Light Cone 2021: Physics of Hadrons on the Light Front
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QCD analysis of pion parton distributions

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Outline

- Motivation(s) for studying pion PDF structure

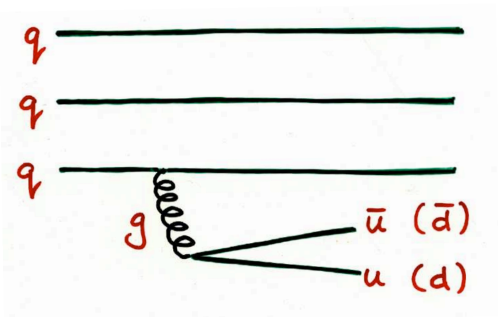
- Consistent description requires simultaneously fitting pion PDFs to Drell-Yan and leading neutron electroproduction data
  - $\longrightarrow$ map out pion structure from low x to high x
  - $\longrightarrow$ constraints on gluon & sea quark PDFs at low x

- Extend analysis to incorporate transverse momentum
  - $\longrightarrow$ sensitivity to gluon PDF at high x
  - $\longrightarrow$ pion TMDs ("3-d structure")

- Global QCD analysis with threshold resummation
  - $\longrightarrow$ consequences for high-x pion PDFs
  - $\longrightarrow$ supports $\sim (1-x)$ behavior at large x

Sea of the proton

- From text-books: perturbative QCD expected to generate symmetric $q\bar{q}$ sea via gluon radiation into $q\bar{q}$ pairs



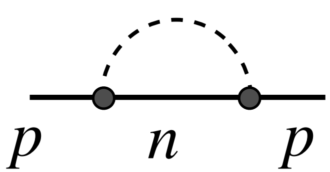
$\rightarrow$ since u and d quarks nearly degenerate, expect flavor-symmetric light-quark sea

$$\bar{d} \approx \bar{u}$$

Ross, Sachrajda (1979)

- (Almost) from text-books: Thomas suggested that chiral symmetry of QCD ("low energy") should have consequences for antiquark PDFs in the nucleon ("high energy")

$\pi^+ \longleftarrow (u\bar{d})$

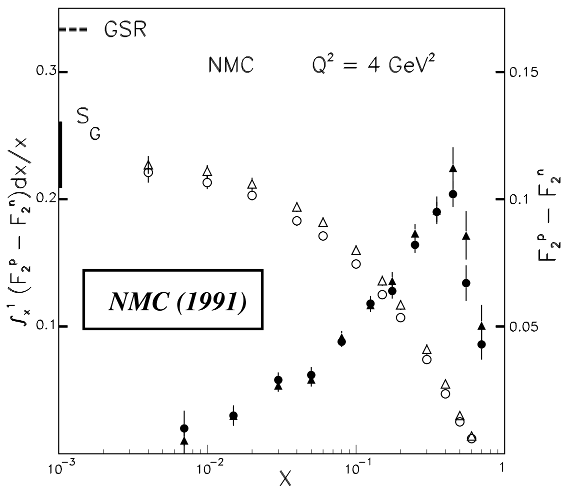


$$\rightarrow \quad \bar{d} > \bar{u}$$

Thomas (1984)

Sea of the proton

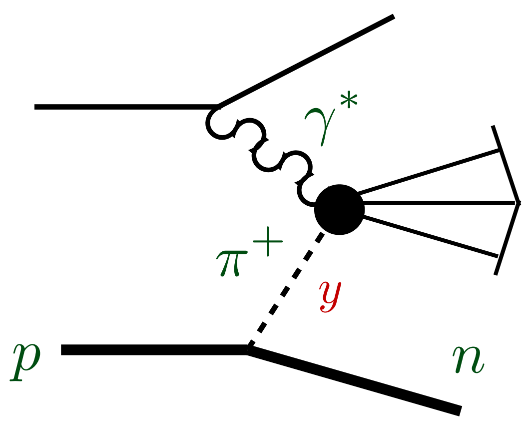
- First clear experimental support for $\bar{d} \neq \bar{u}$ came from measurement of Gottfried sum observed by NMC



$$\int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,(\bar{d} - \bar{u})$$
$$= 0.235(26)$$

→ violation of "Gottfried sum rule"

- Sullivan process — DIS from pion cloud of the nucleon



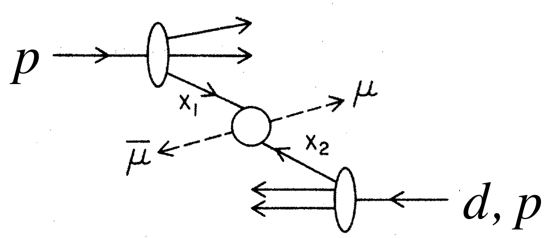
Sullivan (1972)

$$(\bar{d} - \bar{u})(x) = \int_x^1 \frac{dy}{y} f_{\pi^+ n}(y)\, \bar{q}_v^\pi(x/y)$$

$p \to \pi^+ n$ splitting function ("flux factor"), computed from chiral effective theory

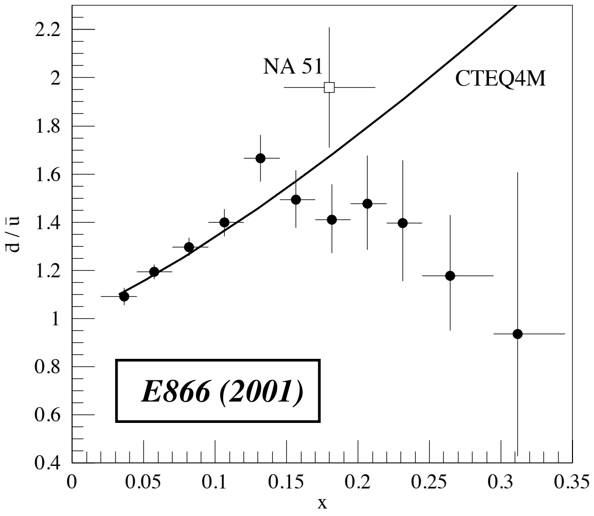
Sea of the proton

- x dependence of $\bar{d} - \bar{u}$ asymmetry established in Fermilab E866 *pp/pd* Drell-Yan experiment



