

Multiloop calculations in supersymmetric theories with the higher covariant derivative regularization

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Abstract. Most calculations of quantum corrections in supersymmetric theories are made with the dimensional reduction, which is a modification of the dimensional regularization. However, it is well known that the dimensional reduction is not self-consistent. A consistent regularization, which does not break the supersymmetry, is the higher covariant derivative regularization. However, the integrals obtained with this regularization can not be usually calculated analytically. We discuss application of this regularization to the calculations in supersymmetric theories. In particular, it is demonstrated that integrals defining the β -function are possibly integrals of total derivatives. This feature allows to explain the origin of the exact NSVZ β -function, relating the β -function with the anomalous dimensions of the matter superfields. However, integrals for the anomalous dimension should be calculated numerically.

1. Introduction

In order to deal with divergent expressions in the quantum field theory, it is necessary to regularize a theory. A proper choice of a regularization can simplify the calculations or reveal some features of quantum corrections. Most calculations in the quantum field theory were made with the dimensional regularization [1] in \overline{MS} -scheme [2]. However, the dimensional regularization breaks the supersymmetry and is not convenient for calculations in supersymmetric theories. That is why most calculations in supersymmetric theories were made with the dimensional reduction [3]. For example, the β -function in supersymmetric theories was calculated up to the four-loop approximation [4, 5, 6, 7, 8]. After a special redefinition of the coupling constant [9, 10] the result coincides with the exact NSVZ β -function, proposed in Refs. [11, 12, 13, 14].

However, it is well known that the dimensional reduction is not self-consistent [15]. Removing the inconsistencies one breaks the explicit supersymmetry [16, 17]. Then the supersymmetry can be broken by quantum corrections in higher loops [18, 19, 20]. In the $N = 2$ SYM theory this already occurs in the three-loop approximation [18, 20], while in the $N = 4$ SYM theory the supersymmetry is not broken even in the four-loop approximation [21]. Thus, a problem of regularization in supersymmetric theories is rather nontrivial [22].

For supersymmetric theories one can use the higher covariant derivative regularization, proposed by A.A.Slavnov [23, 24]. Different versions of this regularization for supersymmetric theories, which do not break the supersymmetry, were proposed in [25, 26]. Unlike the dimensional reduction, the higher covariant derivative regularization is consistent. However, it

was not often applied to explicit calculations of quantum corrections, because the corresponding integrals have very complicated structure, and it is not easy to calculate them analytically, especially in higher loops. Moreover, some theoretical subtleties can raise nontrivial questions even in the simplest calculations [27, 28, 29].

However, we argue that for supersymmetric theories this regularization has some very attractive features and can be used for the calculations. Namely, the integrals defining the β -function are integrals of double total derivatives [30, 31, 32, 33], and one of them can be calculated analytically. As a result, the β -function is related with the anomalous dimension, producing the exact NSVZ β -function without redefinition of the coupling constant. In this paper we demonstrate how this can be proved in $N = 1$ SQED in all loops and for the general renormalizable $N = 1$ SYM in the two-loop approximation.

2. Quantum corrections in $N=1$ SQED, regularized by higher derivatives

2.1. Higher derivative regularization

The action of the massless $N = 1$ SQED in terms of superfields [34, 35] is written as

$$S = \frac{1}{4e^2} \text{Re} \int d^4x d^2\theta W_a C^{ab} W_b + \frac{1}{4} \int d^4x d^4\theta \left(\phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right). \quad (1)$$

The theory is regularized by adding the term with the higher derivatives

$$S_\Lambda = \frac{1}{4e^2} \text{Re} \int d^4x d^2\theta W_a C^{ab} \left(R \left(\frac{\partial^2}{\Lambda^2} \right) - 1 \right) W_b, \quad (2)$$

where $R(0) = 1$ and $R(\infty) = \infty$. For example, it is possible to choose $R = 1 + \partial^{2n}/\Lambda^{2n}$. The gauge is fixed by adding

$$S_{\text{gf}} = -\frac{1}{64e^2} \int d^4x d^4\theta \left(V R \left(\frac{\partial^2}{\Lambda^2} \right) D^2 \bar{D}^2 V + V R \left(\frac{\partial^2}{\Lambda^2} \right) \bar{D}^2 D^2 V \right). \quad (3)$$

