overline{MS},NNLO}(M_Z^2) = 0.1171^{+0.014}_{-0.014}$$

- **CTEQ, NNPDF, HERAPDF** choose standard values and uncertainties
- world average (**PDG 2009**)

$$\alpha_S^{\overline{MS}}(M_Z^2) = 0.1184 \pm 0.0007$$

NLO $\alpha_S(M_Z^2)$ values used by different PDF groups

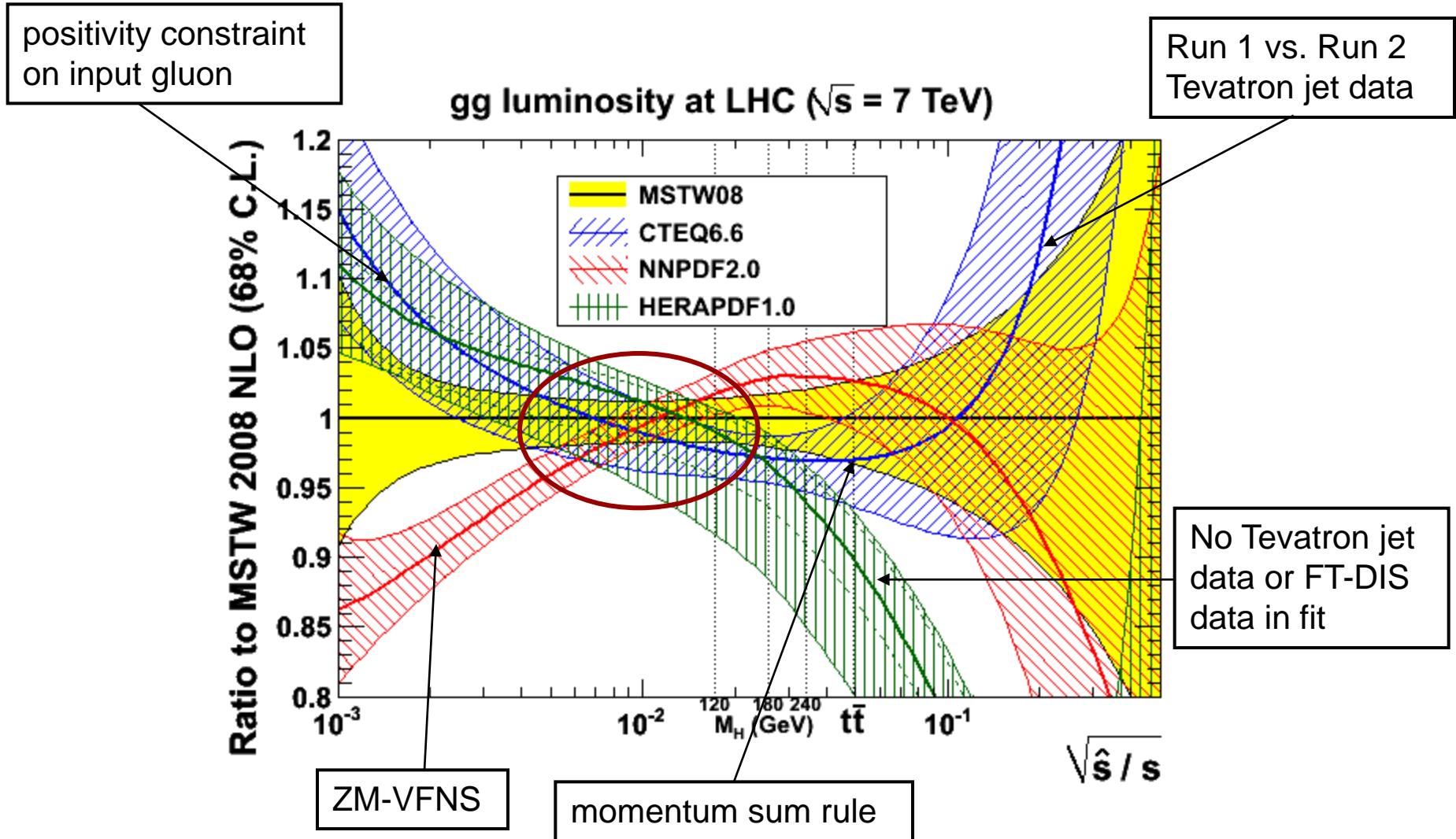


- note that the pdfs and α_S are **correlated!**
- e.g. **gluon** – α_S anticorrelation at small x and **quark** – α_S anticorrelation at large x

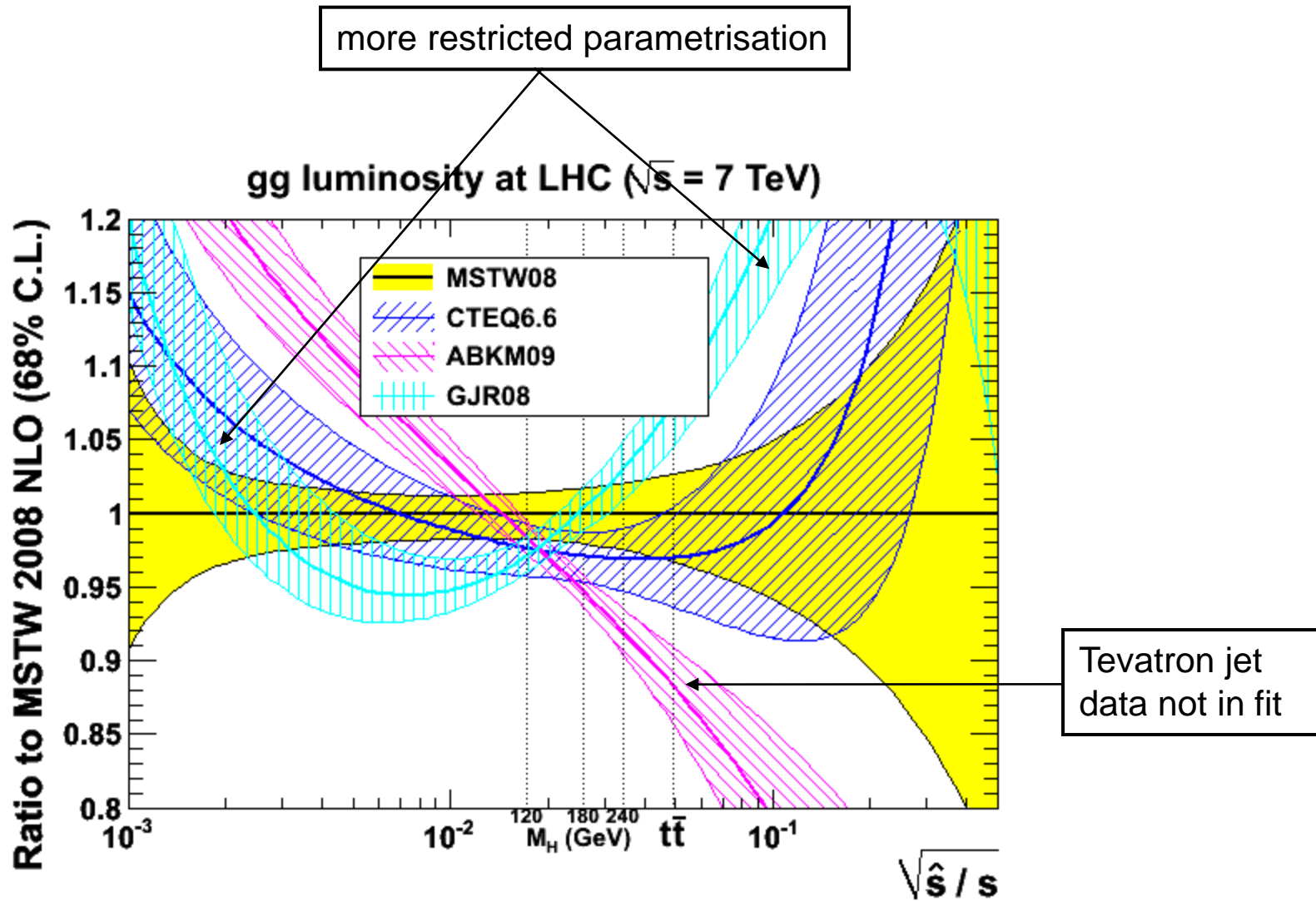
4

LHC benchmark cross sections

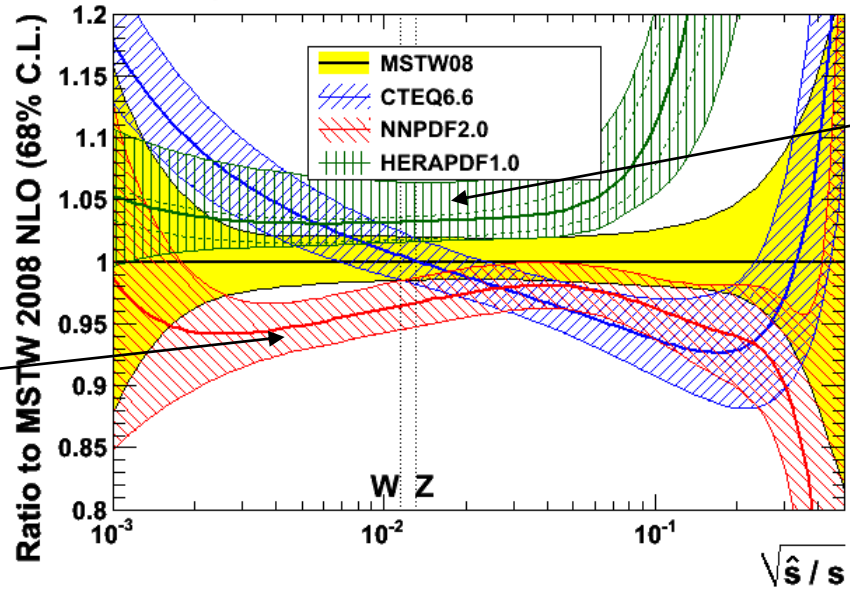
parton luminosity comparisons



Luminosity and cross section plots from Graeme Watt (MSTW, in preparation), available at projects.hepforge.org/mstwpdf/pdf4lhc



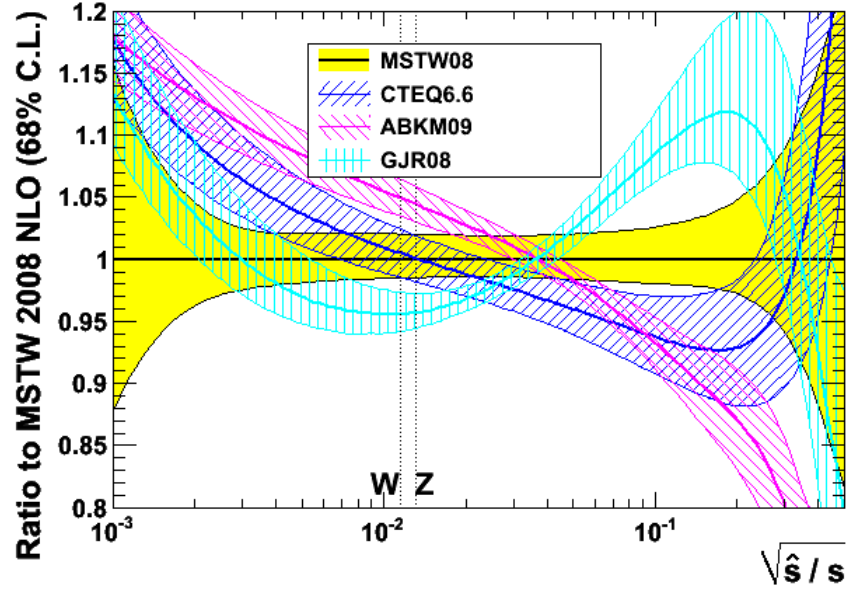
$\Sigma_q(q\bar{q})$ luminosity at LHC ($\sqrt{s} = 7$ TeV)