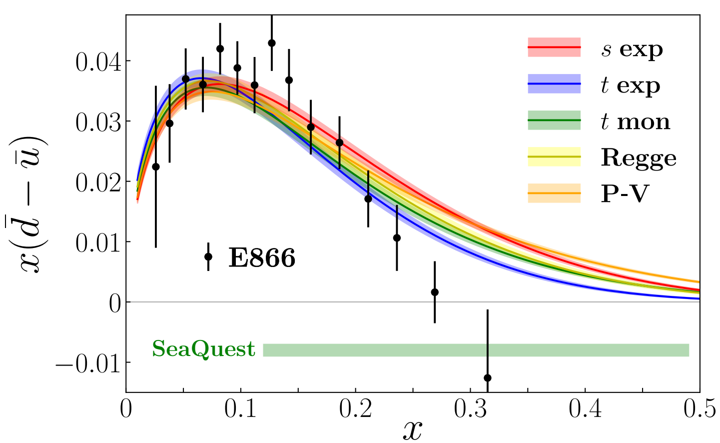
$$\frac{d\sigma}{dx_1 dx_2} \sim \sum_q e_q^2\, q(x_1)\, \bar{q}(x_2) \;+\; (x_1 \leftrightarrow x_2)$$

$$\frac{\sigma^{pd}}{\sigma^{pp}} \approx 1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)} \quad \text{for } x_1 \gg x_2$$



→ data can be well described within chiral EFT / pion cloud framework

→ need to know pion PDF!

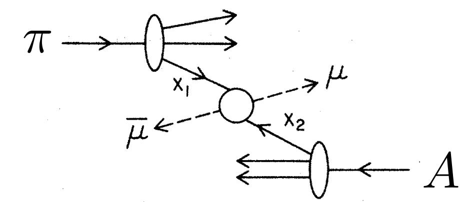


Barry, Sato, WM, C.-R. Ji
PRL 121, 152001 (2018)

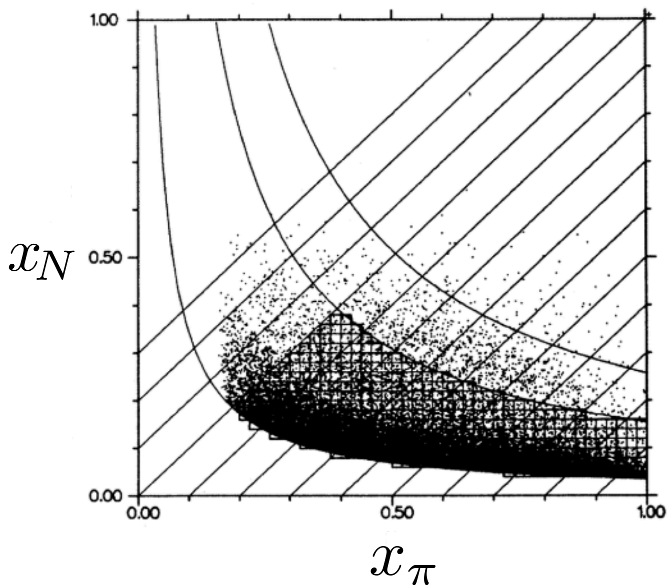
PDFs in the pion — Drell-Yan

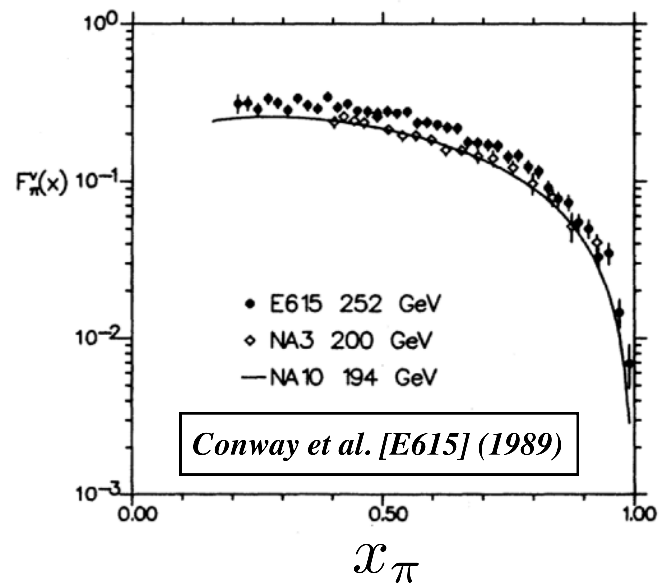
- PDFs in the pion difficult to study experimentally

  ⟶ most information has come from pion-tungsten Drell-Yan data (CERN, Fermilab)



