14-th International Workshop
on Advanced Computing and Analysis Techniques
in Physics Research
Uxbridge, London, September 5 – 9, 2011.

K.V.Stepanyantz

Moscow State University
Department of Theoretical Physics

Multiloop calculations in supersymmetric theories with the higher covariant derivative regularization

Regularization in supersymmetric theories

In order to calculate quantum corrections it is necessary to regularize a theory. Although the physical results do not depend on regularization, a proper choice of a regularization can simplify the calculations or reveal some features of quantum corrections.

Most calculations in QFT were made with the dimensional regularization.

G.t'Hooft, M.Veltman, Nucl.Phys. B44 (1972), 189.

However, DREG breaks the supersymmetry and is not convenient for calculations in supersymmetric theories. That is why most calculations in supersymmetric theories were made with the dimensional reduction.

W.Siegel, Phys.Lett. 84B (1979), 193.

For example, the β-function in supersymmetric theories was calculated up to the four-loop approximation:

S.Ferrara, B.Zumino, Nucl.Phys. **B79** (1974) 413; D.R.T.Jones, Nucl.Phys. **B87** (1975) 127; L.V.Avdeev, O.V.Tarasov, Phys.Lett. **112 B** (1982) 356; I.Jack, D.R.T.Jones, C.G.North, Phys.Lett **B386** (1996) 138; Nucl.Phys. **B 486** (1997) 479; R.V.Harlander, D.R.T.Jones, P.Kant, L.Mihaila, M.Steinhauser, JHEP **0612** (2006) 024.

Regularization in supersymmetric theories

It is well known that the dimensional reduction is not self-consistent.

W.Siegel, Phys.Lett. 94B (1980), 37.

Removing of inconsistencies leads to the loss of the supersymmetry in the higher loops.

L.V.Avdeev, Phys.Lett. B117 (1982), 317; L.V.Avdeev, A.A.Vladimirov, Nucl.Phys. B219 (1983), 262. V.N.Velizhanin, Nucl.Phys. B818 (2009), 95.

Therefore, regularization of supersymmetric theories is not a trivial problem.

I.Jack, D.R.T.Jones, Regularisation of supersymmetric theories, hep-ph/9707278.

For supersymmetric theories one can use the higher covariant derivative regularization.

A.A.Slavnov, Nucl.Phys., **B31**, (1971), 301; Theor.Math.Phys. **13**, (1972), 1064. V.K.Krivoshchekov, Theor.Math.Phys. **36**, (1978), 745; P.West, Nucl.Phys. B268, (1986), 113.

This regularization is self-consistent and does not break the supersymmetry.

Higher covariant derivative regularization

The main idea: To modify a theory in such a way that the inverse propagators will be proportional to higher degrees of the momentum in the UV region, e.g.

$$\frac{1}{k^2} \to \frac{1}{k^2(1+k^{2n}/\Lambda^{2n})}.$$

However, it was not frequently applied for the explicit calculations because after such a modification loop integrals will have very complicated structure, and it is very difficult (if possible) to calculate them analytically.

In this talk application of the higher derivative regularization to calculations in supersymmetric theories is discussed. It is argued that one of the loop integrals defining the β-function can be always calculated analytically. The reason is that the integrands are total derivatives (and even double total derivatives).

A.Soloshenko, K.S., hep-th/0304083;
A.V.Smilga, A.I.Vainshtein, Nucl.Phys. **B 704**, (2005), 445.

For $N = 1$ SQED this can be proved in all loops. As a consequence the exact β-function is related with the anomalous dimension. (This means that in this case the exact NSVZ β-function is obtained).

