

# Progress in automated Next-to-Leading-Order calculations

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**Abstract.** We review the recent progress towards automation in the computation of the next-to-leading corrections to scattering amplitudes. Such progress allows for the construction of quite general, flexible and fully automated packages that would be of major importance for the Higgs boson and beyond the Standard Model physics searches at high energy particle colliders.

## 1. Introduction

The search for the Higgs boson and signals of physics beyond the Standard Model of elementary particles (SM) is underway at the CERN Large Hadron Collider (LHC) with considerable efficiency. The Higgs boson is an essential ingredient of the SM, but its existence has not been proved yet. Radiative corrections are however sensitive to the Higgs boson mass and the combination of the precision electroweak tests and the LEP limit set the upper bound  $m_H < 185 \text{ GeV}$  at the 95% of confidence level. On the other hand, more and more severe exclusion limits are delivered by the Tevatron at the Fermilab and the Large Hadron Collider at CERN and the Higgs boson discovery could be around the corner.

The high luminosity and high center of mass energy of the collisions at the LHC make of course discoveries possible, but through very challenging searches. This is because of the many background processes that have an experimental signature which overlaps with the one of the signal processes and that are hard to disentangle, but also for the possible underlying events in proton-proton collisions and the pile up of collisions happening during the same bunch crossing. From the phenomenological perspective the need of precise predictions obtainable through flexible tools able to produce distributions or events for any desired process and kinematic cuts is mandatory.

Many predictions are currently performed with tools merging the exact leading-order (LO) calculation of the partonic matrix elements (ME) with the evolution, provided by so-called shower Monte Carlo (MC) codes, of the partonic shower (PS) and the subsequent hadronization of the partons into physical hadrons [1, 2, 3, 4]. This is because they can readily be tuned with data and because there are many very flexible LO tools on the market. Further, the parton shower performs a resummation of the collinear emission giving a first reliable approximation in certain regions of phase space for important measured kinematical distributions where fixed order calculations fails. On the other hand the Next-to-Leading-Order (NLO) computations give more precise predictions on total rates, a better control of the jet structure of events and more reliable estimation of the theoretical error related to the renormalization and factorization

scale dependences. Also, the uncertainty related to the Parton Distributions Functions can be estimated more reliably within NLO computations. Therefore NLO matrix element generators at parton level paired with a parton shower represent the ideal instruments to address the phenomenology of the colliders. The Next-to-Next-to-Leading-Order computations are quite far from automation because the structure of the divergences has still not allowed for an easy and fully general implementation of a subtraction scheme and, as a result, only very few differential computations are available for hadron colliders processes. There exist a number of excellent NLO public tools containing many hard-coded processes and a full setup to produce phenomenological results, among them MCFM [5, 6], VBF@NLO [7, 8], MC@NLO [9, 10] and POWHEG [11, 12, 13]. Nevertheless, nowadays it is possible to flank these packages with fully automated tools able to generate any process up to NLO accuracy within one and the same framework, as it is already possible at LO with MADGRAPH/MADEVENT [14, 15] for example. Such automated codes are under construction by several groups partly interfacing well tested tools for subtasks of the full package. In the following, I will describe corrections within Quantum Chromodynamics (QCD), as these are the most relevant at present hadron colliders, although the most recent developments are quite general and apply to other one-loop computations as well.

## 2. Ingredients of NLO computations

The actual description of the hadron collisions characterized by a hard scale or a large momentum transfer among the hadron constituents, is based on the factorization formula:

$$\sigma_{AB} = \int dx_a dx_b f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \hat{\sigma}_{ab \rightarrow X}. \quad (1)$$

This equation expresses a dynamical property of the QCD that, when there is a hard collision, meaning a collision at large momentum transfer, the cross section  $\sigma$  for the scattering of initial hadrons  $A$  and  $B$ , can be computed as the convolution of two factors: the product of the parton distribution probability functions PDF's inside the hadrons  $f_{a/A} f_{b/B}$ , that represents the result of the low momentum scale dynamic, and the hard scale cross section among the partons  $\hat{\sigma}$ , summed over all the possible parton pairs that could scatter in the specific hadron-hadron collision experiment. The convolution is over the fractional momentum carried by the partons inside the hadrons. The actual knowledge of QCD does not allow to derive the PDF's, they have to be evaluated through dedicated experiments, but the formula is general so that the PDF's measured in an experiment can be used to assess prediction for another experiment. On the other hand the partonic cross section is calculable in perturbation theory as a series expansion of the QCD coupling constant.