  ⟶ constrains valence PDFs at $x \gg 0$

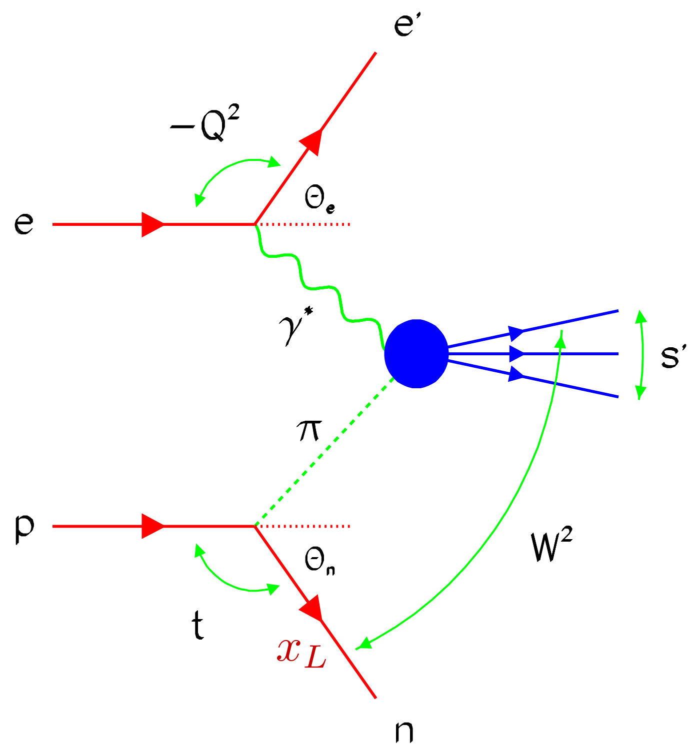


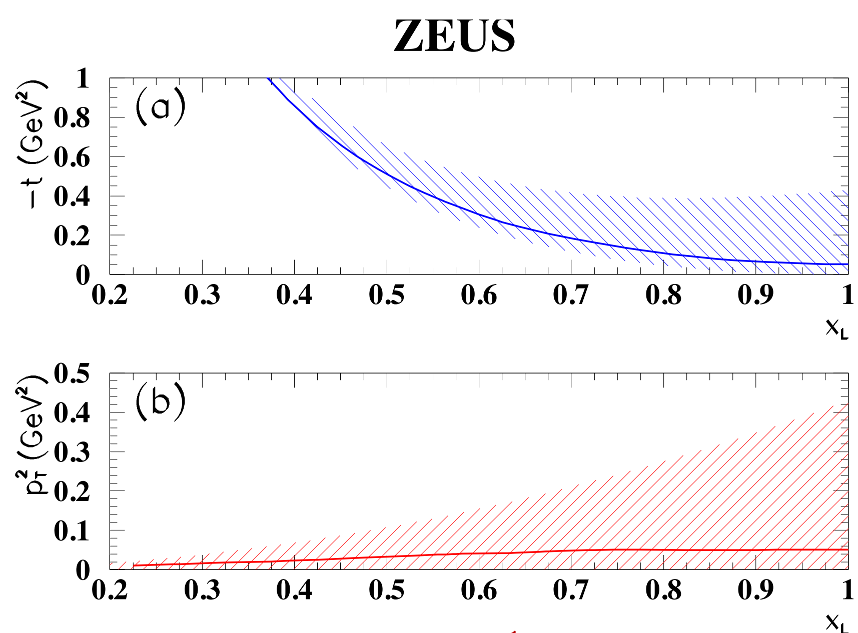


… but pion sea quark & gluon PDFs at small x unconstrained

PDFs in the pion — leading neutrons

- ZEUS & H1 collaborations at HERA measured spectra of neutrons produced at very forward angles, $\theta_n < 0.8$ mrad



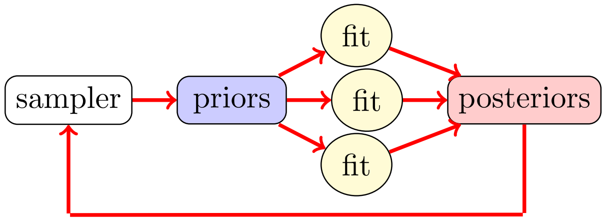


→ can data be described within Sullivan process?

→ first simultaneous fit performed by JAM Collaboration…

JAM global QCD analysis

- **Theoretical framework**
  - — collinear factorization (NLO)
  - — iterative Monte Carlo
  - — Bayesian sampling of parameter space



- **Traditional functional form for PDFs**

$$f(x) = N\, x^{\alpha} (1-x)^{\beta}\, P(x)$$

polynomial, neural net, …

→ iterate until convergence (posteriors = priors)

- **"Bayesian master formulas" for expectation values and variances for $\mathcal{O}$ with parameters $\vec{a}$**

$$E[\mathcal{O}] = \int d^n a\, \mathcal{P}(\vec{a}|\text{data})\, \mathcal{O}(\vec{a}) \qquad V[\mathcal{O}] = \int d^n a\, \mathcal{P}(\vec{a}|\text{data})\, \big[\mathcal{O}(\vec{a}) - E[\mathcal{O}]\big]^2$$

probability distribution

$$\mathcal{P}(\vec{a}|\text{data}) \propto \mathcal{L}(\text{data}|\vec{a})\, \pi(\vec{a})$$

likelihood function

prior distribution

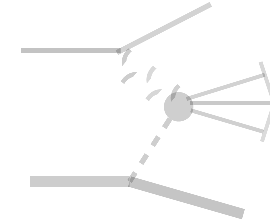
$$\mathcal{L}(\text{data}|\vec{a}) = \exp\left[-\tfrac{1}{2}\chi^2(\vec{a})\right]$$

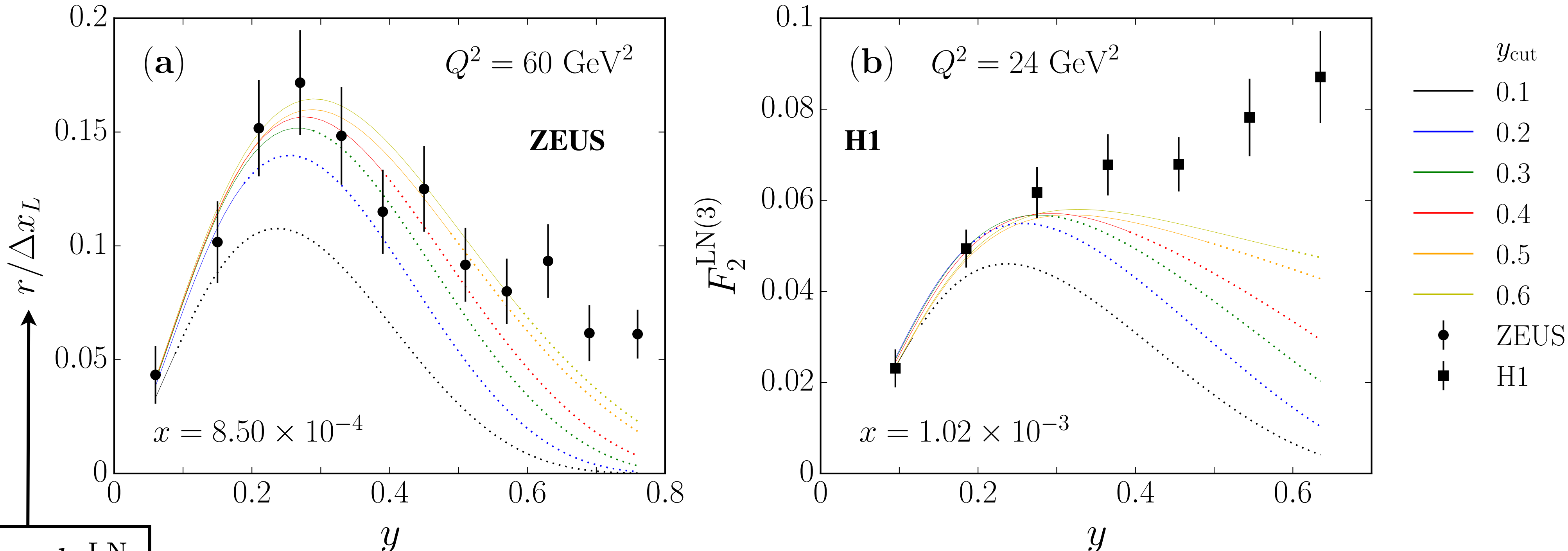
Impact of leading neutrons

- **Measured LN differential cross section** (integrated over p_T)