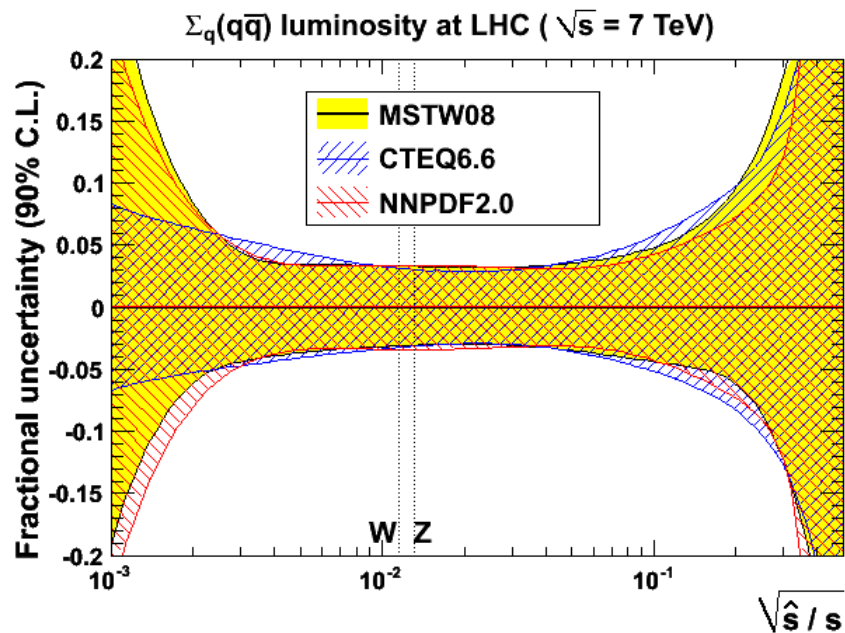
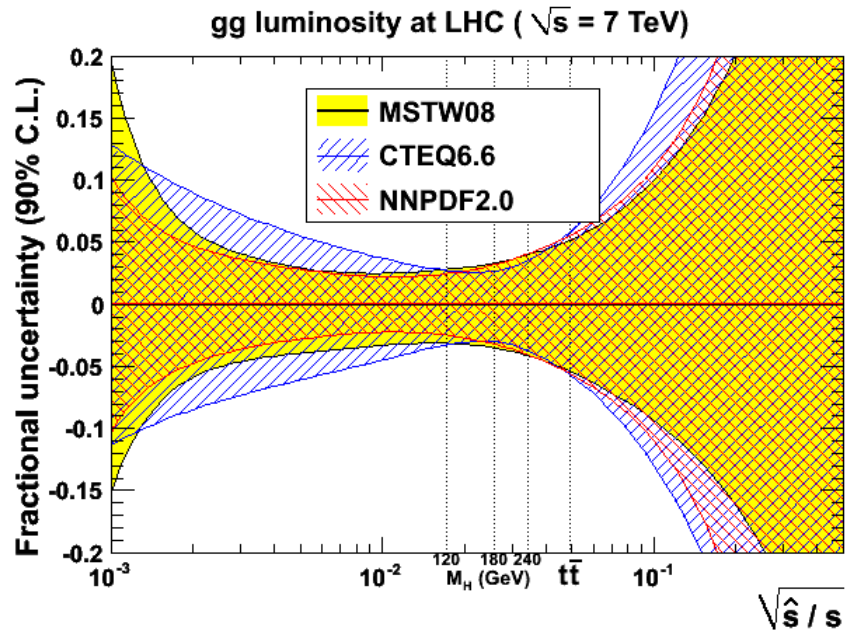
ZM-VFNS

new combined
HERA SF data

$\Sigma_q(q\bar{q})$ luminosity at LHC ($\sqrt{s} = 7$ TeV)

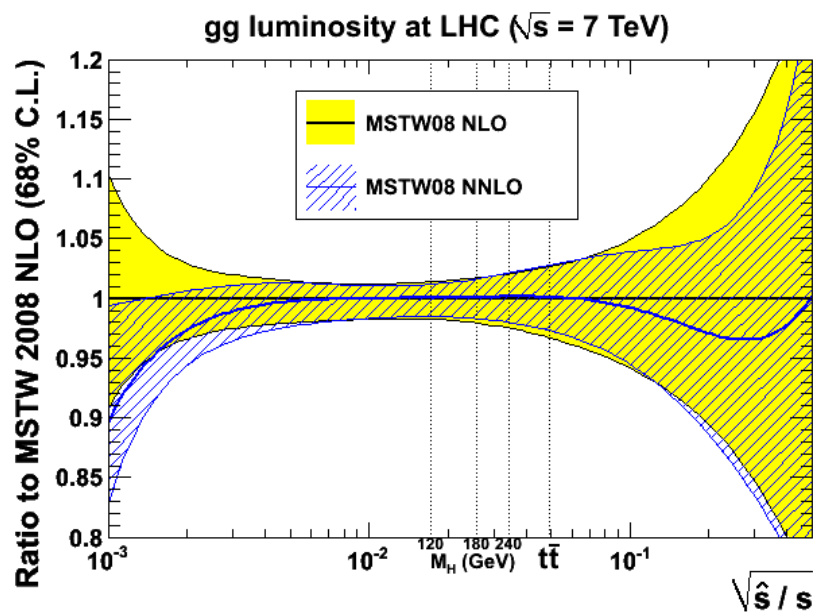
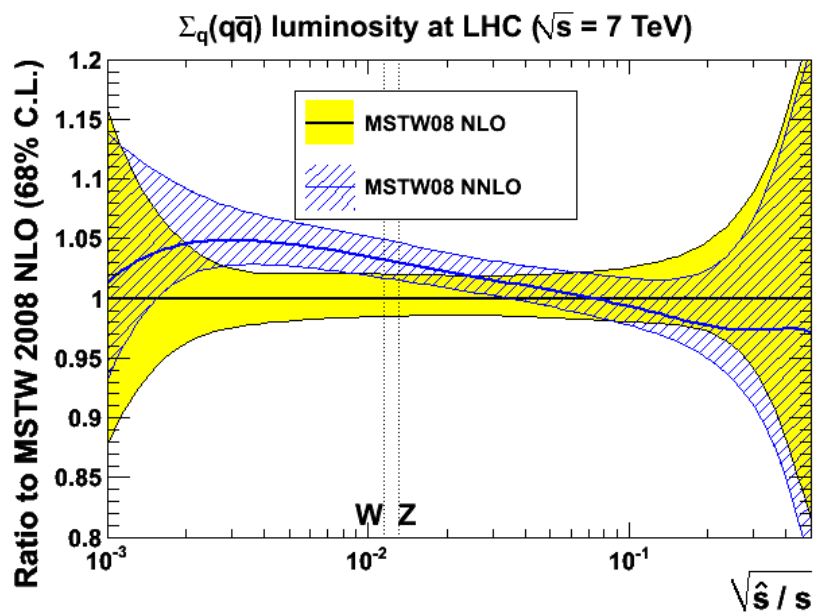


fractional uncertainty comparisons



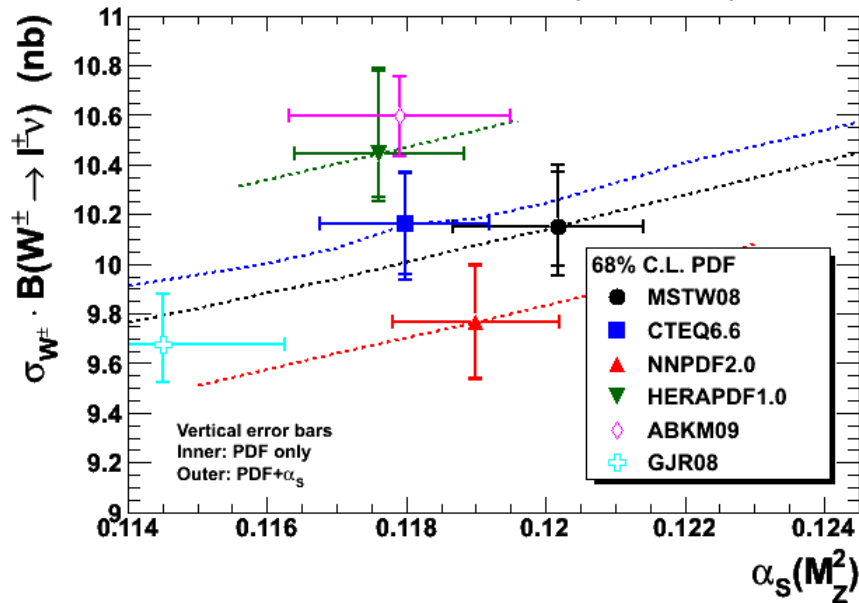
remarkably similar
considering the
different definitions of
pdf uncertainties used
by the 3 groups!

NLO and NNLO parton luminosity comparisons

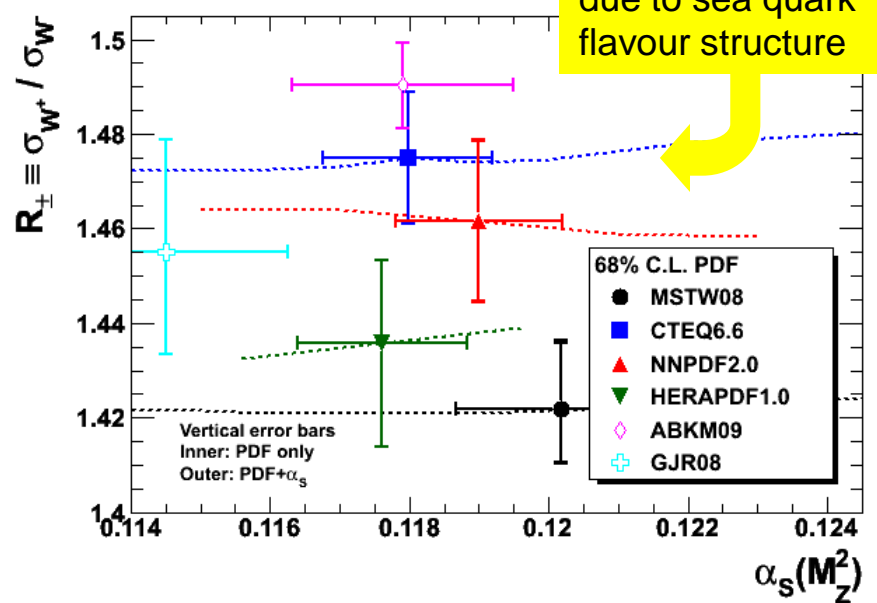


benchmark W,Z cross sections

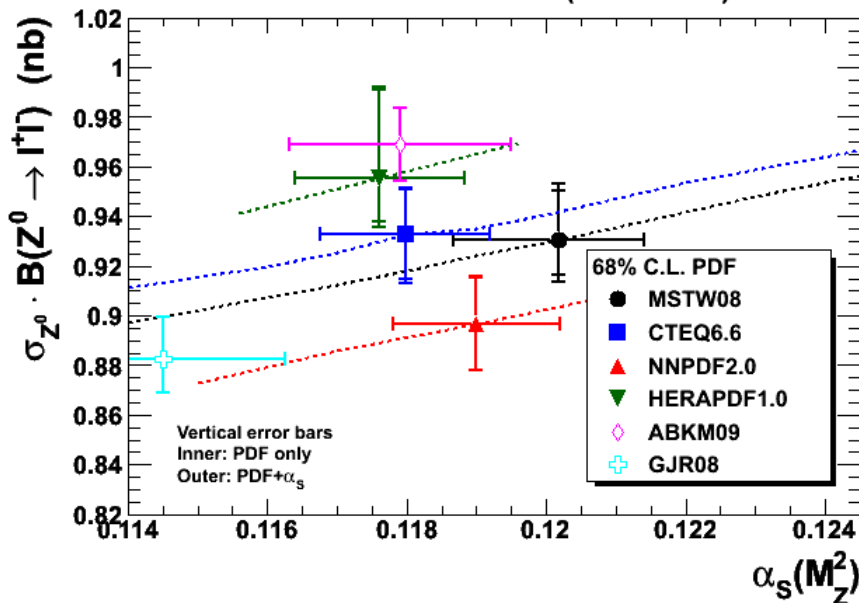
NLO $W^\pm \rightarrow l^\pm \nu$ at the LHC ($\sqrt{s} = 7$ TeV)



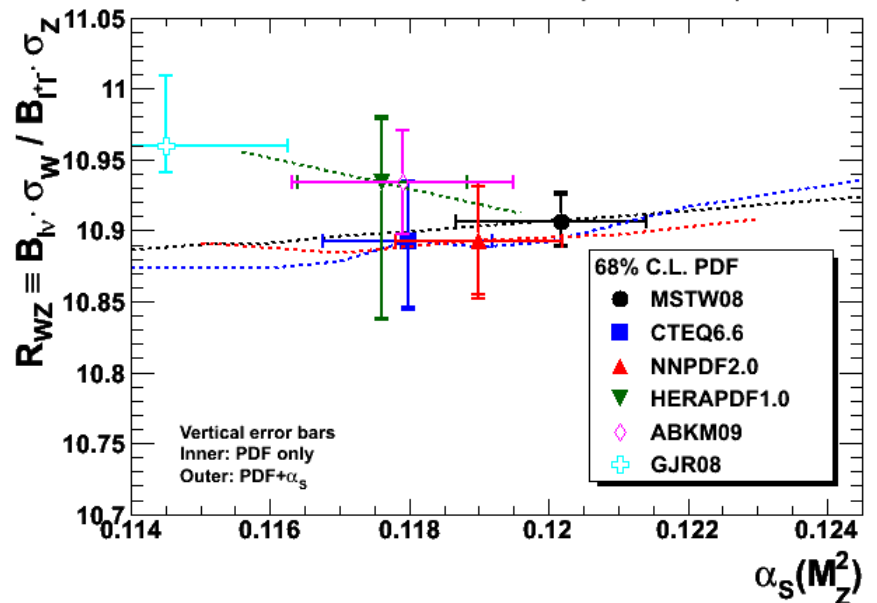
NLO W^+/W^- ratio at the LHC



NLO $Z^0 \rightarrow l^+ l^-$ at the LHC ($\sqrt{s} = 7$ TeV)

