Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare

Impact of *pdfs* and α_s uncertainties on Higgs production via gluon fusion

A critical application of the PDF4LHC recipe to the calculation of the Higgs total production cross section via gluon fusion

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Motivations and content of the talk

- application of the PDH4LHC recipe for the combination of pdf+alphas uncertainties
  to the Higgs production via gluon fusion total cross section
  at NLO-QCD and at NNLO-QCD

- some comments
  on the weak correlation between pdf and alphas uncertainties
  on the problem of alphas(MZ)
  on the practical implementation of the PDF4LHC recipe

The hadronic cross section via gluon fusion

$$\sigma(h_1 h_2 \to H + X) = \sum_{a,b} \int_0^1 dx_1 dx_2 \; f_{a,h_1}(x_1, \mu_F^2) f_{b,h_2}(x_2, \mu_F^2) \times \int_0^1 dz \; \delta\left(z - \frac{\tau_H}{x_1 x_2}\right) \hat{\sigma}_{ab}(z)$$

The results in this talk at

NLO-QCD (code GGSCA by Aglietti, Bonciani, Degrassi, AV)
NNLO-QCD (code GGSCA plus NNLO-QCD by Anastasiou et al.)

- best knowledge of the matrix element: the basic request to obtain a reliable prediction
  NNLO-QCD still very large, soft-gluon resummation and NLO-EW not negligible
  stabilization from further higher orders

- this process strongly depends on alphas

$$\sigma_{tot} = \alpha_s^2 \sigma_0 + \alpha_s^3 \sigma_1 + \ldots \qquad\qquad \sigma_0 \sim \alpha_s \sigma_1$$

and the sensitivity to a variation of alphas is roughly approximated by

$$\boxed{\frac{\Delta\sigma_{\alpha_s}}{\sigma} \sim 2.5 \; \frac{\delta\alpha_s}{\alpha_s}}$$

- the gluon and alphas are correlated but the relevant quantity is the gg-luminosity

- we want to study the pdf+alphas interplay
  ➔ we need parton sets extracted with different alphas reference values

The PDF4LHC recipe

NLO-QCD

- compute the Higgs xsec using CTEQ66, MSTW2008, NNPDF20
- for each set, use the preferred alphas(MZ) value (0.118, 0.1207, 0.119)
- compute the pdf+alphas uncertainty band, according to the rules of each pdf group
- take the envelope of the three resulting bands
- compute the mid-point and call uncertainty the distance of the envelope from it

NNLO-QCD

- from the NLO-QCD exercise determine the percentage uncertainty from the mid point of the envelope.
- determine the ratio of this percentage uncertainty to MSTW2008nlo
- compute central value and pdf+alphas uncertainty with MSTW2008nnlo
- rescale the MSTW2008nnlo percentage uncertainty by the above ratio

Which central value and 1-σ error for $\alpha_s(m_Z)$?

World average (PDG) $\alpha_s(m_Z) = 0.1176(20)$

World average (Bethke) $\alpha_s(m_Z) = 0.1184(7)$
(new value on the PDG web update)

$\alpha_s(m_Z) =$

CTEQ6.6	0.118	} PDG values
NNPDF2.0	0.119	
MSTW2008nlo	0.12018	MSTW determination

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As a compromise for the 1-σ error we take

$\delta_{\alpha_s}^{(90)} =$ 0.002 as a 90% C.L.

$\delta_{\alpha_s}^{(68)}$ = 0.002/1.64885 = 0.0012 as a 68% C.L.

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$\delta_{\alpha_s}^{(68)}$ = 0.002/1.64885 = 0.0012 as a 68% C.L.

It is crucial to use the same variation of alphas when computing the uncertainty bands

Combination of *pdfs* and α_s uncertainties : full correlation

- CTEQ6.6 recommends to sum pdf and alphas uncertainties in quadrature

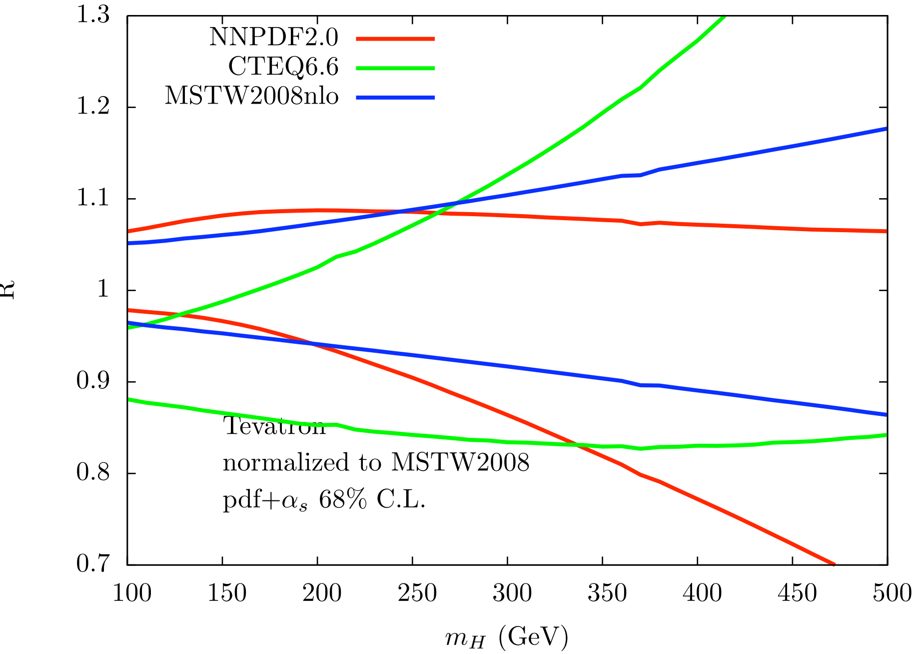
  In arXiv:1004.4624 it has been demonstrated that in the Hessian approach the sum in quadrature is exact.

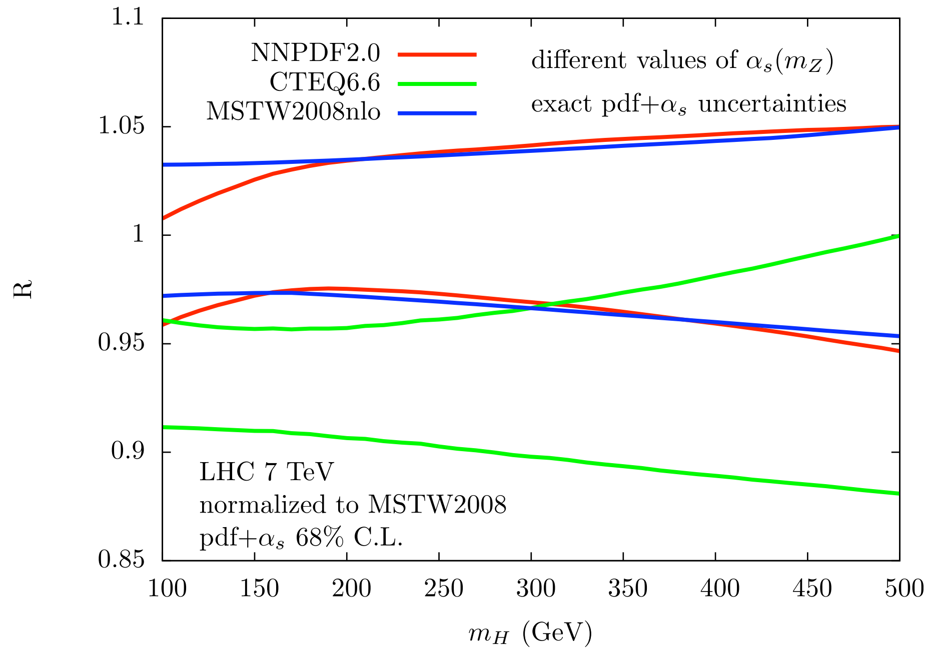
- MSTW2008 recipe described in **Eur.Phys.J.C64:653-680,2009. arXiv:0905.3531**
  a variation of α_s by $\delta_{\alpha_s}^{MSTW} = 0.12018^{+0.0012}_{-0.0015}\ (68\%)\ {}^{+0.0032}_{-0.0039}\ (90\%)$
  is used in the different *pdf* sets

- NNPDF2.0 recommends to treat α_s as a gaussian variable
  and to combine accordingly the replicas extracted with different
  in a Montecarlo way

(note that Hessian approach is not used
so the sum in quadrature is not necessarily exact)

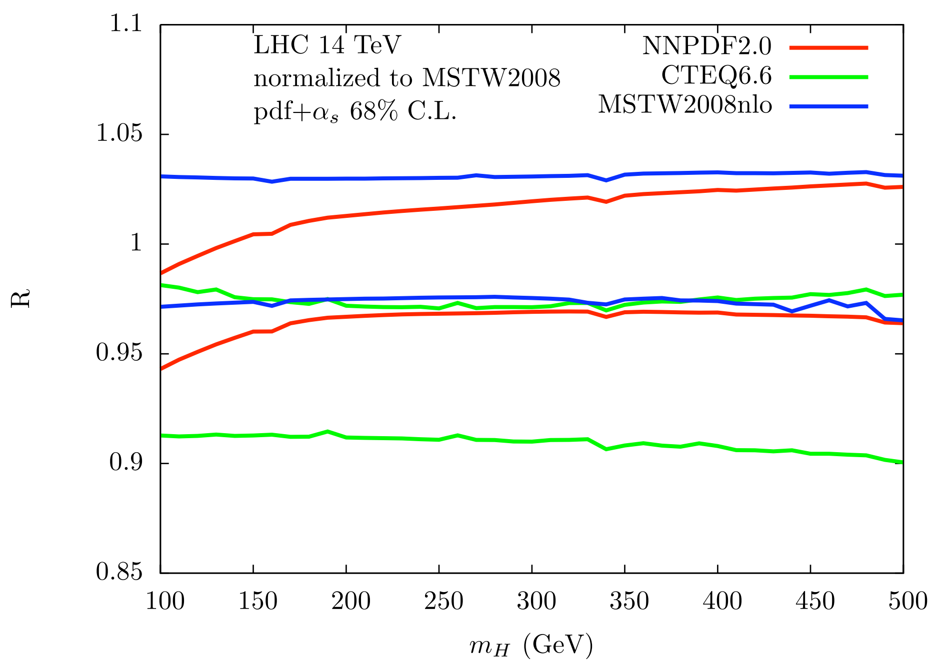
Bands including pdf + as uncertainty (normalized to MSTW2008)



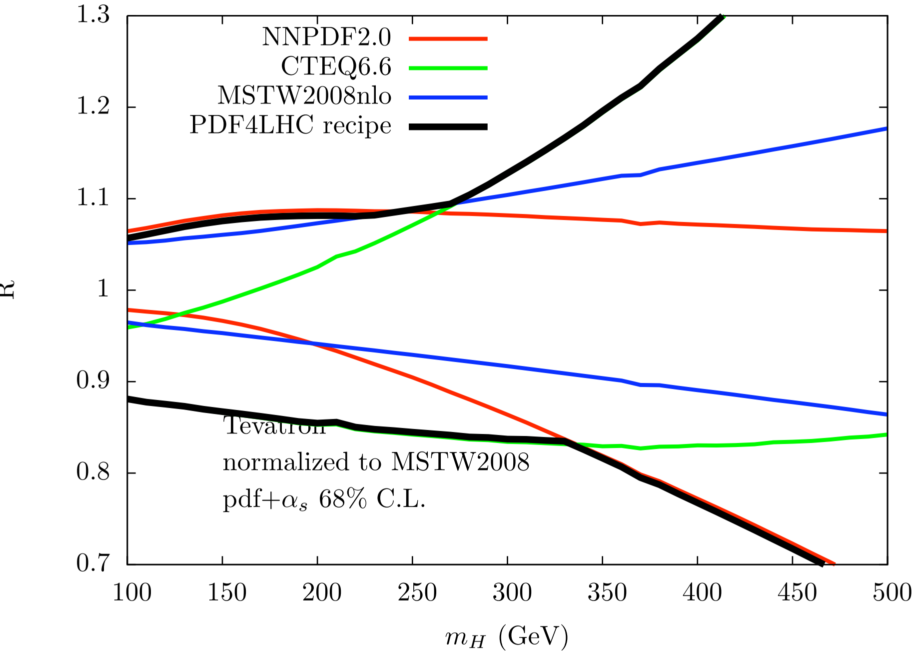


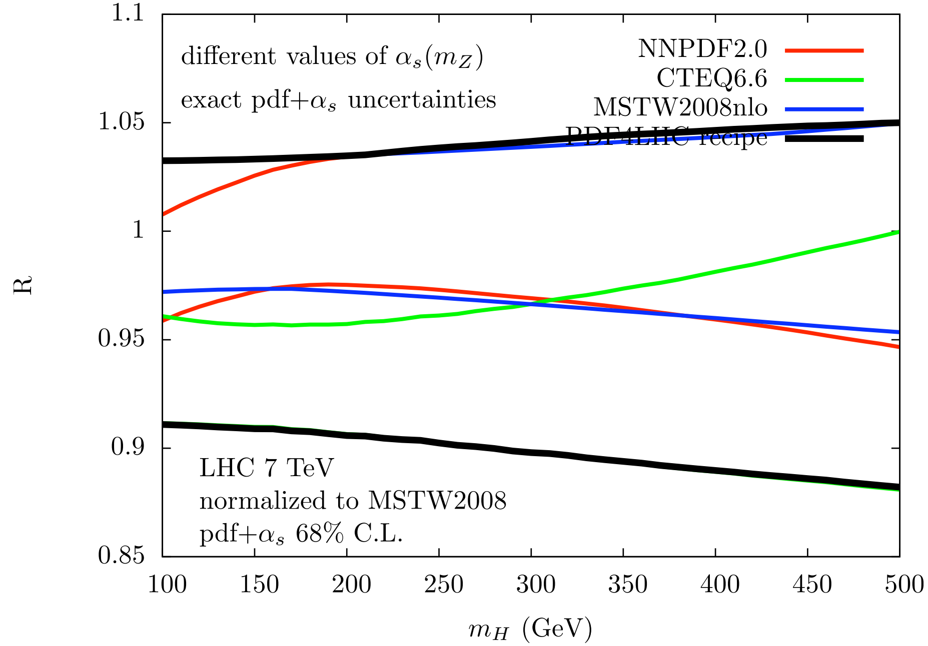
the preferred alphas(MZ) values have been used

$$\delta\alpha_s^{NLO}(68\%) = 0.0012$$

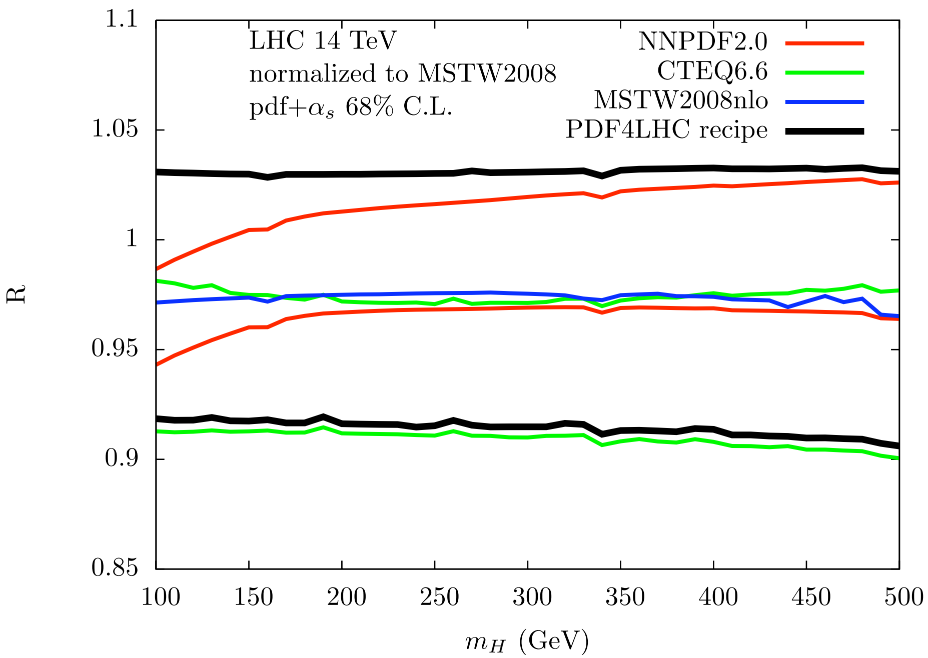


Bands including pdf + as uncertainty (normalized to MSTW2008)