Then the propagator will contain large degrees of the momentum in the denominator, and all loop diagrams beyond the one-loop approximation become convergent. The remaining one-loop diagrams are regularized by inserting the Pauli–Villars determinants [36]

$$\prod_I \left(\int D\phi_I^* D\phi_I e^{iS_I} \right)^{-c_I} \quad (4)$$

into the generating functional, where

$$S_I = \frac{1}{4} \int d^4x d^4\theta \left(\phi_I^* e^{2V} \phi_I + \tilde{\phi}_I^* e^{-2V} \tilde{\phi}_I \right) + \left(\frac{1}{2} \int d^4x d^4\theta M_I \phi_I \tilde{\phi}_I + \text{h.c.} \right) \quad (5)$$

and $\sum c_I = 1$, $\sum c_I M_I^2 = 0$. It is important that the masses M_I are proportional to the parameter Λ .

2.2. Three-loop β -function

In order to find the β -function we consider

$$\Gamma_V^{(2)} = -\frac{1}{16\pi} \int \frac{d^4p}{(2\pi)^4} d^4\theta V(-p) \partial^2 \Pi_{1/2} V(p) d^{-1}(\alpha, \mu/p) \quad (6)$$

and calculate

$$\frac{d}{d \ln \Lambda} \left(d^{-1}(\alpha_0, \Lambda/p) - \alpha_0^{-1} \right) \Big|_{p=0} = -\frac{d\alpha_0^{-1}}{d \ln \Lambda} = \frac{\beta(\alpha_0)}{\alpha_0^2}. \quad (7)$$

In the three-loop approximation the result can be written as ($R_k \equiv R(k^2/\Lambda^2)$)

$$\begin{aligned} \frac{\beta(\alpha_0)}{\alpha_0^2} &= 2\pi \frac{d}{d \ln \Lambda} \sum_I c_I \int \frac{d^4 q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \frac{\ln(q^2 + M^2)}{q^2} + 4\pi \frac{d}{d \ln \Lambda} \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{e^2}{k^2 R_k^2} \\ &\times \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left(\frac{1}{q^2(k+q)^2} - \sum_I c_I \frac{1}{(q^2 + M_I^2)((k+q)^2 + M_I^2)} \right) \left[R_k \left(1 + \frac{e^2}{4\pi^2} \ln \frac{\Lambda}{\mu} \right) \right. \\ &- 2e^2 \left(\int \frac{d^4 t}{(2\pi)^4} \frac{1}{t^2(k+t)^2} - \sum_J c_J \int \frac{d^4 t}{(2\pi)^4} \frac{1}{(t^2 + M_J^2)((k+t)^2 + M_J^2)} \right) \left. \right] \\ &+ 4\pi \frac{d}{d \ln \Lambda} \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{d^4 l}{(2\pi)^4} \frac{e^4}{k^2 R_k l^2 R_l} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \left(-\frac{2k^2}{q^2(q+k)^2(q+l)^2(q+k+l)^2} \right. \right. \\ &+ \left. \left. \frac{2}{q^2(q+k)^2(q+l)^2} \right) - \sum_I c_I \left(-\frac{2(k^2 + M_I^2)}{(q^2 + M_I^2)((q+k)^2 + M_I^2)((q+l)^2 + M_I^2)} \right. \right. \\ &\left. \left. \frac{1}{((q+k+l)^2 + M_I^2)} + \frac{2}{(q^2 + M_I^2)((q+k)^2 + M_I^2)((q+l)^2 + M_I^2)} - \frac{1}{(q^2 + M_I^2)^2} \right. \right. \\ &\left. \left. \times \frac{4M_I^2}{((q+k)^2 + M_I^2)((q+l)^2 + M_I^2)} \right) \right\}. \quad (8) \end{aligned}$$