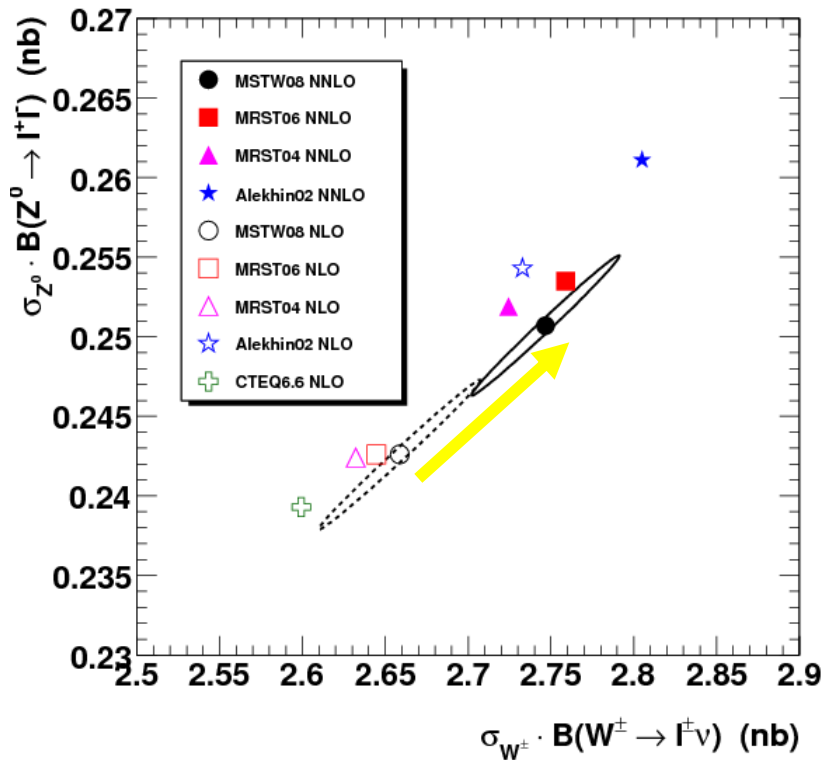


NLO W/Z ratio at the LHC ($\sqrt{s} = 7$ TeV)

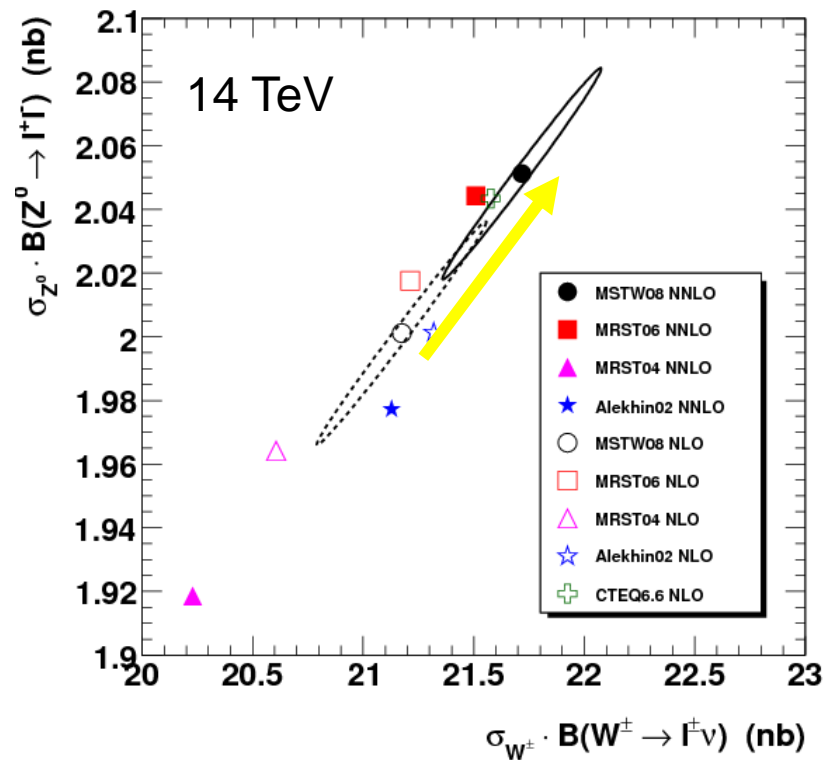


predictions for $\sigma(W,Z)$ @ Tevatron, LHC: NLO vs. NNLO

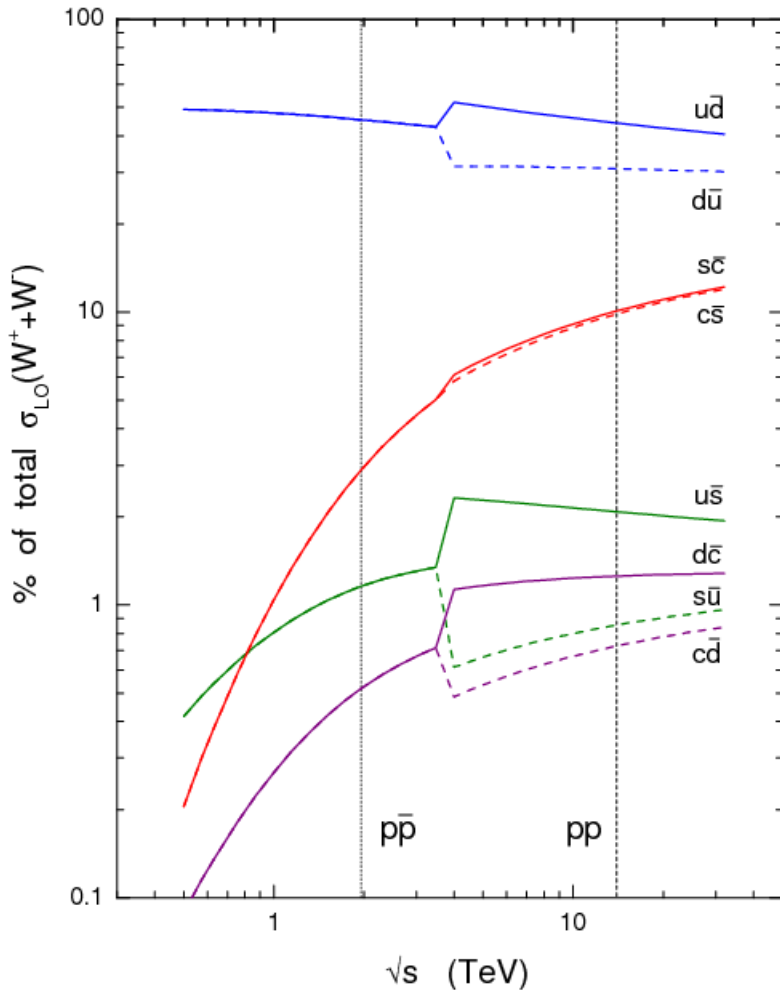
W and Z total cross sections at the Tevatron



W and Z total cross sections at the LHC



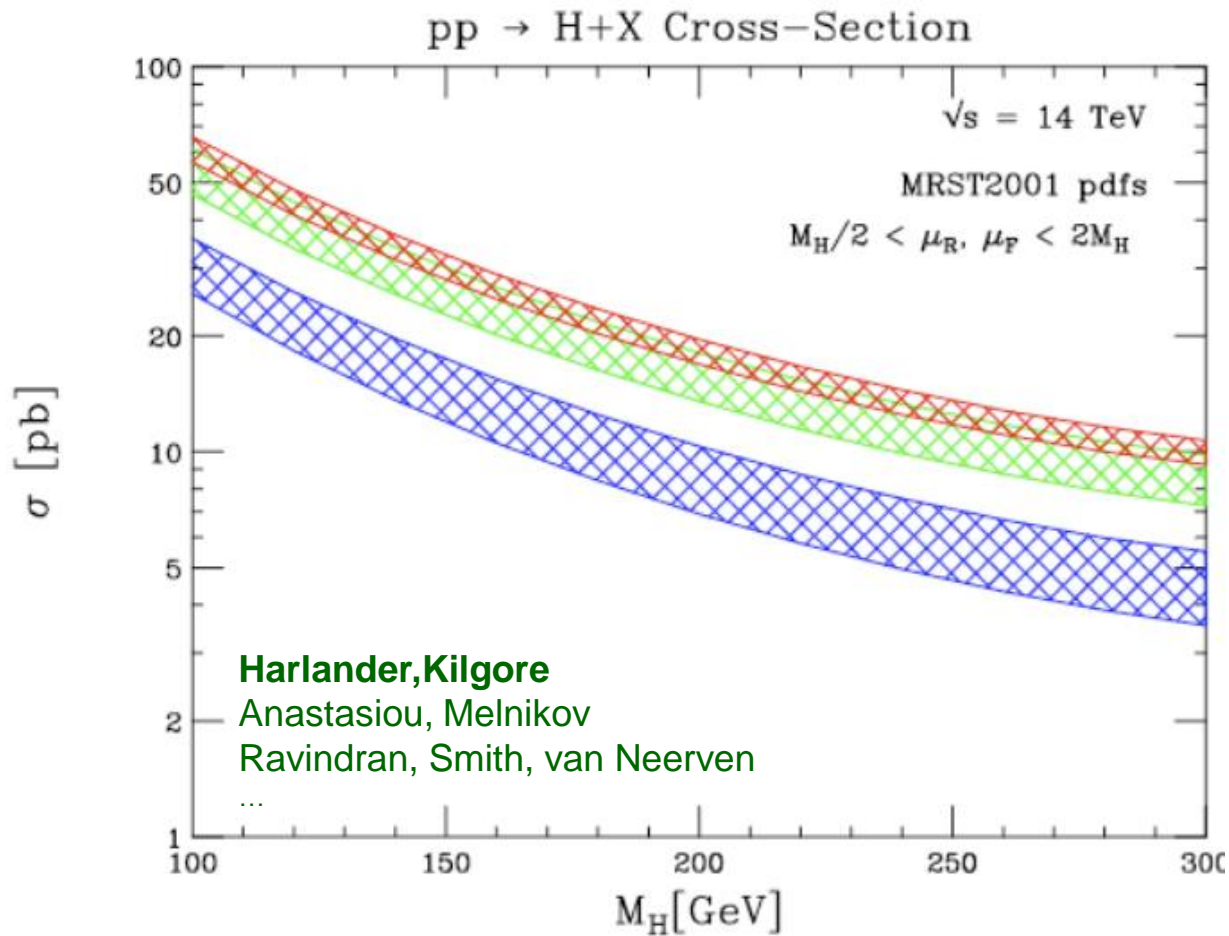
flavour decomposition of W cross sections



impact of sea quarks on the NLO W charge asymmetry ratio at 7 TeV:

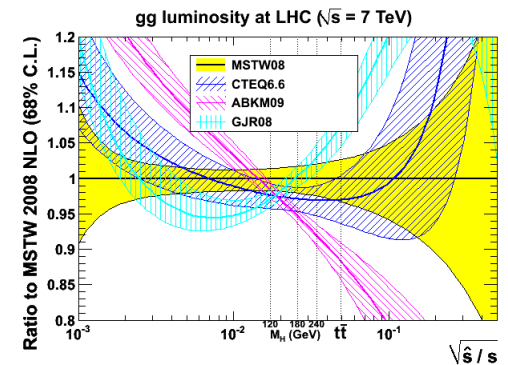
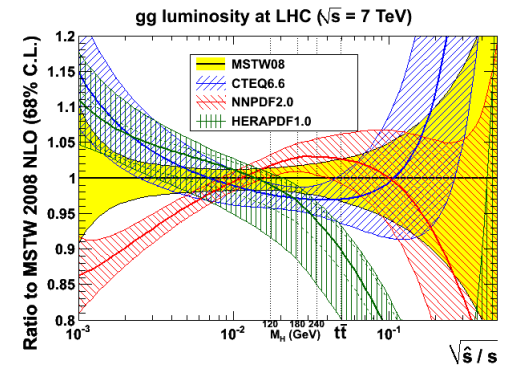
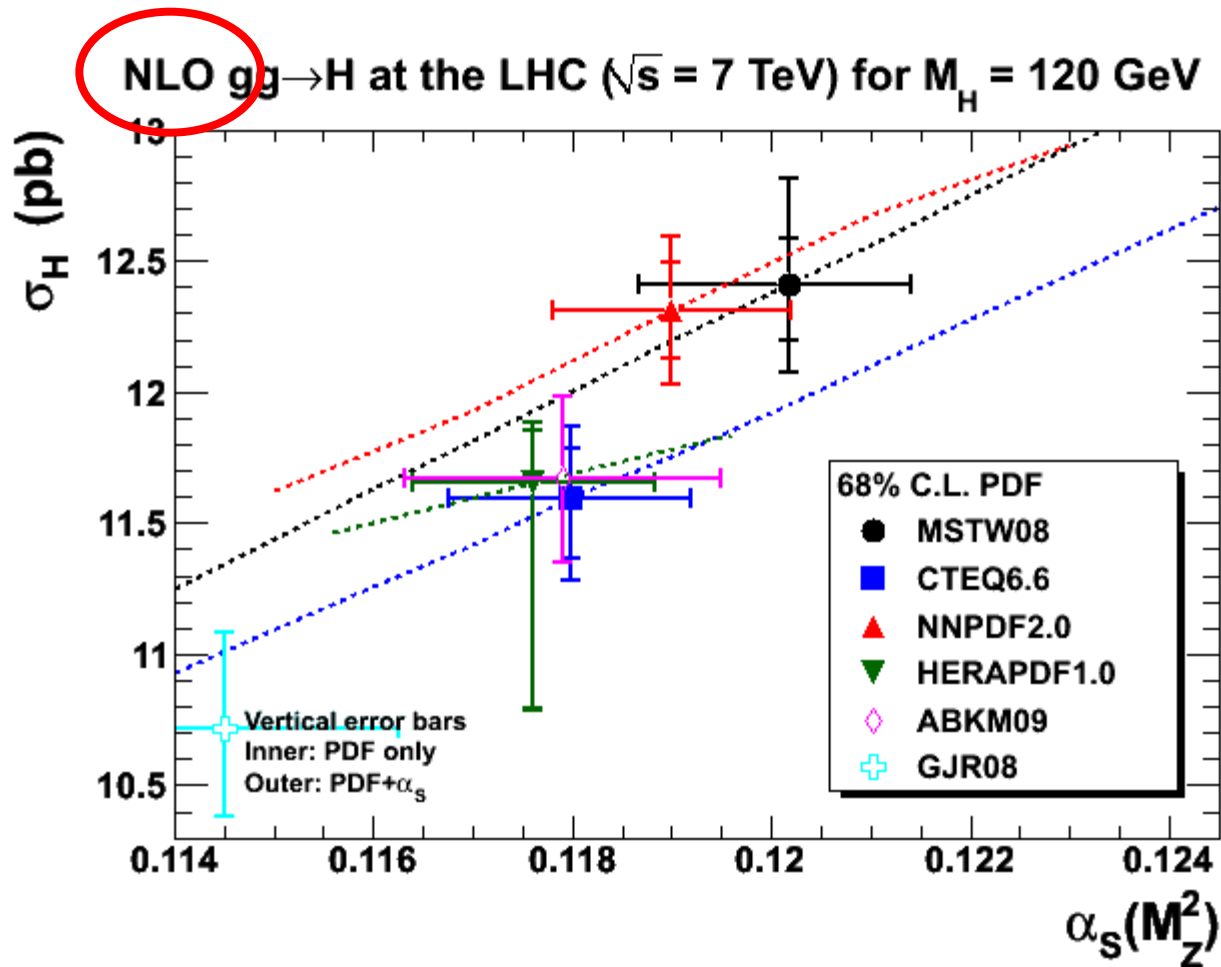
pdfs	R(W ⁺ /W ⁻)
{udg} only	1.53
{udscbg} = MSTW08	1.42 ± 0.02
{udscbg} _{sea} only	0.99
{udscbg} _{sym.sea} only	1.00