$N = 1$ supersymmetric electrodynamics (SQED), regularized by higher derivatives

The N=1 SQED in the massless case is described by the action

$$S = \frac{1}{4e^2}\text{Re}\int d^4x\, d^2\theta\, W_a C^{ab} W_b + \frac{1}{4}\int d^4x\, d^4\theta\, \Big(\phi^* e^{2V}\phi + \widetilde{\phi}^* e^{-2V}\widetilde{\phi}\Big),$$

where ϕ_i and $\widetilde{\phi}$ are chiral matter superfields, V is a real gauge superfield, and

$$W_a = \frac{1}{4}\bar{D}^2 D_a V.$$

We add the term with higher derivatives

$$S_{reg} = \frac{1}{4e^2}\text{Re}\int d^4x\, d^2\theta\, W_a C^{ab} R(\partial^2/\Lambda^2) W_b$$
$$+\frac{1}{4}\int d^4x\, d^4\theta\, \Big(\phi^* e^{2V}\phi + \widetilde{\phi}^* e^{-2V}\widetilde{\phi}\Big)$$

where $R(\partial^2/\Lambda^2)$ is a regulator, e.g. $R = 1 + \partial^{2n}/\Lambda^{2n}$.

The higher derivative regularization and quantization

The gauge is fixed by adding:

$$S_{gf} = -\frac{1}{64e^2} \int d^4x\, d^4\theta \Big(V R D^2 \bar{D}^2 V + V R \bar{D}^2 D^2 V \Big).$$

After adding the term with the higher derivatives divergences remain only in the one-loop approximation. In order to remove them we insert in the generating functional the Pauli–Villars determinants.

L.D.Faddeev, A.A.Slavnov, Gauge fields, introduction to quantum theory, Benjamin, Reading, 1990.

$$Z[J,\mathbf{\Omega}] = \int D\mu \prod_I \Big(\det PV(V, M_I) \Big)^{c_I} \exp\Big\{ iS_{reg} + \text{Sources} \Big\},$$

$\sum\limits_I c_I = 1$; $\sum\limits_I c_I M_I^2 = 0$; $M_I = a_I \Lambda$. (Λ is the only dimensionful parameter.)

$$\det PV(V,M) = \Big(\int D\Phi^* D\Phi e^{iS_{PV}} \Big)^{-1},$$

$$S_{PV} = \frac{1}{4} \int d^4x\, d^4\theta \Big(\Phi^* e^{2V} \Phi + \widetilde{\Phi}^* e^{-2V} \widetilde{\Phi} \Big) + \Big(\frac{1}{2} \int d^4x\, d^4\theta\, M \Phi \widetilde{\Phi} + .. \Big).$$

Calculation of the β-function

The notation is

$$\Gamma^{(2)} = \int \frac{d^4p}{(2\pi)^4}\, d^4\theta \Big(-\frac{1}{16\pi}\mathbf{V}(-p)\,\partial^2\Pi_{1/2}\mathbf{V}(p)\, d^{-1}(\alpha,\mu/p) + $$
$$+\frac{1}{4}(\phi^*)^i(-p,\theta)\,\phi_j(p,\theta)\,(ZG)_i{}^j(\alpha,\mu/p)\Big).$$

We calculate

$$\frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0,\Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0} = -\frac{d}{d\ln\Lambda}\alpha_0^{-1}(\alpha,\mu/\Lambda) = \frac{\beta(\alpha_0)}{\alpha_0^2}$$

The main result: (It was obtained as the equality of some well defined integrals due to the factorization of integrands into total derivatives)

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{1}{\pi}\Big(1 - \frac{d}{d\ln\Lambda}\ln G(\alpha_0,\Lambda/q)\Big|_{q=0}\Big) = \frac{1}{\pi} + \frac{1}{\pi}\frac{d}{d\ln\Lambda}\Big(\ln ZG(\alpha,\mu/q)$$
$$-\ln Z(\alpha,\Lambda/\mu)\Big)\Big|_{q=0} = \frac{1}{\pi}\Big(1-\gamma\Big(\alpha_0(\alpha,\Lambda/\mu)\Big)\Big).$$

(Without any redefinition of the coupling constant.)