The computation of the parton level matrix element up to NLO consists of several pieces: first there is of course the LO matrix element made of tree level Feynman diagrams, to get the NLO correction to this, diagrams with one extra parton in the final state have to be included, as well as virtual one-loop diagrams with the same number of external particles as the Born process. These two processes are of the same order in the coupling constants and in general separately divergent. The virtual part is both ultraviolet and infrared divergent, the UV divergences being removed through renormalization. The infrared divergences of the virtual part are cancelled by the ones of the real part leaving only the initial state mass singularities to be subtracted “renormalising” the parton distribution functions of the partons in the incoming hadrons. A complication here is related to the fact that the cancellation among the infrared singularities involves integrals over different phase space. One way to overcome this problem is the implementation of the so called subtraction method [16, 17, 18, 19], in which one designs counter matrix elements that map the singularities of the real part point-wise and that are analytically integrable over the extra radiation degrees of freedom, to be added to the virtual part as poles times born-like matrix

elements. So that numerically the NLO contribution amounts to construct two distinguished integrals well defined and numerically integrable in four dimensions.

Tools for the computation of tree level matrix elements contributing to the Born and the real correction processes have existed for many years and to a very high level of automation. The full formulation of the subtraction method has been known since many years and recently several packages have been constructed which, for a given process, perform the full generation of the counter terms and their integrated version [20, 21, 22, 23, 24, 25].

The need for an automation of NLO virtual calculations has been noticed some time ago and lead to public programs like FeynArts[26] and QGraf[27] for diagram generation and FormCalc/LoopTools [28, 29] and GRACE [30] for the automated calculation of NLO corrections, primarily in the electroweak sector. Nevertheless, until recently the computation with more than two particles in the final state were still an involved case by case computation. The number of particles in the final state sets the difficulty of NLO computations for basically two reasons. First the virtual computation even if conceptually straightforward becomes very extensive and difficult to keep under proper control. Second, the integration over the real matrix element phase space that has the extra parton becomes more and more time consuming. The problems related to the virtual part is however still considered the bottleneck for the automated next to leading order computation. The recent development in this last bit of automation will be the subject of the next section.

### 3. Recent developments

Improved reduction techniques for the tensor integrals have been proposed in [31] and [32], the first group also released a public library for the numerical computation of loop tensor integrals recently updated in [33]. It was then possible to construct quite general algorithms [34, 35] to reconstruct the tensor structure of a loop integrand in terms of the external particle kinematic tensors convoluted with the loop tensor integrals and then express the latter numerically thanks to the tensor loop integral libraries. Further, we have recently experienced major advances in the activity of constructing packages for fully automated one-loop calculations, see e.g. [36, 37, 38, 39, 40, 41]. The concepts that lead to these advances have been recently reviewed in [42]. Among the most important developments are the integrand-reduction technique [43, 44] and the generalized unitarity in dimensional regularization [45]. Their main outcome is a numerical reconstruction of a representation of the tensor structure of any one-loop integrand where the multi-particle pole configuration is manifest. As a consequence, decomposing one-loop amplitudes in terms of basic integrals becomes equivalent to reconstructing the polynomial forms of the residues to all multi-particle cuts. Within this algorithm, the integrand of a given scattering amplitude, carrying complete and explicit information on the chosen dimensional-regularisation scheme, is the only input required. In fact, the integration is substituted by a much simpler operation, namely by polynomial fitting, which requires the sampling of the integrand on the solutions of generalised on-shell conditions.