at LHC, ~30% of W and Z total cross sections involves s,c,b quarks



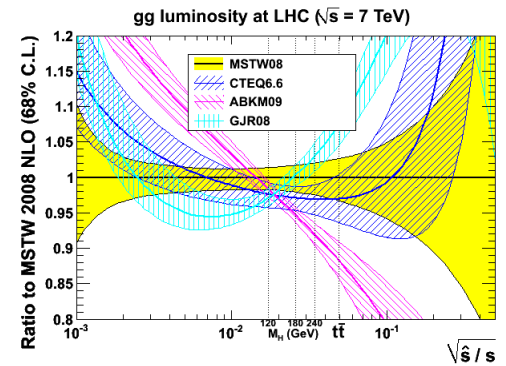
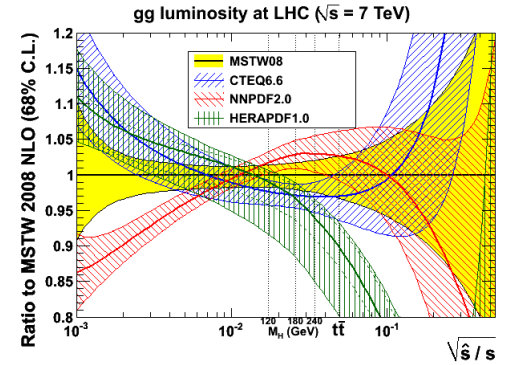
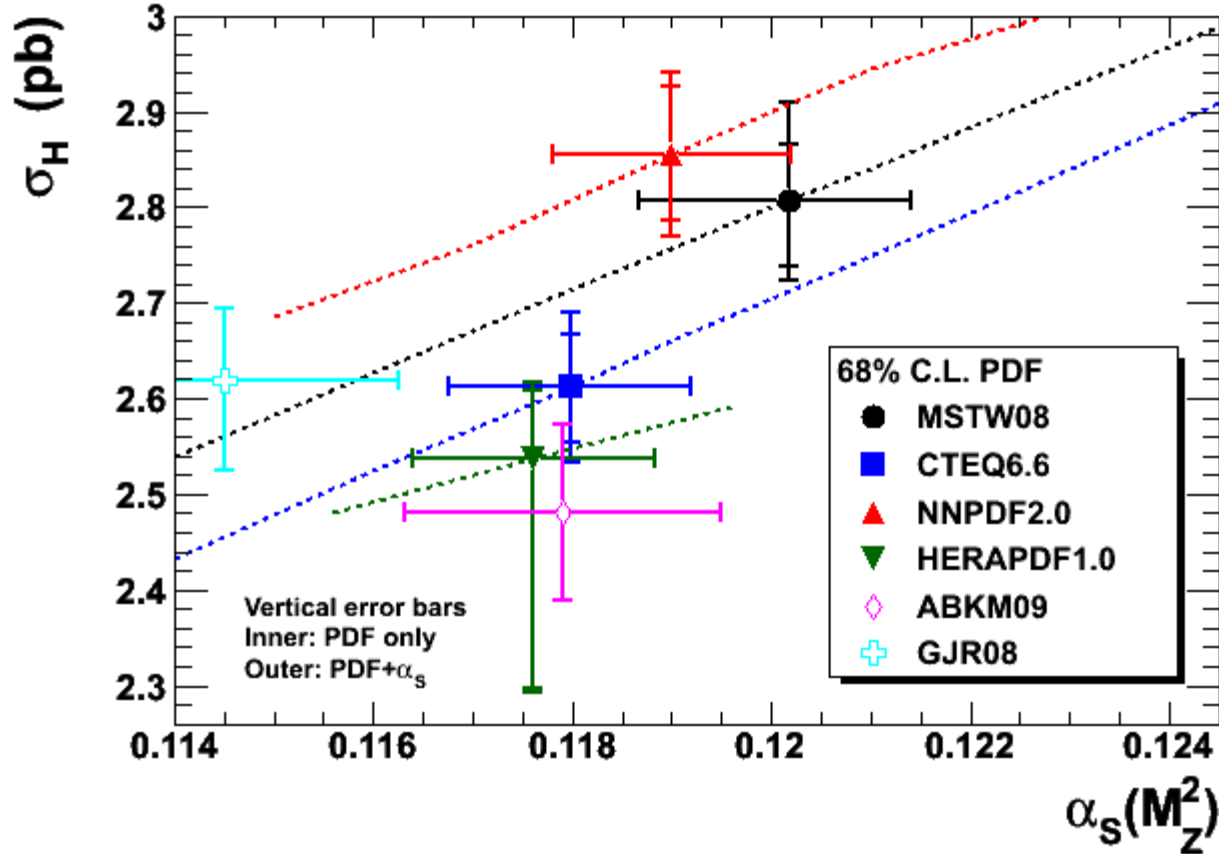
- only scale variation uncertainty shown
- central values calculated for a *fixed* set pdfs with a *fixed* value of $\alpha_S(M_Z)$

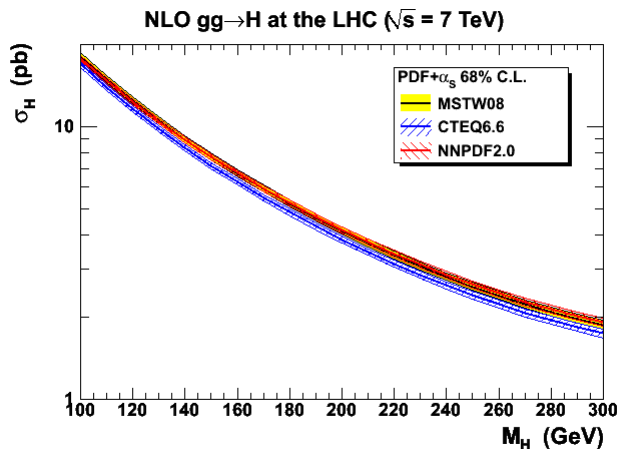
benchmark Higgs cross sections



... differences from both pdfs AND α_s !

NLO $gg \rightarrow H$ at the LHC ($\sqrt{s} = 7$ TeV) for $M_H = 240$ GeV

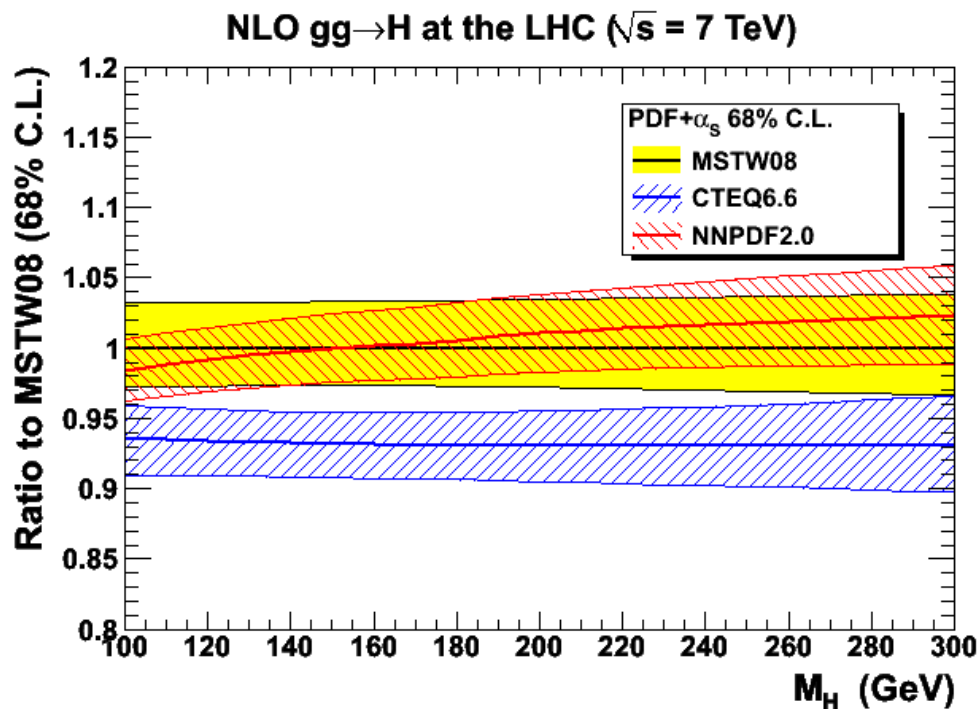




How to define an overall 'best theory prediction'?! See LHC Higgs Cross Section Working Group meeting, 5-6 July, higgs2010.to.infn.it

small print

Central predictions use the values of $\alpha_s(M_Z)$ favoured by each PDF group, i.e. 0.1202 for MSTW08, 0.1180 for CTEQ6.6 and 0.1190 for NNPDF2.0. For MSTW08, $\alpha_s(M_Z)$ was determined simultaneously with the PDFs in the global fit. The *experimental* uncertainties on $\alpha_s(M_Z)$ are +0.0012/-0.0015 at 68% C.L. The uncertainties on $\alpha_s(M_Z)$ for CTEQ6.6 and NNPDF2.0 are taken to be ± 0.0012 at 68% C.L. The combined PDF+ α_s uncertainty is calculated following the prescription recommended by each group, i.e. α_s uncertainties are simply added in quadrature for CTEQ6.6, while for NNPDF2.0 the exact prescription is used as explained in arXiv:1004.0962.



Note: (i) for MSTW08, uncertainty band similar at NNLO
(ii) everything here is at fixed scale $\mu = M_H$!

summary

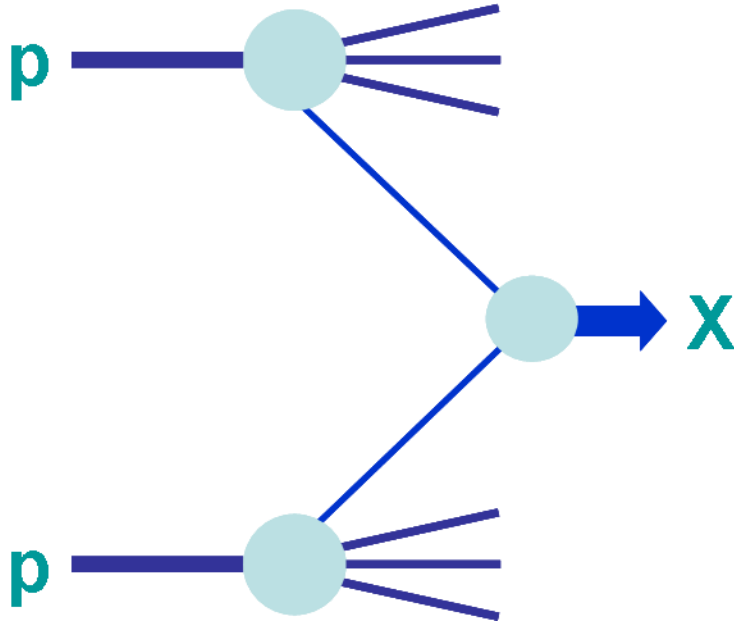
- two major advances in the past few years:
 - the NLO revolution – more and more SM background processes now known (see talks by Lance Dixon and others)
 - better understanding of and continuing convergence between the various pdf sets (see the PDF discussions this week)
 - inclusion of combined HERA data (not yet in all fits)
 - difficulty of reconciling **Run II Tevatron W asymmetry data**
 - proper assessment of uncertainties due to treatment of heavy quark flavours (GM-VFNS optimal but not uniquely defined)
 - beyond NNLO? e.g. influence of $[\alpha_S \ln(1/x)]^n$ contributions, higher twists, ...
 - ‘QED pdfs’ (e.g. **MRST 2004**)
- other issues ‘solved’: PS-MCs at NLO, robust jet algorithms, ...
- NNLO is the other major frontier: good for standard candles but no “+ jet” processes yet
- ‘scale variation uncertainty’ still a big issue: consensus needed
- eagerly awaiting *precision* cross sections at 7 TeV
- ... and don’t forget other more novel applications of pQCD (hard diffraction, multiple parton interactions, etc.