From this expression we see that the β -function is defined by the integrals of (double) total derivatives [30, 31, 32]. Such a structure allows to calculate one of the loop integrals analytically and, thus, to reduce a number of integrations. This can be done using the identity

$$\int \frac{d^4 q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left(\frac{f(q^2)}{q^2} \right) = \lim_{\varepsilon \rightarrow 0} \int_{S_\varepsilon} \frac{dS_\mu}{(2\pi)^4} \frac{(-2)q^\mu f(q^2)}{q^4} = \frac{1}{4\pi^2} f(0), \quad (9)$$

where f is a nonsingular function which rapidly tends to 0 at the infinity. It is equivalent to the identity

$$\int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \frac{d}{dq^2} f(q^2) = \frac{1}{16\pi^2} (f(\infty) - f(0)) = -\frac{1}{16\pi^2} f(0). \quad (10)$$

(This is a total derivative in the four-dimensional spherical coordinates.) Taking the integrals in Eq. (8) and comparing the result with the expression for the two-loop anomalous dimension of the matter superfield

$$\begin{aligned} \gamma(\alpha_0) &= -2e^2 \int \frac{d^4 k}{(2\pi)^4} \frac{d}{d \ln \Lambda} \frac{1}{k^4 R_k^2} \left[R_k \left(1 + \frac{e^2}{4\pi^2} \ln \frac{\Lambda}{\mu} \right) - \int \frac{d^4 t}{(2\pi)^4} \frac{2e^2}{t^2(k+t)^2} \right. \\ &+ \left. \sum_I c_I \int \frac{d^4 t}{(2\pi)^4} \frac{2e^2}{(t^2 + M_I^2)((k+t)^2 + M_I^2)} \right] - \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 l}{(2\pi)^4} \frac{d}{d \ln \Lambda} \frac{4e^4 k_\mu l_\mu}{k^4 R_k l^4 R_l (k+l)^2}, \quad (11) \end{aligned}$$

in the considered approximation we obtain the exact NSVZ β -function for the $N = 1$ SQED [14]:

$$\beta(\alpha_0) = \frac{\alpha_0^2}{\pi} (1 - \gamma(\alpha_0)) + O(\alpha_0^5). \quad (12)$$

This equality of the well-defined integrals is obtained without any redefinitions of the coupling constant. Thus, with the higher derivative regularization the NSVZ scheme can be naturally defined. However, it is much more complicated problem to calculate analytically the integrals for the anomalous dimension. Possibly, in the lowest orders this can be done analytically, but it seems that in higher loops for this purpose one should use numerical methods.

2.3. The exact result

For the $N = 1$ SQED it is possible to demonstrate that the features discussed in the previous section take place in all loops. In particular, the integrals defining the β -function are integrals of double total derivatives, and the β -function coincides with the NSVZ expression without a redefinition of the coupling constant [37].

For this purpose, first, we make the substitution

$$V \rightarrow \bar{\theta}^a \bar{\theta}_a \theta^b \theta_b \equiv \theta^4, \quad (13)$$

which allows to extract the function d^{-1} in Eq. (6). Then we will try to present the sum of Feynman diagrams as integrals of total derivatives, which in the coordinate representation are given by

$$\text{Tr}([x^\mu, \text{Something}]) = 0. \quad (14)$$

We start with the expression for the part of the effective action corresponding to the two-point Green function of the gauge superfield [37]

$$\Delta\Gamma_{\mathbf{V}}^{(2)} = \left\langle -2i \left(\text{Tr}(\mathbf{V} J_0 *) \right)^2 - 2i \text{Tr}(\mathbf{V} J_0 * \mathbf{V} J_0 *) - 2i \text{Tr}(\mathbf{V}^2 J_0 *) \right\rangle + \text{terms with } \tilde{*} + (PV), \quad (15)$$

where

$$* \equiv \frac{1}{1 - (e^{2V} - 1) \bar{D}^2 D^2 / 16 \partial^2}, \quad \tilde{*} = \frac{1}{1 - (e^{-2V} - 1) \bar{D}^2 D^2 / 16 \partial^2} \quad (16)$$

encode sequences of vertexes and propagators on the matter line, and $J_0 = e^{2V} \bar{D}^2 D^2 / 16 \partial^2$ is the effective vertex. (PV) denotes contributions of the Pauli–Villars fields. The first term in Eq. (15) is a sum of diagrams in which external lines are attached to different loops of the matter superfields. The second term is a sum of diagrams in which external lines are attached to a single line of the matter superfields. The last term is not transversal. The sum of such terms vanishes due to the Ward identities.

