Parton distribution functions: Heavy Quarks

Pedro Jimenez-Delgado

[in collaboration with E. Reya]

International Workshop "QCD at the LHC", ECT* Trento

Universität Zürich UZH

technische universität dortmund

PHYSICS AT THE TERA SCALE Helmholtz Alliance

Parton distribution functions: Heavy Quarks

Entering in the Large Hadron Collider (LHC) era; a 14 TeV p-p collider:

$$\sigma(P_1,P_2) = \sum_{i,j} \int dx_1\, dx_2\, \hat{\sigma}_{ij}\big(x_1,x_2,\ln\tfrac{M^2}{\mu^2},a_s(\mu^2)\big) f_i(x_1,\mu^2) f_j(x_2,\mu^2)$$

Parton distribution functions (PDFs) are determined by global fits

The primary input for these analyses is DIS (semi)inclusive data

Heavy quarks (HQs) contribute considerably to the dominant $F_2(x,Q^2)$:

$$\mathbf{F_2^c \lesssim 30\%}, \qquad \mathbf{F_2^b \lesssim 3\%}$$

Thus their appropriate treatment is *vital* for a correct interpretation of the upcoming results at high-energy colliders

There are several formalisms (“schemes”): FFNS, VFNS, GM-VFNS’s

When is which scheme adequate? Is this clear or are there uncertainties?

Parton distribution functions: Heavy Quarks

FOPT heavy-quark contributions: FFNS

Massive coefficient functions

Perturbative stability of the FFNS

Effective heavy-quark PDFs: VFNS

Relevance of the VFNS

General-mass VFNS's

Plausibility of the GM-VFNS's

Present HQ treatments in DIS

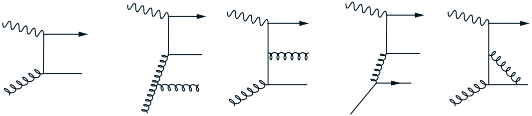
FOPT heavy-quark contributions: FFNS

Experiment: **No** (< 1%) **intrinsic** heavy-quark content in the nucleon

HQ generated in hard collisions, *not collinearlly*, short "lifetime" ($\neq$ parton)

$\Rightarrow$ **Final state** $\equiv$ **extrinsic** heavy-quark content; **fully massive calculations**

$\Rightarrow$**FOPT** initiated by gluons and light (u,d,s) quarks $\equiv$ **FFNS** (in this context)



$$F^h_{k=2,L}(x,Q^2,m^2) = \frac{\alpha_s(\mu^2)Q^2}{4\pi^2 m^2} \sum_{f=g,q,\bar{q}} \int_{z_{th}}^{1} dz\, f(z,\mu^2)\, \hat{F}^h_{k=2,L}(\tfrac{x}{z},\mu^2,Q^2,m^2), \quad z_{th} = x(1+\tfrac{4m^2}{Q^2})$$

Gluon dominated (starts at LO), therefore "small-x"-dominated:

about 80% originates in the region $z_{th} \leq z \leq 3z_{th}$ [A.Vogt 96]

$\Rightarrow$ *threshold region is always important* (irrespective of Q^2)

Massive coefficient functions

Theoretical status:

The (inclusive) coefficient functions are known at LO [Witten 75, Glück and Reya 79] and NLO [Laenen,Riemersma,Smith,van Neerven 93]:

$$\hat{F}^{h}_{k=2,L}(\tfrac{x}{z},\mu^2,Q^2,m^2) = e_h^2\delta_{fg}\, c^{(0)}_{k,g} + 4\pi\alpha_s(\mu^2)\left[e_h^2\left(c^{(1)}_{k,f}+\bar{c}^{(1)}_{k,f}\ln\frac{\mu^2}{m^2}\right)+e_f^2\left(d^{(1)}_{k,f}+\bar{d}^{(1)}_{k,f}\ln\frac{\mu^2}{m^2}\right)\right]$$

There is a fully exclusive NLO calculation [Harris and Smith 95]: **HVQDIS**, **in which all the experimental analysis at HERA is based**

At NNLO only the asymptotic ($Q^2 \gg m^2$) coefficients [Bierenbaun,Blümlein,Klein 09] exist:

There is no complete NNLO (massive) calculation of HQ contributions in DIS

Some *approximations* can be made using small-x [Catani, Cialfoni, Hautmann 91] and threshold [Laenen and Moch 99] resummations

The coefficient functions contain potentially large $\ln\frac{\mu^2}{m^2}$'s (**not mass divergences**):

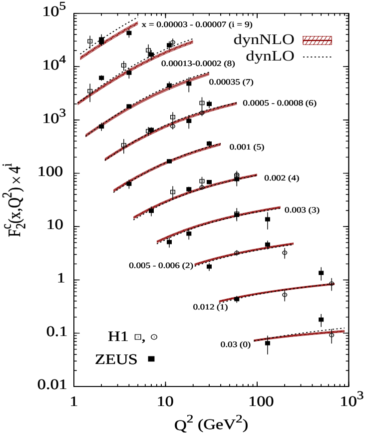
Are these terms dangerous or is the FFNS stable for DIS phenomenology?