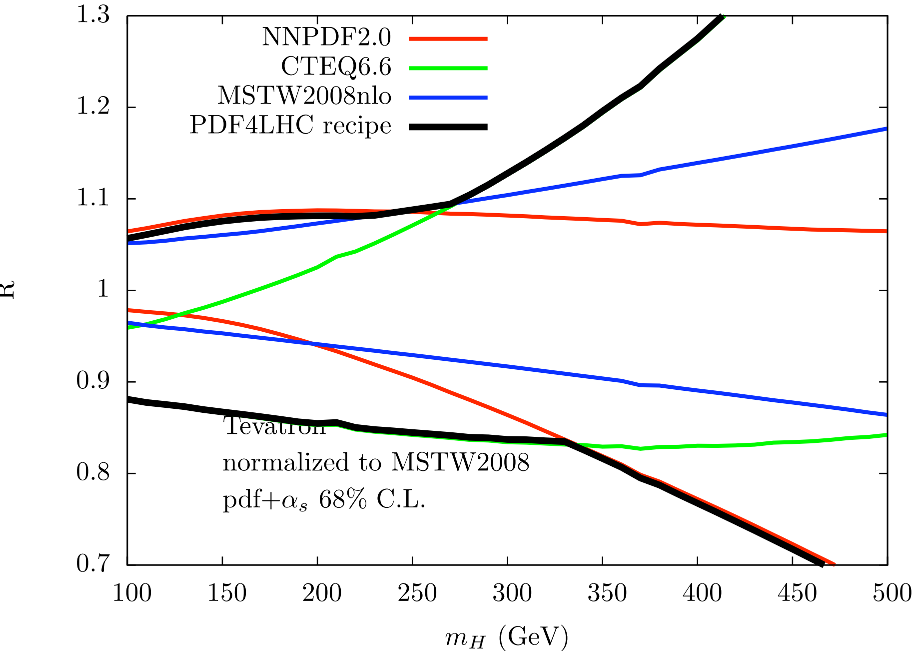


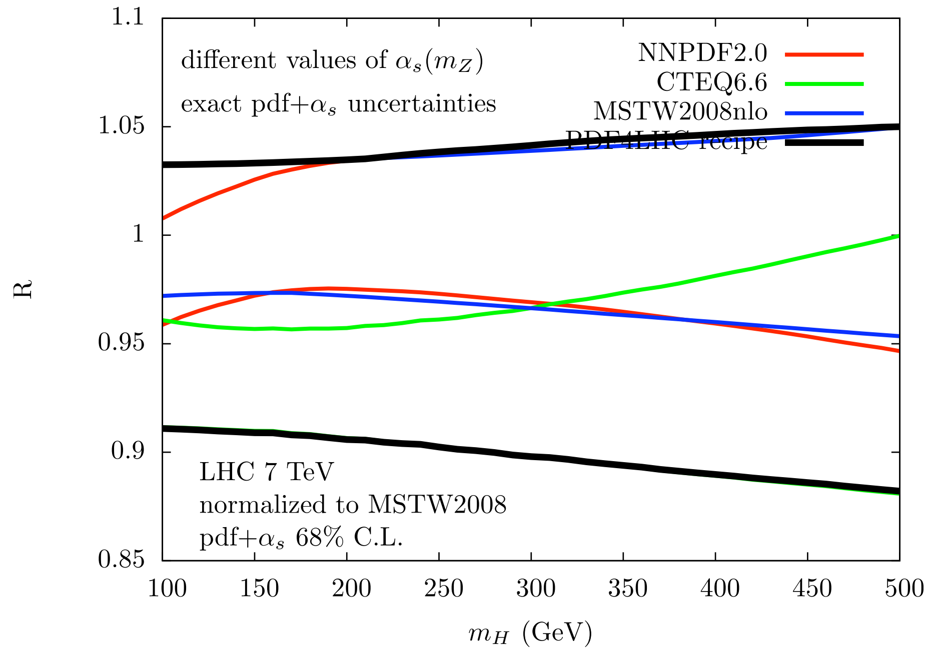


the envelope represents
the result of the PDF4LHC recipe



Bands including pdf + as uncertainty (normalized to MSTW2008)





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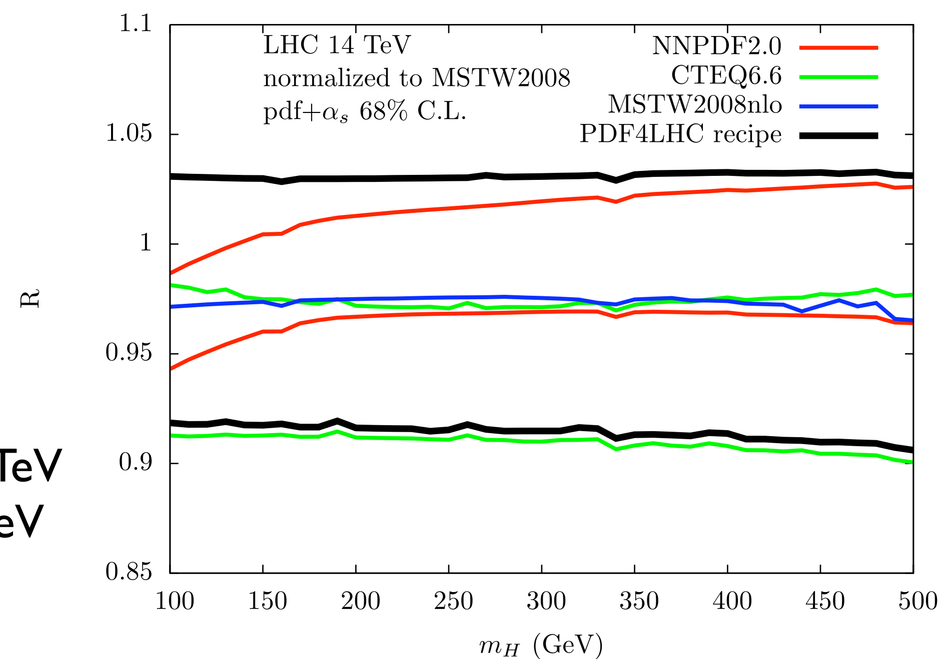
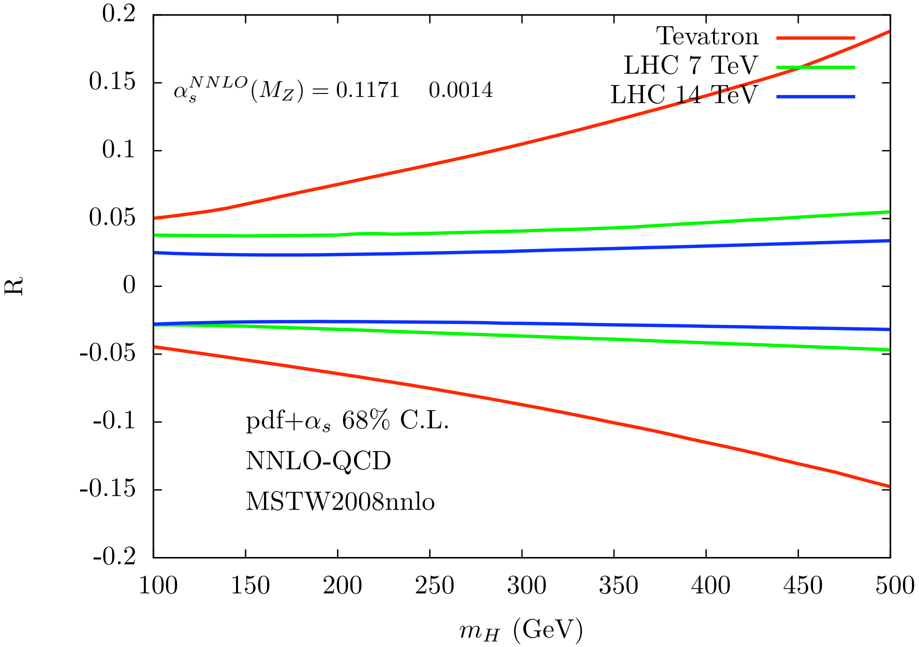


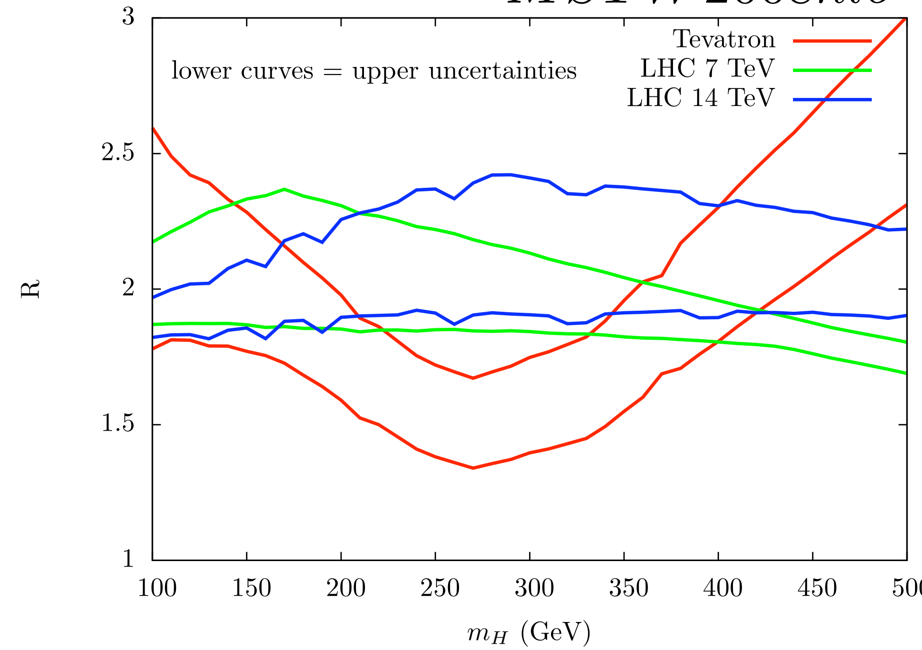
from ± 9% (MH=100) to ± 12% (MH=250) Tevatron
from ± 5.5% (MH=100) to ± 6.5% (MH=250) LHC 7 TeV
from ± 5% (MH=100) to ± 5% (MH=250) LHC 14 TeV

Uncertainty pdf+alphas bands at NNLO-QCD (MSTW2008nnlo)

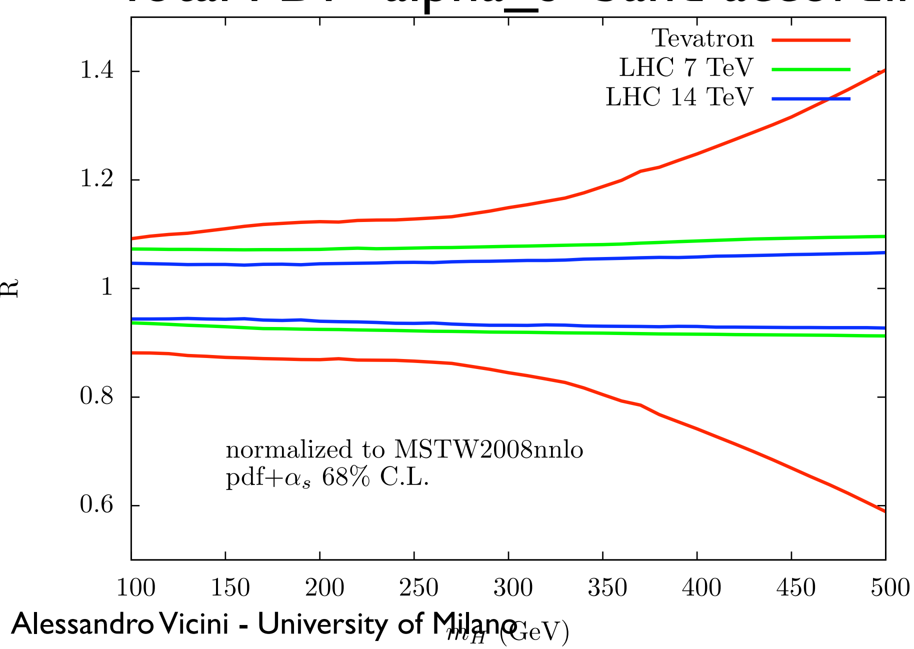
MSTW2008nnlo PDF+alpha_s



Rescaling ratio : $\frac{NLO-QCD \quad envelope}{MSTW2008nlo}$



Total PDF+alpha_s band according to PDF4LHC recipe



	130 GeV	160 GeV	250 GeV
Tevatron	+10 -12%	+11 -12%	±13%
LHC 7	+7.2 -6.8%	+7.2 -7.1%	+7.4 -7.8%
LHC 14	+4.5 -5.6%	+4.5 -5.7%	+4.8 -6.4%

Practical implementation of the PDF4LHC recipe

- The calculation of the combined PDF + alpha_s uncertainty requires the evaluation of a large number of PDF sets

CTEQ66	46	sets (44 + 2 sets)	(alpha_s = central, ± 1σ)
MSTW2008	200	sets (5 x 40 sets)	(alpha_s = central, ± 1σ ,±0.5 σ)
NNPDF20	310	sets (1+5+27+72+100+72+27+5+1)	(alpha_s=0.115-0.123)

- The calculation can be speed up by reweigthing each phase-space point with all the necessary PDFs