After substitution (13) and some algebraic transformations [37] the first term in Eq. (15) gives the contribution

$$-2i \frac{d}{d \ln \Lambda} \left\langle \left(\text{Tr} \left(-2\theta^c \theta_c \bar{\theta}^d [\bar{\theta}_d, \ln(*)] - \ln(\tilde{*}) \right) + i \bar{\theta}^c (\gamma^\nu)_c{}^d \theta_d [y_\nu^*, \ln(*)] - \ln(\tilde{*}) \right) \right\rangle + (PV)^2. \quad (17)$$

Similarly, the second term in Eq. (15) gives

$$i \frac{d}{d \ln \Lambda} \text{Tr} \left\langle \theta^4 \left[y_\mu^*, \left[(y^\mu)^*, \ln(*) + \ln(\tilde{*}) \right] \right] \right\rangle + (PV) - \text{terms with a } \delta\text{-function}. \quad (18)$$

The third term in Eq. (15) vanishes after substitution (13). From expressions (17) and (18) we see that in all orders the β -function is given by integrals of double total derivatives. A different method to see this [31] is based on the covariant Feynman rules in the background field method [38, 39].

Expressions (17) and (18) can be calculated explicitly in the three-loop approximation. The result coincides with Eq. (8).

Terms with the δ -function in Eq. (18) appear due to the identity

$$[x^\mu, \frac{\partial_\mu}{\partial^4}] = [-i \frac{\partial}{\partial p_\mu}, -\frac{i p^\mu}{p^4}] = -2\pi^2 \delta^4(p_E) = -2\pi^2 i \delta^4(p). \quad (19)$$

Due to this δ -function one of loop integrals can be calculated and a number of integrations is reduced. Qualitatively, we consider all diagrams in which two external gauge lines are attached to the same graph. Integration of the δ -function corresponds to cutting a matter line in this graph [31]. This gives diagrams with two external matter lines, defining the anomalous dimension. Thus, the β -function in a certain loop is reduced to the anomalous dimension in the previous loop. These arguments can be formulated rigorously [37] in all orders and allow to obtain the exact NSVZ β -function

$$\beta(\alpha) = \frac{\alpha^2}{\pi} (1 - \gamma(\alpha)). \quad (20)$$

Note that deriving Eq. (20) one does not redefine the coupling constant, as in the case of the dimensional reduction [9, 10]. Therefore, with the higher derivative regularization we can naturally define the NSVZ scheme for the $N = 1$ SQED.

3. Two-loop β -function with the higher covariant derivative regularization in the non-Abelian case

Let us consider a general renormalizable $N = 1$ supersymmetric Yang–Mills theory with a gauge group G and matter superfields ϕ_i in a representation R , in the massless limit:

$$S = \frac{1}{2e^2} \text{Re tr} \int d^4x d^2\theta W_a C^{ab} W_b + \frac{1}{4} \int d^4x d^4\theta (\phi^*)^i (e^{2V})_i{}^j \phi_j + \left(\frac{1}{6} \int d^4x d^2\theta \lambda^{ijk} \phi_i \phi_j \phi_k + \text{h.c.} \right) \quad (21)$$

This theory is invariant under the gauge transformation if

$$(T^A)_m{}^i \lambda^{mjk} + (T^A)_m{}^j \lambda^{imk} + (T^A)_m{}^k \lambda^{ijm} = 0. \quad (22)$$

Below we assume that this condition is satisfied. In order to introduce the higher covariant derivative regularization and calculate the β -function in this case we will use the background field method. In the supersymmetric case [34, 35] we split the gauge superfield (which is below denoted by V') into the quantum part V and the background field Ω according to the prescription $e^{2V'} \equiv e^{\Omega^+} e^{2V} e^\Omega$. Then the gauge can be fixed without breaking the background gauge invariance:

$$S_{\text{gf}} = -\frac{1}{32e^2} \text{tr} \int d^4x d^4\theta (V D^2 \bar{D}^2 V + V \bar{D}^2 D^2 V). \quad (23)$$

(In our notation D , \bar{D} , and D_α are the background covariant derivatives.) Certainly, the gauge fixing procedure also requires introducing the Faddeev–Popov and Nielsen–Kallosh ghosts.

