Three-Loop Calculation of the Higgs Boson Mass in Supersymmetry

Philipp Kant
Humboldt-Universität zu Berlin
E-mail: philipp.kant@physik.hu-berlin.de

Abstract. A Key feature of the minimal supersymmetric extension of the Standard Model (MSSM) is the existence of a light Higgs boson, the mass of which is not a free parameter but an observable that can be predicted from the theory. Given that the LHC is able to measure the mass of a light Higgs with very good accuracy, a lot of effort has been put into a precise theoretical prediction.

We present a calculation of the SUSY-QCD corrections to this observable to three-loop order. We perform multiple asymptotic expansions in order to deal with the multi-scale three-loop diagrams, making heavy use of computer algebra and keeping a keen eye on the numerical error introduced.

We provide a computer code in the form of a Mathematica package that combines our three-loop SUSY-QCD calculation with the literature of one- and two-loop corrections to the Higgs mass, providing a state-of-the-art prediction for this important observable.

1. Introduction

The minimal supersymmetric extension of the Standard Model (MSSM) is a promising candidate for physics beyond the Standard Model. Its Higgs sector is a two-Higgs doublet model with the additional constraint that supersymmetry relates the quartic Higgs couplings to the gauge couplings of the theory. This increases the predictiveness of the model and allows the Higgs sector to be parametrised by just two new parameters, the mass M_A of the pseudoscalar Higgs and the ratio $\tan\beta$ of the vacuum expectation values of the Higgs doublets.

In particular, the mass M_h of the light Higgs boson is not a free parameter, but can be predicted. At tree level, M_h is bounded above by M_Z, but radiative corrections shift the value significantly. Since M_h will be a precision observable once the Higgs is found at the Large Hadron Collider (LHC), it is imperative to have a precise theoretical prediction. Consequently, a lot of effort has been put into the calculation of radiative corrections to M_h at the one- and two-loop level (see, for example [1, 2, 3, 4, 5, 6, 7, 8]). The remaining theoretical uncertainty has been estimated to be about $3 - 5\,\mathrm{GeV}$ [9, 10], which is confirmed by a study of the leading and next-to-leading terms in $\ln(M_{SUSY}/M_t)$, where M_{SUSY} is the typical scale of SUSY particle masses, at three-loop order [11].

This uncertainty of the theoretical prediction justifies a calculation of the next term in the perturbative expansion. A study of the corrections to M_h shows that the contributions from loops of top quarks and their superpartners, the stops, are dominant at the one- and two-loop level. In [12, 13], we have calculated three-loop SUSY-QCD corrections to these diagrams. The calculation of these terms is the subject of this talk.

2. Calculation of the Three-Loop Corrections

Motivated by the discussion above, we restrict the calculation at the three-loop level to the SUSY-QCD corrections where the Higgs couples to a top quark or its superpartners. In the perturbative expansion of M_h, these are the terms of $\mathcal{O}\left(\alpha_t \alpha_s^2\right)$, where α_s is the strong coupling and α_t denotes the coupling of the Higgs to the top quark.

We thus have to evaluate three-loop propagator diagrams in SUSY-QCD, which faces us with two complications. First, we need a convenient regulator that respects supersymmetry, and second, a lot of masses appear in our diagrams.

2.1. Regularisation by Dimensional Reduction

The regularisation workhorse of multi-loop calculations is dimensional regularisation (DREG) [14], where the number of spacetime dimensions is altered from 4 to $d = 4 - 2\varepsilon$, and the divergences of the loop integrals are manifest as poles in ε.