- The estimate of the combined uncertainty can be obtained, introducing some approximations, with a smaller amount of CPU

given the weak correlation between the parton luminosity and alpha_s
MSTW suggests to sum in quadrature
PDF uncertainty and alpha_s uncertainty estimated using the central PDF sets with ± 1σ alpha_s variations
→ 42 sets

the number of replicas that have to be used with NNPDF can be reduced
according to the gaussian distribution chosen for alpha_s

e.g. (0+1+4+12+16+12+4+1+0) (alpha_s=0.115-0.123) → 50 sets

increasing the statistical uncertainty in the detrmination of the central values

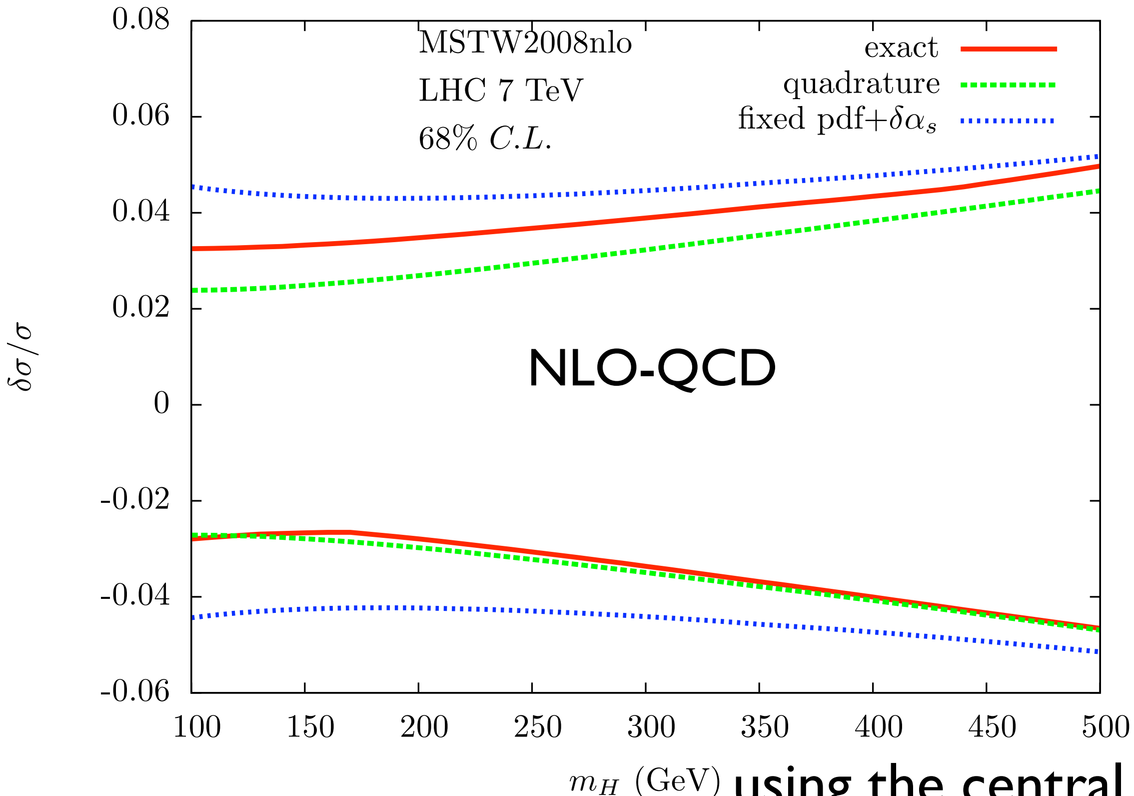
Combination α_s + *pdfs*: MSTW2008nlo

1) full correlation

sum the two uncertainties in quadrature with

2) variation of α_s in the partonic xsec AND in the *pdfs*

3) variation of α_s in the partonic xsec, keeping fixed the *pdfs*



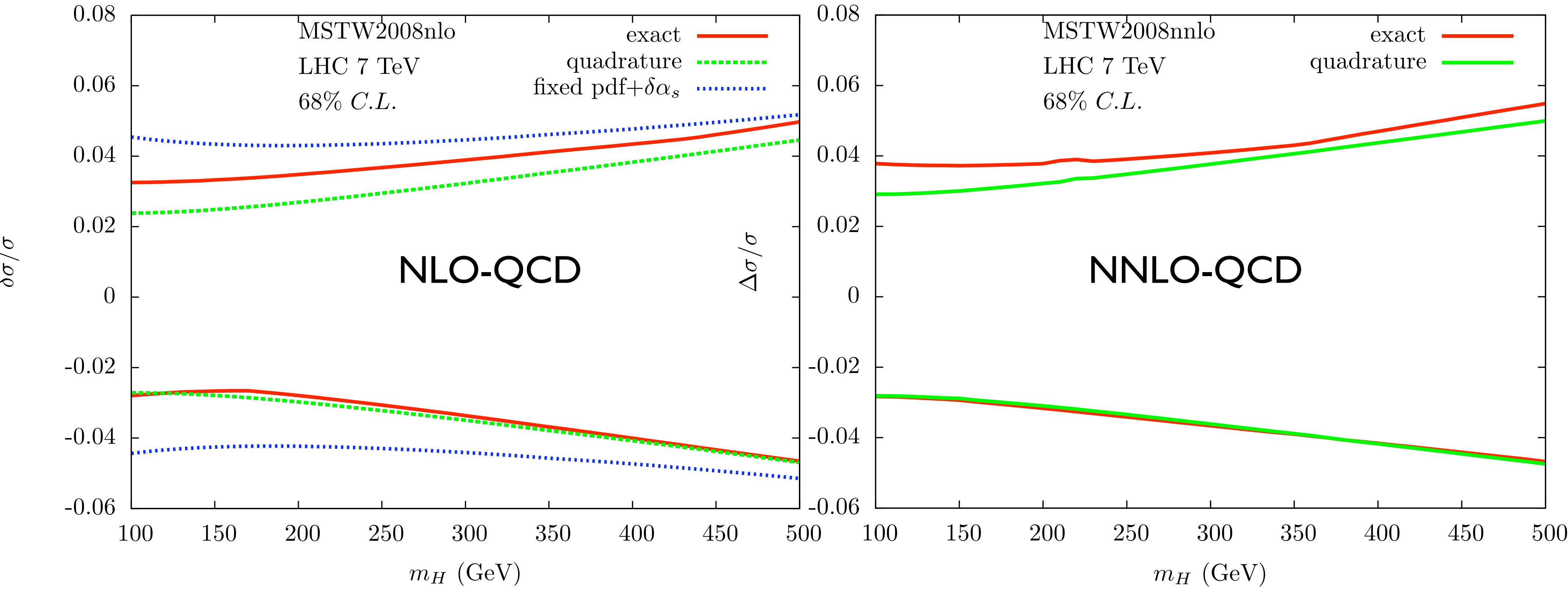
68% C.L.

$$\delta^{(68)}_{\alpha_s} = ^{+0.0012}_{-0.0015}$$

using the central values of MSTW2008nlo68cl_asmz+68cl.LHgrid
MSTW2008nlo68cl_asmz-68cl.LHgrid
in

The sum in quadrature is a quite good approximation of the full correlation recipe

pdf+alphas: quadrature *vs* exact, MSTW2008 NLO *vs* NNLO



A possible problem

The PDF4LHC recipe double counts at 68% C.L. the alphas uncertainty:

the 3 preferred values are already distant ~ 1σ w.r.t. 0.119

and

the pdf+alphas bands account for a 1σ variation of alphas about the preferred value

The recipe is conservative (it includes 2σ alphas spread)

→

can we discuss separately pdf and alphas issues
and add, at the end, the alphas uncertainty only once ?

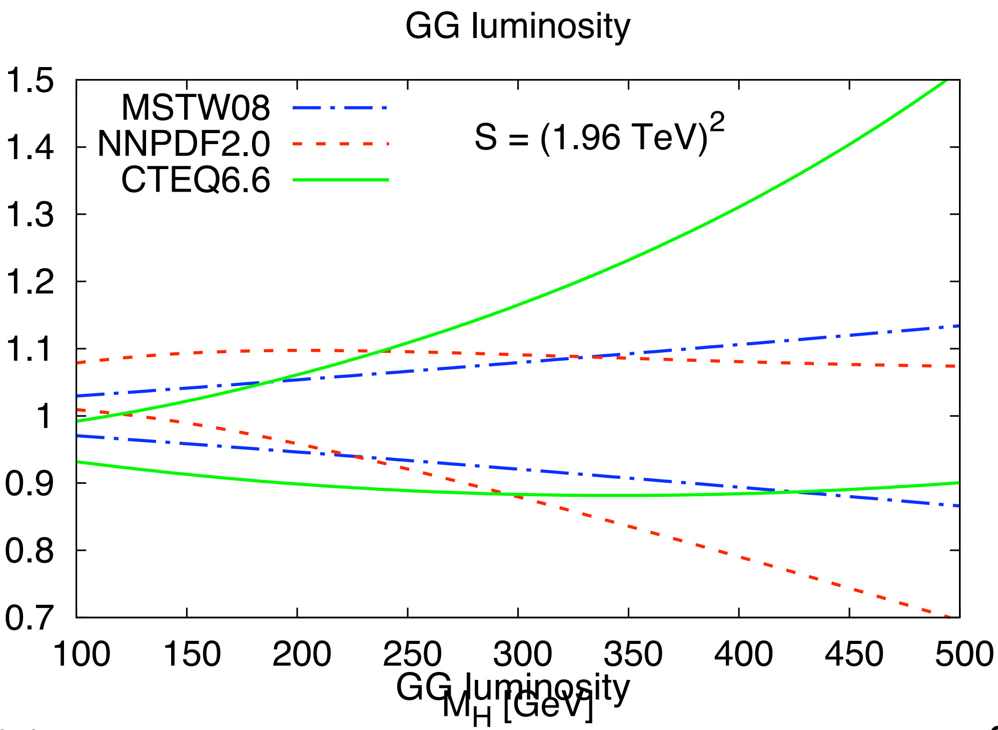
Few facts about the pdf and about the alphas uncertainties

The purely pdf part of the uncertainty can be read from the gluon-gluon luminosities, which show

1) agreement of the central values of the 3 groups
2) good agreement of the size of the relative pdf uncertainty bands

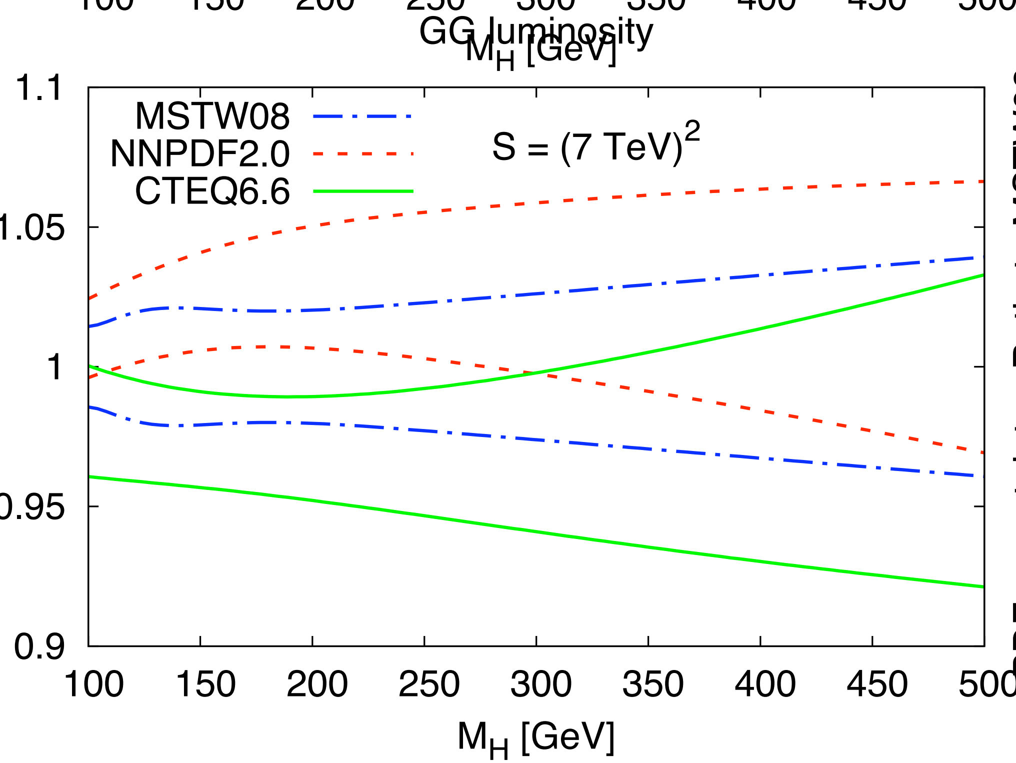
Non trivial statement, given the very different parametrizations, methods,...

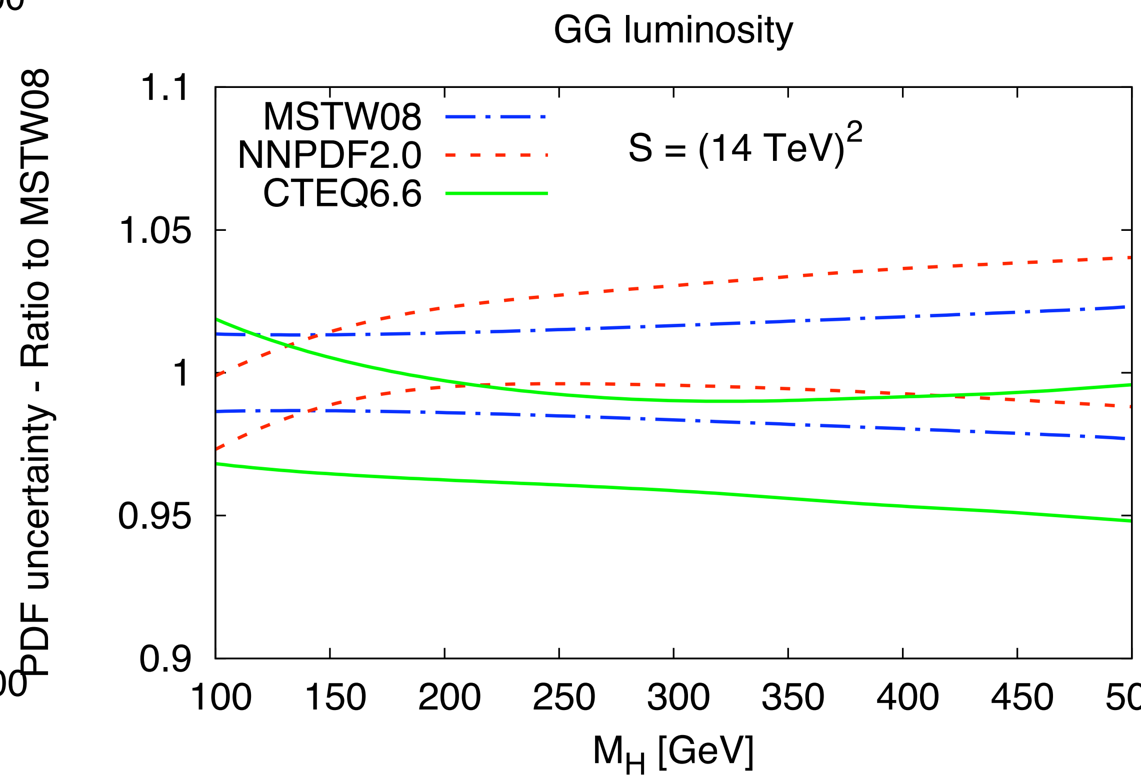
Gluon-gluon luminosity (normalized to MSTW2008) with NNPDF2.0



the gluon-gluon luminosity
with 68% C.L. *pdf* error bands

the gg-lumi bands have been computed
with the preferred alphas(MZ)
and DO overlap





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Non trivial statement, given the very different parametrizations, methods,...