The higher covariant derivative regularization can be also introduced without breaking the background gauge invariance. This can be done by different ways. For example, it is possible to add

$$S_\Lambda = \frac{1}{2e^2} \text{tr Re} \int d^4x d^4\theta V \frac{(\mathbf{D}_\mu^2)^{n+1}}{\Lambda^{2n}} V + \frac{1}{8} \int d^4x d^4\theta \left((\phi^*)^i \left[e^{\Omega^+} e^{2V} \frac{(\mathbf{D}_\alpha^2)^m}{\Lambda^{2m}} e^\Omega \right]_i^j \phi_j + \right. \\ \left. + (\phi^*)^i \left[e^{\Omega^+} \frac{(\mathbf{D}_\alpha^2)^m}{\Lambda^{2m}} e^{2V} e^\Omega \right]_i^j \phi_j \right) \quad (24)$$

to the action, assuming that $n > m$. (It is important that for a theory with a nontrivial cubic superpotential a term with the higher covariant derivatives should be also introduced for the matter superfields.)

As in the case of $N = 1$ SQED, the higher covariant derivative term does not remove divergences in the one-loop approximation, and in order to regularize them the Pauli–Villars determinants should be inserted into the generating functional. The Pauli–Villars fields should be introduced for the matter superfields and all ghosts. (A contribution of the gauge superfields in the one-loop approximation vanishes.) Masses of the Pauli–Villars superfields ϕ_I are proportional to the parameter Λ : $M_I^{ij} = a_I^{ij} \Lambda$ and satisfy the relation $M_I^{ij} (M_I^*)_{jk} = M_I^2 \delta_k^i$.

Using the background gauge invariance it is possible to choose $\Omega = \Omega^+ = \mathbf{V}$. Then the function d^{-1} is defined by

$$\Gamma_V^{(2)} = -\frac{1}{8\pi} \text{tr} \int \frac{d^4p}{(2\pi)^4} d^4\theta \mathbf{V}(-p) \partial^2 \Pi_{1/2} \mathbf{V}(p) d^{-1}(\alpha, \lambda, \mu/p). \quad (25)$$

In order to find the β -function we again use prescription (7) and find the derivative of d^{-1} with respect to $\ln \Lambda$ in the limit of the vanishing external momentum. In the non-Abelian case the result is given by

$$\beta(\alpha, \lambda) = -\frac{3\alpha^2}{2\pi} C_2 + \alpha^2 T(R) I_0 + \alpha^3 C_2^2 I_1 + \frac{\alpha^3}{r} C(R)_i^j C(R)_j^i I_2 \\ + \alpha^3 T(R) C_2 I_3 + \alpha^2 C(R)_i^j \frac{\lambda_{jkl}^* \lambda^{ikl}}{4\pi r} I_4 + \dots, \quad (26)$$

where

$$\text{tr}(T^A T^B) \equiv T(R) \delta^{AB}; \quad (T^A)_i^k (T^A)_k^j \equiv C(R)_i^j; \\ f^{ACD} f^{BCD} \equiv C_2 \delta^{AB}; \quad r \equiv \delta_{AA}. \quad (27)$$