Unfortunately, DREG does not respect supersymmetry. An easy way to see this is that changing the number of spacetime dimensions also changes the number of degrees of freedoms of the vector fields, and supersymmetry requires an equal number of bosonic and fermionic degrees of freedom. While it is possible (but tedious) to manually construct finite counterterms that restore the supersymmetric ward identities, a more convenient approach was suggested by Siegel under the name of dimensional reduction (DRED) [15]. The main idea is that the change of dimensions need only affect momenta in order to regularise the loop integrals. In DRED, one splits the four dimensional space into orthogonal spaces of dimension 2ε and $d = 4 - 2\varepsilon$, and restricts the loop momenta to the d dimensional subspace while all gauge fields are kept four dimensional. The 2ε components of the gauge fields transform as a 2ε tuple of scalar fields and are called ε-scalars.

The consistent formulation of DRED is more involved than sketched here [16], and the question whether the inclusion of the ε-scalars truly restores the supersymmetric ward identities in all cases is not yet resolved. However, DRED has successfully been applied in many multi-loop calculations[17, 18, 19, 20, 21, 22, 23, 24, 25, 26], so far without observing a violation of the ward identities.

2.2. Asymptotic Expansion in the Masses

The difficulty of calculating a Feynman diagram rises with the number of loops, the number of external momenta and the number of masses in the diagram. To obtain the $\mathcal{O}\left(\alpha_t \alpha_s^2\right)$ terms of M_h, we have to evaluate diagrams with three loops, two legs and a lot of masses: treating the light quarks as massless and their superpartners as mass degenerate with mass $M_{\tilde{q}}$, we are left with five masses: the top quark mass M_t, the masses of its superpartners $M_{\tilde{t}_1}$, $M_{\tilde{t}_2}$, the mass of the gluino $M_{\tilde{g}}$ and $M_{\tilde{q}}$. Calculating these diagrams without any approximations is not feasible with contemporary methods.

Given the mild dependence of the one-loop corrections to M_h on the external momentum, it is reasonable to expand the diagrams in small external momentum, reducing the problem to the evaluation of vacuum diagrams. A further simplification is possible by performing nested asymptotic expansions [27, 28, 29, 30], which expresses the multi-mass diagrams as a series in ratios of the masses and logarithms of these ratios. The coefficients in the series involve only one-scale integrals. Assuming that the ratios of the masses are small, the series should give good approximations to the original diagrams.

Of course, before carrying out the asymptotic expansions, one has to identify which mass ratios are small. Given that so far none of the superpartner masses are known, this is an undecidable problem. So, instead of calculating with a fixed, known, hierarchy among the masses, one has to systematically consider various possible mass hierarchies and carry out the calculation for each of these. To get a prediction of M_h for a specific point in the parameter

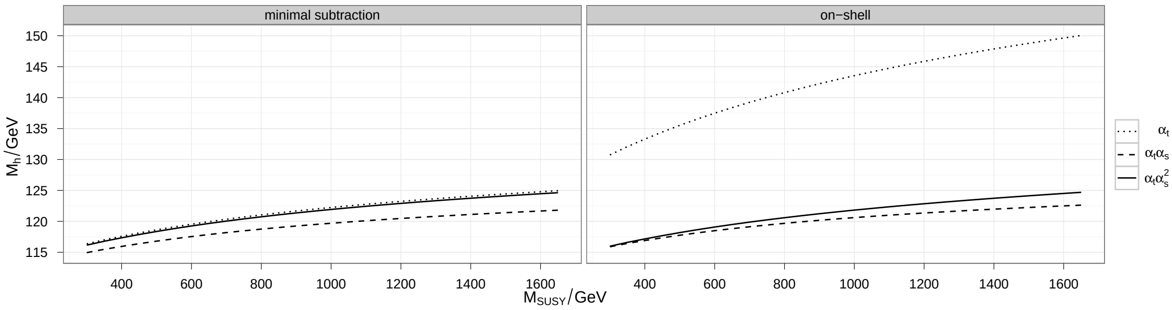

Figure 1. Behaviour of the perturbative series for M_h using the on-shell and the minimal subtraction scheme (for degenerate superpartner masses). The short-dashed, long-dashed and solid lines are the one-, two-, and three-loop approximation to M_h, respectively. In the on-shell scheme, the distance between one- and two-loop approximation is much larger than in the $\overline{\text{DR}}$ scheme.

space of the MSSM, where the masses of the superpartners have specific values, one has to choose whichever hierarchy fits these values best.