The pdf and the alphas uncertainties are weakly correlated

1) the sum in quadrature of the two uncertainties
   is close to the exact prediction according to each group combination rule
   even for NNPDF where it is not necessarily the case
2) the statement holds at NLO-QCD and at NNLO-QCD

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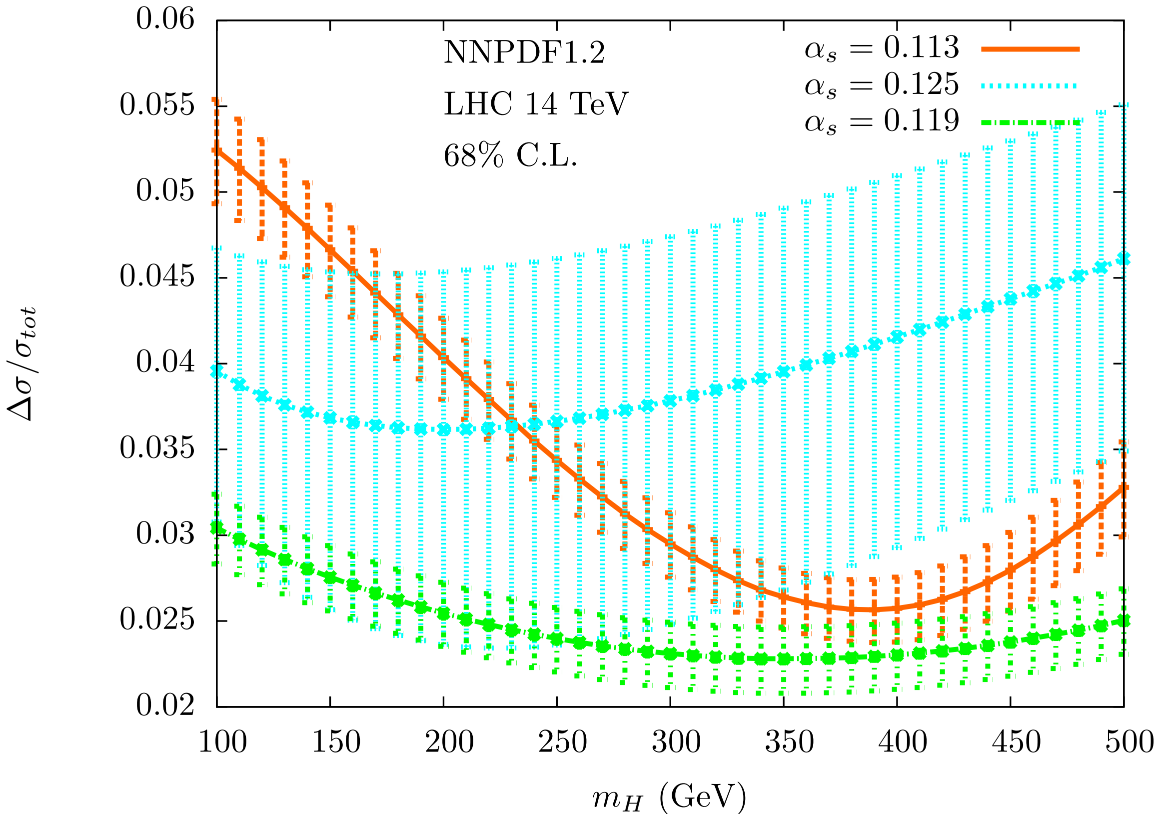
3) the precise definition of alphas uncertainty (i.e.which pdfs)must be specified

The definition of when exact and approximated results are sufficiently close can be taken from uncertainty which affect the pdf uncertainty band.

Uncertainty of the *pdf* uncertainty

The width of the *pdf* uncertainty band is an observable whose variance can be computed

$$\sigma^2[\sigma^2] = \frac{1}{N_{\rm rep}}\left[m_4[q] - \frac{N_{\rm rep}-3}{N_{\rm rep}-1}\left(\bar{\sigma}^2\right)^2\right]$$



The compatibility of *pdf* uncertainty bands corresponding to different alphas can be checked comparing the overlap of $\sqrt{N_{\rm rep}}\;\sigma[\sigma^2]$ where σ is plotted in the figure

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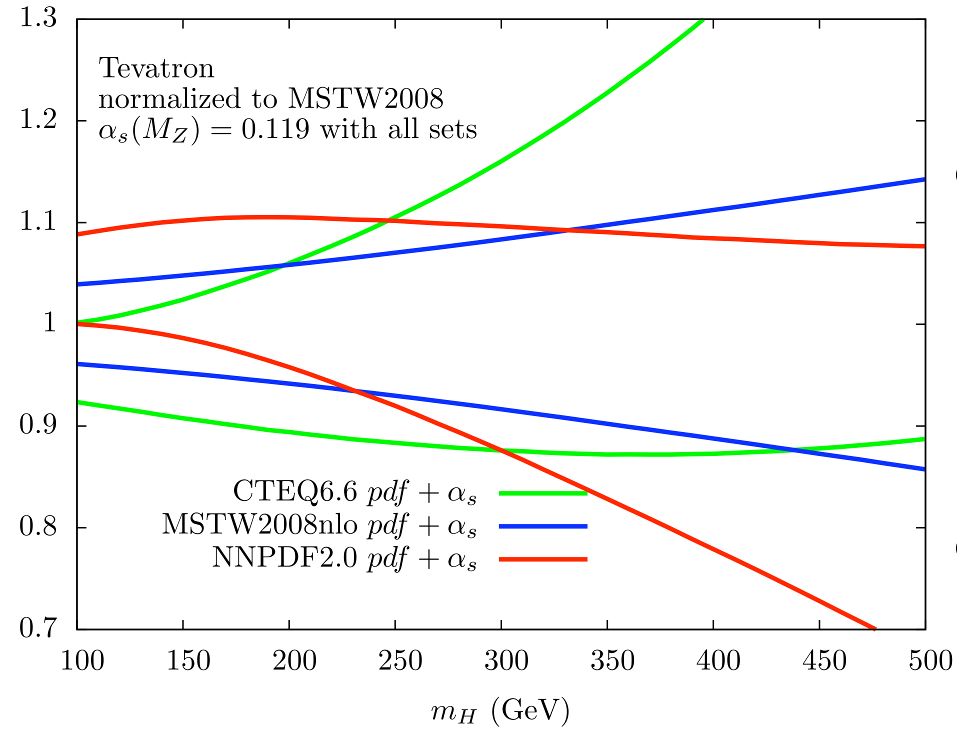
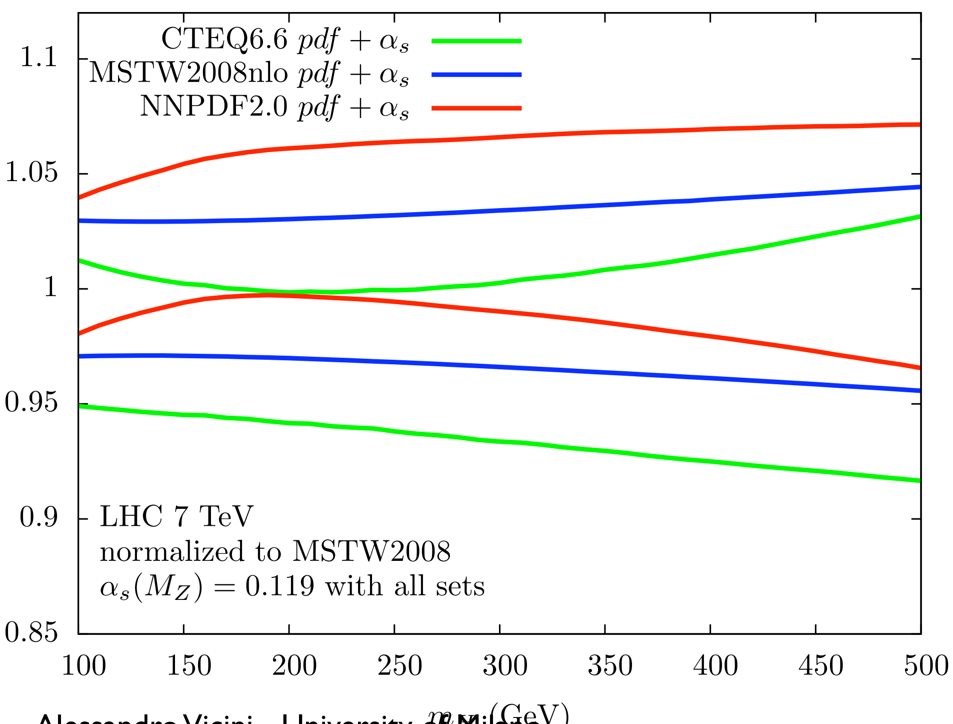
The definition of when exact and approximated results are sufficiently close
can be taken from uncertainty which affect the pdf uncertainty band.

my conclusion:

We can discuss separately the pdf issues (quite in good shape)
the alphas problems (central value, error)

Common alphas=0.119 recipe





- Evaluate the partonic cross section and the pdf uncertainty bands with alphas=0.119 using the best *pdf* sets by the 3 groups

- Keep the *pdf* sets fixed, and vary alphas by ±0.0012; from the difference of the central values derive the 68% C.L. alphas uncertainty

- Sum in quadrature 68% C.L. *pdf* and alphas uncertainties

- Take the envelope of the 3 results

Common alphas recipe

one common value for alphas

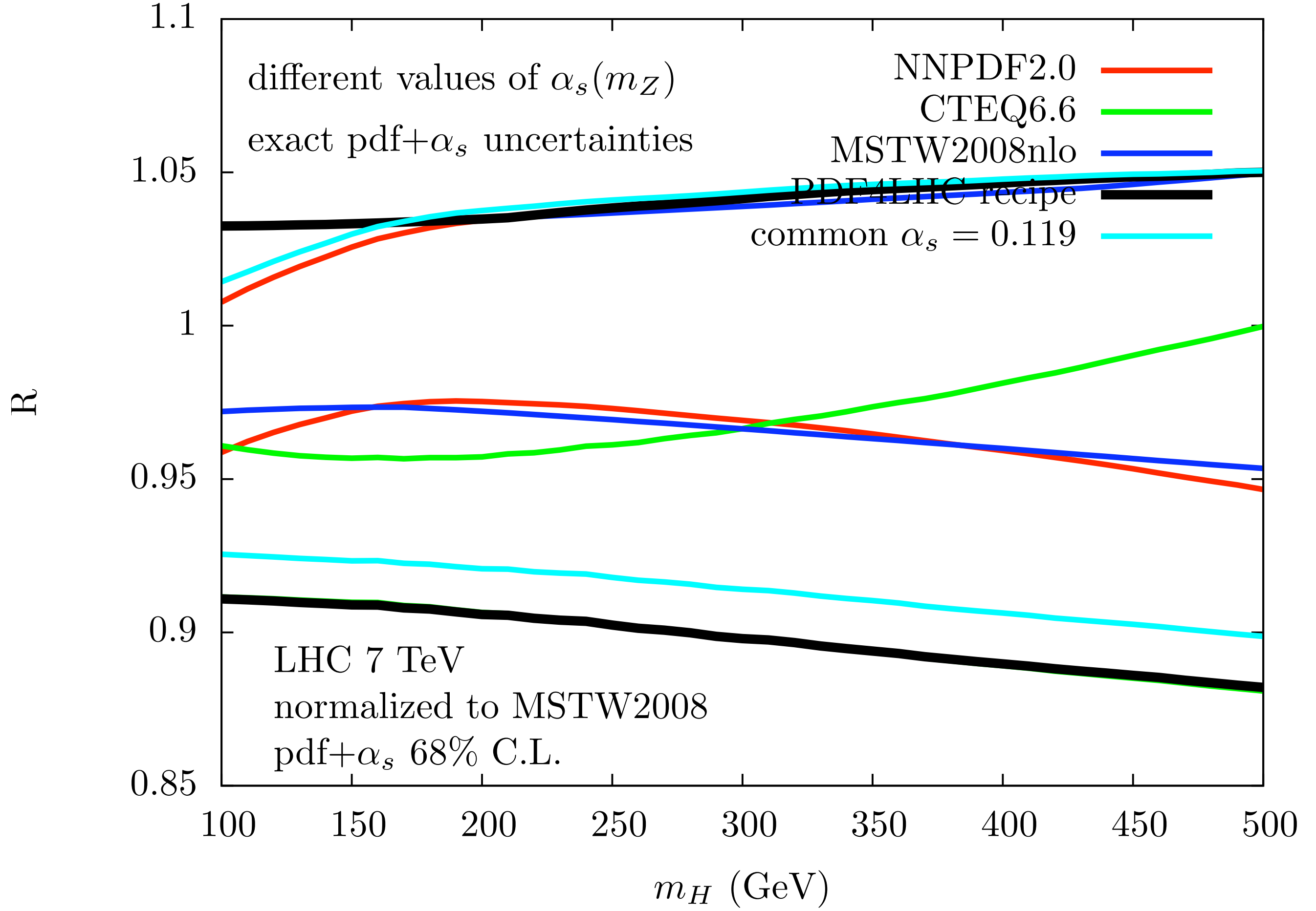
- separates the issue of the uncertainty from alphas,
  from the choice made by individual pdf groups

- the best alphas value should also include information not used in pdf fits
  (LEP data, lattice,...)

the discussion on the error on alphas is the most difficult one
(from 7 to 20 to 44 parts 10^-4 (Bethke, old PDG, Baglio-Djouadi))
but
given a value of the 68% C.L. error on alphas, it is straightforward to compute
the corresponding uncertainty and to add it in quadrature to the pdf unc.

the use of a pdf set extracted with a "non-optimal" alphas value or
of the best pdf set with a different alphas value
introduces a small error in the final result
compatible with the uncertainty on the pdf-uncertainty

“PDF4LHC” *vs* “common alphas=0.119 envelope” recipes



The PDF4LHC yields a slightly larger uncertainty band than the common alphas=0.119 recipe because of the different treatment of the alphas uncertainty

Conclusions

- the PDF4LHC recipe yields results with uncertainties that range

  from ± 9% (MH=100) to ± 12% (MH=250) Tevatron
  from ± 5.5% (MH=100) to ± 6.5% (MH=250) LHC 7 TeV
  from ± 5% (MH=100) to ± 5% (MH=250) LHC 14 TeV

- a common alphas approach would (in my opinion)
  simplify the discussion on the alphas treatment
  without affecting significantly the accuracy of the prediction of the total uncertainty