$$\frac{d^3\sigma^{\mathrm{LN}}}{dx\,dQ^2\,dy} \sim F_2^{\mathrm{LN}(3)}(x, Q^2, y)$$

$2 f_{\pi N}(y)\, F_2^{\pi}(x/y, Q^2)$ for π exchange





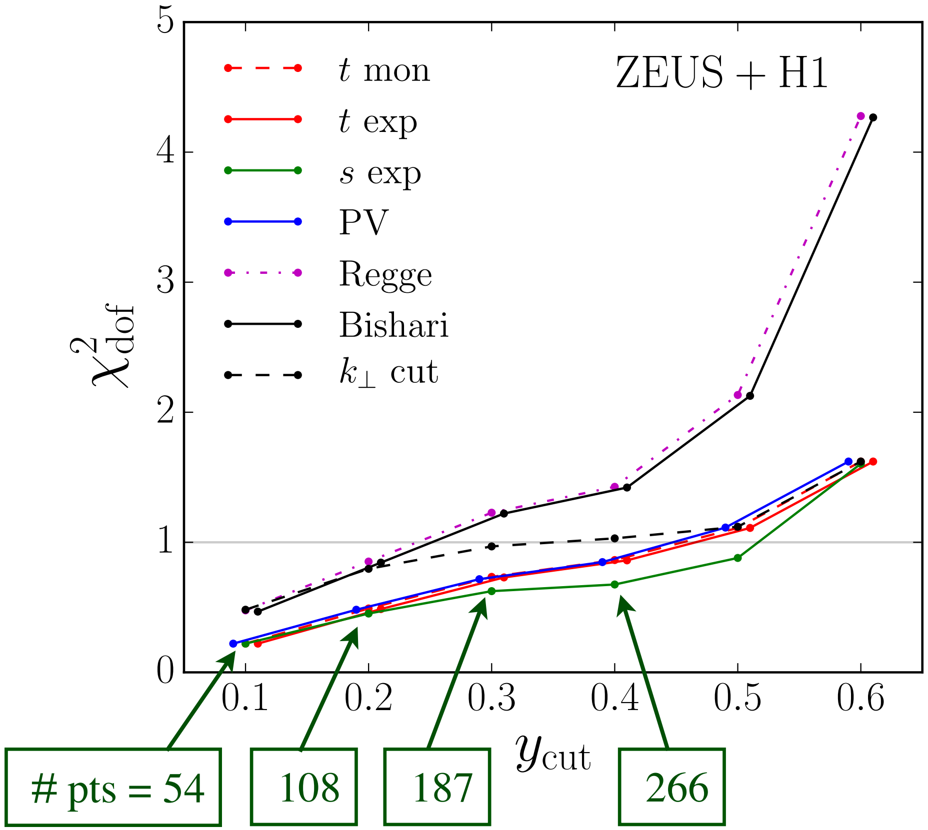
$$r = \frac{d\sigma^{\mathrm{LN}}}{d\sigma^{\mathrm{inc}}}$$