Three-loop calculation for SQED

$$
\begin{aligned}
\frac{\beta(\alpha_0)}{\alpha_0^2} &= 2\pi \frac{d}{d\ln\Lambda}\Big\{ \sum_I c_I \int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu} \frac{\ln(q^2+M^2)}{q^2} + 4\pi \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{e^2}{k^2R_k^2} \\
&\times \frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu}\Big(\frac{1}{q^2(k+q)^2} - \sum_I c_I \frac{1}{(q^2+M_I^2)((k+q)^2+M_I^2)}\Big)\Big[R_k\Big(1+\frac{e^2}{4\pi^2}\ln\frac{\Lambda}{\mu}\Big) \\
&-2e^2\Big(\int \frac{d^4t}{(2\pi)^4}\frac{1}{t^2(k+t)^2} - \sum_J c_J \int \frac{d^4t}{(2\pi)^4}\frac{1}{(t^2+M_J^2)((k+t)^2+M_J^2)}\Big)\Big] \\
&+4\pi \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{d^4l}{(2\pi)^4}\frac{e^4}{k^2R_k l^2 R_l}\frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu}\Big\{\Big(-\frac{2k^2}{q^2(q+k)^2(q+l)^2(q+k+l)^2} \\
&+\frac{2}{q^2(q+k)^2(q+l)^2}\Big) - \sum_I c_I \Big(-\frac{2(k^2+M_I^2)}{(q^2+M_I^2)((q+k)^2+M_I^2)((q+l)^2+M_I^2)} \\
&\times \frac{1}{((q+k+l)^2+M_I^2)} + \frac{2}{(q^2+M_I^2)((q+k)^2+M_I^2)((q+l)^2+M_I^2)} - \frac{1}{(q^2+M_I^2)^2} \\
&\times \frac{4M_I^2}{((q+k)^2+M_I^2)((q+l)^2+M_I^2)}\Big)\Big\}
\end{aligned}
$$

NSVZ β-function for $N=1$ SQED in the three-loop approximation

The integrals can be calculated using the identity

$$\int \frac{d^4q}{(2\pi)^4}\frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu}\Big(\frac{f(q^2)}{q^2}\Big) = \lim_{\varepsilon\to 0}\int\limits_{S_\varepsilon} \frac{dS_\mu}{(2\pi)^4}\frac{(-2)q^\mu f(q^2)}{q^4} = \frac{1}{4\pi^2}f(0)$$

where f is a nonsingular function, which rapidly decreases at the infinity. It is equivalent to the identity

$$\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\frac{d}{dq^2}f(q^2) = \frac{1}{16\pi^2}\Big(f(\infty)-f(0)\Big) = -\frac{1}{16\pi^2}f(0).$$

(This is a total derivative in the four-dimensional spherical coordinates.)

Comparing the result with the expression for the two-loop anomalous dimension of the matter superfield we obtain

$$\beta(\alpha) = \frac{\alpha^2}{\pi}\Big(1-\gamma(\alpha)\Big).$$

M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, JETP Lett. 42, (1985), 224;
Phys.Lett. 166B, (1986), 334.

Two-loop anomalous dimension of the matter superfield

$$\gamma(\alpha_0) = -2e^2 \int \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{1}{k^4 R_k^2} \Bigg[R_k \Big(1 + \frac{e^2}{4\pi^2} \ln\frac{\Lambda}{\mu}\Big)$$

$$- \int \frac{d^4t}{(2\pi)^4} \frac{2e^2}{t^2(k+t)^2} + \sum_I c_I \int \frac{d^4t}{(2\pi)^4} \frac{2e^2}{(t^2+M_I^2)((k+t)^2+M_I^2)} \Bigg]$$

$$- \int \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{4e^4 k_\mu l_\mu}{k^4 R_k\, l^4 R_l (k+l)^2}.$$

It is much more complicated problem to calculate these integrals analytically.

In general, it seems that these (or similar) integrals should be calculated numerically. However, in some simplest cases, e.g.

$$R = 1 + \partial^{2n}/\Lambda^{2n},$$

analytical calculation using the four-dimensional spherical coordinates can be also possible.