Back-up slides

Combination of the *pdf*s and of the α_s uncertainties: MSTW2008

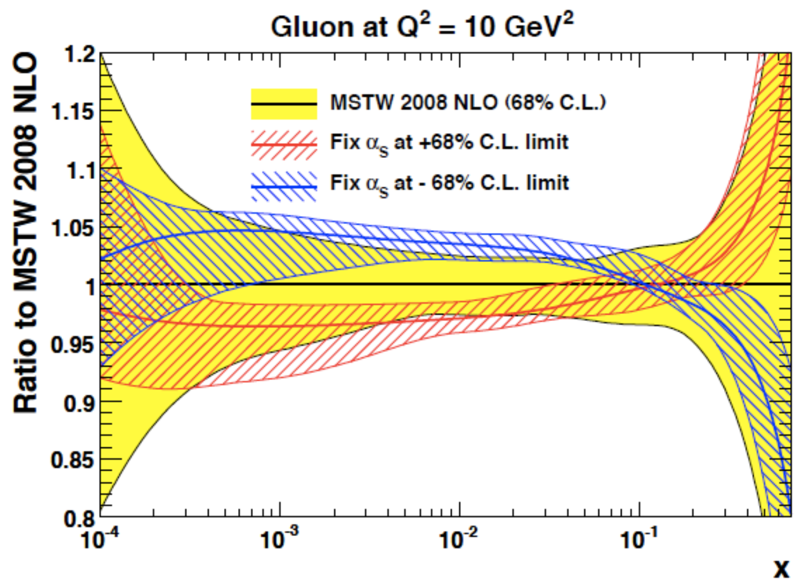
For MSTW, the uncertainty on the *pdf*s and on alpha_s are correlated

$$\alpha_s \in [\alpha_s^0 - 1\sigma,\ \alpha_s^0 + 1\sigma] \qquad \alpha_s^0 \equiv \alpha_s(m_Z) = 0.1202^{+0.0012}_{-0.0015}$$

For each of the 5 values: $\alpha_s^0 - 1\sigma,\ \alpha_s^0 - 0.5\sigma,\ \alpha_s^0,\ \alpha_s^0 + 0.5\sigma,\ \alpha_s^0 + 1\sigma$

there are 40 pdf sets

MSTW2008nlo68cl_asmz+68cl.LHgrid
MSTW2008nlo68cl_asmz+68clhalf.LHgrid
MSTW2008nlo68cl.LHgrid
MSTW2008nlo68cl_asmz-68clhalf.LHgrid
MSTW2008nlo68cl_asmz-68cl.LHgrid



Some *pdf*s spreads are much smaller that the central-value spread

Combination of the *pdfs* and of the α_S uncertainties: MSTW2008

For each of the 5 values compute the *pdf* spread (not necessarily symmetric)

$$(\Delta F_{\rm PDF}^{\alpha_S})_+ = \sqrt{\sum_{k=1}^{n} \left\{ \max \left[F^{\alpha_S}(S_k^+) - F^{\alpha_S}(S_0),\ F^{\alpha_S}(S_k^-) - F^{\alpha_S}(S_0),\ 0 \right] \right\}^2},$$

$$(\Delta F_{\rm PDF}^{\alpha_S})_- = \sqrt{\sum_{k=1}^{n} \left\{ \max \left[F^{\alpha_S}(S_0) - F^{\alpha_S}(S_k^+),\ F^{\alpha_S}(S_0) - F^{\alpha_S}(S_k^-),\ 0 \right] \right\}^2},$$

The (pdf+alpha_s) spread is obtained as follows

$$(\Delta F_{{\rm PDF}+\alpha_S})_+ = \max_{\alpha_S} \left(\{ F^{\alpha_S}(S_0) + (\Delta F_{\rm PDF}^{\alpha_S})_+ \} \right) - F^{\alpha_S^0}(S_0),$$

$$(\Delta F_{{\rm PDF}+\alpha_S})_- = F^{\alpha_S^0}(S_0) - \min_{\alpha_S} \left(\{ F^{\alpha_S}(S_0) - (\Delta F_{\rm PDF}^{\alpha_S})_- \} \right),$$

Combination of the *pdfs* and of the α_s uncertainties: NNPDF2.0

$$\sigma_{\mathcal{F}} = \left(\frac{1}{N_{set}-1} \sum_{j=1}^{N_\alpha} \sum_{k_j=1}^{N_{rep}^{\alpha_s^{(j)}}} \left(\mathcal{F}[\{q^{(k_j,j)}\}] - \mathcal{F}[\{q^{(0)}\}] \right)^2 \right)^{1/2}$$

N_α number of distinct values of α_s used

$$N_{rep}^{\alpha_s^{(j)}} \propto \exp\left(- \frac{\left(\alpha_s^{(j)} - \alpha_s^{(0)}\right)^2}{2\left(\delta_{\alpha_s}^{(68)}\right)^2} \right)$$

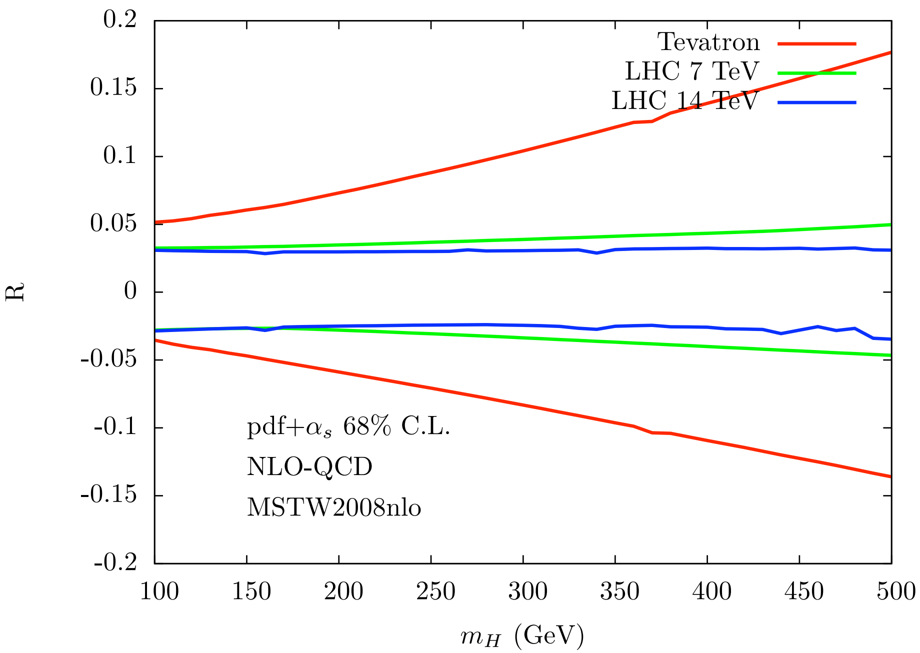
number of replicas used extracted with $\alpha_s^{(j)}$

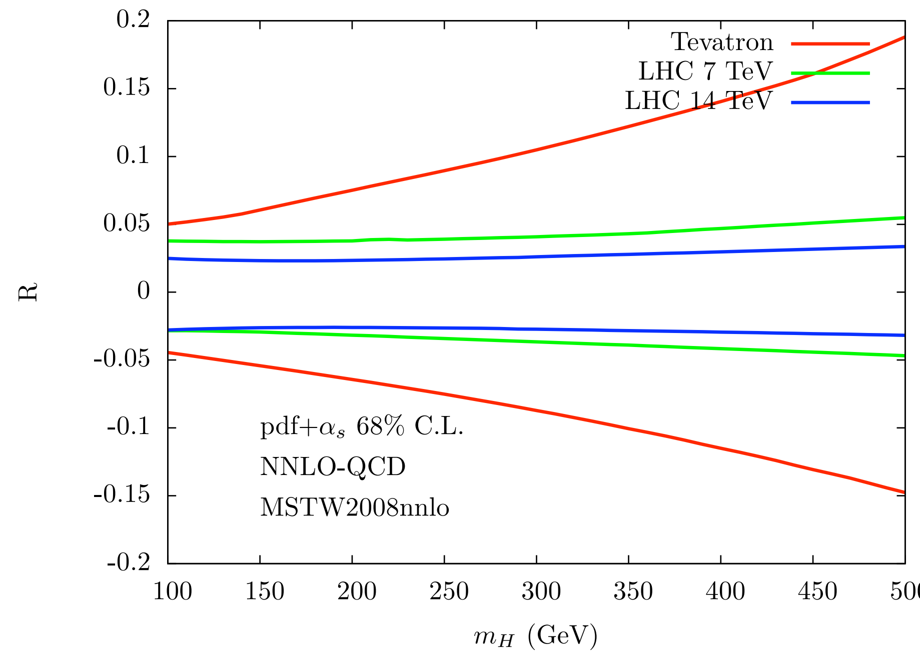
$$N_{set} = \sum_j N_{rep}^{\alpha_s^{(j)}}$$

total number of replicas

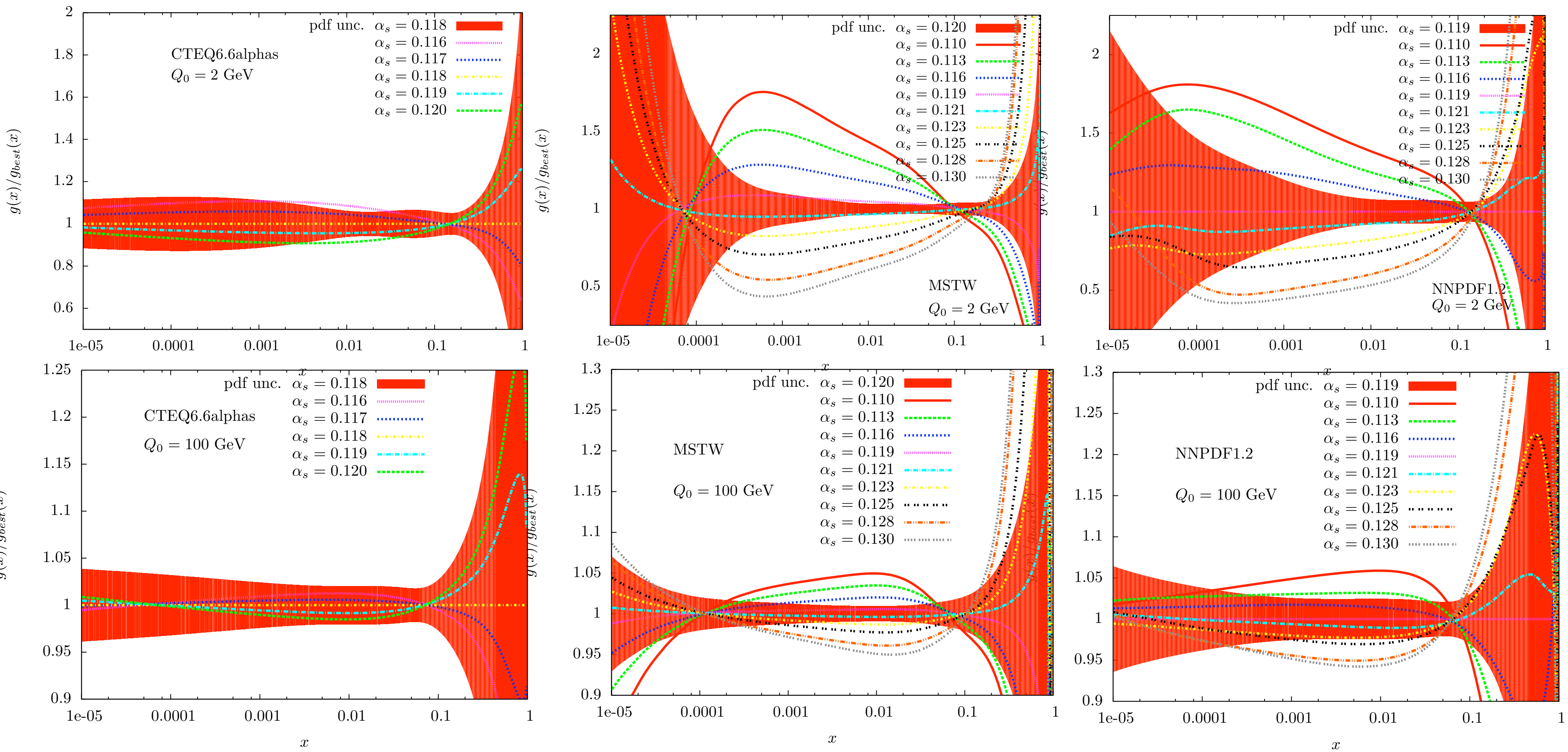
$$\delta_{\alpha_s}^{(68)} = 0.0012$$

pdf+alphas uncertainties at 68% C.L. MSTW2008 NLO vs NNLO





Comparison of gluon densities extracted with different values of α_s



Red bands: gluon *pdf* uncertainty, normalized the corresponding central value

Colored lines: gluon central values, extracted with different α_s , normalized to the best central value

gluon density and α_s at small-*x* are anticorrelated

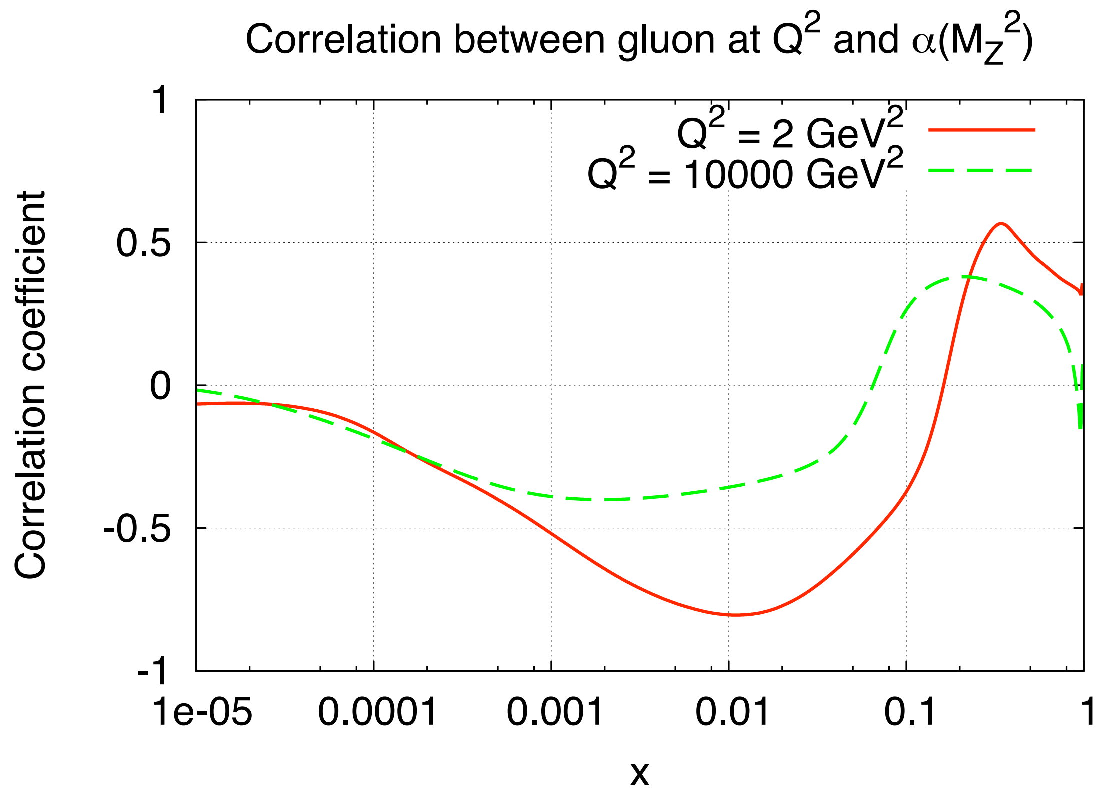
Correlation of gluon density and α_s

$$\rho\left[\alpha_s\left(M_Z^2\right), g\left(x,Q^2\right)\right] = \frac{\left\langle \alpha_s\left(M_Z^2\right) g\left(x,Q^2\right)\right\rangle_{\rm rep} - \left\langle \alpha_s\left(M_Z^2\right)\right\rangle_{\rm rep}\left\langle g\left(x,Q^2\right)\right\rangle_{\rm rep}}{\sigma_{\alpha_s\left(M_Z^2\right)}\sigma_{g(x,Q^2)}}$$

