Different Forms of the Generalised Crewther Relation in QCD : Concrete Consequences of Analytical Multiloop Calculations

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Why analytical perturbative calculations (APC) important:

1. Precise check of predictions of gauge theories realised in nature (QCD, QED, EW);

a) Comparison with the data of e^+e^- colliders (Novosibirsk, Beijing, Frascati); data of LEP still exist;

b) Lepton-hadron DIS (CEBAF-JLAB working; lots of HERA, NOMAD (CERN) data still not studied, COMPASS (CERN); νN DIS Minerva (Fermilab) works;

c) Tevatron (existing) and new LHC data are appearing

2. New Theoretical Effects/Relations found in APC

Among them Generalized quark-parton Crewther Relation (72) in $\bar{M}S$-scheme **Broadhurst, Kataev (93)** in NNLO; **Crewther (97)** all orders proof; **Baikov, Chetyrkin, Kuhn (10)** in N^3LO; **Kataev, Mikhailov (10)** N^3LO $\bar{M}S$ resummed representation.

Physics which follows from this Generalized Relation between Characteristics of e^+e^- and DIS and Axial-Vector–Vector–Vector Triangle Diagram

a) Discovery of the Consequences of the Conformal Symmetry-relations between concrete parts of APC ;

b) Manifestation of Conformal Symmetry Breaking in QCD, related to exp. detectable Effect of **Asymptotic Freedom** in QCD ; Responsible for **Definition of Energy Momentum Tensor** in High Orders PC- Important for Self-Consistent Formulation of Gauge Theories ; Both are encoded in Factor $\beta(a_s)/a_s$ $a_s = \alpha_s/\pi$.

Notion of Conformal Symmetry and its PT Breaking

Conformal Symmetry (CS) is the generalisation of Poincaré symmetry ; namely the symmetry under following transformations

- Space-time translations $x^{'\ \mu} = x^\mu + \alpha^\mu$;
- Lorentz transformations $x^{'\ \mu} = \Lambda^\mu_\nu x^\nu$;
- Special conformal transformations $x^{'\ \mu} = \frac{x^\mu + \beta^\mu x^2}{1+2\beta\ x + \beta^2 x^2}$
- Scale transformations $x^{'\ \mu} = \rho\ x^\mu$

In PT this symmetry is broken by the procedure of renormalizations and and **non-zero** RG β-function.

Where in higher orders of APC one can discover the support of importance of both CS and CSB effects ???

Example of guess using notion on structure of CSB effects

Consider $D^V_A(a_s) = Q^2\int_0^\infty \frac{R^{e^+e^-}(s)}{(s+Q^2)^2}ds = D^{NS}_A(a_s) + D^{SI}_A(a_s)$

First part evaluated analytically up to a_s^4 in $SU(N_c)$ (**BChK (10)**)

$D^{NS}_A(a_s) = d_R(\sum_F Q_F^2)C^{NS}_A(a_s) = d_R(\sum_F Q_F^2)(1+\sum_{n=1}^4 d_n a_s^n + ...)$

5-loop diagrams for Green function of VV currents ;

$D^{SI}_A(a_s) = \big(\sum_F Q_F\Big)^2 C^{SI}_A(a_s) = \big(\sum_F Q_F\big)\big(\sum_{n=3}^4 d_n^{SI} a_s^n\big)$

SI-contributions to Green function of VV currents, d_3^{SI} known;

Gorishny,Kataev,Larin (91); Surguladze, Samuel (91)

!! Example of guess on structure of CSB effects in generalized Crewther relation !! Theoretical prediction **Kataev (11)**

$d_4^{SI} = d^{abc}d^{abc}\Big(C_F(-\frac{13}{64}-\frac{\zeta_3}{4}+\frac{5\zeta_5}{8})+C_A(\frac{481}{1152}-\frac{971}{1152}\zeta_3+\frac{295}{576}\zeta_5-\frac{11}{32}\zeta_3^2)$
$+(T_F N_F)(-\frac{119}{1152}+\frac{67}{288}\zeta_3-\frac{35}{144}\zeta_5+\frac{1}{8}\zeta_3^2)\Big)$ from guess on CSB effects and **Baikov, Chetyrkin, Kuhn (10)** evaluations of a_s^4 corrections to the Gross-Llewellyn Smith sum rule for νN DIS

Waiting for the direct APC by BChK- work in progress !!