5

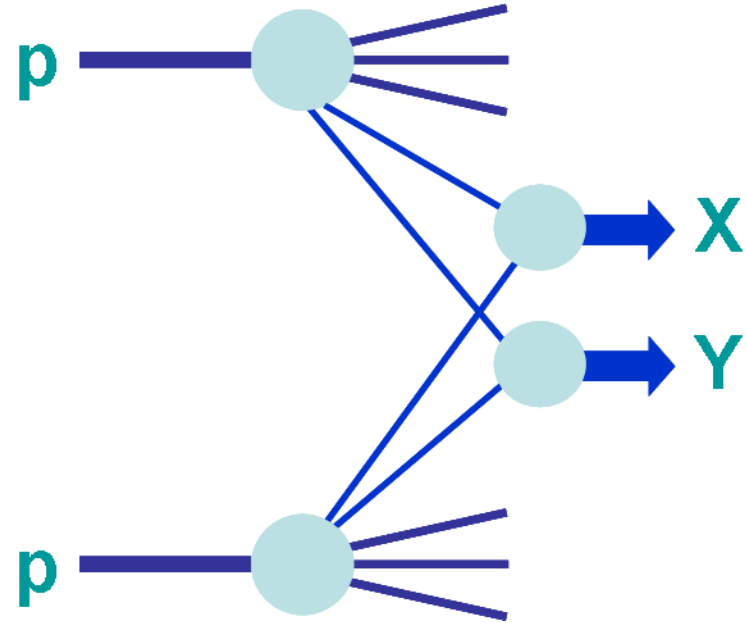
double parton distributions

For a `state of the art' overview of Multiple Parton Interactions, see the talks at the recent DESY workshop: indico.desy.de/conferenceDisplay.py?confId=3241

single and double hard parton scattering



$$f_{a/A}(x; Q^2)$$



$$f_{ab/A}(x_1, x_2; Q_X^2, Q_Y^2)$$

e.g. $X = Y = W$, $Q_X^2 \sim Q_Y^2 \sim M_W^2$

double parton scattering: rates and topologies

- if we assume that the dPDFs factorise, i.e.

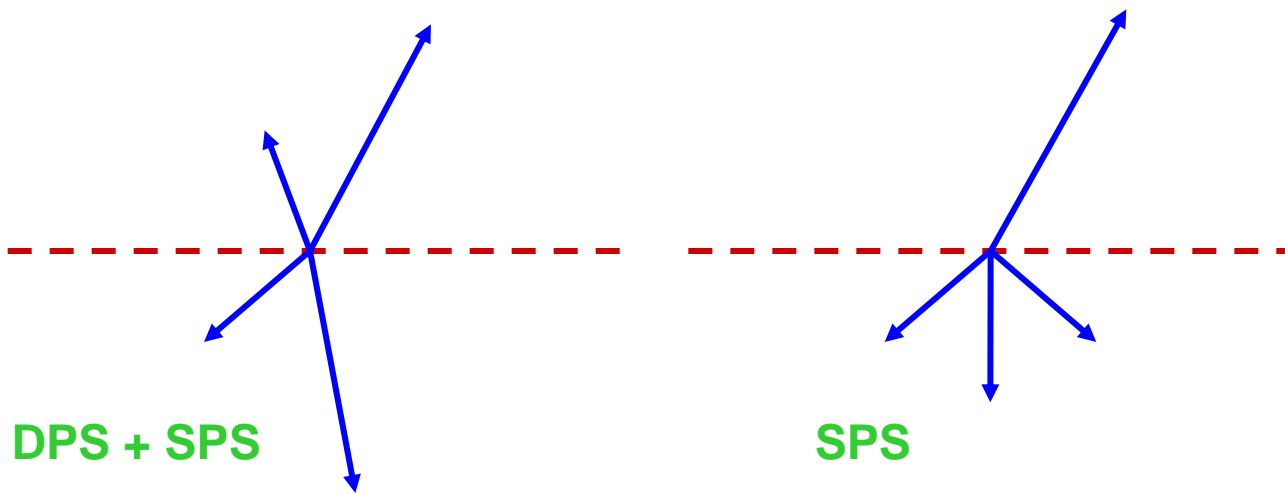
$$f_{ab/A}(x_1, x_2; Q_1^2, Q_2^2) = f_a(x_1, Q_1^2) f_b(x_2, Q_2^2)$$

- then we obtain

$$\sigma^{\text{DPS}}(X, Y) = \frac{m}{2} \frac{\sigma_X \sigma_Y}{\sigma_{\text{eff}}}$$

X, Y distinct: $m=2$
X, Y same: $m=1$

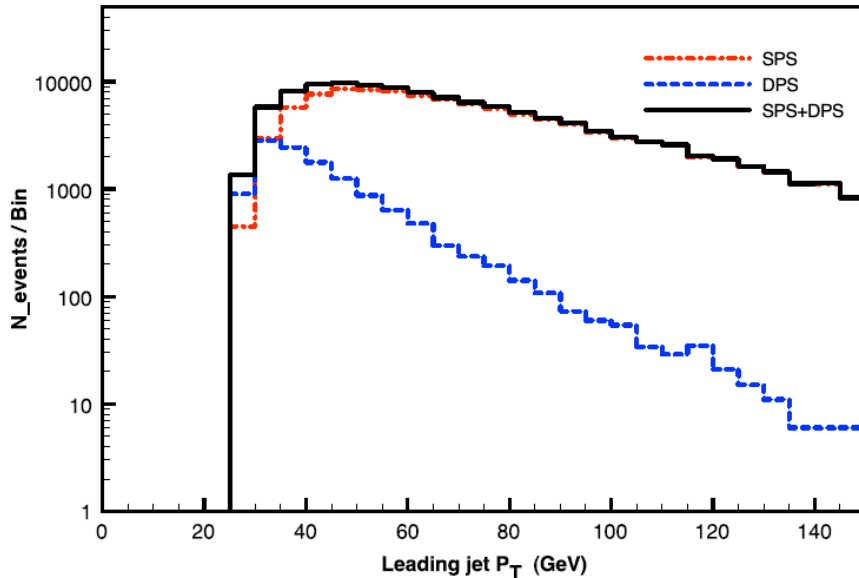
- studies of $\gamma+3j$ production by CDF and D0 suggest $\sigma_{\text{eff}} \approx 15 \text{ mb}$
- but there is generally a SPS ‘background’, $a+b \rightarrow XY$



- use 'pairwise transverse momentum balance' (shape variable) as a signal for double parton scattering
- many final states have been studied*: $X, Y = \gamma j, 2j, W, bb, tt, H, \dots$
- interesting example: same-sign W at LHC

*Del Fabbro, Treleani, Cattaruzza; Berger, Jackson, Shaughnessy; Maina; Hussein; Gaunt, Kom, Kulesza, S; ...

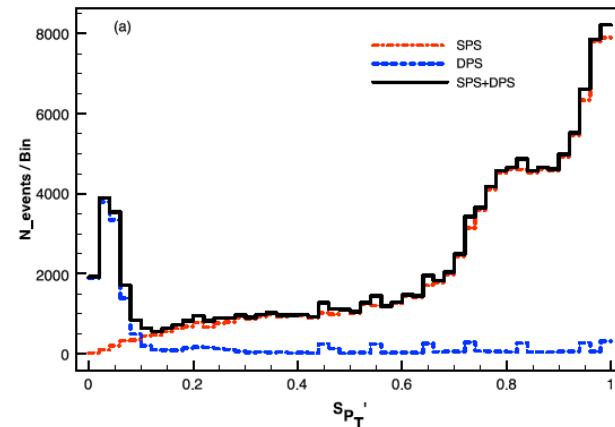
DPS in bbjj production at LHC



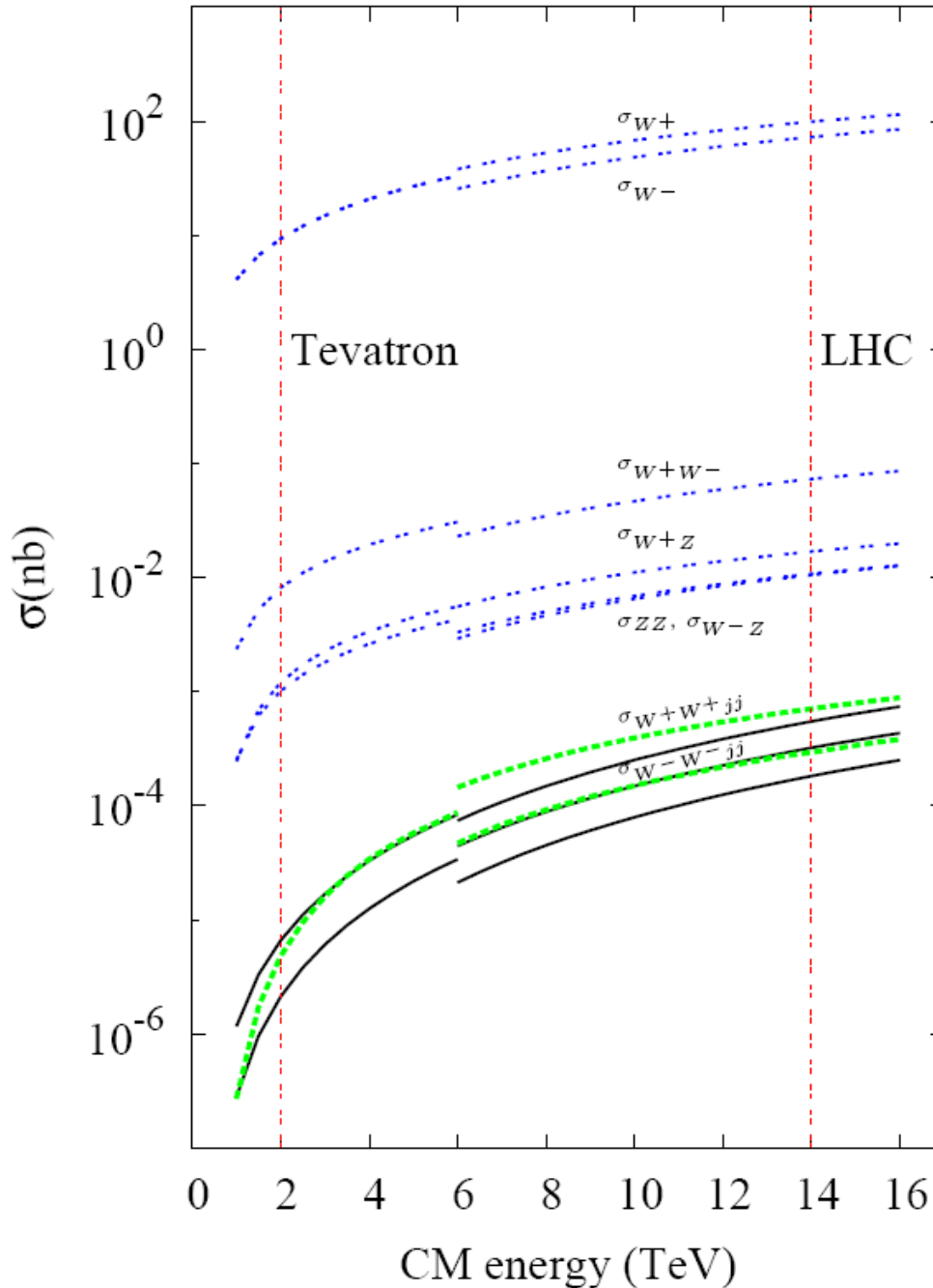
← DPS dominates at low $p_T(\text{jet})$

$$S'_{p_T} = \frac{1}{\sqrt{2}} \sqrt{\left(\frac{|p_T(b_1, b_2)|}{|p_T(b_1)| + |p_T(b_2)|} \right)^2 + \left(\frac{|p_T(j_1, j_2)|}{|p_T(j_1)| + |p_T(j_2)|} \right)^2}$$

easy to separate DPS using shape (imbalance) variable →



Berger, Jackson, Shaughnessy (2009)



Kulesza, S (1999)
 Maina (2009)
 Gaunt, Kom, Kulesza, S (2010)

Note:

$$a + b \rightarrow W^+W^-$$

but

$$a + b \not\rightarrow W^+W^+$$

instead

$$q + q \rightarrow W^+W^+ + q'q'$$

$$\begin{aligned} &\sigma_{W^+W^-}^{DPS} \\ &\sigma_{W^+W^+}^{DPS} \\ &\sigma_{W^-W^-}^{DPS} \end{aligned}$$

so same-sign W production could be a good place to look for DPS (with a lot of luminosity!)