Obtained with NNPDF1.2

Gaussian distribution assumed for α_s

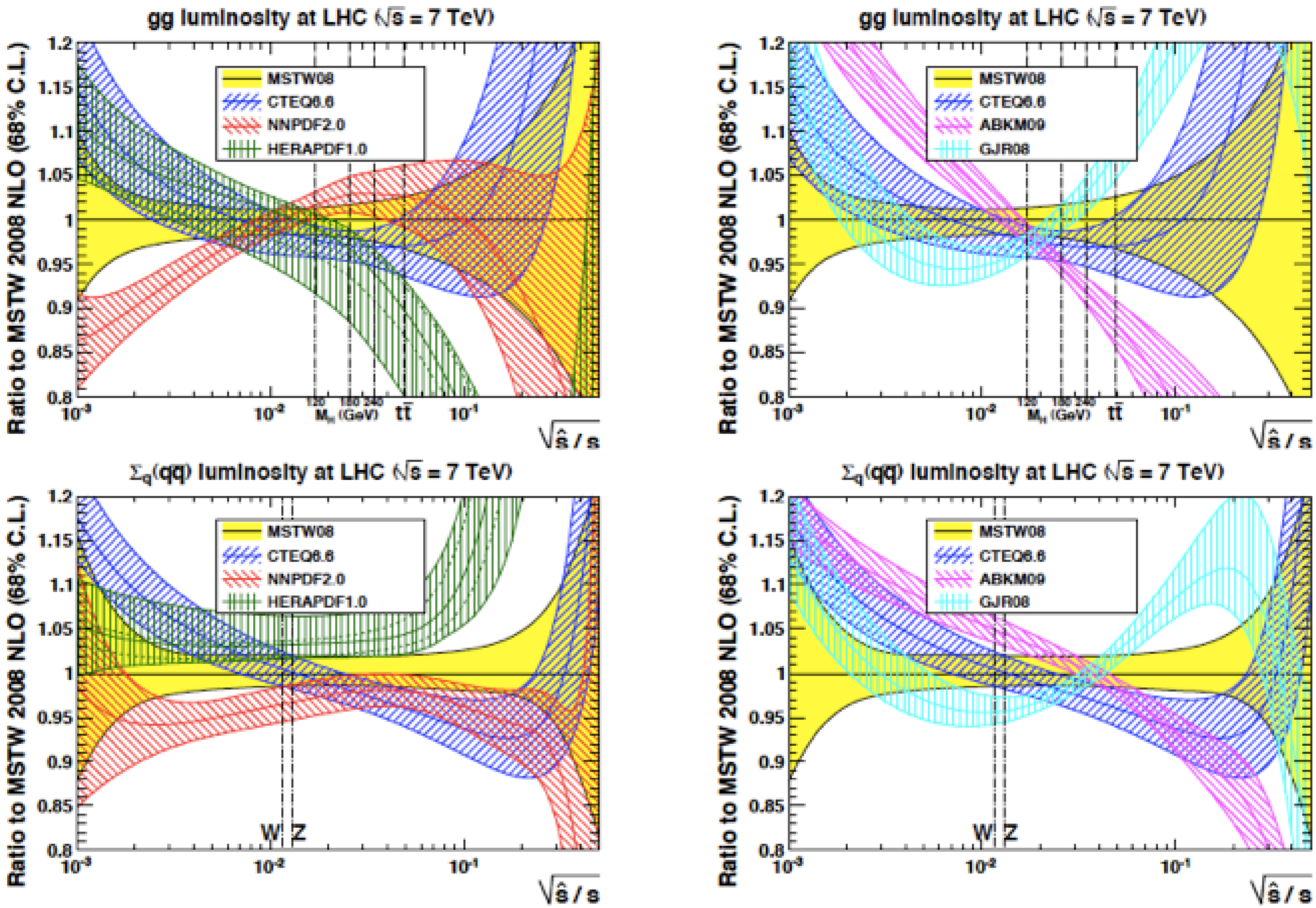
Both the various central gluons and their uncertainties enter in $\rho\left[\alpha_s\left(M_Z^2\right), g\left(x,Q^2\right)\right]$



The evolution de-correlates the gluon from the strong coupling

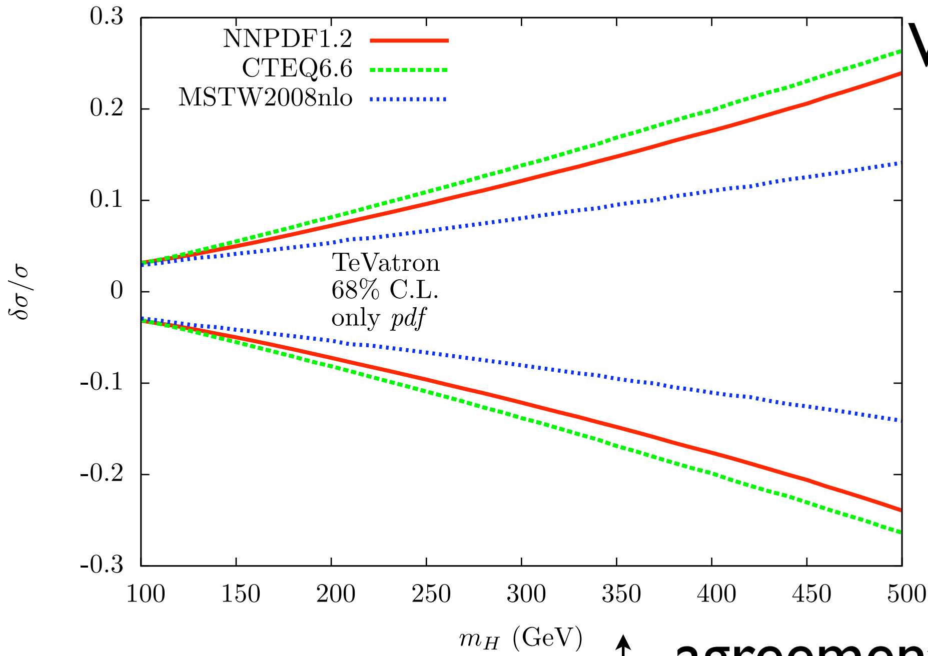
Parton luminosities

Predictions by various groups - parton luminosities – NLO. Plots by G. Watt.



MSTW including all data, full GM-VFNS and sufficiently flexible parameterisation seems to lead to it never being the outlying PDF. (Outlying might not be wrong but surely requires explanation).

Uncertainties due only to the *pdfs* 68% C.L.

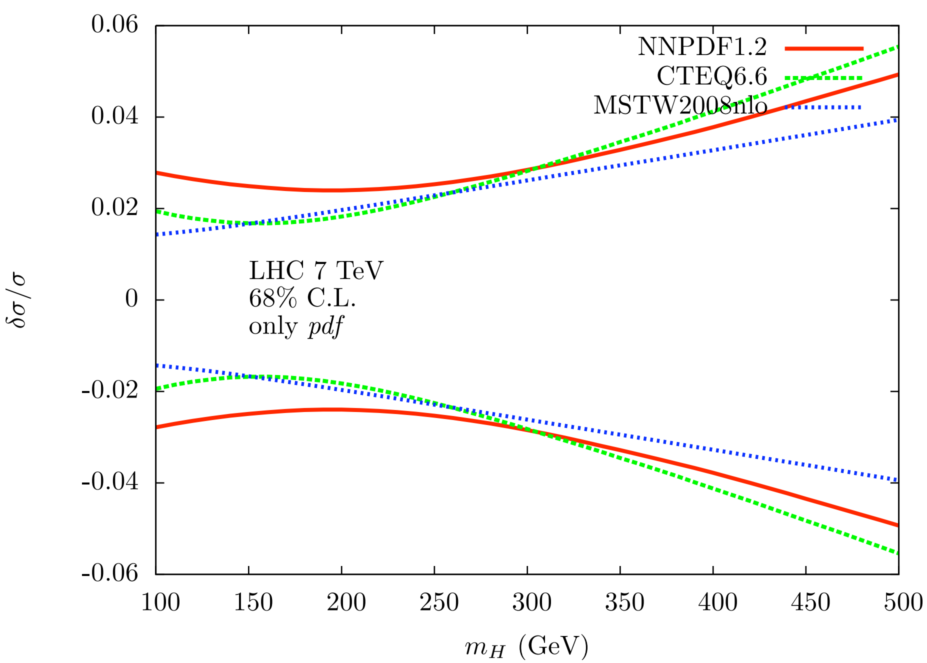


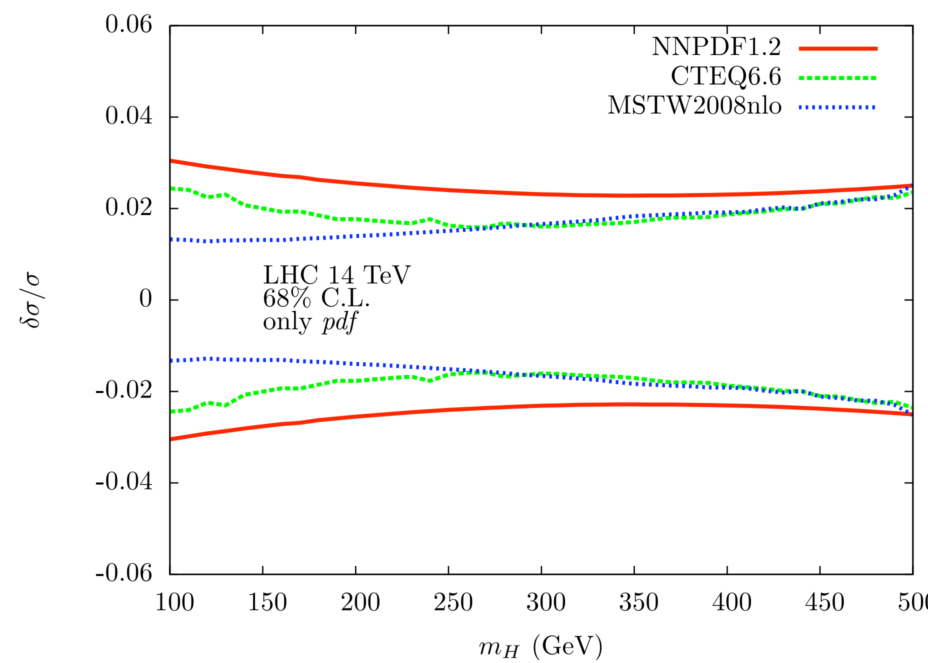
Width of the uncertainty bands
normalized to the respective central value

We used:
MSTW2008nlo68cl.LHgrid
NNPDF12_100.LHgrid
cteq6.6.LHgrid with rescaling of error
band from 90% to 68% C.L.