→ quality of fit depends on range of y fitted

→ expect more non-pionic contributions at larger y

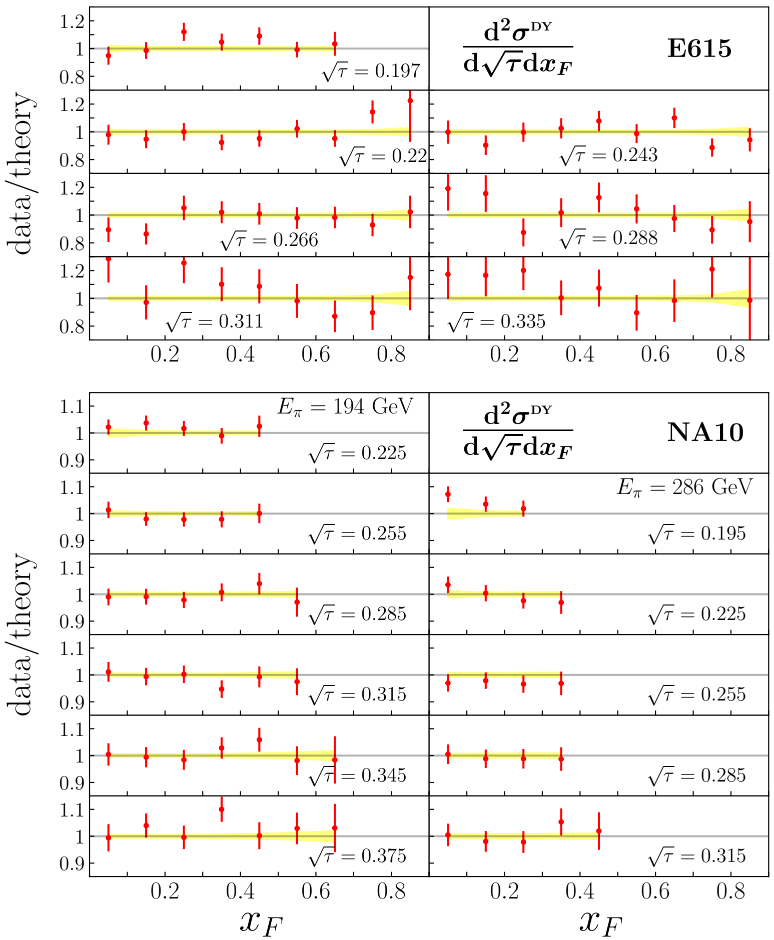
Impact of leading neutrons

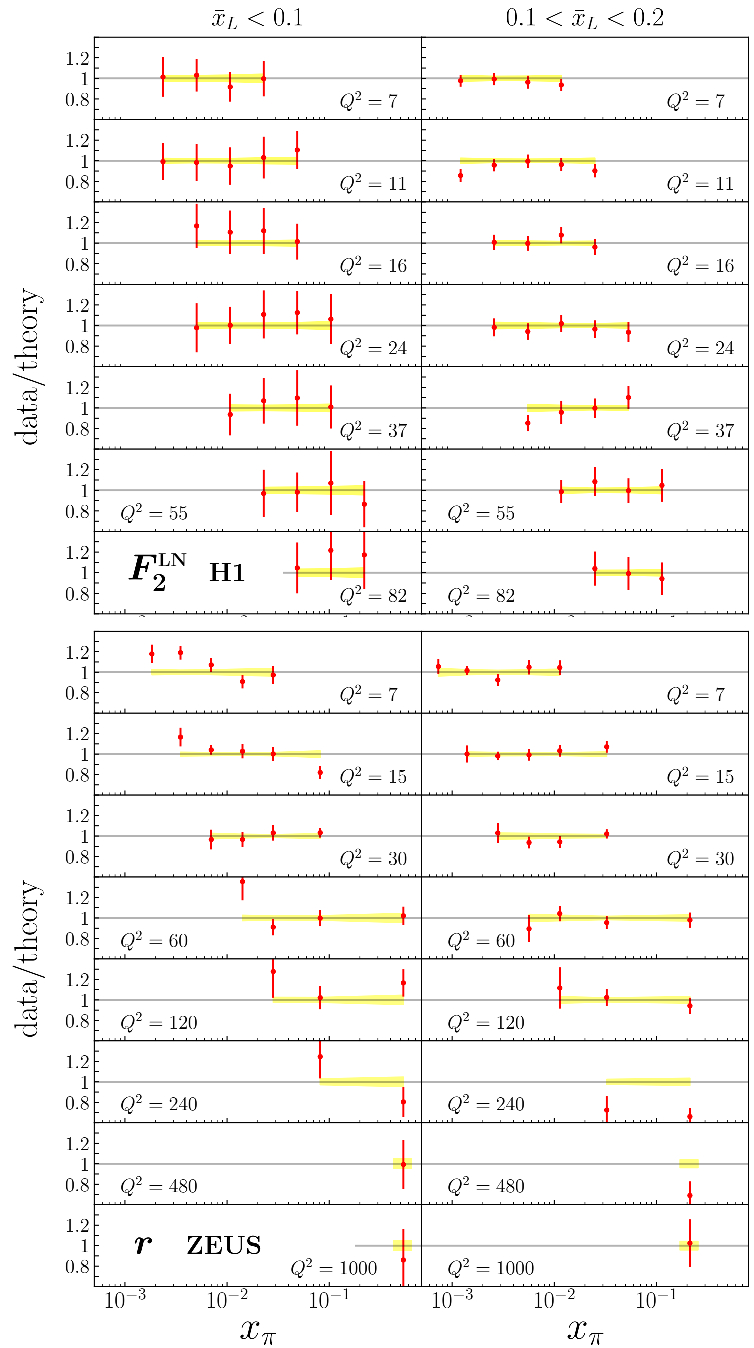
- Combined fit to HERA LN and Drell-Yan data



→ optimize $\chi^2_{\rm dof}$ with maximum number of points that can be described

Impact of leading neutrons



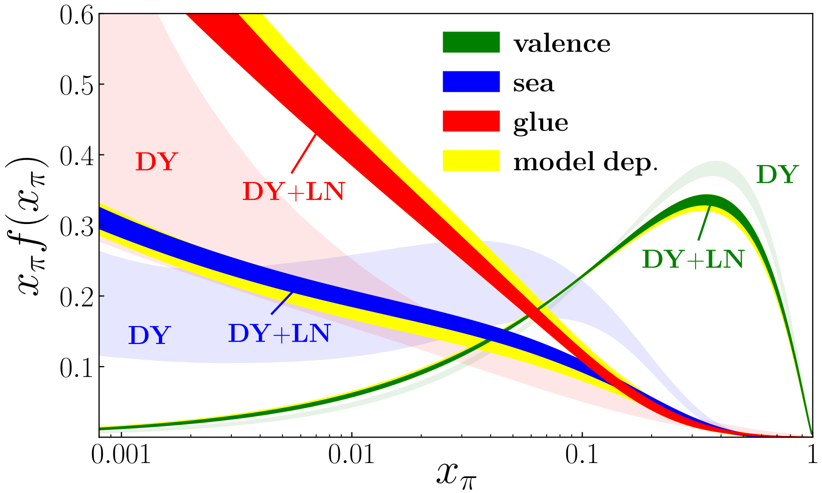


$$\bar{x}_L = 1 - x_L = y$$

Process	Experiment (observables)	N_{dat}	χ^2_{dat}	n_e
DY	E615 (x_F, Q)	61	0.85	1.08
	NA10 (194 GeV) (x_F, Q)	36	0.52	0.88
	NA10 (286 GeV) (x_F, Q)	20	0.78	0.83
	E615 (Q, p_T)	34	1.08	0.83
	E615 (x_F, p_T)	49	0.85	0.50
LN	H1	58	0.38	1.26
	ZEUS	50	1.51	0.95
Total		308	0.85	

Impact of leading neutrons

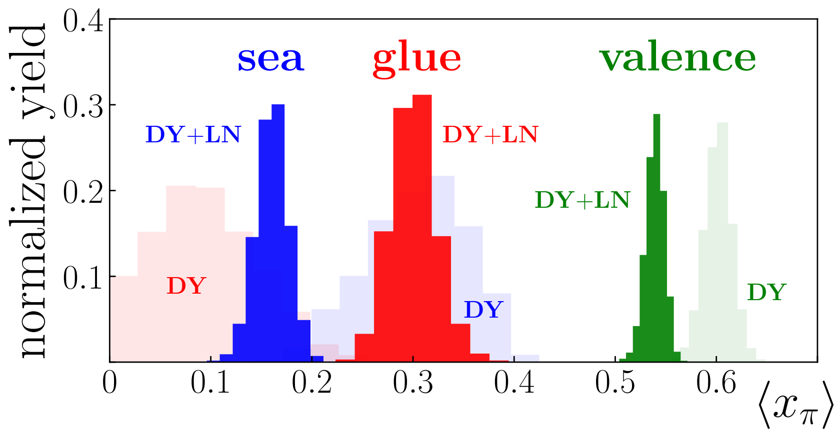
- MC analysis combining pQCD with chiral EFT to fit πN Drell-Yan + leading neutron electroproduction data from HERA