dDGLAP

- the dPDFs satisfy a 'double DGLAP' equation

$$\begin{aligned} \frac{dD_h^{j_1 j_2}(x_1, x_2; t)}{dt} = & \frac{\alpha_s(t)}{2\pi} \left[\sum_{j'_1} \int_{x_1}^{1-x_2} \frac{dx'_1}{x'_1} D_h^{j'_1 j_2}(x'_1, x_2; t) P_{j'_1 \rightarrow j_1} \left(\frac{x_1}{x'_1} \right) \right. \\ & + \sum_{j'_2} \int_{x_2}^{1-x_1} \frac{dx'_2}{x'_2} D_h^{j_1 j'_2}(x_1, x'_2; t) P_{j'_2 \rightarrow j_2} \left(\frac{x_2}{x'_2} \right) \\ & \left. + \sum_{j'} D_h^{j'}(x_1 + x_2; t) \frac{1}{x_1 + x_2} P_{j' \rightarrow j_1 j_2} \left(\frac{x_1}{x_1 + x_2} \right) \right] \end{aligned}$$

Kirschner 1979
 Shelest, Snigirev, Zinovjev 1982
 Snigirev 2003
 Korotkikh, Snigirev 2004
 Cattaruzza et al. 2005

- and note that $f_{ab}(x_1, x_2; Q_1^2, Q_2^2) = f_a(x_1, Q_1^2) f_b(x_2, Q_2^2)$ is **not** a solution, i.e. factorisation is broken (in fact this must be true since must have $x_1 + x_2 < 1$ for momentum conservation)

Snigirev 2003
 Korotkikh, Snigirev 2004
 Cattaruzza et al. 2005

- the dPDFs and sPDFS are related by sum rules, e.g.

$$\sum_a \int_0^{1-x_2} dx_1 x_1 f_{ab}(x_1, x_2; Q^2) = (1-x_2) f_b(x_2, Q^2)$$

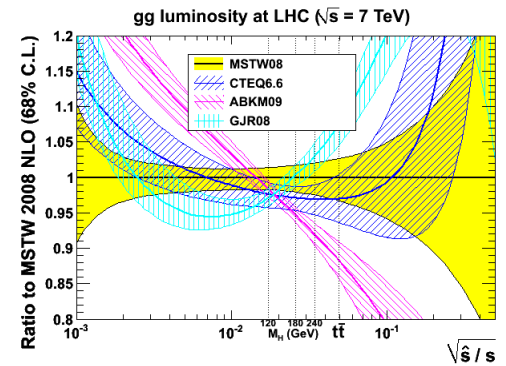
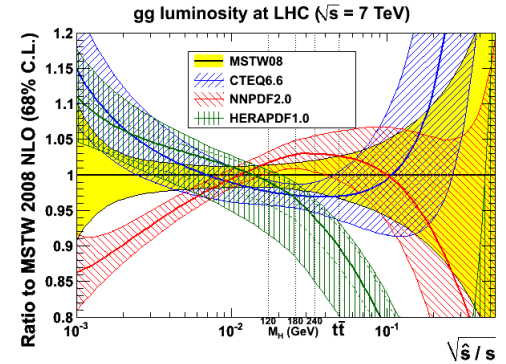
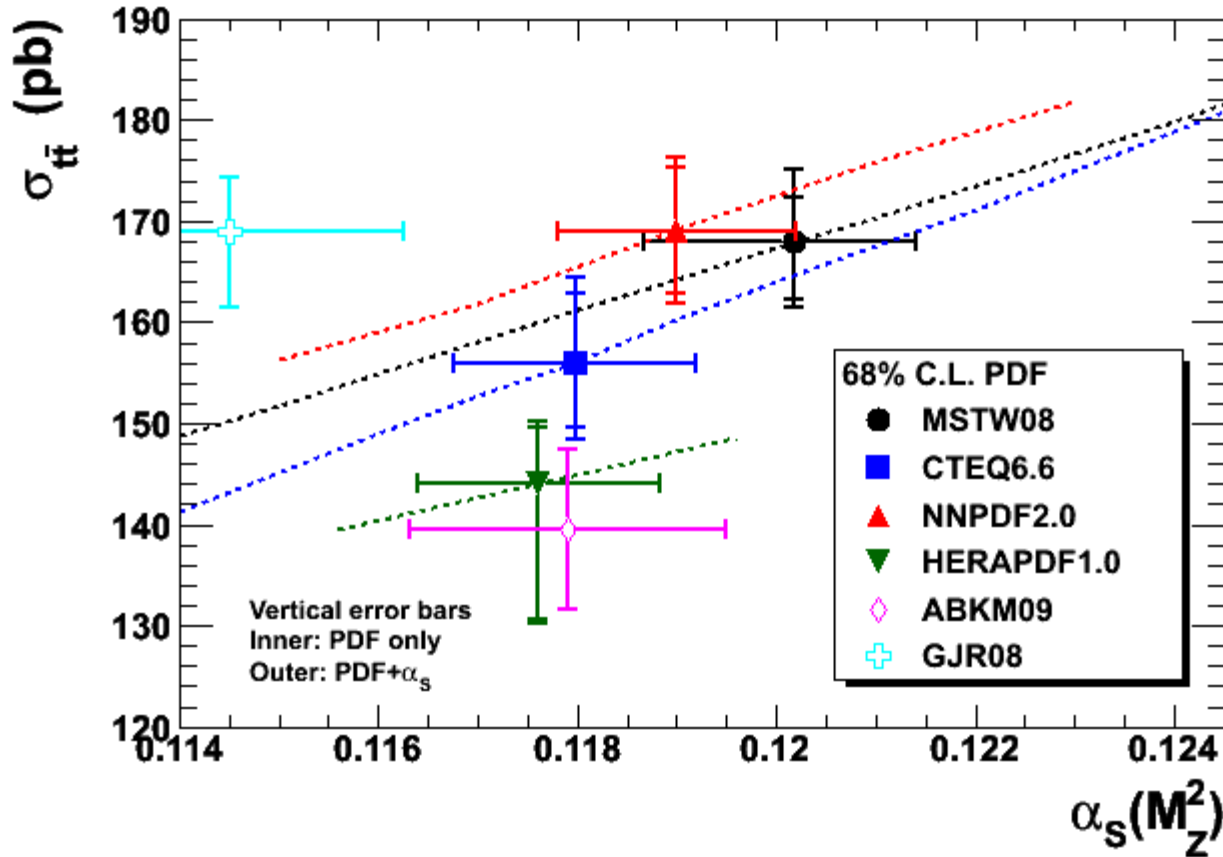
- a consistent LO package (GS09) is available

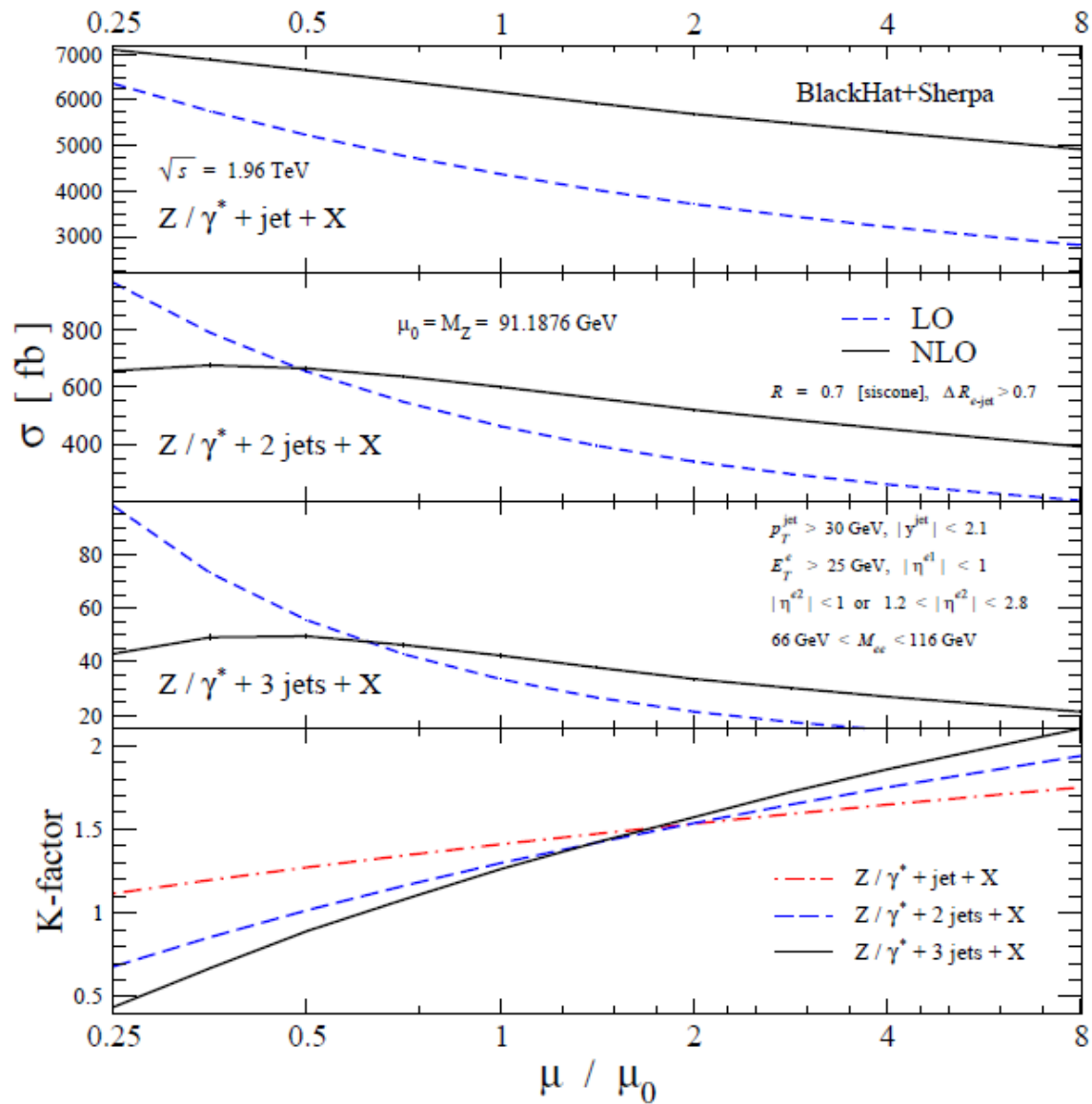
Gaunt, S 2009

extra slides

benchmark top cross sections

NLO $t\bar{t}$ cross sections at the LHC ($\sqrt{s} = 7$ TeV)





examples of data sets used in fits*

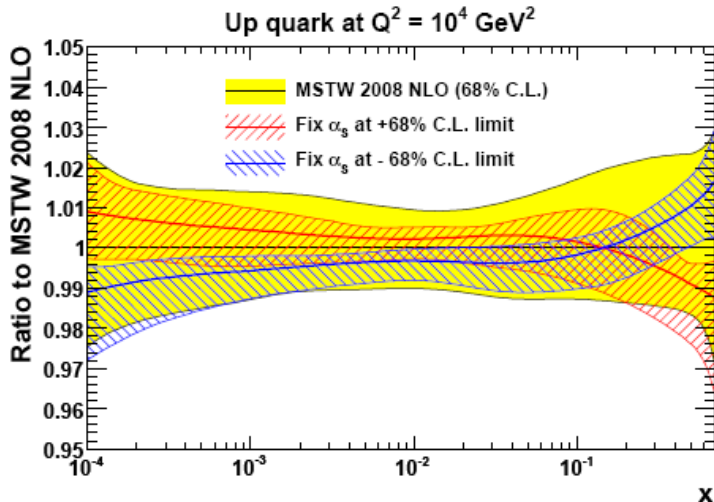
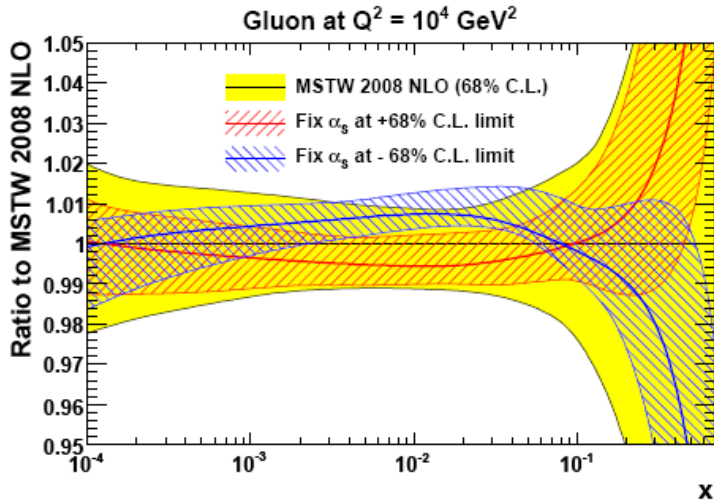
Data set	$N_{\text{pts.}}$
H1 MB 99 e^+p NC	8
H1 MB 97 e^+p NC	64
H1 low Q^2 96–97 e^+p NC	80
H1 high Q^2 98–99 e^-p NC	126
H1 high Q^2 99–00 e^+p NC	147
ZEUS SVX 95 e^+p NC	30
ZEUS 96–97 e^+p NC	144
ZEUS 98–99 e^-p NC	92
ZEUS 99–00 e^+p NC	90
H1 99–00 e^+p CC	28
ZEUS 99–00 e^+p CC	30
H1/ZEUS $e^\pm p F_2^{\text{charm}}$	83
H1 99–00 e^+p incl. jets	24
ZEUS 96–97 e^+p incl. jets	30
ZEUS 98–00 $e^\pm p$ incl. jets	30
DØ II $p\bar{p}$ incl. jets	110
CDF II $p\bar{p}$ incl. jets	76
CDF II $W \rightarrow l\nu$ asym.	22
DØ II $W \rightarrow l\nu$ asym.	10
DØ II Z rap.	28
CDF II Z rap.	29