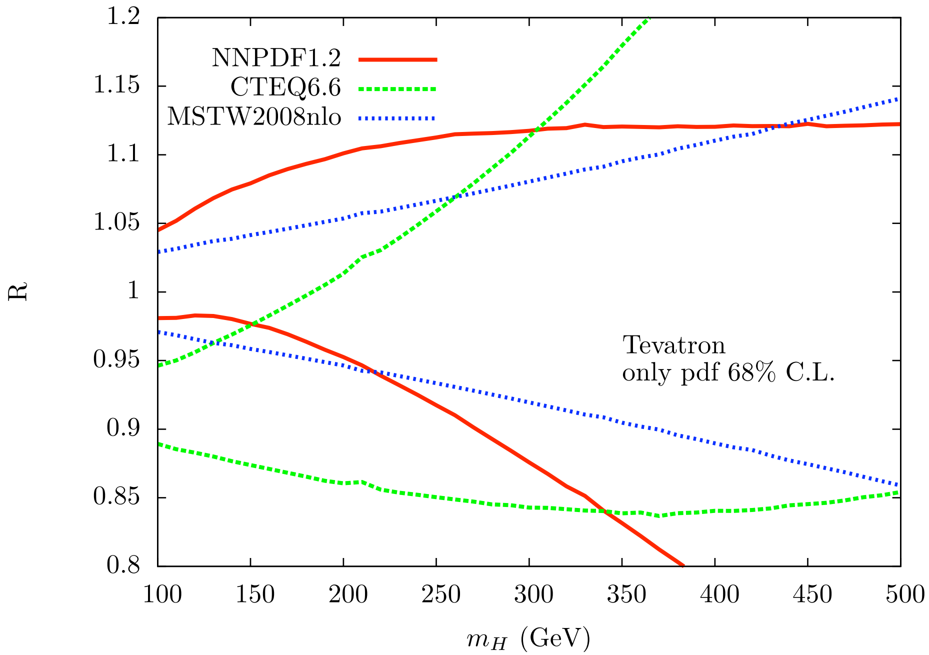
plot scales are different

agreement of CTEQ and MSTW for medium-large mh
deviation at LHC for small mh



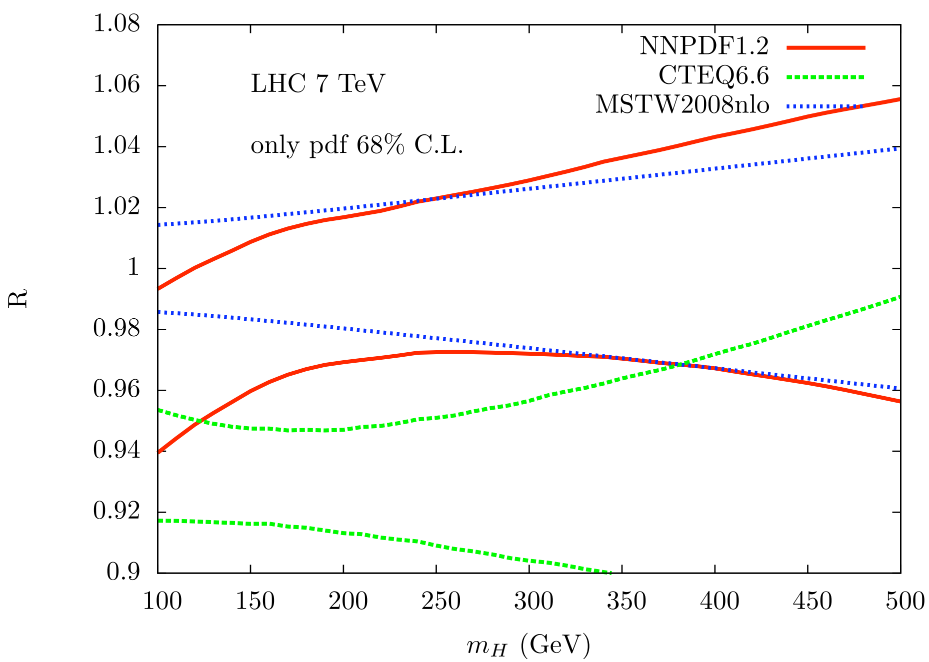


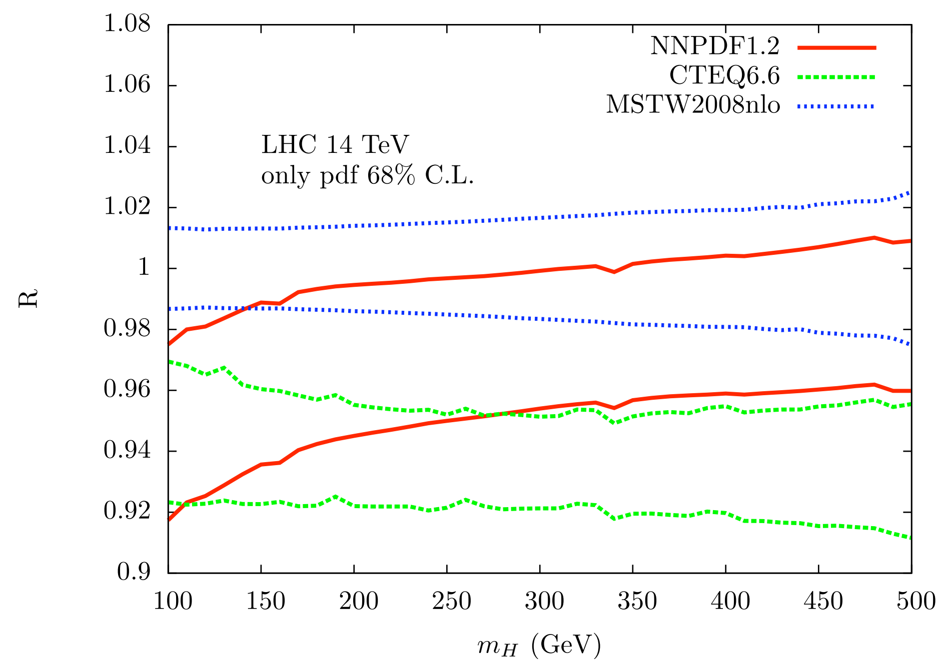
Comparison only-*pdf* bands (normalized to MSTW2008) 68% C.L.



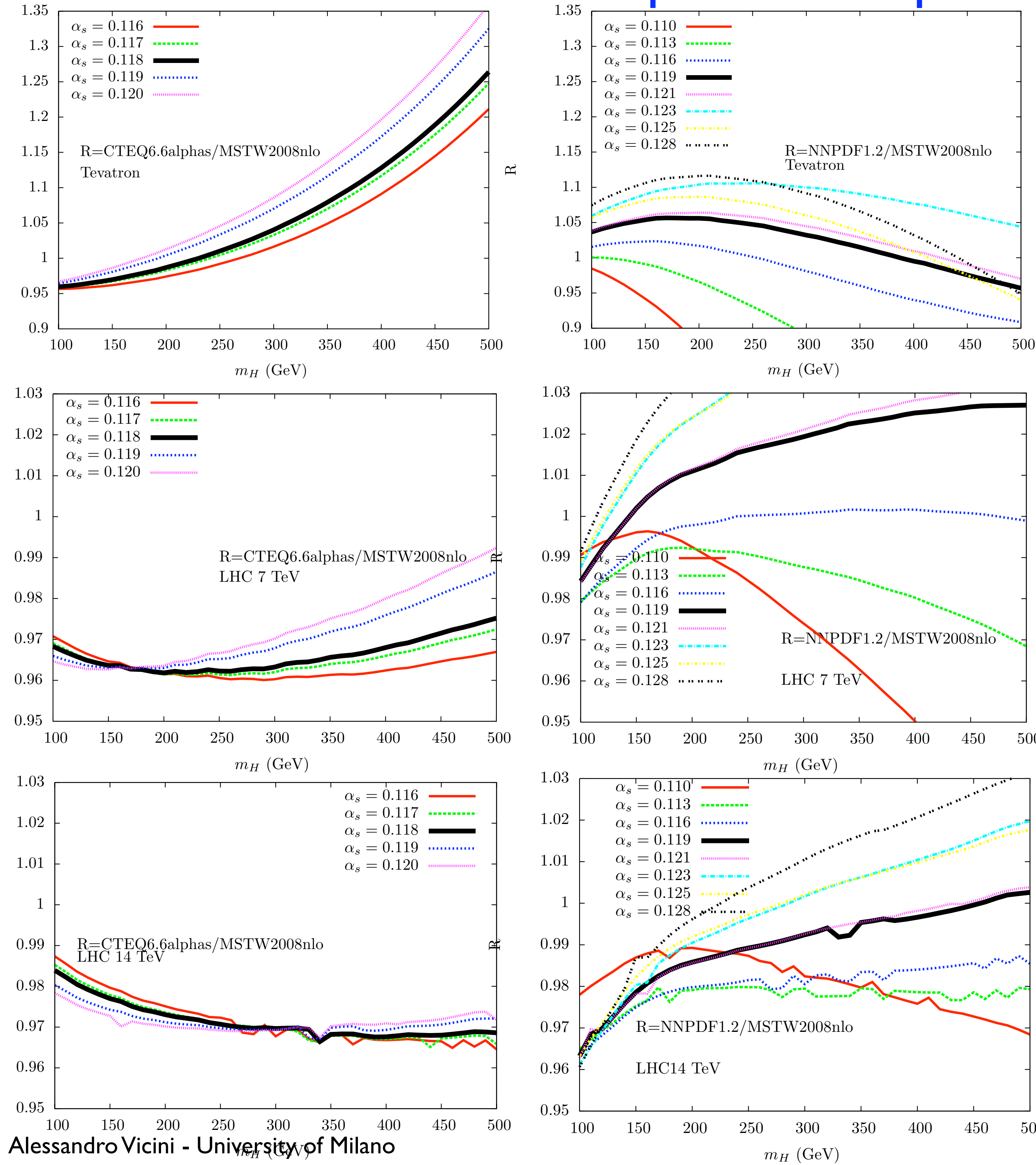
Central values by CTEQ6.6 and MSTW2008 differ between
6% (LHC 14 TeV) and 9% (Tevatron)
Uncertainty bands do not overlap

$\alpha_s(m_Z) =$	0.118	CTEQ6.6
	0.119	NNPDF1.2
	0.12018	MSTW2008nlo





Cross sections with same alphas in the pdfs and in the partonic xsec



Cross sections computed with
a given alphas in the partonic xsec
and with
pdf sets extracted using the very same alphas

In this comparison
discrepancies are only due to intrinsic differences in the pdf sets:
different data sets,
parametrizations,...

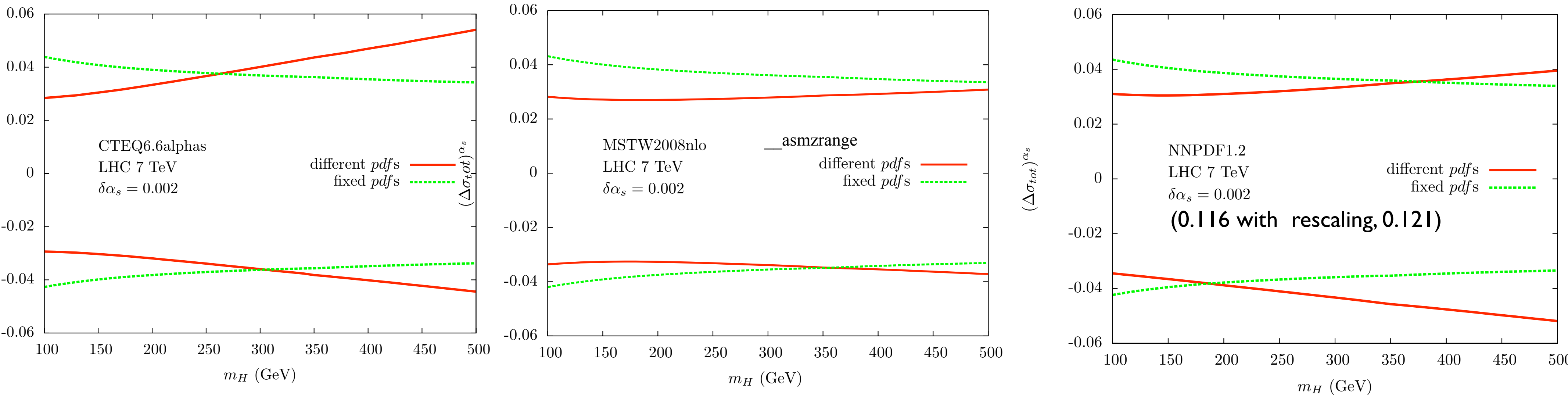
Discrepancies

- do not exceed the 4% level
- are of the same size or smaller than the *pdf* uncertainty bands

it is precisely the order of magnitude
we expect for these discrepancies

Evaluation of the uncertainty associated to α_s

$$\Delta\sigma^{\pm}_{\alpha_s} = \frac{\sigma(\alpha_s^0 \pm \delta\alpha_s)}{\sigma(\alpha_s^0)} - 1 \qquad \delta\alpha_s^{(90)} = 0.002$$



variation of α_s in the partonic xsec AND in the *pdfs*

- The spread is measured with respect to the central value of the best set

pdfs fixed to their central value, α_s changed only in the partonic xsec

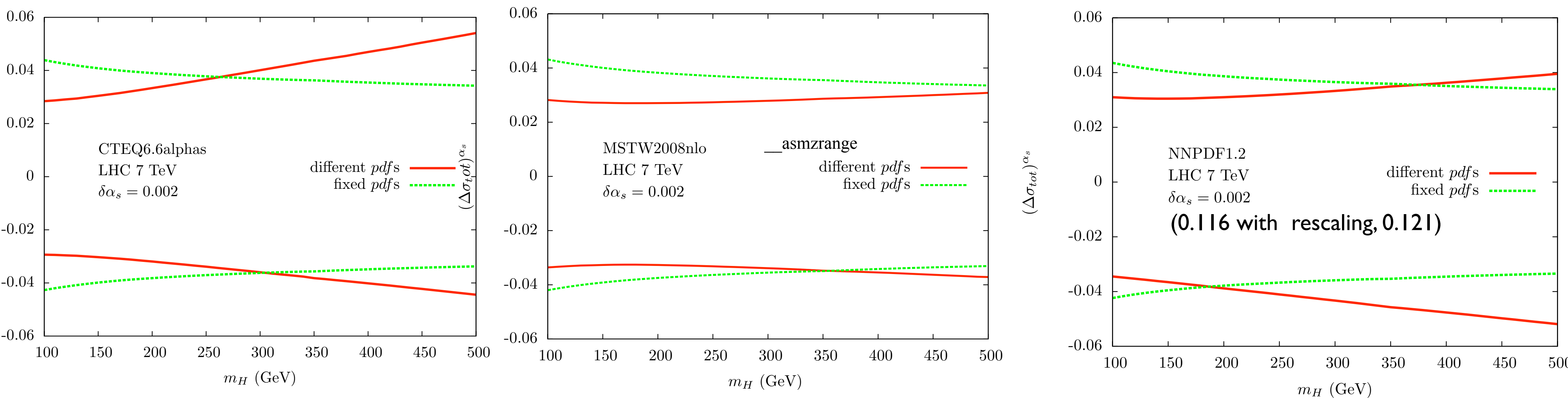
- A change of α_s only in the partonic xsec, keeping the *pdfs* fixed to their central value, (green lines) overestimates the effect by at most 35%

$$\frac{\Delta\sigma_{\alpha_s}}{\sigma} \sim 2.5\ \frac{\delta\alpha_s}{\alpha_s}$$

Evaluation of the uncertainty associated to α_s

$$\Delta\sigma^{\pm}_{\alpha_s} = \frac{\sigma(\alpha_s^0 \pm \delta\alpha_s)}{\sigma(\alpha_s^0)} - 1$$

$$\delta\alpha_s^{(90)} = 0.002$$



(red line) variation of α_s in the partonic xsec AND in the *pdfs*

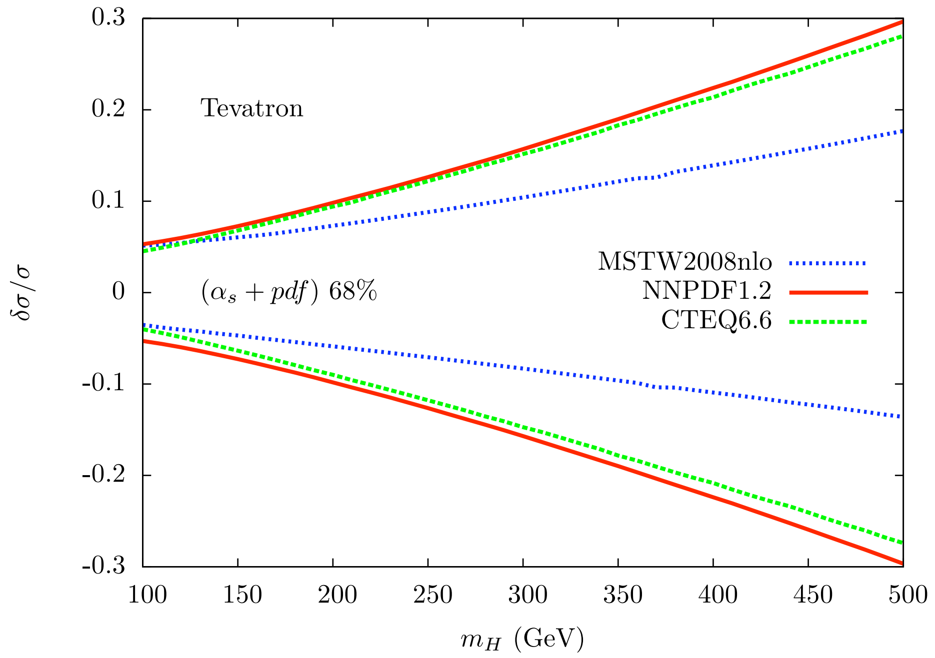
- The spread is measured with respect to the central value of the best set

(green line) *pdfs* fixed to their central value, α_s changed only in the partonic xsec