This kind of algorithm is very flexible, allowing for the construction of the integrand both by sewing of tree level on-shell amplitudes or through Feynman diagrams. The first approach works with gauge invariant building blocks that in principle allows for the construction of more stable and faster codes. Nevertheless it is still not clear how to automate the construction of the rational terms in general. The integrands constructed from Feynman diagrams contains of course all the information on the rational terms and so can easily be used for general purposes matrix element generators, but single diagrams are of course not gauge invariant and in principle there could be large cancellations among different diagrams.

The Ngluon [46] code is publically available and represents an example of the efficiency of the methods discussed above for the computation of pure QCD one loop virtual amplitudes, see the contribution by Biederman at this conference. The Blackhat [47] and Rocket [48] codes are

**Table 1.** Sample of processes for which GoSAM has been compared to the literature.

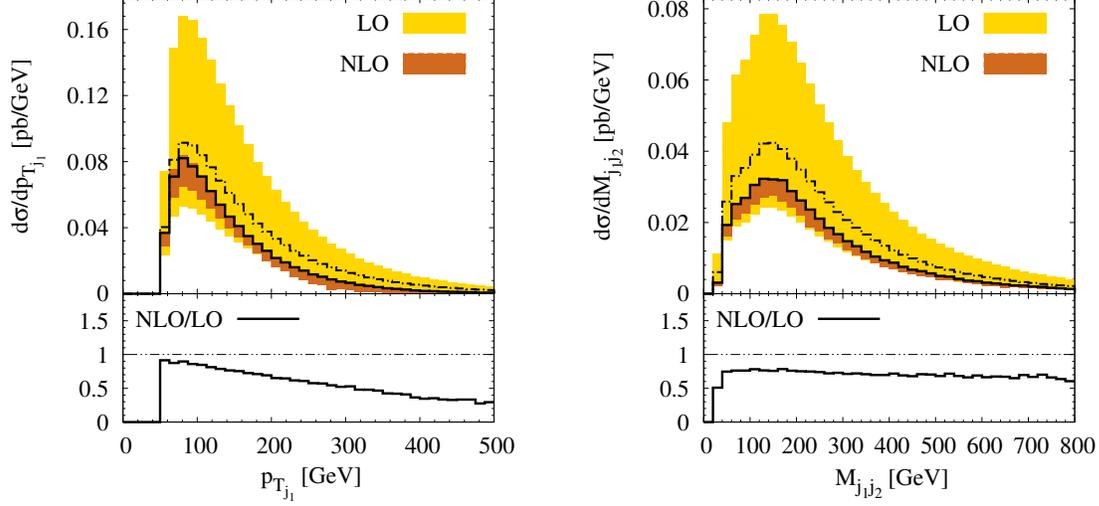
process	checked with Ref.
$e^+e^- \rightarrow u\bar{u}$	[50]
$e^+e^- \rightarrow t\bar{t}$	[51, 52], own analytic calculation
$u\bar{u} \rightarrow d\bar{d}$	[53, 37]
$gg \rightarrow gg$	[54]
$gg \rightarrow gZ$	[55], own analytic calculation
$pp \rightarrow t\bar{t}$	[37], [5, 6]
$bg \rightarrow Hb$	[56, 37]
$\gamma\gamma \rightarrow \gamma\gamma$ (W loop)	[57]
$pp \rightarrow W^\pm j$ (QCD corr.)	[5, 6]
$pp \rightarrow W^\pm j$ (EW corr.)	for IR poles: Eqs. (67),(70) of [58], [59]
$pp \rightarrow W^\pm t$	[5, 6]
$pp \rightarrow W^\pm jj$	[5, 6]
$pp \rightarrow W^\pm b\bar{b}$ (massive b's)	[5, 6]
$e^+e^- \rightarrow e^+e^-\gamma$ (QED)	[60]
$pp \rightarrow t\bar{t}H$	[37]
$pp \rightarrow t\bar{t}Z$	[40]
$\gamma\gamma \rightarrow \gamma\gamma\gamma\gamma$ (fermion loop)	[62]
$pp \rightarrow W^+W^+jj$	[61, v3]
$pp \rightarrow b\bar{b}b\bar{b}$	[63, 64]
$pp \rightarrow W^+W^-b\bar{b}$	[36, 37]
$pp \rightarrow t\bar{t}b\bar{b}$	[36, 37]
$u\bar{d} \rightarrow W^+ggg$	[36]