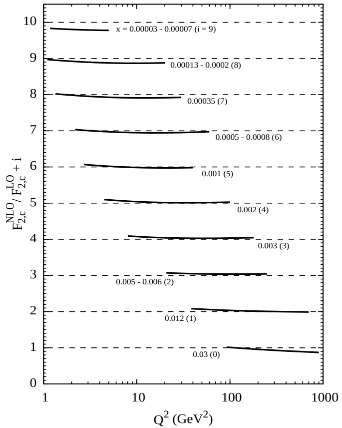
Perturbative stability of the FFNS

It should be clear since (again):

all the experimental analysis at HERA is based on the FFNS (HVQDIS)

Nevertheless we can have a look at the (semi)inclusive calculation (used in the fits):





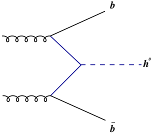
FFNS gets trough *all* “stability tests”!!

No need to resum *supposedly* “large logarithms”... why are there other schemes then?

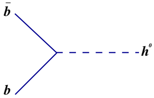
Effective heavy-quark PDFs: VFNS

The only *drawback* of the FFNS is the calculational difficulty, for instance:

FFNS: $g\,g \to b\,\bar{b}\,H$



VFNS: $b\,\bar{b} \to H$



We can construct *effective* heavy-quark PDFs from the the asymptotic limit of the massive calculation [Buza, Matiouine, Smith, van Neerven 98]:

$$H^{Q^2 \gg m^2}\left(\frac{Q^2}{\mu^2}, \frac{\mu^2}{m^2}\right) = A\left(\frac{\mu^2}{m^2}\right) \otimes C\left(\frac{Q^2}{\mu^2}\right) \quad \Rightarrow \quad f_j^{VFNS} = \sum_k A_{jk} \otimes f_k^{FFNS}$$

A's=massive OME's, process independent $\Rightarrow$ **preserves universality!!**
C's=light-parton coefficient functions

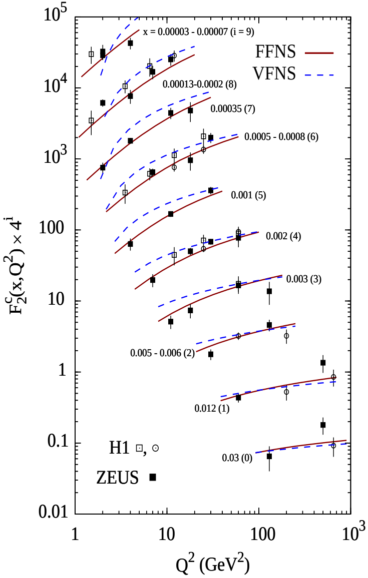
In practice: **massless evolution** with increasing n_f at unphysical "thresholds" $\mu^2 \simeq m^2$

This resums (RGE) the $\ln \frac{\mu^2}{m^2}$'s of the final-state contributions $\neq$ intrinsic HQs

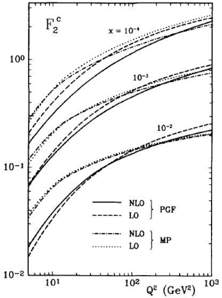
The effective VFNS HQ-PDFs are *assumed* to be correct asymptotically but:

Is this scheme relevant for DIS phenomenology?

Relevance of the VFNS



Even though $W^2 \gg m_c^2$ for *all* data:
At small-x (small Q^2): **gross overestimate**
It does better at larger x (larger Q^2)



[Glück,Reya, Stratmann 94]

It never reduces to the exact (FFNS) result
(not even at very large Q^2):
dropped terms ($\propto \frac{m^2}{Q^2}$) *are always relevant*

The VFNS should not be used for global analyses!! (this is well-known since a long time)

Relevance of the VFNS

VFNS reliable for large invariant mass of the produced system: $W_{th}^2 \gg m^2$

$\Rightarrow$ *non-relativistic* ($\beta = \frac{|\vec{p}|}{E} \lesssim 0.9$) *threshold effects supressed* [Glück,Reya,PJD 08]

For charm production in neutral-current DIS: $\frac{W_{\rm th}}{m_c} = 2 \Rightarrow$ VFNS fails

For the previous example, Higgs-boson production in $b\bar{b}$ fusion:

$$\frac{W_{\rm th}}{m_b} = \frac{2m_b + m_H}{m_b} \simeq \frac{m_H}{m_b} \gg 1 \Rightarrow \text{VFNS should work}$$

Note that *we can* generate VFNS PDFs from our FFNS PDFs (3-flavor input)

Input determined using DIS data and the FFNS!!