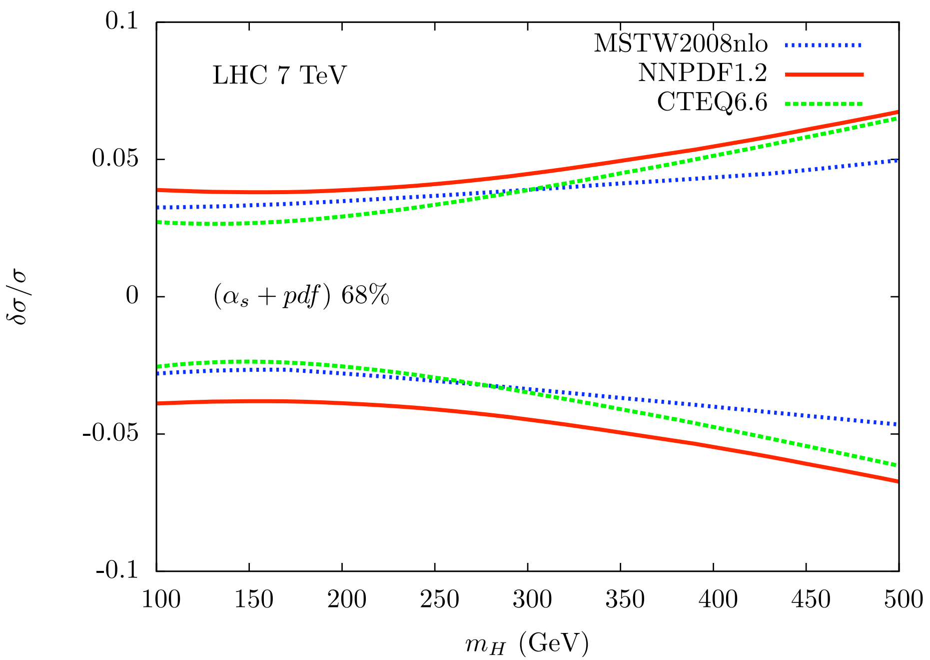
- A change of α_s only in the partonic xsec, keeping the *pdfs* fixed to their central value, (green lines) overestimates the effect by at most 35%

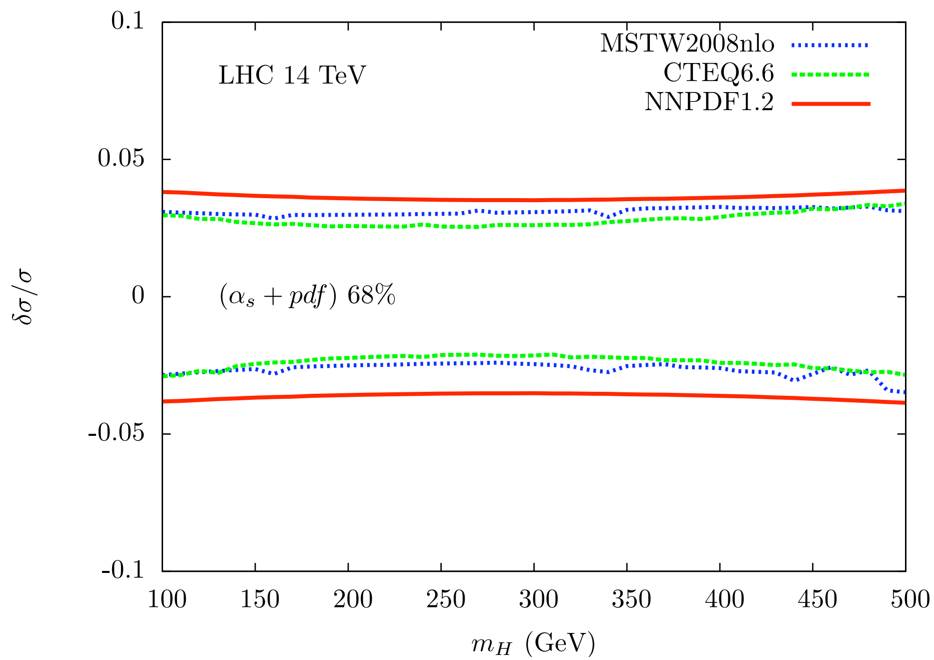
$$\frac{\Delta\sigma_{\alpha_s}}{\sigma} \sim 2.5\ \frac{\delta\alpha_s}{\alpha_s}$$

Combined *pdf*+alphas uncertainties at 68% C.L.

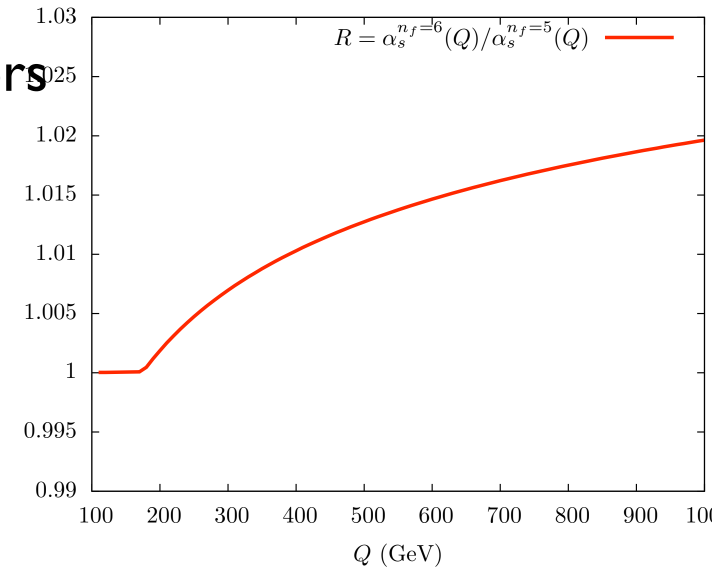


relative uncertainty
normalized to the central value
of each group





The running of α_s



The running of α_s depends on the number of active flavors

The routines for α_s, provided by CTEQ and by MSTW, implemented in LHAPDF, use $Q > m_b \quad n_f = 5$

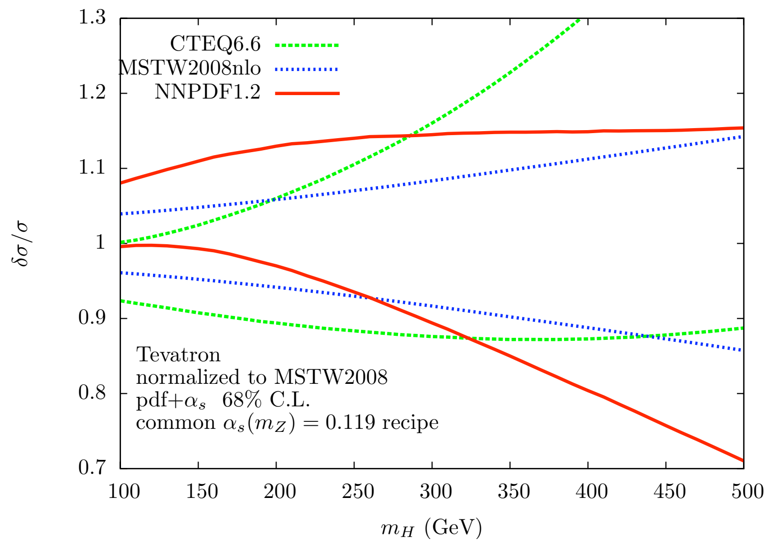
The routines for α_s, provided by NNPDF, implemented in LHAPDF, use the variable nf

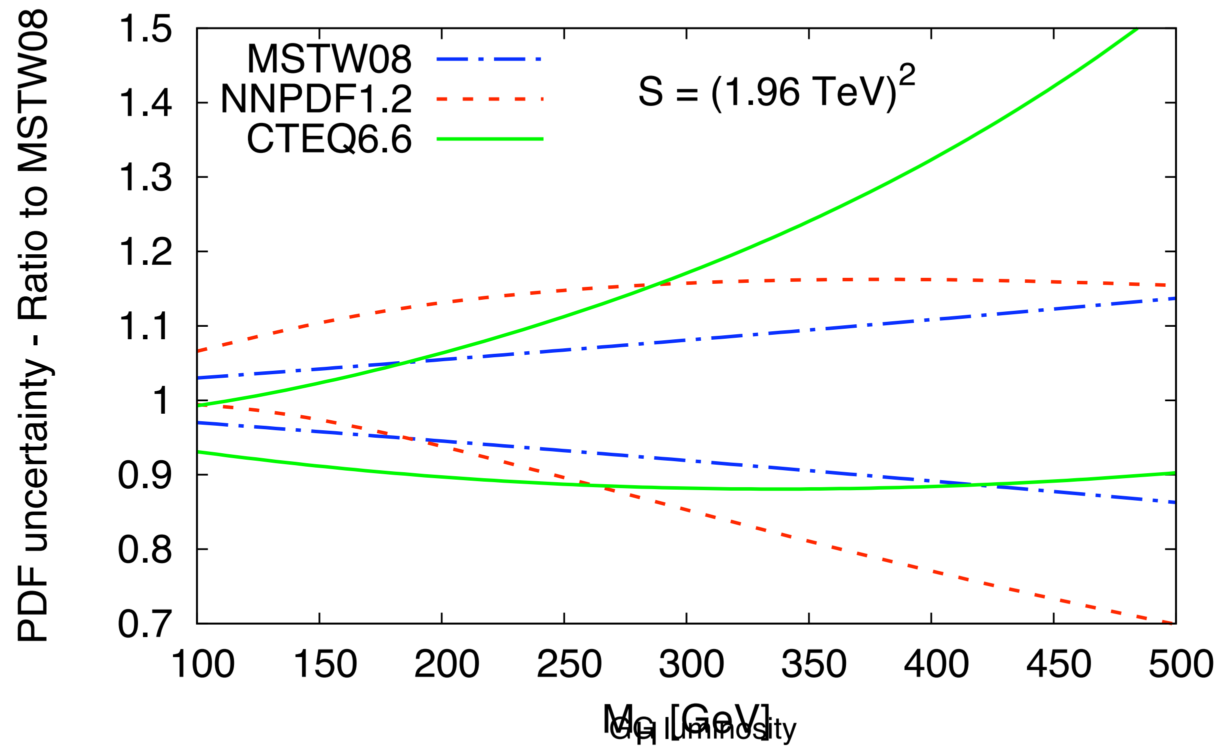
$$m_b < Q < m_t \quad n_f = 5$$
$$Q > m_t \quad n_f = 6$$

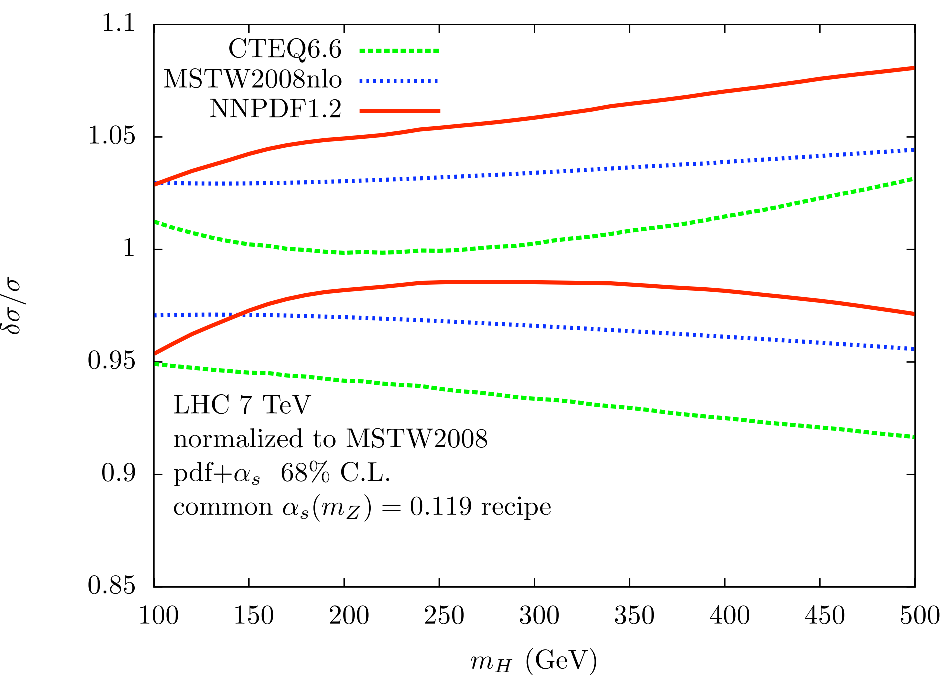
In a code like HIGLU (or like GGSCA) where the top mass is renormalized on-shell the variable number of active flavours has to be adopted
The cross sections in the two cases might differ at the percent level !

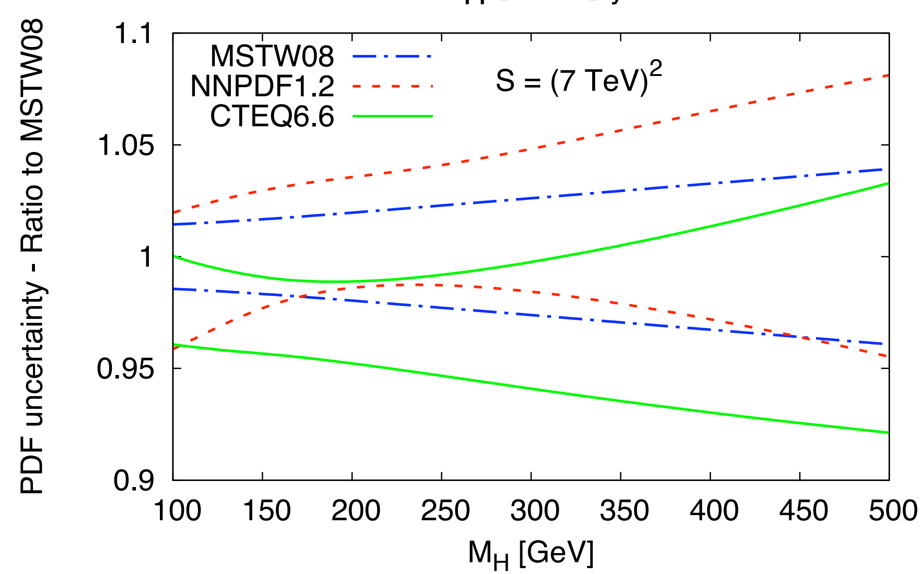
In the rest of the talk all the cross-sections evaluated with nf=6 above the top mass
There will be a missing cancellation of α_s running effects between partonic xsec and *pdf* evolution, when using CTEQ or MSTW *pdfs*

Cross sections "common alphas" *vs* gg-luminosities









The ABKM09 pdf set (NNLO-QCD)

based on DIS and fixed-target DY data

the alphas value is fitted from the data together with the pdfs

$$\alpha_s(m_Z) = \begin{array}{ll} 0.1135 \pm 0.0014 & \text{ABKM09-5-nnlo} \\ 0.1171 \pm 0.0014 & \text{MSTW2008nnlo} \end{array}$$

(2.6 σ difference)

the use of a common alphas explains the discrepancy between ABKM09 and MSTW2008nnlo:

- at small MH the difference is completely due to the choice of alphas
- at large MH, the difference is likely to be due to the fact that the large-x gluon is constrained by Tevatron jet data (not included by ABKM09),