The integrals defining the β -function are given by

$$I_i = I_i(0) - \sum_I c_I I_i(M_I), \quad i = 0, 2, 3, \quad (28)$$

where for simplicity we do not write the ghost contributions (they are also given by integrals of double total derivatives) and

$$I_0(M) = -\pi \int \frac{d^4q}{(2\pi)^4} \frac{d}{d \ln \Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \frac{1}{q^2} \ln \left(q^2 (1 + q^{2m}/\Lambda^{2m})^2 + M^2 \right) \right\}; \quad (29)$$

$$I_1 = -12\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial k^\mu} \frac{\partial}{\partial k_\mu} \left\{ \frac{1}{k^2(1+k^{2n}/\Lambda^{2n})q^2(1+q^{2n}/\Lambda^{2n})(q+k)^2} \right. \\ \left. \times \frac{1}{(1+(q+k)^{2n}/\Lambda^{2n})} \right\}; \quad (30)$$

$$I_2(M) = 2\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \frac{(2+(q+k)^{2m}/\Lambda^{2m}+q^{2m}/\Lambda^{2m})^2}{k^2(1+k^{2n}/\Lambda^{2n})} \right. \\ \left. \times \frac{(1+q^{2m}/\Lambda^{2m})(1+(q+k)^{2m}/\Lambda^{2m})}{(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2)((q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})^2+M^2)} \right\}; \quad (31)$$

$$I_3(M) = 2\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial k_\mu} \left\{ \frac{(2+k^{2m}/\Lambda^{2m}+q^{2m}/\Lambda^{2m})^2}{(k+q)^2(1+(q+k)^{2n}/\Lambda^{2n})} \right. \\ \left. \times \frac{(1+k^{2m}/\Lambda^{2m})(1+q^{2m}/\Lambda^{2m})}{(k^2(1+k^{2m}/\Lambda^{2m})^2+M^2)(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2)} \right\}; \quad (32)$$

$$I_4 = -8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \frac{1}{k^2(1+k^{2m}/\Lambda^{2m})q^2(1+q^{2m}/\Lambda^{2m})(q+k)^2} \right. \\ \left. \times \frac{1}{(1+(q+k)^{2m}/\Lambda^{2m})} \right\}. \quad (33)$$

Thus, we see that the β -function is given by integrals of double total derivatives. In the considered (two-loop) approximation they can be easily calculated analytically using Eq. (9):

$$\beta = -\frac{\alpha^2}{2\pi} (3C_2 - T(R)) + \frac{\alpha^3}{(2\pi)^2} \left(-3C_2^2 + T(R)C_2 + \frac{2}{r} C(R)_i{}^j C(R)_j{}^i \right) - \frac{\alpha^2 C(R)_i{}^j \lambda_{jkl}^* \lambda^{ikl}}{8\pi^3 r} + \dots \quad (34)$$

This expression should be compared with the one-loop anomalous dimension

$$\gamma_i{}^j(\alpha) = -\frac{\alpha C(R)_i{}^j}{\pi} + \frac{\lambda_{ikl}^* \lambda^{jkl}}{4\pi^2} + \dots \quad (35)$$

Then we see that in the considered approximation the β -function agrees with the exact NSVZ β -function

$$\beta(\alpha) = -\frac{\alpha^2 [3C_2 - T(R) + C(R)_i{}^j \gamma_j{}^i(\alpha)/r]}{2\pi(1 - C_2\alpha/2\pi)}. \quad (36)$$

4. Conclusion

Although it is generally believed that the integrals appearing with the higher covariant derivative regularization have too complicated structure, we see that in the supersymmetric case some of them can be calculated analytically. This makes possible analytical multiloop calculations in supersymmetric theories with this regularization. In principle, in the lowest loops it is not very difficult to construct integrals corresponding to various Green functions.

A very attractive feature of the higher covariant derivative regularization is that all integrals defining the β -function in the supersymmetric case seem to be integrals of double total derivatives. As a consequence, one of them can be calculated analytically. (For $N = 1$ SQED this was proved exactly in all loops. For the general renormalizable $N = 1$ SYM theory this was so far verified only in the two-loop approximation.) In both considered cases the factorization of integrands into total derivatives allows to obtain the exact NSVZ β -function, and for $N = 1$ SQED for this purpose it is not necessary to redefine the coupling constant. (In the non-Abelian case we cannot so far make this conclusion, because the calculation was made only in the two-loop approximation, where the β -function is scheme-independent.)

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