Data set	$N_{\text{pts.}}$
BCDMS $\mu p F_2$	163
BCDMS $\mu d F_2$	151
NMC $\mu p F_2$	123
NMC $\mu d F_2$	123
NMC $\mu n/\mu p$	148
E665 $\mu p F_2$	53
E665 $\mu d F_2$	53
SLAC $ep F_2$	37
SLAC $ed F_2$	38
NMC/BCDMS/SLAC F_L	31
E866/NuSea pp DY	184
E866/NuSea pd/pp DY	15
NuTeV $\nu N F_2$	53
CHORUS $\nu N F_2$	42
NuTeV $\nu N xF_3$	45
CHORUS $\nu N xF_3$	33
CCFR $\nu N \rightarrow \mu\mu X$	86
NuTeV $\nu N \rightarrow \mu\mu X$	84
All data sets	2743

red font = new wrt MRST2006 fit

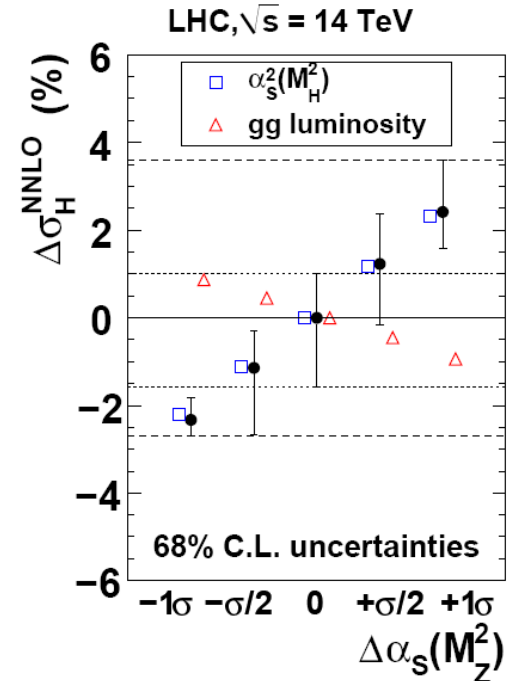
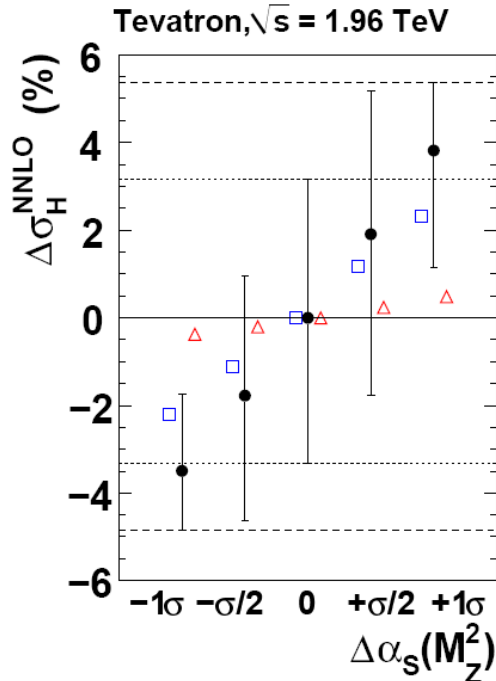
*MSTW2008

α_S - pdf correlations



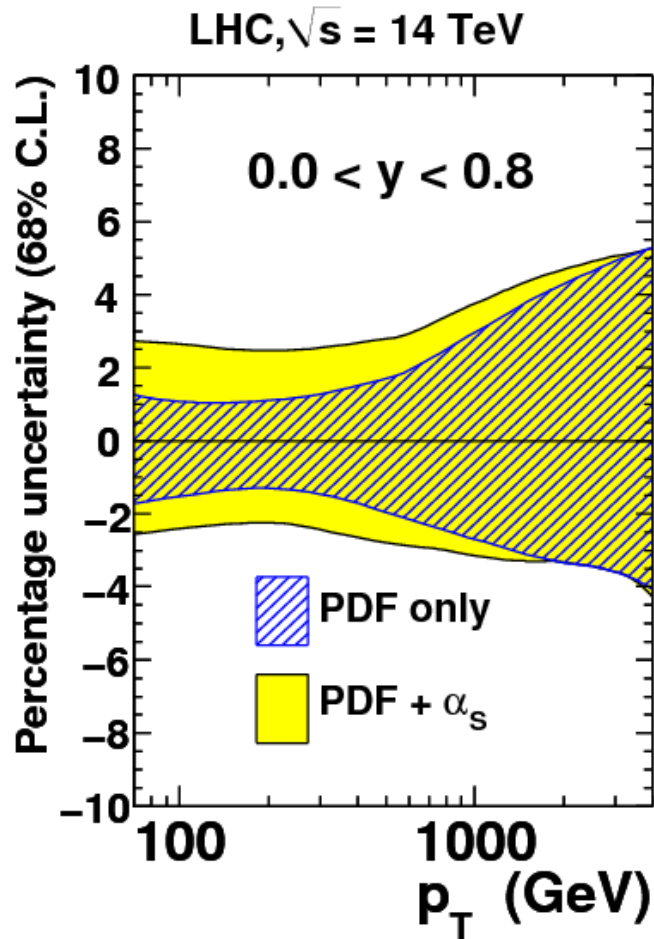
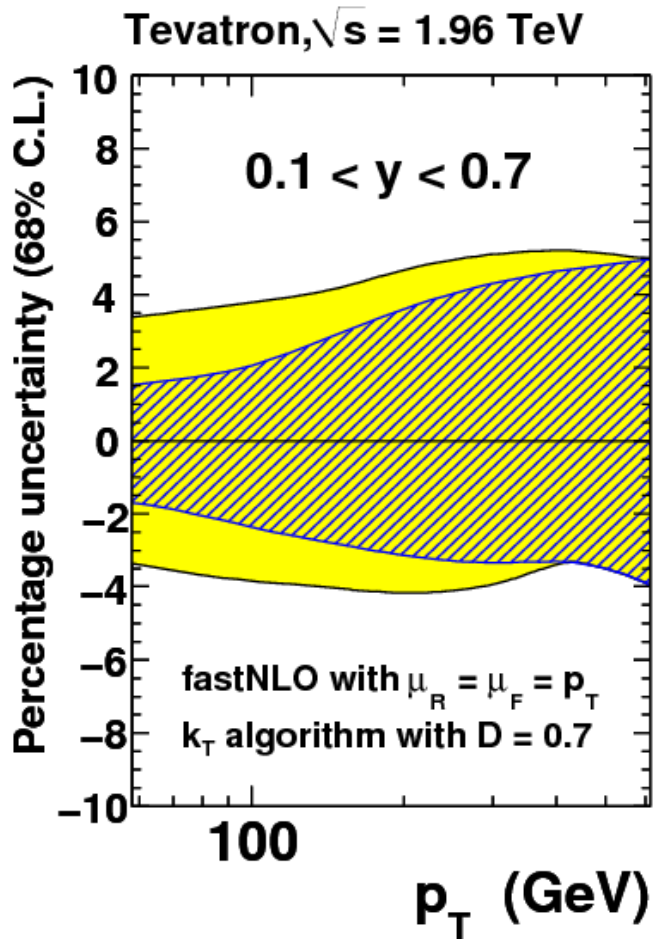
- care needed when assessing impact of varying α_S on cross sections $\sim (\alpha_S)^n$

Higgs ($M_H = 120 \text{ GeV}$) with MSTW 2008 NNLO PDFs

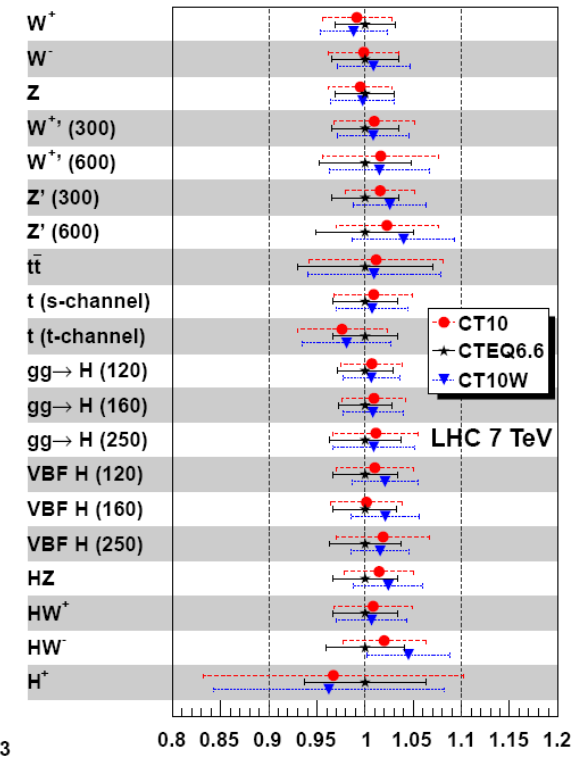
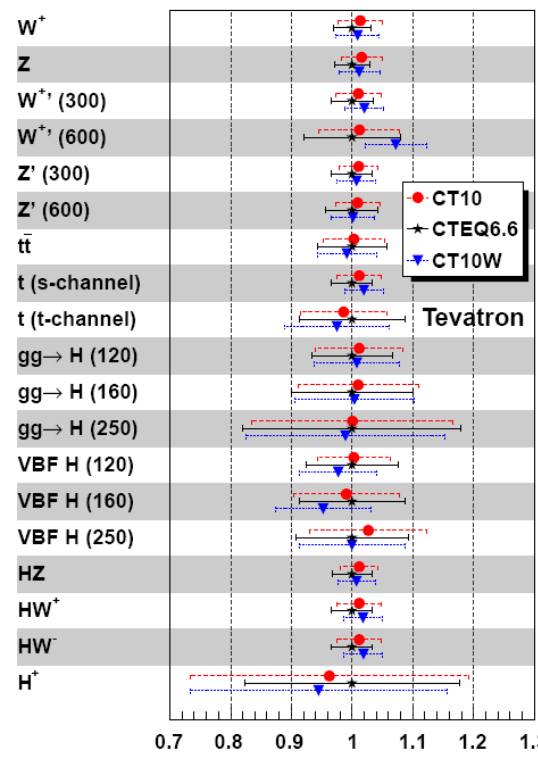
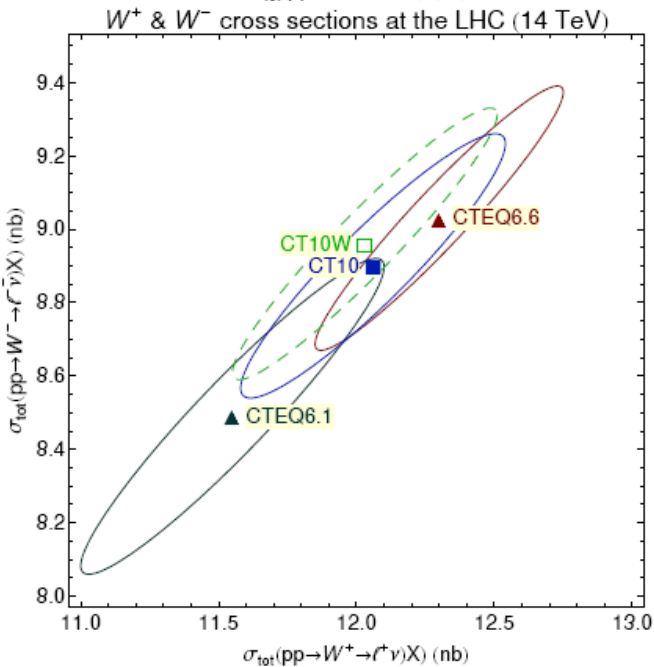
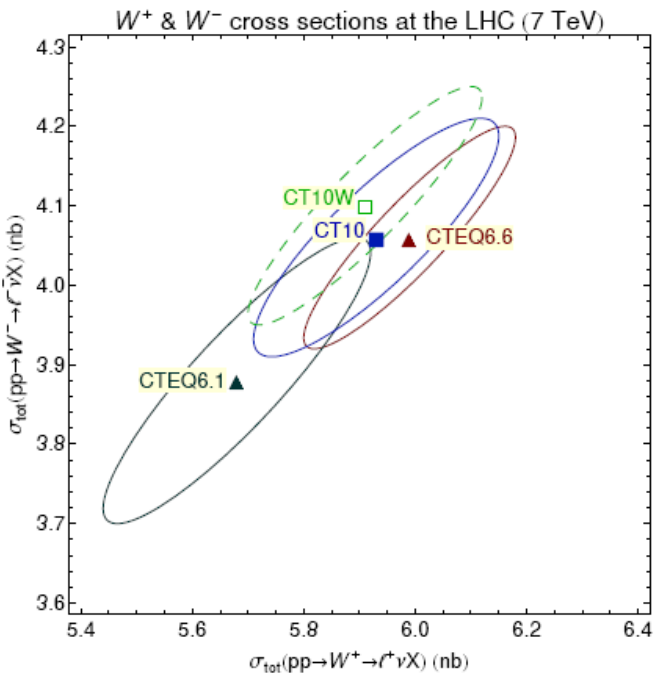


pdf + α_S uncertainties in jet cross sections

Inclusive jet cross sections with MSTW 2008 NLO PDFs



CTEQ6.6 vs. CT10, CT10W (NLO)