other examples. Further, other three codes for the automated generation of the virtual matrix elements have been recently made available: the HELAC-NLO [40] and the GoSam [49] codes have been released, while comparison with the MadLoop [37] code can be done through a web interface. In table 1 there is a list of SM processes tested with the GoSam code generator, see the contribution by G. Heinrich at this conference for the GoSam collaboration for more details.

#### 4. Selection of recent full physical results obtained exploiting the new developments

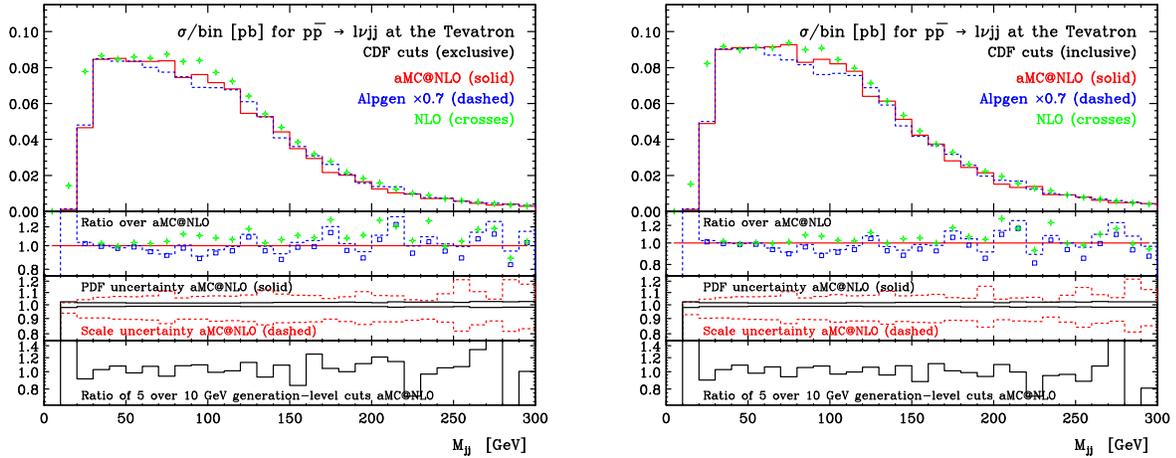
The big progress in the technical treatment of one loop correction allowed to perform computations of incredible complexity. Among the most relevant NLO results there has been the production of  $t\bar{t}b\bar{b}$  by two collaborations [65, 66], of  $W^+W^-$  plus two jets [67] and  $W^+W^-$  plus two b-jets [68, 69] at the hadron colliders. All these processes include six-point one loop diagrams. Further, the Blackhat collaboration pioneered the computation of the NLO QCD corrections to processes with seven external particles, producing precise predictions for the associate production of an electroweak gauge boson  $W/Z/\gamma^*$  and four jets [70, 71].

Regarding the results obtained through fully automated setup I will mention three recent achievements. The HELAC-NLO group produced a full study for the top-antitop plus two jets final state [72]. A sample of the long list of distributions presented in [72] is given in figure 1. These results have been obtained with a fully automated setup constructed out a suite of programs that perform the generation of the matrix elements, the needed subtractions and integrate everything to produce physical predictions. The second example is the study of the



**Figure 1.** (a)  $p_T$  of the first jet (top) and (b) dijet mass (bottom) distributions for the process  $pp \rightarrow t\bar{t}jj$  at the LHC with  $\sqrt{s} = 7 \text{ TeV}$ , see [72] for the cut selection and the other details of the analysis.