We carry out the calculation using the following setup: In a first step, all diagrams contributing to M_h at $\mathcal{O}\left(\alpha_t\alpha_s^2\right)$ are found using the program QGRAF [31]. There are 30.717 diagrams. For each hierarchy, these diagrams are expanded asymptotically using Q2E/EXP [32]. The one-scale integrals are then calculated using the FORM [33] program MATAD [34].

2.3. Renormalisation

For the renormalisation of the parameters entering our calculation, we adopt a variation of the $\overline{\text{DR}}$-scheme, i.e. minimal subtraction using dimensional reduction. This leads to a much better behaviour of the perturbative series compared to using on-shell renormalisation (see Fig. 1).

3. Computer Code

In order to provide a precise prediction for the value of M_h within the MSSM, our $\mathcal{O}\left(\alpha_t\alpha_s^2\right)$ terms have to be combined with the wealth of corrections from other sectors of the MSSM at one- and two-loop level that are available in the literature [1, 2, 3, 4, 5, 6, 7, 8]. Also, the choice of hierarchy mentioned in 2.2 should be automatised. In [13], we have presented the MATHEMATICA package H3M, which is publicly available, to address these points. For convenience, it implements the Susy Les Houches Accord SLHA [35] and has an interface to the spectrum generators SOFTSUSY [36], SuSPECT [37] and SPHENO [38].

The program uses FEYNHIGGS [39, 9] to get a two-loop approximation of M_h. An obstacle for adding our terms of $\mathcal{O}\left(\alpha_t\alpha_s^2\right)$ to the result delivered by FEYNHIGGS is the usage of on-shell renormalisation in FeynHiggs. In order to be consistent, we have to convert the $\mathcal{O}\left(\alpha_t\right)$ and $\mathcal{O}\left(\alpha_t\alpha_s\right)$ terms that contain on-shell parameters to the $\overline{\text{DR}}$ scheme. Thanks to [7], which gives a compact expression for these terms both in the $\overline{\text{DR}}$ and in the on-shell scheme, this obstacle is easily overcome.

The corrections to M_h depend very strongly on the mass M_t of the top quark and the strong coupling α_s. These parameters must be known, within SUSY-QCD renormalised in the $\overline{\text{DR}}$ scheme, as precisely as possible. In [40], the relation between the top mass in the $\overline{\text{DR}}$- and the on-shell scheme in SUSY-QCD has been calculated to $\mathcal{O}\left(\alpha_s^2\right)$. Using the library TSIL [41], this relation can be used to obtain $M_t^{\overline{\text{DR}}}$ from the known value of the top quark pole mass.

We determine α_s in SUSY-QCD from the value of α_s in five-flavour QCD at the Z mass following [42] by running, within five-flavour QCD, to the decoupling scale where we perform the transition to the full theory. We then run, within SUSY-QCD, to the desired value of the renormalisation scale.

In order to choose the most appropriate hierarchy and get an estimate for the error introduced by the asymptotic expansion, we compute an approximation to the two-loop corrections of $\mathcal{O}(\alpha_t \alpha_s)$ using asymptotic expansions in the mass ratios and compare this approximation to the result of [7]. The hierarchy that matches the exact two-loop result best is chosen for the calculation of the three-loop term, and the error at two-loop level is recorded to get a handle on the error in the three-loop result (see Fig. 4).

4. Numerical Results

In this chapter, we present a state-of-the-art numerical prediction for M_h in the MSUGRA model that has been obtained with H3M. Fig. 2 shows the dependence of M_h on the parameters $m_{1/2}$, m_0, $\tan\beta$, and A_0 for $\mu > 0$. The shaded bands around the individual curve show the variation of M_h when M_t is varied between 171.4 GeV and 174.4 GeV. We restrict the plot to values of $m_{1/2} > 300$ GeV in light of the latest exclusions from the LHC experiments [43, 44, 45, 46].