Some useful tricks

Two main purposes:

1. How the factorization of the integrands into total derivatives can be proven exactly in all loops?

2. How one can obtain NSVZ β-function exactly to all loops?

In order to simplify the calculations (in the limit $p \to 0$) and find the β-function it is possible to substitute

$$\mathbf{V} \to \bar\theta^a \bar\theta_a \theta^b \theta_b$$

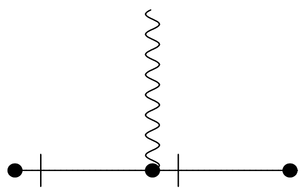
An integral of a total derivative in the coordinate representation is given by

$$\mbox{Tr}\Big([x^\mu, \mbox{Something}]\Big) = 0.$$

We will try to reduce the sum of diagrams to such commutators.

Summation of subdiagrams

In order to extract integrals of total derivatives we consider the following sum of subdiagrams:


$+$


$$= -\theta^a\theta_a\bar\theta^b\frac{\bar D_b D^2}{4\partial^2} + \theta^a\theta_a\frac{D^2}{4\partial^2}$$

$$+ i\bar\theta^b(\gamma^\mu)_b{}^a\theta_a\frac{\bar D^2 D^2\partial_\mu}{\partial^4} - i\theta^a(\gamma^\mu)_a{}^b\frac{\bar D_b D^2\partial_\mu}{4\partial^4} + \frac{\bar D^2 D^2}{16\partial^4}$$

Only the terms written by the blue color give nontrivial contributions to the two-point function of the gauge superfield.

Really, finally it is necessary to obtain

$$\int d^4\theta\,\theta^a\theta_a\bar\theta^b\bar\theta_b,$$

and calculating the θ-part of the graph can not produce powers of θ or $\bar\theta$.

Integration over the matter superfields

Let us formally perform Gaussian integration over the matter superfields:

$$Z = \int DV \prod_I \Big(\det PV(V, M_I)\Big)^{c_I}$$

$$\times \exp\left\{ i\int d^8x \left(\frac{1}{4e^2} V\partial^2 R(\partial^2/\Lambda^2)V - j\frac{D^2}{4\partial^2} * \frac{\bar{D}^2}{4\partial^2} j^* - \widetilde{j}\frac{D^2}{4\partial^2}\widetilde{*}\frac{\bar{D}^2}{4\partial^2}\widetilde{j}^*\right)\right\},$$

where

$$* \equiv \frac{1}{1-(e^{2V}-1)\bar{D}^2D^2/16\partial^2}, \qquad \widetilde{*} = \frac{1}{1-(e^{-2V}-1)\bar{D}^2D^2/16\partial^2}$$

encode chains of propagators and vertexes.

This allows to write the sum of Feynman diagrams for the two-point function of the gauge superfield in the limit $p \to 0$ in the form of an integral of double total derivatives. Details can be found in

K.S., Nucl.Phys. B 852 (2011), 71.

Notation: $y^*_\mu = x_\mu - i\bar{\theta}^a(\gamma_\mu)_a{}^b\theta_b$.

The result

$$\Delta\Gamma^{(2)}_{\mathbf{V}} = \Big\langle -2i\Big(\mathrm{Tr}(\mathbf{V}J_0*)\Big)^2 - 2i\mathrm{Tr}(\mathbf{V}J_0 * \mathbf{V}J_0*) - 2i\mathrm{Tr}(\mathbf{V}^2 J_0*)\Big\rangle$$

$+$terms with $\widetilde{*} + (PV)$, where $J_0 = e^{2V}\dfrac{\bar{D}^2 D^2}{16\partial^2}$ is the effective vertex.

External lines are attached to different matter loops

$$-2i\frac{d}{d\ln\Lambda}\Big\langle\Big(\mathrm{Tr}\Big(-2\theta^c\theta_c\bar{\theta}^d[\bar{\theta}_d, \ln(*) - \ln(\widetilde{*})] + i\bar{\theta}^c(\gamma^\nu)_c{}^d\theta_d[y^*_\nu, \ln(*) - \ln(\widetilde{*})]\Big)$$
$$+(PV)\Big)^2\Big\rangle$$

0

External lines are attached to a single matter loop

$$i\frac{d}{d\ln\Lambda}\mathrm{Tr}\Big\langle\theta^4\Big[y^*_\mu, \Big[(y^\mu)^*, \ln(*) + \ln(\widetilde{*})\Big]\Big]\Big\rangle + (PV) -$$ terms with a δ-function,

These expressions are evidently integrals of double total derivatives.