Barry, Sato, WM, C.-R. Ji
PRL 121, 152001 (2018)

→ significant reduction of glue and sea quark PDF uncertainties

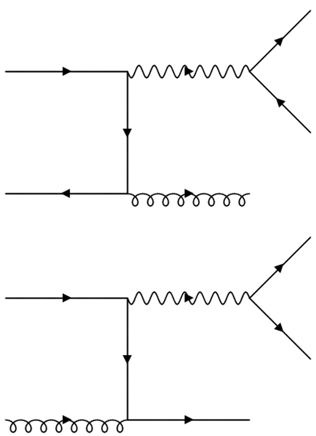
→ larger gluon fraction in the pion than without LN constraint



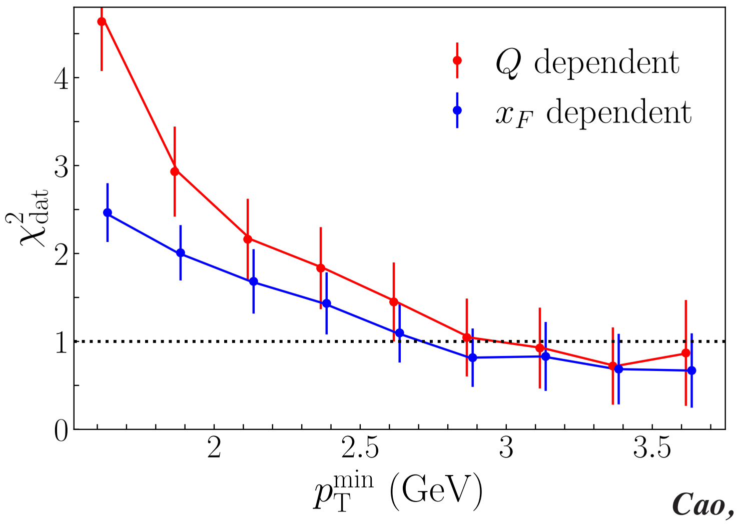
Transverse momentum dependence

- E615 also collected data differential in transverse momentum…
— never before included in global QCD analysis

→ large-p_T photon requires hard gluon to recoil against, offering sensitivity to gluon PDF in pion at high x