* **CT10W**: attempt to include recent D0 lepton asymmetry data in global fit
 → slightly different d/u

improved LO pdfs

- conventional wisdom is to match pQCD order of pdfs with that of MEs
- but, in practice,
 - $\sigma_{LO} = \text{PDFs(LO)} \otimes \text{ME(LO)}$ can be different from $\sigma_{NLO} = \text{PDFs(NLO)} \otimes \text{ME(NLO)}$, in both shape and normalisation
 - LO pdfs have very poor χ^2 in (LO) global fit (no surprise: NLO corrections at large and small x are significant and preferred by the data)
- momentum conservation limits how much additional glue can be added to LO partons to compensate for missing NLO pQCD corrections (e.g. to get correct evolution rate of small- x quarks)
- therefore relax momentum conservation and redo LO fit; study the impact of this on χ^2 , partons and cross sections
- e.g. **Thorne & Shertsnev 2007**: LO* partons
 - χ^2 : 3066/2235 \rightarrow 2691/2235, momentum conservation: 100% \rightarrow 113%

heavy quarks: charm, bottom, ...

considered sufficiently massive to allow pQCD treatment: $g \rightarrow Q\bar{Q}$

distinguish two regimes:

- (i) $Q^2 \sim m_H^2$ include full m_H dependence to get correct threshold behaviour
- (ii) $Q^2 \gg m_H^2$ treat as \sim massless partons to resum $\alpha_s^n \log^n(Q^2/m_H^2)$ via DGLAP

FFNS: OK for (i) only **ZM-VFNS:** OK for (ii) only

consistent **GM(=general mass)-VFNS** now available (e.g. ACOT(χ), RT, BMSN,...) which interpolates smoothly between the two regimes

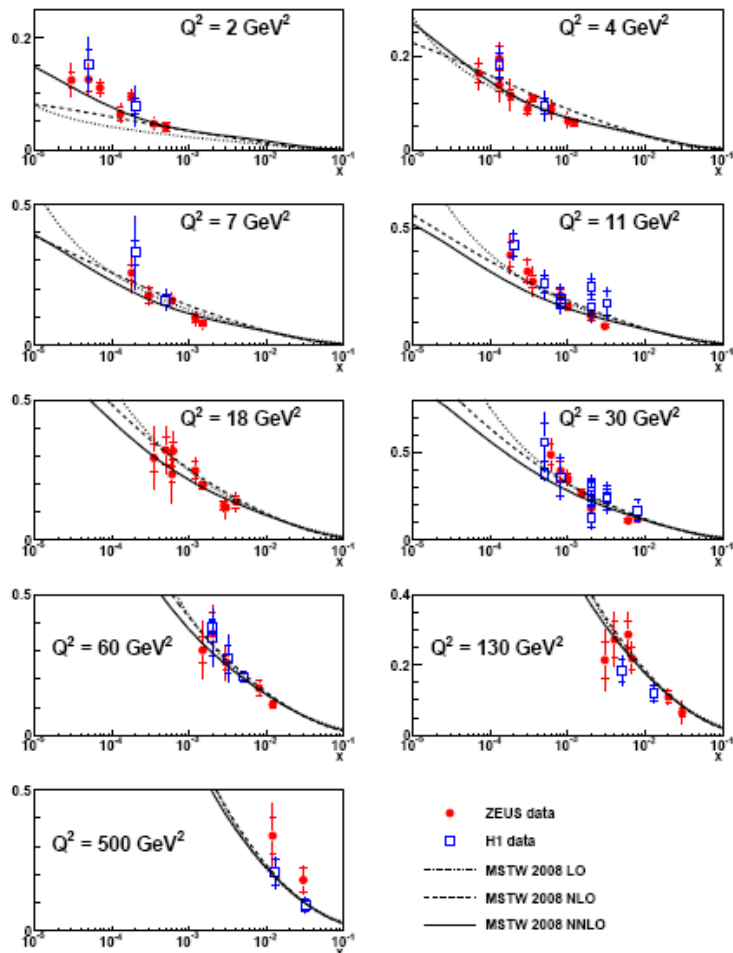
Aivazis, Collins, Olness, Tung; Roberts, Thorne; Buza, Matiounine, Smith, Migneron, van Neerven, ...

Note:

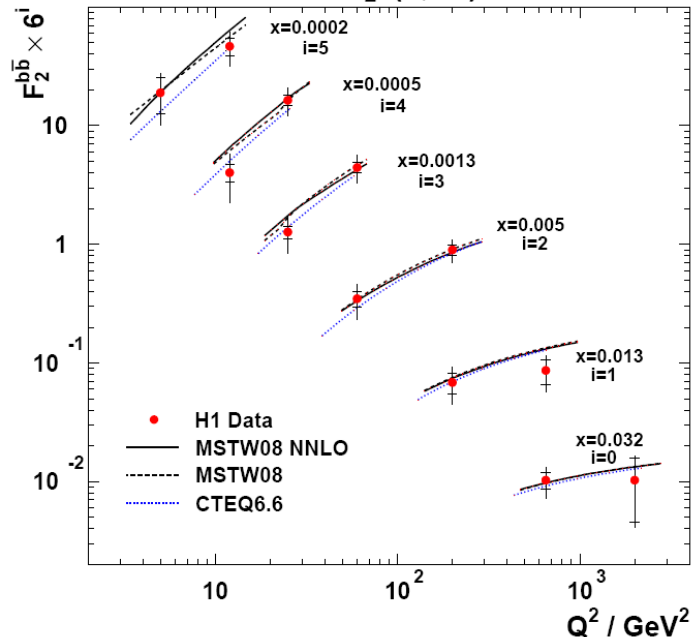
- (i) the definition of these is tricky and non-unique (ambiguity in assignment of $O(m_H^2/Q^2)$ contributions), and the implementation of improved treatment (e.g. in going from MRST2004 \rightarrow MRST 2006 or CTEQ 6.1 \rightarrow 6.5) can have a big effect on light partons
- (ii) the *true* uncertainty on e.g. LHC predictions coming from ambiguities in the heavy quark treatment has yet to be quantified

charm and bottom structure functions

Charm structure function, $F_2^{c\bar{c}}(x, Q^2)$

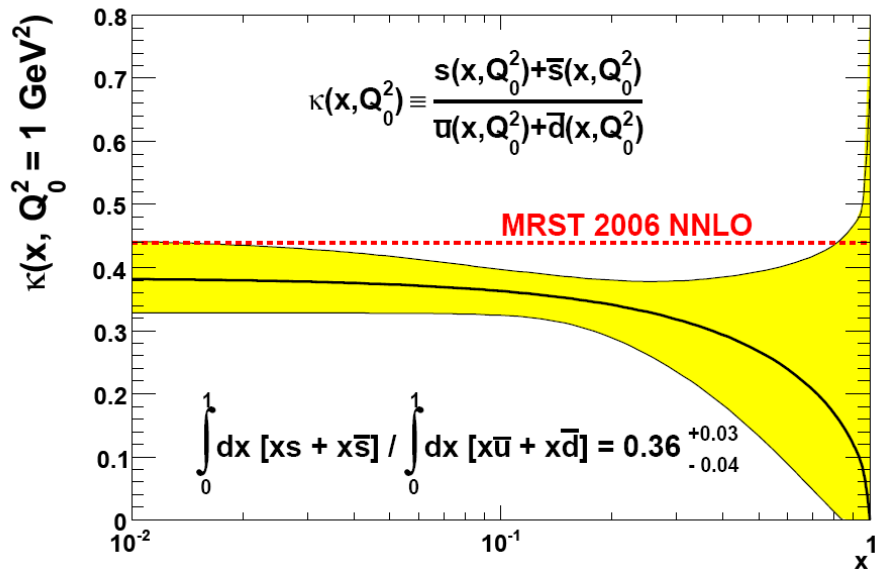


H1 $F_2^{b\bar{b}}(x, Q^2)$

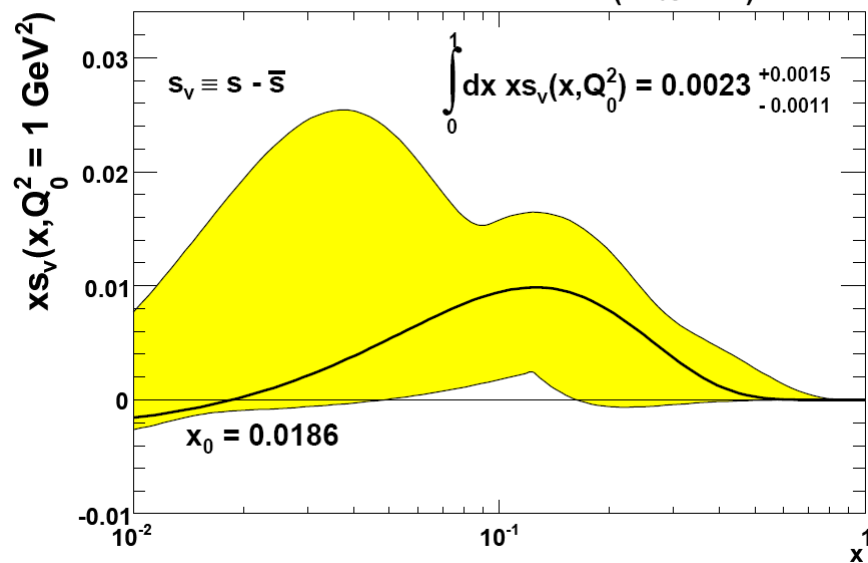


- MSTW 2008 uses *fixed* values of $m_c = 1.4 \text{ GeV}$ and $m_b = 4.75 \text{ GeV}$ in a GM-VFNS
- currently studying the sensitivity of the fit to these values, and impact on LHC cross sections

MSTW 2008 NNLO PDF fit (68% C.L.)

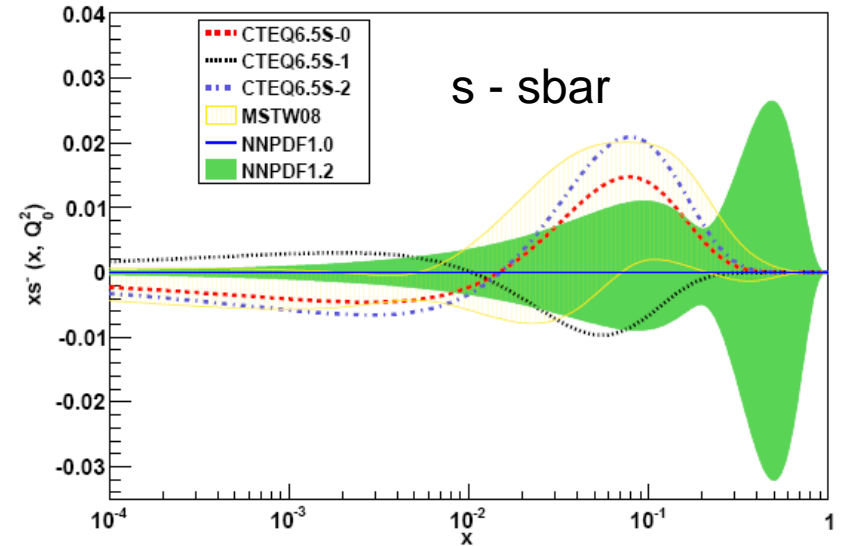
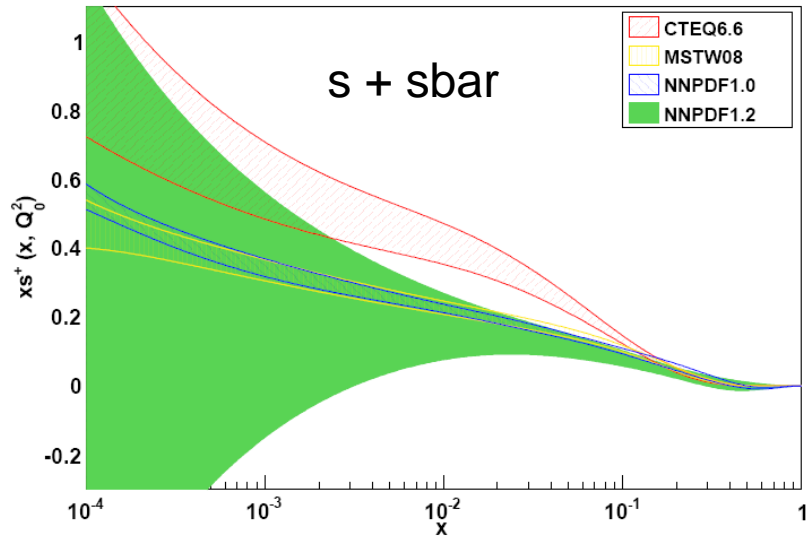


MSTW 2008 NNLO PDF fit (68% C.L.)



MSTW

strange quark in NNPDF



Note:

MSTW: assume u, d, s quarks have same x^δ behaviour as $x \rightarrow 0$

NuTeV $\sin^2\theta_W$ anomaly largely removed

F_L

- an independent measurement of the small-x **gluon**
- a test of the assumptions in the DGLAP LT pQCD analysis of small-x F_2
- higher-order $\ln(1/x)$ and higher-twist contributions could be important

$$\frac{\partial F_2}{\partial \ln Q^2} \simeq \alpha_S P^{qg} \otimes g + \dots$$

$$F_L \simeq \alpha_S C_{Lg} \otimes g + \dots$$