The result

In order to derive this result it is necessary to use the identity

$$\mathrm{Tr}\Big(\theta^a\theta_a\bar\theta^b\bar\theta_b\Big((\gamma_\mu)^{ab}[y_\mu^*,A][\bar\theta_b,B\}[\theta_a,C\}+(\gamma_\mu)^{ab}(-1)^{P_A}[\theta_a,B\}[\bar\theta_b,C\}$$

$$\times[y_\mu^*,A]-4i[\theta^a,[\theta_a,A\}\}[\bar\theta^b,B\}[\bar\theta_b,C\}\Big)\Big)+\text{cyclic perm. of } A,\ B,\ C$$

$$=\frac{1}{3}\mathrm{Tr}\Big(\theta^a\theta_a\bar\theta^b\bar\theta_b(\gamma_\mu)^{ab}\Big[y_\mu^*, A[\bar\theta_b,B\}[\theta_a,C\}+(-1)^{P_A}[\theta_a,B\}[\bar\theta_b,C\}A\Big]\Big)$$

$$+\text{cyclic perm. of } A,\ B,\ C$$

where A, B, and C are operators constructed from the supersymmetric covariant derivatives and usual derivatives which do not explicitly depend on θ and $\bar\theta$.

Thus, the sum of diagrams is given by integrals of double total derivatives. However, if the external lines are attached to a single matter line, the result does not vanish due to appearing of δ-functions.

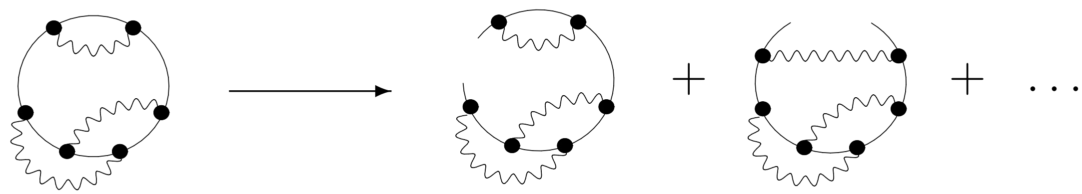
Obtaining the exact NSVZ β-function

The δ-functions come from the identity

$$[x^\mu, \frac{\partial_\mu}{\partial^4}] = [-i\frac{\partial}{\partial p_\mu}, -\frac{ip^\mu}{p^4}] = -2\pi^2\delta^4(p_E) = -2\pi^2 i\delta^4(p).$$

Qualitatively these δ-functions correspond to cutting the matter loop

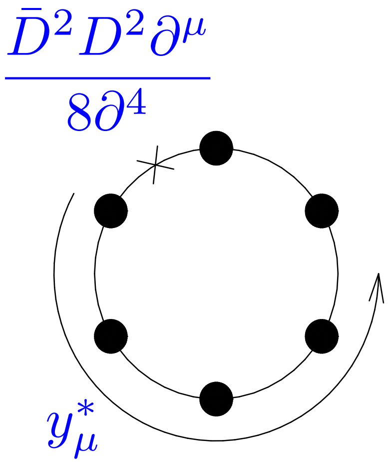
A.V.Smilga, A.I.Vainshtein, Nucl.Phys. **B 704**, (2005), 445.