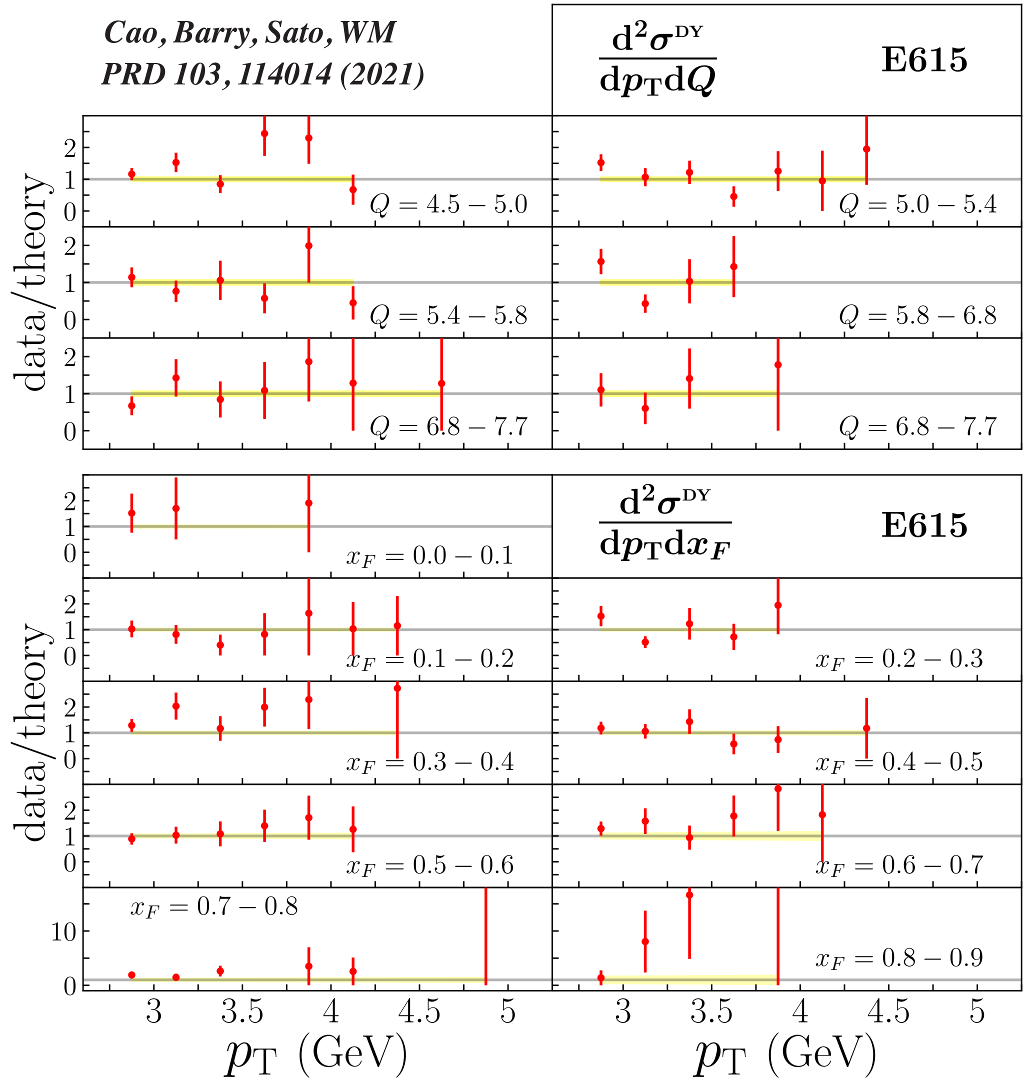


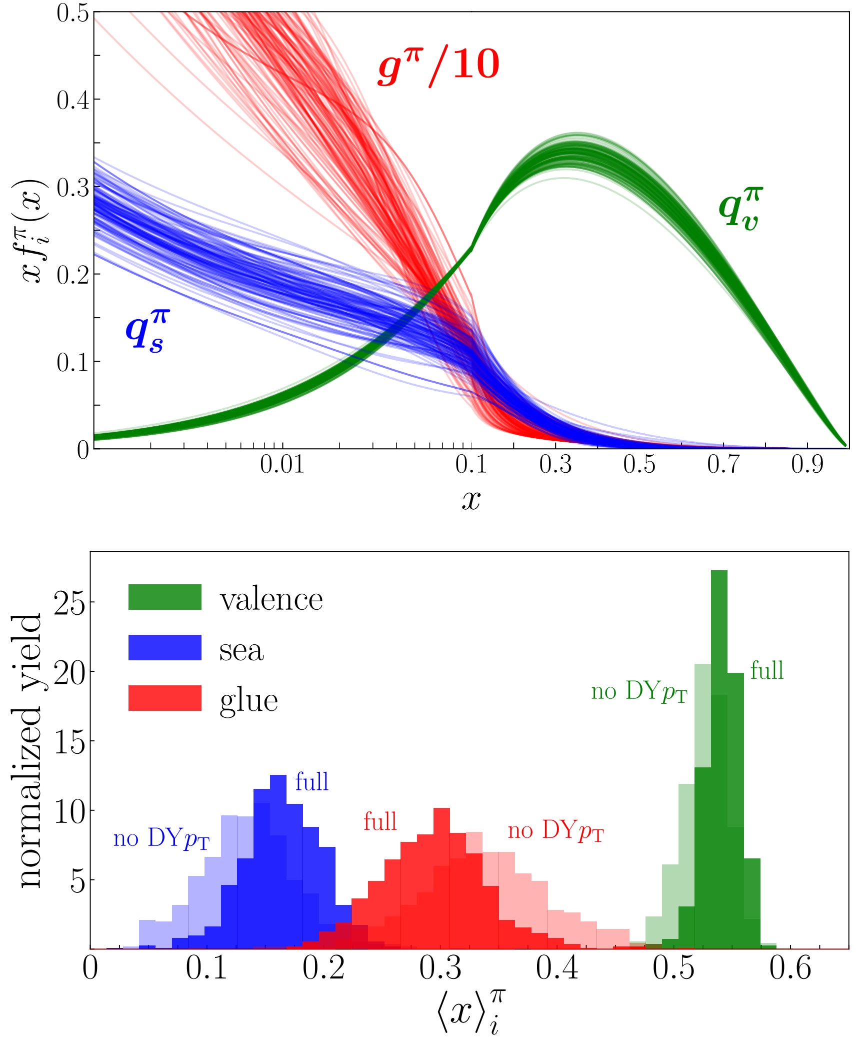
→ what is range of applicability of pQCD description of p_T distribution?



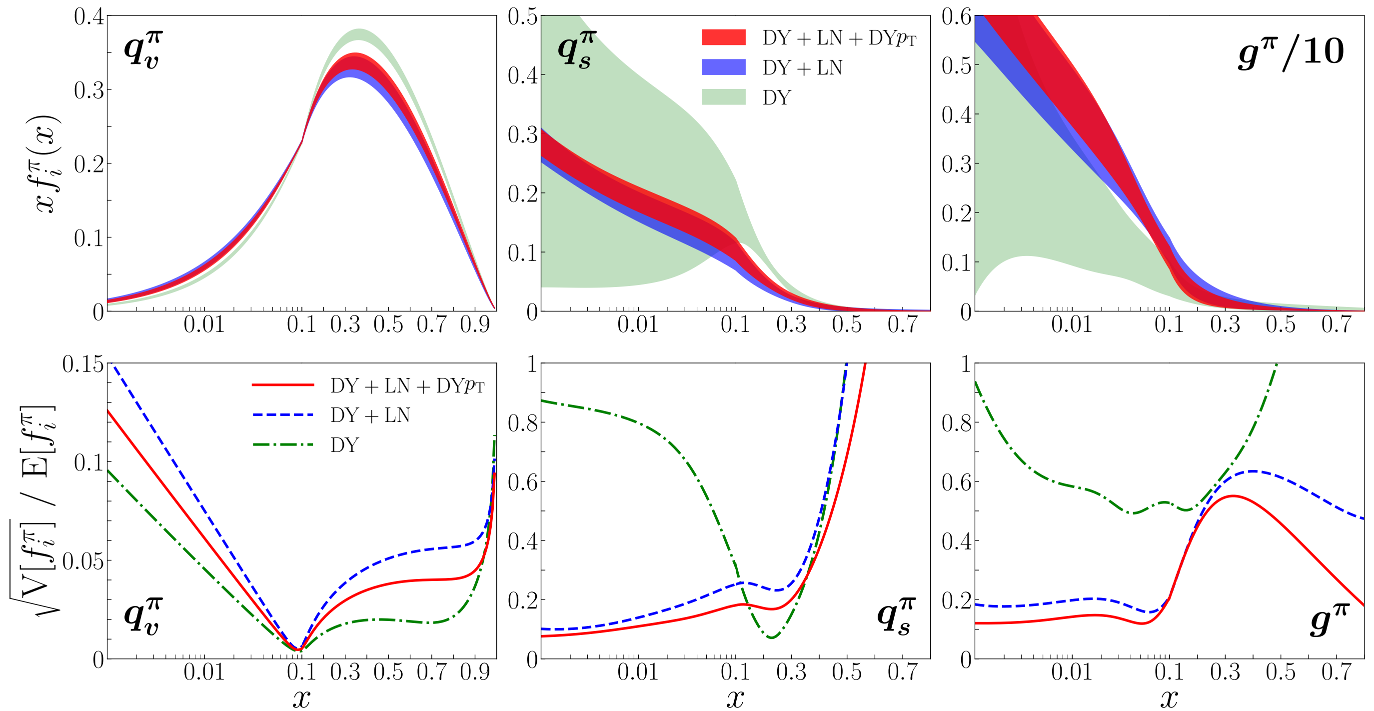
Cao, Barry, Sato, WM
PRD 103, 114014 (2021)