This combines the virtues of both **FFNS + VFNS** schemes

Typical scheme-choice uncertainties? Example, W production at *LHC*:

$$\sigma^{\rm NLO}(pp \to W^+ + W^- + X) = \begin{cases} 186.5 \pm 4.9_{\rm pdf} \, {}^{+4.8}_{-5.5}\,|_{\rm scale} \text{ nb} & \text{(VFNS)} \\ 192.7 \pm 4.7_{\rm pdf} \, {}^{+3.8}_{-4.8}\,|_{\rm scale} \text{ nb} & \text{(FFNS)} \end{cases}$$

VFNS sufficiently accurate ($\approx 10\%$) for LHC and Tevatron energies.

General-mass VFNS's

Phenomenological *models* for HQ contributions in *inclusive* DIS; idea:

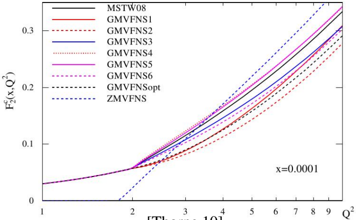
Interpolation between FFNS and VFNS

Constructed (as the VFNS) over the FFNS: **no new information** (e.g. no complete NNLO)

Often goes something like: $F^{GM} \equiv F^{FF} + F^{ZM} - F^{\text{model}}$ with:

$F^{\text{mod}} \to F^{\text{FF}}$ for $Q^2 \gg m^2$

In the intermediate region there is a lot of "freedom" $F^{\text{mod}} \to F^{\text{ZM}}$ for $Q^2 \simeq m^2$



[Thorne 10]

... and correspondingly many implementations:

ACOT [Aivazis,Collins,Olness,Tung] + variations

BMSN [Buza,Matiounine,Smith,van Neerven]

CSN [Chuvakin,Smith,van Neerven]

RT [Roberts,Thorne] + variations

FONNL [Forte,Laenen,Nason,Rojo]

(Although some of them are known "not to work" properly)

Scheme choices affect the PDFs and in turn the predictions for physical observables

Plausibility of the GM-VFNS's

In GM-VFNS's, DIS mass dependences are somewhat absorbed into the PDFs:

Process-dependent PDFs? **What happened with universality?**

GM-VFNS's are *unnecesary* for HERA (FFNS) and for Tevatron or LHC (FF+VF):

What is the advantage of interpolation models?

My opinion (there are different opinions in the audience):

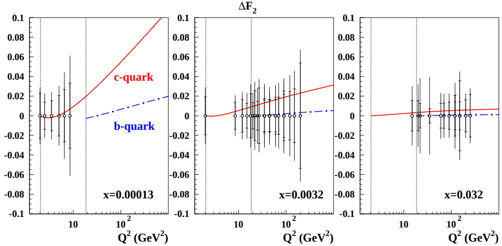
We should not try to model HQ contributions, but stick to schemes unequivocally defined on solid theoretical grounds

Anyway, arguing about which is the best scheme is rather superfluous:

How do the differences compare with the experimental errors?

We can compare $F^{h,FFNS} - F^{h,BMSN}$ with ΔF_2^{tot} [Alekhin,Blümlein,Klein,Moch 10]

$\Rightarrow$ generally effects are smaller (but in some regions in the limit)



Present HQ treatments in DIS

Traditionally the most "popular choice" for global fits was the **VFNS**, with the exception of the GRV group, which used the FFNS *already* for GRV95

This changed after the release of CTEQ6.5 (2007), where a GM-VFNS was used and the effects of HQ masses in the predictions at hadron colliders were "re-discovered": **today their importance is generally recognized**

Current choices of the (main) PDF groups are:

- CTEQ: ACOT-like
- MSTW and HERAPDF: TR-like
- NNPDF: VFNS (switching to FONLL)
- ABKM: both FFNS and BMSN
- (G)JR: FFNS and VFNS (generated from the FFNS)

The experimental analyses use the FFNS (exclusive calculations needed)

Summary and conclusions

An appropriate treatment of heavy quarks in global PDF analyses is *vital* for future results in high-energy colliders

The current theoretical status is (and will be for a while) NLO

The **FFNS is a stable and reliable framework** for the treatment of HQ contributions (there is no need to resum *supposedly* "large logarithms")

The **VFNS is** not valid for global analyses but **sufficiently accurate for LHC and Tevatron** energies (the input distributions must be generated in the FFNS)

Other "popular" choices (which we do *not recommend*) are the so-called GM-VFNS's (model/process dependence)

Schemes choices are a source of theoretical uncertainties in the predictions for high-energy colliders which cannot (and should not) be hidden "by convention"