DIS characteristics: Bjorken polarized sum rule
$\int_0^1 g_1^{lp-ln}(x,Q^2)dx = \frac{g_a}{6}C^{NS}_{Bjp}(a_s) = \frac{g_a}{6}\big(1+\sum_n^4 a_s^n c_n + ...)$
Measured at CEBAF at intermediate and low Q^2; Like the
coefficients d_1-d_4 of e^+e^--annihilation characteristic $D_A^{NS}(a_s)$
c_1-c_4 known analytically, depend from powers of $SU(N_c)$ structures
$C_F\ , C_A\ , T_F N_F, C_F^2, C_F C_A, C_F T_F N_F$, and $\frac{d_F^{abcd}d_A^{abcd}}{d_R}$, $\frac{N_F d_F^{abcd}d_F^{abcd}}{d_R}$
(in c_4 and d_4)
GLS sum rule : $\frac{1}{2}\int_0^1 F_3^{\nu p+\overline{\nu}p}(x,Q^2)dx = 3C_{GLS}(a_s)$
$C_{GLS}(a_s) = C^{NS}_{GLS}(a_s) + C^{SI}_{GLS}(a_s) = C^{NS}_{Bjp}(a_s) + C^{SI}_{GLS}(a_s)$,
$C^{SI}_{GLS} = \sum_{n=3}^4 c_n^{SI}a_s^n + ...$ (**BChK (10)**)
Ellis-Jaffe polarized sum rule
$\int_0^1 g_1^{lp}(x,Q^2)dx = C^{NS}_{Bjp}(a_s)(\frac{1}{12}a_3 + \frac{1}{36}a_8) + C^{SI}_{EJp}(a_s)\frac{1}{9}\Delta\Sigma(Q^2)$
where a_3, a_8 and $\Delta\Sigma$ in the $\bar{M}S$ are defined through polarized
parton distributions Δu, Δd, Δs, $C^{SI}_{EJP}(a_s) = 1 + \sum_{n=1}^3 c_n^{EJ,\ SI}a_s^3$
Larin, van Ritbergen and Vermaseren (97) - expansion of
$\int_0^{a_s}(\gamma_{SI}(a)/\beta(a))da$ taken into account in defining $c_n^{EJ,\ SI}$.

Generalized $\bar{M}S$-scheme Crewther relation(s)

$C_A^{NS}(a_s(Q^2)) \times C_{Bjp}^{NS}(a_s(Q^2)) = 1 + \Delta_{CSB}(a_s(Q^2))$

$\Delta_{\rm CSB}(a_s) = \left(\frac{\beta(a_s)}{a_s}\right) \mathcal{P}(a_s)$ where $\mathcal{P}(a_s) = \sum_{m\geq 1} K_m a_s^m$ (Note $(\beta(a_s)/a_s)$ enters into conformal anomaly- anomalous dimension of Trace of Energy Momentum Tensor!) $K_1 = K_1[1,0,0]{\rm C_F}$,

$K_2 = K_2[2,0,0]{\rm C_F^2} + K_2[1,1,0]{\rm C_F C_A} + K_2[1,0,1]{\rm C_F T_F N_F}$ notice ${\rm T_F N_F}$-dependence (!) **Broadhurst, Kataev (93)**

$K_3 = K_3[3,0,0]{\rm C_F^3} + K_3[2,1,0]{\rm C_F^2 C_A} + K_3[1,2,0]{\rm C_F C_A^2} + K_3[2,0,1]{\rm C_F^2 T_F N_F} + K_3[1,1,1]{\rm C_F C_A T_F N_F} + K_3[1,0,2]{\rm C_F T_F^2 N_F^2}$

BChK (10), contain $K_3[1,0,2]$-term evaluated in **BK (93)**

K_1 contain rational numbers, ζ_3; K_2- rational, ζ_3, ζ_5,

K_3- rational, ζ_3, ζ_5, ζ_7.

In this approximation $\beta(a_s)$ is 3-loop QCD β-function, evaluated by **Tarasov, Vladimirov, Zharkov (1980)**.