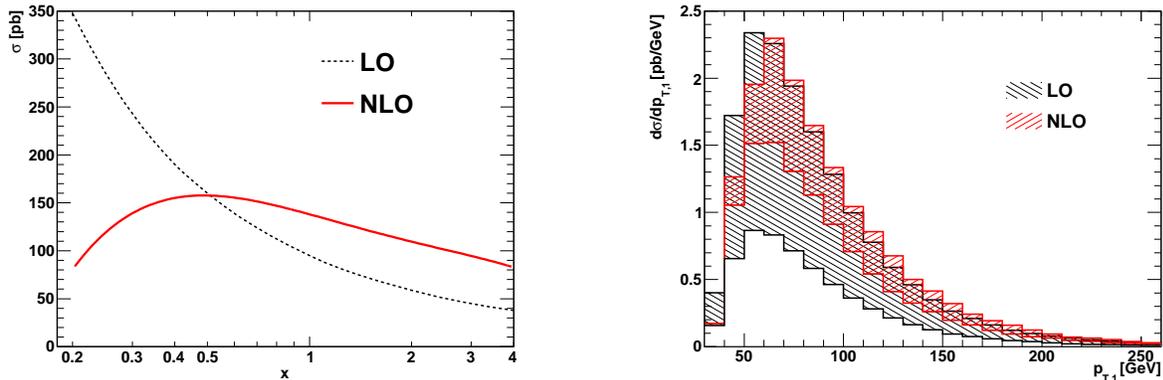
product of  $Wjj$  final state at the Tevatron by the aMC@NLO collaboration [73]. In this case the fully automated NLO parton level code produced merging MadLoop and MadFKS has been automatically interfaced with MC@NLO to produce event samples. The same strategy linking automatically Helac-NLO [40] with the Powheg-box [13] has been followed in [74] producing top-antitop final states in association with other particles, the most complicated example is the one including a  $Z$  boson [74]. As the last example I mention the four  $b$  quark jet production at



**Figure 2.** Dijet invariant mass distribution for the process  $p\bar{p} \rightarrow Wjj$  at the Tevatron energy studied by the aMC@NLO collaboration, see [73] for the detail of the analysis, in the bottom panel (b) a veto on extra jet radiation present in the top (a) is removed.

the LHC energies reported in [64]. In this case the generation of the virtual matrix elements has been done through the GoSam package, while the tree level matrix elements, the subtractions and the integration has been worked out with MadGraph/MadEvent and MadDipole. In this case the simplicity of the Binoth Les Houchs interface [75] allowed for the interface of the GoSam

virtual code generator with the MadGraph/MadEvent/MadDipole suite of programs.



**Figure 3.** (a) Scale variation (top) and (b)  $p_T$  distribution of the hardest jet (bottom) for the process  $pp \rightarrow b\bar{b}b\bar{b}$ , see [64] for the details of the analysis.

## 5. Conclusions

Thanks to the recent progress in the computation of the virtual corrections for scattering amplitudes, the full automation for packages performing the NLO computation of distributions related to multiparticle final states in hadronic collisions is now feasible. Several groups are working towards this goal. Once such generators are publically available they allow to perform every desired analysis at the NLO accuracy for both the signal and the main background processes to the experimental signature under study. The bottleneck for this achievement has been the automated computation of the virtual matrix elements. Nowadays, thanks to the implementation of modern techniques, examples of full automation in the production of NLO computations have been provided by the aMC@NLO, the HELAC-NLO and the GoSam+MadEvent+MadDipole combination of tools respectively. With perfect timing, extraordinarily powerful tools are being developed that hopefully will be of help to disentangle New Physics signals from the large source of backgrounds at the LHC.

## Acknowledgments

The author wishes to acknowledge N. Glover and G. Heinrich for the invitation to give this presentation.