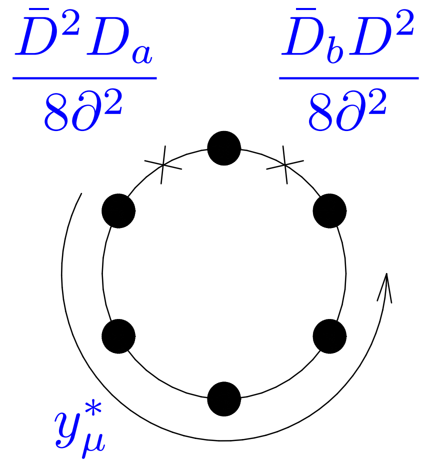
It is possible to calculate all contributions of δ-functions and compare them with the two-point Green function of the matter superfield. The result is the exact NSVZ β-function

$$\beta(\alpha) = \frac{\alpha^2}{\pi}\Big(1-\gamma(\alpha)\Big).$$

Obtaining the exact NSVZ β-function



$+ \quad (\gamma^\mu)^{ab}$



(for $p = 6$)

$$G^{-1} = (1 + \Delta G)^{-1} = \sum_{p=0}^{\infty} (-1)^p (\Delta G)^p$$

$$-\sum_{p=1}^{\infty} \frac{(-1)^p (p-1)}{p} (\Delta G)^p$$

$$1 + \sum_{p=1}^{\infty} \frac{(-1)^p}{p} (\Delta G)^p = 1 - \ln G$$

Non-Abelian $N = 1$ supersymmetric theories

N=1 supersymmetric Yang-Mills theory with matter in the massless case is described by the action

$$S = \frac{1}{2e^2}\mathrm{Re\,tr}\int d^4x\, d^2\theta\, W_a C^{ab} W_b + \frac{1}{4}\int d^4x\, d^4\theta\, (\phi^*)^i (e^{2V})_i{}^j \phi_j +$$
$$+\Big(\frac{1}{6}\int d^4x\, d^2\theta\, \lambda^{ijk}\phi_i\phi_j\phi_k + \mathrm{h.c.}\Big),$$

where ϕ_i are chiral scalar matter superfields, V is a real scalar gauge superfield, and the supersymmetric gauge field stress tensor is given by

$$W_a = \frac{1}{8}\bar{D}^2\Big[e^{-2V} D_a e^{2V}\Big].$$

The action is invariant under the gauge transformations

$$e^{2V} \to e^{i\Lambda^+} e^{2V} e^{-i\Lambda}; \qquad \phi \to e^{i\Lambda}\phi$$

if $(T^A)_m{}^i \lambda^{mjk} + (T^A)_m{}^j \lambda^{imk} + (T^A)_m{}^k \lambda^{ijm} = 0.$

Higher derivative regularization

For the calculation we use the background field method.

The gauge is fixed by adding the following term:

$$S_{gf} = -\frac{1}{32e^2}\,\mathrm{tr}\int d^4x\,d^4\theta\,\Big(V\boldsymbol{D}^2\bar{\boldsymbol{D}}^2 V + V\bar{\boldsymbol{D}}^2\boldsymbol{D}^2 V\Big).$$

To regularize the theory we add the following term with the higher covariant derivatives:

$$S_\Lambda = \frac{1}{2e^2}\mathrm{tr}\,\mathrm{Re}\int d^4x\,d^4\theta\,V\,\frac{(\boldsymbol{D}_\mu^2)^{n+1}}{\Lambda^{2n}}V + \frac{1}{4}\int d^4x\,d^4\theta\,(\phi^*)^i\Big[e^{\boldsymbol{\Omega}^+}\frac{(\boldsymbol{D}_\mu^2)^m}{\Lambda^{2m}}e^{\boldsymbol{\Omega}}\Big]_i{}^j\phi_j.$$

where $\boldsymbol{D}$, $\bar{\boldsymbol{D}}$, and $\boldsymbol{D}_\mu$ are background covariant derivatives.

In order to regularize the remaining one-loop divergences, it is necessary to introduce Pauli-Villars determinants into the generating functional. As earlier, we assume that $M_I = a_I\Lambda$, where a_I are constants. (Therefore, there is the only dimensionful parameter Λ.)