How to use for checks: 1) Strong check of validity of complicated calculations of a_s^4 coefficients in $C_A^{NS}(a_s(Q^2))$ and $C_{BJP}^{NS}(a_s(Q^2))$ by **BChK (10)**)

a) cancellation of $\frac{d_F^{abcd}d_A^{abcd}}{d_R}$, $\frac{N_F d_F^{abcd}d_F^{abcd}}{d_R}$ and C_F^4 and of some other contributions to d_4 and c_4 - consequence of Crewther relation, valid in the Conformal Invariant Limit ($\beta(a_s)=0$)

b) Possibility to compare with resumming $\mathbf{T_F N_F}$-contributions to $\mathcal{P}(a_s)$ polynomial representation of generalized Crewther

$\Delta_{\rm CSB} = \sum_{n\geq 1}\sum_{r\geq 1}\left(\frac{\beta(a_s)}{a_s}\right)^n \; P_n^{(r)}[\mathrm{k},\mathrm{m}]C_F^{\mathrm{k}}C_A^{\mathrm{m}}a_s^r$, $r=\mathrm{k}+\mathrm{m}$

Kataev, Mikhailov (10) valid at a_s^4-level In high-orders may (or may not) get additional $d^{abcd}d^{abcd}$ contributions. c): Consequence: fixation of $\beta(N_F=n_0)=0$, namely choice $\mathrm{T_F N_F}=(11/4)\mathrm{C_A}$ leads to $\mathrm{d}_4(n_0)+\mathrm{k}_4(n_0) = -\frac{333}{1024}\mathrm{C_F^4}+\mathrm{C_A C_F^3}\left(-\frac{1661}{3072}+\frac{1309}{128}\zeta_3-\frac{165}{16}\zeta_5\right)+$
$+\mathrm{C_A^2 C_F^2}\left(-\frac{3337}{1536}+\frac{7}{2}\zeta_3-\frac{105}{16}\zeta_5\right)+\mathrm{C_A^3 C_F}\left(-\frac{28931}{12288}+\frac{1351}{512}\zeta_3\right)$

This expression agrees with result, which follows from **BChK (10)** - additional confirmation of the result of complicated calculations

How to use for predictions: 1) The comparison of generalized Crewther relations

$C_A^{NS}(a_s(Q^2)) \times C_{Bjp}^{NS}(a_s(Q^2)) = 1 + \Delta_{csb}^{NS}(a_s(Q^2))$

$C_A^{V}(a_s(Q^2)) \times C_{GLS}(Q^2) = 1 + \Delta_{csb}^{V,GLS}(a_s(Q^2))$

and the guess of "universality"

$C_A^{NS}(a_s(Q^2)) \times C_{Bjp}^{NS}(a_s(Q^2)) = C_A^{V}(a_s(Q^2)) \times C_{GLS}(Q^2)$

which seems to follow from application of OPE method to AVV triangle amplitude allows to get already described theoretical prediction for a_s^4 correction to SI contribution to

$D_A^V(a_s) = Q^2 \int_0^\infty \frac{R^{e^+e^-}(s)}{(s+Q^2)^2} ds = D_A^{NS}(a_s) + D_A^{SI}(a_s)$ **Kataev (11)**

2) Application of the generalized Crewther relation in the SI channel gives possibility to prove the following identity in the**CI-limit** $C_A^{SI}(a_s) \times C_{EJp}^{SI}(a_s) = C_A^{NS}(a_s) \times C_{Bjp}^{NS}(a_s)$

which gives $C_{EJp}^{SI}(a_s)|_{CI-limit} = C_{Bjp}^{NS}(a_s)|_{CI-limit}$ **Kataev (10)**

Planned calculations by **Baikov, Chetyrkin, Kuhn** of a_s^4 terms to Ellis-Jaffe sum rule have prediction for control of future APC.

Conclusion

There are a lot of interesting problems for future APC (analytical perturbative calculations):

1) Study of the consequences of Conformal Symmetry and Conformal Symmetry Breaking

2) Closely related to study of the possibility of the evaluation of high order corrections to Triangle Diagrams with two large external momentum (at present some studies already exist, but at 2-loop level- (**Braguta and Onishchenko (04)**; **F. Jegerlehner and O. V. Tarasov (06)**), etc.- CI-limit.

3) Studies of generalized Crewther relation may give additional insight on some special features of schemes and scheme-dependence problem (**Garkusha and Kataev (11)**)