We observe that of all MSUGRA parameters, varying $m_{1/2}$ has by far the largest impact on the Higgs mass. We also observe that the present uncertainty of the top mass directly translates to a parametric uncertainty of about one GeV of the Higgs mass.

To estimate the effect of unknown higher orders, Fig. 3 shows the two- and three-loop corrections to M_h. We observe a partial cancellation between the two- and three-loop correction. We also observe that while the higher order corrections do decrease in magnitude, they do not do so by a large factor. Thus, we should be careful when estimating the magnitude of the missing higher order contributions. We choose to assign 50 % of the three-loop contribution as a theoretical error. This amounts to a hundred MeV for low values of $m_{1/2}$ and to about one GeV for $m_{1/2}$ above one GeV. Using the scale variation as an error estimate would lead to a smaller error [13].

By performing asymptotic expansions, we have introduced an additional source of uncertainty to the prediction of M_h. With Fig. 4, we can analyse how large this uncertainty is. It shows the error that we would have introduced had we made the same approximation at the two-loop level. We see that this error is typically at or below 100 MeV, or 200 MeV for low values of $m_{1/2}$ and $\tan\beta$. Since the three-loop corrections are smaller than the two-loop corrections, we expect the actual error due to asymptotic expansions to be below 100 MeV.

5. Conclusions

We have presented a calculation of the $\mathcal{O}(\alpha_t \alpha_s^2)$ corrections to the mass M_h of the light Higgs boson in the MSSM. These contributions shift the value of M_h by $1 - 3$ GeV, depending on the mass spectrum of the superpartners.

The results have been implemented in the program H3M, which is freely available [13].

Using our results, we improve the theoretical error significantly. The theoretical error is now on the order of about 100 MeV for light and 1 GeV for heavy superpartner masses. This is comparable to the parametric uncertainty with the top mass and α_s.

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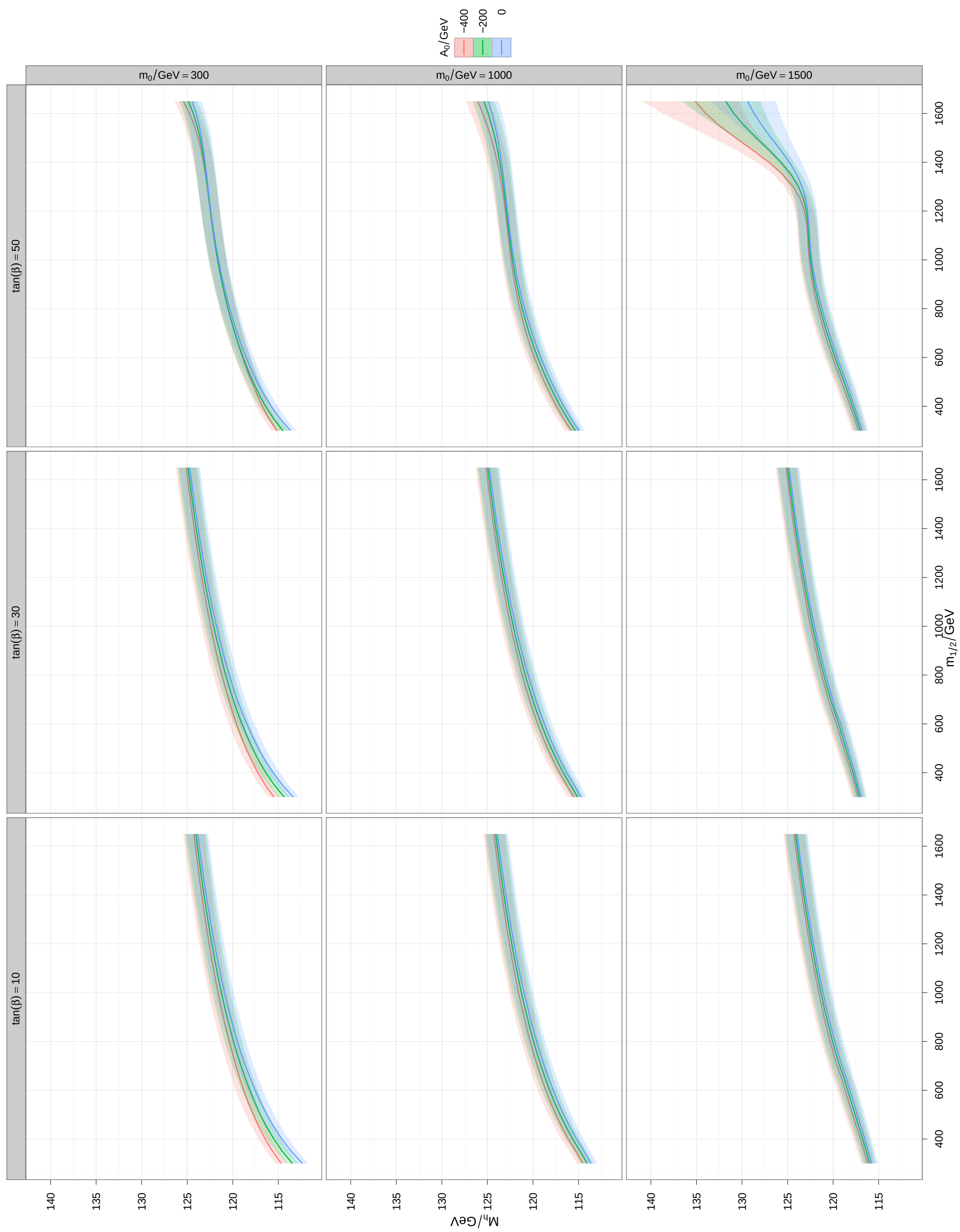