## References

- [1] Mangano M L, Moretti M, Piccinini F, Pittau R and Polosa A D 2003 *JHEP* **0307** 001 (*Preprint hep-ph/0206293*)
- [2] Sjostrand T, Mrenna S and Skands P Z 2008 *Comput.Phys.Commun.* **178** 852 (*Preprint 0710.3820*)
- [3] Corcella G, Knowles I G, Marchesini G, Moretti S, Odagiri K, Richardson P, Seymour M H and Webber B R 2001 *JHEP* **0101** 010 (*Preprint hep-ph/0011363*)
- [4] Catani S, Krauss F, Kuhn R and Webber B R 2001 *JHEP* **0111** 063 (*Preprint hep-ph/0109231*)
- [5] Campbell J M and Ellis R K 1999 *Phys.Rev.* **D60** 113006 (*Preprint hep-ph/9905386*)
- [6] Campbell J M, Ellis R K, Williams C 2011 *JHEP* **1107** 018 (*Preprint 1105.0020*)
- [7] Arnold K, Bahr M, Bozzi G, Campanario F, Englert C, *et al.* 2009 *Comput.Phys.Commun.* **180** 1661-1670 (*Preprint 0811.4559*)
- [8] Arnold K, Bellm J, Bozzi G, Brieg M, Campanario F, *et al.* 2011 (*Preprint 1107.4038*)
- [9] Frixione S and Webber B R 2002 *JHEP* **0206** 029 (*Preprint hep-ph/0204244*)
- [10] Frixione S, Stoeckli F, Torrielli P, Webber B R, and White C D 2010 (*Preprint 1010.0819*)