Two-loop β-function for $N=1$ supersymmetric Yang-Mills theory

Two-loop calculation gives the following result:

$$\beta(\alpha) = -\frac{3\alpha^2}{2\pi}C_2 + \alpha^2 T(R) I_0 + \alpha^3 C_2^2 I_1 + \frac{\alpha^3}{r} C(R)_i{}^j C(R)_j{}^i I_2 +$$
$$+\alpha^3 T(R) C_2 I_3 + \alpha^2 C(R)_i{}^j \frac{\lambda^*_{jkl}\lambda^{ikl}}{4\pi r} I_4 + \ldots,$$

where we do not write the integral for the one-loop ghost contribution and the integrals I_0–I_4 are given below, and the following notation is used:

$$\mathrm{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$
$$f^{ACD} f^{BCD} \equiv C_2 \delta^{AB}; \qquad r \equiv \delta_{AA}.$$

Taking into account Pauli–Villars contributions,

$$I_i = I_i(0) - \sum_I I_i(M_I), \qquad i = 0,2,3$$

where I_i are given by

$$I_0(M) = -\pi \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \frac{1}{q^2} \ln\Big(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2 \Big) \right\};$$

$$I_1 = -12\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial k^\mu} \frac{\partial}{\partial k_\mu} \left\{ \frac{1}{k^2(1+k^{2n}/\Lambda^{2n})q^2(1+q^{2n}/\Lambda^{2n})} \right.$$

$$\left. \times \frac{1}{(q+k)^2(1+(q+k)^{2n}/\Lambda^{2n})} \right\};$$

$$I_2(M) = 8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \frac{1}{k^2(1+k^{2n}/\Lambda^{2n})} \right.$$

$$\left. \times \frac{(1+q^{2m}/\Lambda^{2m})(1+(q+k)^{2m}/\Lambda^{2m})}{(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2)((q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})^2+M^2)} \right\};$$

$$I_3(M) = 8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial k_\mu} \left\{ \frac{1}{(k+q)^2(1+(q+k)^{2n}/\Lambda^{2n})} \right.$$

$$\left. \times \frac{(1+k^{2m}/\Lambda^{2m})(1+q^{2m}/\Lambda^{2m})}{(k^2(1+k^{2m}/\Lambda^{2m})^2+M^2)(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2)} \right\};$$

$$I_4 = -8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \frac{1}{k^2(1+k^{2m}/\Lambda^{2m})q^2(1+q^{2m}/\Lambda^{2m})} \right.$$

$$\left. \times \frac{1}{(q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})} \right\}.$$

Two-loop β-function for $N=1$ supersymmetric Yang-Mills theory

The result for the two-loop β-function is given by

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\Big(3C_2 - T(R)\Big) + \frac{\alpha^3}{(2\pi)^2}\Big(- 3C_2^2 + T(R)C_2 +$$
$$+\frac{2}{r}C(R)_i{}^j C(R)_j{}^i\Big) - \frac{\alpha^2 C(R)_i{}^j \lambda^*_{jkl}\lambda^{ikl}}{8\pi^3 r} + \ldots.$$

Comparing the result with the one-loop anomalous dimension

$$\gamma_i{}^j(\alpha) = -\frac{\alpha C(R)_i{}^j}{\pi} + \frac{\lambda^*_{ikl}\lambda^{jkl}}{4\pi^2} + \ldots,$$

gives the exact NSVZ β-function in the considered approximation.

$$\beta(\alpha) = -\frac{\alpha^2\Big[3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha)/r\Big)\Big]}{2\pi(1 - C_2\alpha/2\pi)}.$$

V.A.Novikov, M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, Nucl.Phys. B 229, (1983), 381; Phys.Lett. 166B, (1985), 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. B 277, (1986), 456.

Conclusion

- ✓ It is possible to make analytical multiloop calculations for supersymmetric theories with the higher covariant derivative regularization. In principle, it is not very difficult to construct integrals defining different Green functions.
- ✓ All integrals defining the β-function in $N=1$ SQED, regularized by higher derivatives, are integrals of double total derivatives. This allows to calculate one of the loop integrals analytically.
- ✓ The factorization of integrands into total derivatives allows to obtain the exact NSVZ β-function without redefinition of the coupling constant.
- ✓ Possibly, the factorization of integrands into double total derivatives is a general feature of supersymmetric theories. At least, this takes place for a general renormalizable $N=1$ supersymmetric theory at the two-loop level.

Thank you for the attention!