Figure 2. Theoretical prediction for the light Higgs mass in the MSUGRA model. We plot M_h over the fermion mass parameter $m_{1/2}$. The panels show, from left to right, an increasing value of $\tan\beta$, and, from top to bottom, of the scalar mass parameter m_0. We show three curves according to different values of the trilinear coupling A_0.

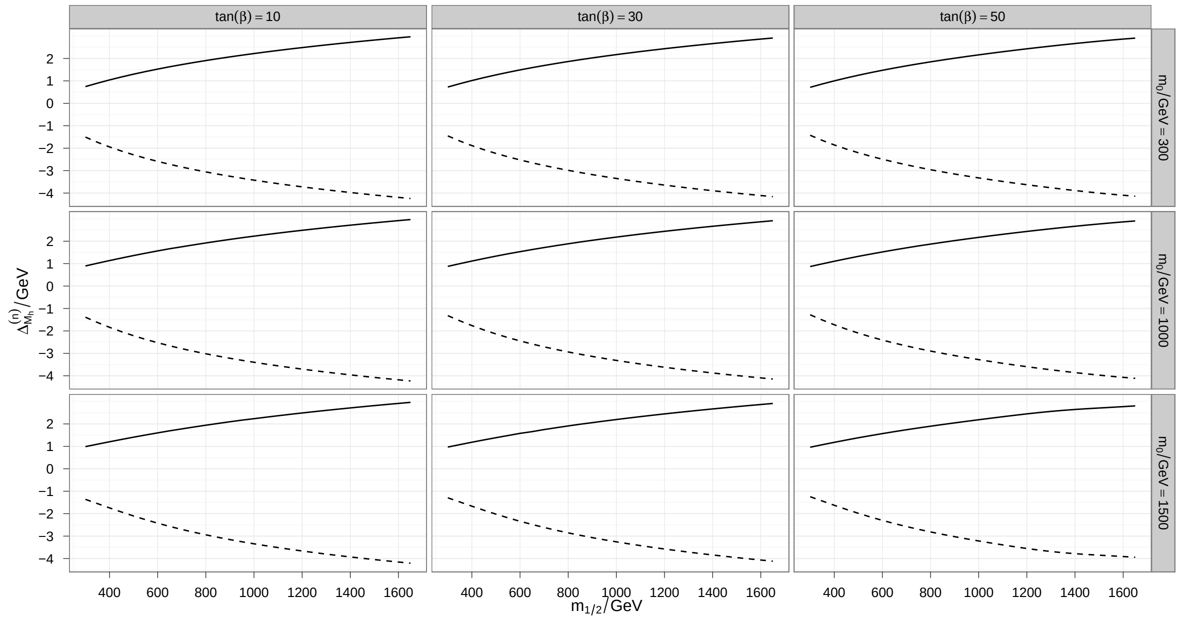

Figure 3. Two-loop (dashed) and three-loop (solid) contributions to M_h. The same conventions as in Fig. 2 are adopted. For lucidity, only the curves for $A_0 = 0$ and $M_t = 