- [11] Nason P 2004 *JHEP* **0411** 040 (*Preprint hep-ph/0409146*)
- [12] Frixione S, Nason P and Oleari C 2007 *JHEP* **0711** 070 (*Preprint 0709.2092*)
- [13] Alioli S, Nason P, Oleari C and Re E 2010 *JHEP* **1006** 043 (*Preprint 1002.2581*)
- [14] Maltoni F and Stelzer T 2003 *JHEP* **0302** 027 (*Preprint hep-ph/0208156*)
- [15] Alwall J, Herquet M, Maltoni F, Mattelaer O and Stelzer T 2011 *JHEP* **1106** 128 (*Preprint 1106.0522*)
- [16] Ellis R K, Ross D A and Terrano A E 1981 *Nucl.Phys.* **B178** 421
- [17] Catani S and Seymour M H 1997 *Nucl.Phys.* **B485** 421, 1998 **B510** 503, (*Preprint hep-ph/9605323*)
- [18] Catani S, Dittmaier S, Seymour M H and Trocsanyi Z 2002 *Nucl.Phys.* **B627** 189 (*Preprint hep-ph/0201036*)
- [19] Frixione S, Kunszt Z and Signer S 1996 *Nucl.Phys.* **B467** 399 (*Preprint hep-ph/9512328*)
- [20] Frederix R, Gehrmann T and Greiner N 2008 *JHEP* **0809** 122 (*Preprint 0808.2128*)
- [21] Frederix R, Gehrmann T and Greiner N 2010 *JHEP* **1006** 086 (*Preprint 1004.2905*)
- [22] Czakon M, Papadopoulos C G and Worek M 2009 *JHEP* **0908** 085 (*Preprint 0905.0883*)
- [23] Hasegawa K, Moch S and Uwer P 2010 *Comput.Phys.Commun.* **181** 1802 (*Preprint 0911.4371*)
- [24] Gleisberg T and Krauss F 2008 *Eur.Phys.J.* **C53** 501 (*Preprint 0709.2881*)
- [25] Frederix R, Frixione S, Maltoni F and Stelzer T 2009 *JHEP* **0910** 003 (*Preprint 0908.4272*)
- [26] Hahn T 2001 *Comput.Phys.Commun.* **140** 418–431 (*Preprint hep-ph/0012260*)
- [27] Nogueira P 1993 *J.Comput.Phys.* **105** 279–289.
- [28] Hahn T and Perez-Victoria M 1999 *Comput.Phys.Commun.* **118** 153–165 (*Preprint hep-ph/9807565*)
- [29] Hahn T, *Talk given at the International Workshop on Advanced Computing and Analysis Techniques in Physics Research (ACAT)*, Uxbridge, London, September 2011
- [30] Belanger G, Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, *et al.* 2006 *Phys.Rept.* **430** 117-209 (*Preprint hep-ph/0308080*)
- [31] Binoth T, Guillet J P, Heinrich G, Pilon E and Schubert C 2005 *JHEP* **0510** 015 (*Preprint hep-ph/0504267*)
- [32] Denner A and Dittmaier S 2006 *Nucl.Phys.* **B734** 62 (*Preprint hep-ph/0509141*)
- [33] Cullen G, Guillet J P, Heinrich G, Kleinschmidt T, Pilon E, Reiter T and Rodgers M 2011 *Comput.Phys.Commun.* **182** 2276 (*Preprint 1101.5595*)
- [34] Heinrich G, Ossola G, Reiter T and Tramontano F 2010 *JHEP* **1010** 105 (*Preprint 1008.2441*)
- [35] Cascioli F, Maierhofer P and Pozzorini S 2011 (*Preprint 1111.5206*)
- [36] van Hameren A, Papadopoulos C and Pittau R 2009 *JHEP* **0909** 106 (*Preprint 0903.4665*)
- [37] Hirschi V, Frederix R, Frixione S, Garzelli M V, Maltoni F *et al.* 2011 *JHEP* **1105** 044 (*Preprint 1103.0621*)
- [38] Mastrolia P, Ossola G, Reiter T and Tramontano F 2010 *JHEP* **1008** 080 (*Preprint 1006.0710*)
- [39] Cullen G, Greiner N, Guffanti A, Guillet J P, Heinrich G, *et al.* 2010 *Nucl.Phys.Proc.Suppl.* **205-206** 67–73 (*Preprint 1007.3580*)
- [40] Bevilacqua G, Czakon M, Garzelli M, van Hameren A, Kardos A, *et al.* 2011 (*Preprint 1110.1499*)
- [41] Reina L and Schutzmeier T 2011 (*Preprint 1110.4438*)
- [42] Ellis R K, Kunszt Z, Melnikov K and Zanderighi G 2011 (*Preprint 1105.4319*)
- [43] Ossola G, Papadopoulos C G and Pittau R 2007 *Nucl.Phys.* **B763** 147–169 (*Preprint hep-ph/0609007*)
- [44] Ossola G, Papadopoulos C G and Pittau R 2007 *JHEP* **07** 085 (*Preprint 0704.1271*)
- [45] Ellis R K, Giele W T, Kunszt Z and Melnikov K 2009 *Nucl.Phys.* **B822** 270–282 (*Preprint 0806.3467*)
- [46] Badger S, Biedermann B and Uwer P 2011 *Comput.Phys.Commun.* **182** 1674 (*Preprint 1011.2900*)
- [47] Berger C F, Bern Z, Dixon L J, Febres Cordero F, Forde D, Ita H, Kosower D A and Maitre D 2008 *Phys.Rev.* **D78** 036003 (*Preprint 0803.4180*)
- [48] Ellis R K, Giele W T, Kunszt Z, Melnikov K and Zanderighi G 2009 *JHEP* **0901** 012 (*Preprint 0810.2762*)
- [49] Cullen G, Greiner N, Heinrich G, Luisoni G, Mastrolia P, Ossola G, Reiter T and Tramontano F 2011 (*Preprint 1111.2034*)
- [50] Ellis R, Stirling W and Webber B 1996 *Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol.* **8** 1–435
- [51] Jersak J, Laermann E and Zerwas P 1982 *Phys.Rev.* **D25** 1218
- [52] Catani S, Dittmaier S, Seymour M H and Trocsanyi Z 2002 *Nucl.Phys.* **B627** 189–265 (*Preprint hep-ph/0201036*)
- [53] Ellis R and Sexton J 1986 *Nucl.Phys.* **B269** 445
- [54] Binoth T, Guillet J and Heinrich G 2007 *JHEP* **0702** 013 (*Preprint hep-ph/0609054*)
- [55] van der Bij J and Glover E 1989 *Nucl.Phys.* **B313** 237
- [56] Campbell J M, Ellis R, Maltoni F and Willenbrock S 2003 *Phys.Rev.* **D67** 095002 (*Preprint hep-ph/0204093*)
- [57] Gounaris G, Porfyriadis P and Renard F 1999 *Eur.Phys.J.* **C9** 673–686 (*Preprint hep-ph/9902230*)
- [58] Kuhn J H, Kulesza A, Pozzorini S and Schulze M 2008 *Nucl.Phys.* **B797** 27–77 (*Preprint 0708.0476*)
- [59] Gehrmann T and Greiner N 2010 *JHEP* **12** 050 (*Preprint 1011.0321*)
- [60] Actis S, Mastrolia P and Ossola G 2010 *Phys.Lett.* **B682** 419–427 (*Preprint 0909.1750*)
- [61] Melia T, Melnikov K, Rontsch R and Zanderighi G 2010 *JHEP* **1012** 053 (*Preprint 1007.5313*)
- [62] Bernicot C 2008 (*Preprint 0804.1315*)

- [63] Binoth T, Greiner N, Guffanti A, Reuter J, Guillet J P *et al.* 2010 *Phys.Lett.* **B685** 293–296 (*Preprint* 0910.4379)
- [64] Greiner N, Guffanti A, Reiter T and Reuter J 2011 *Phys.Rev.Lett.* **107** 102002 (*Preprint* 1105.3624)
- [65] Bredenstein A, Denner A, Dittmaier S and Pozzorini S 2009 *Phys.Rev.Lett.* **103** 012002 (*Preprint* 0905.0110)
- [66] Bevilacqua G, Czakon M, Papadopoulos C G, Pittau R and Worek M 2009 *JHEP* **0909** 109 (*Preprint* 0907.4723)
- [67] Melia T, Melnikov K, Rontsch R and Zanderighi G 2011 *Phys.Rev.* **D83** 114043 (*Preprint* 1104.2327)
- [68] Denner A, Dittmaier S, Kallweit S and Pozzorini S 2011 *Phys.Rev.Lett.* **106** 102002 (*Preprint* 1012.3975)
- [69] Bevilacqua G, Czakon M, van Hameren A, Papadopoulos C G and Worek M 2011 *JHEP* **1102** 083 (*Preprint* 1012.4230)
- [70] Berger C F, Bern Z, Dixon L J, Febres Cordero F, Forde D, Gleisberg T, Ita H, Kosower D A *et al.* 2011 *Phys.Rev.Lett.* **106** 092001 (*Preprint* 1009.2328)
- [71] Ita H, Bern Z, Dixon L J, Febres Cordero F, Kosower D A and Maitre D 2011 (*Preprint* 1108.2229)
- [72] Bevilacqua G, Czakon M, Papadopoulos C G and Worek M 2011 (*Preprint* 1108.2851)
- [73] Frederix R, Frixione S, Hirschi V, Maltoni F, Pittau R and Torrielli P 2011 (*Preprint* 1110.5502)
- [74] Kardos A, Papadopoulos C and Trocsanyi Z 2011 (*Preprint* 1111.0610)
- [75] Binoth T, Boudjema F, Dissertori G, Lazopoulos A, Denner A *et al.* 2010 *Comput.Phys.Commun.* **181** 1612–1622 dedicated to the memory of, and in tribute to, Thomas Binoth, who led the effort to develop this proposal for Les Houches 2009 (*Preprint* 1001.1307)