172.9\,\text{GeV}$ are shown. We observe that the three-loop terms are smaller than the two-loop terms, but not by a large factor. This suggests a conservative approach when estimating higher order effects.

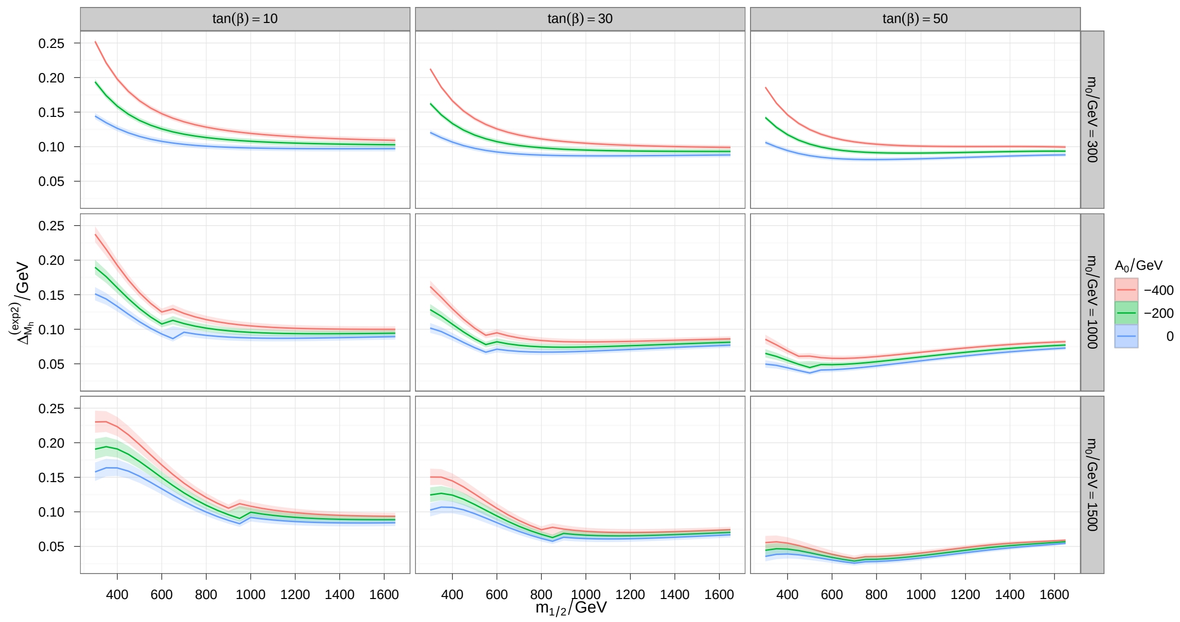

Figure 4. Deviation of the expansion in masses from the exact result at two loops. The conventions are the same as in Fig. 2. The error introduced by the asymptotic expansions is well under control. The little bumps in the curves are due to a change from one hierarchy to another.

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