Transverse momentum dependence





Transverse momentum dependence



Cao, Barry, Sato, WM
PRD 103, 114014 (2021)

⟶ first time that one has been able to describe p_T spectra $(p_T > 2.7\,\mathrm{GeV})$ spectra in terms of collinear PDFs

⟶ opens path to pion TMD studies

Pion TMDs

TMD factorization in Drell-Yan

- In small-p_T region, Use the CSS formalism for TMD evolution

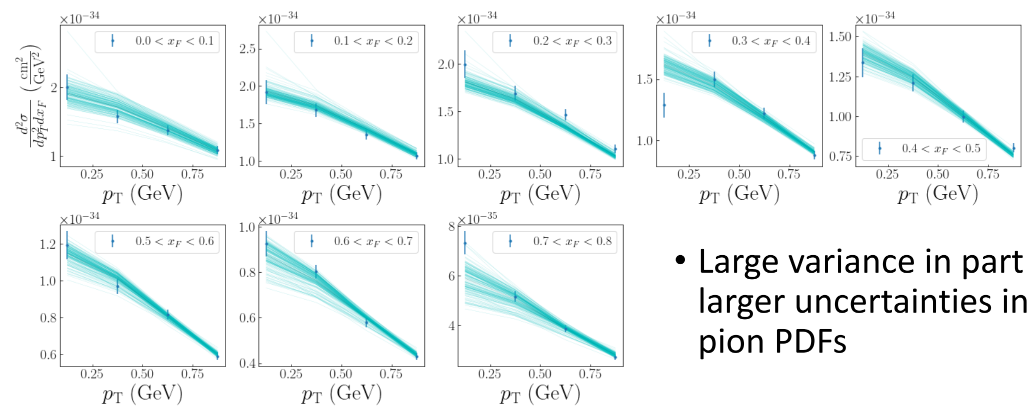
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} H^{\mathrm{DY}}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}$$
$$\times\, e^{-g_{j/A}(x_A,b_T;b_{\max})} \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right)$$
$$\times\, e^{-g_{\bar{j}/B}(x_B,b_T;b_{\max})} \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right)$$
$$\times\, \exp\left\{-g_K(b_T;b_{\max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*}) \ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Collinear pion PDF

Non-perturbative TMDs to extract

- Fit non-perturbative TMDs to pion-induced E615 data

Monte Carlo extraction of pion TMDs

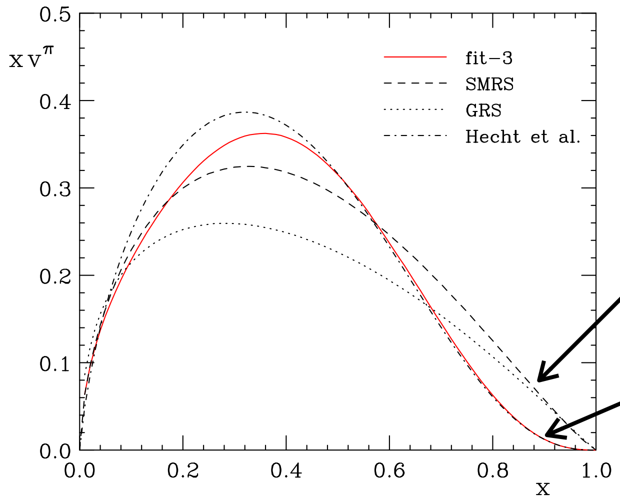


- Large variance in part due to larger uncertainties in collinear pion PDFs
- Aim to perform a simultaneous extraction of pion collinear PDFs and TMDPDFs

→ ***Patrick Barry***

Pion PDFs with threshold resummation

- $x \to 1$ behavior of pion PDF is controversial: $\sim (1-x)$ or $(1-x)^2$?

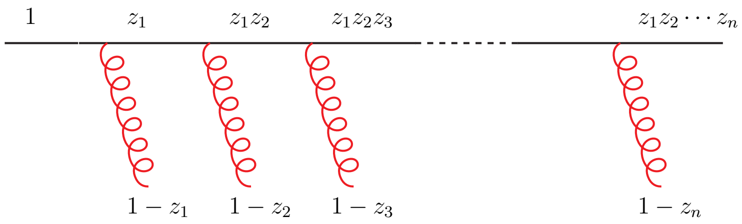


Aicher, Schafer, Vogelsang (2010)

no resummation: more consistent with $\sim (1-x)$

with resummation: more consistent with $\sim (1-x)^2$

- Hard scattering coefficient function kinematically enhanced when $z \to 1$ because of (soft) gluon emissions



→ ***Patrick Barry***

→ effect of resummation on phenomenology?

Pion PDFs with threshold resummation

- Two ways to construct Mellin moments of differential cross section

→ Mellin-Fourier

Mukherjee, Vogelsang (2006)
Bolzoni (2006)
Bonvini, Forte, Ridolfi (2011)

$$\sigma_{\mathrm{MF}}(N,M) \equiv \int_0^1 \mathrm{d}\tau\, \tau^{N-1} \int_{\log\sqrt{\tau}}^{\log\frac{1}{\sqrt{\tau}}} \mathrm{d}Y\, e^{iMY} \frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau\mathrm{d}Y}$$

$\tau = Q^2/s$ $\quad$ $x^0_{\pi,A} = \sqrt{\tau}\, e^{\pm Y}$

→ double Mellin

Westmark, Owens (2017)
Lustermans, Michel, Tackmann (2019)

$$\sigma_{\mathrm{DM}}(N,M) \equiv \int_0^1 \mathrm{d}x^0_\pi\, (x^0_\pi)^{N-1} \int_0^1 \mathrm{d}x^0_A\, (x^0_A)^{M-1} \frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau\mathrm{d}Y}$$

- For MF method, Fourier transform of threshold log $\delta(\hat{Y} - \frac{1}{2}\log(x_\pi/x_A))$ gives factor $\cos\left(M\log(1/\sqrt{z})\right)$

$\hat{Y} = Y - \frac{1}{2}\log(x_\pi/x_A)$
$z = Q^2/x_\pi x_A S$

→ expand cosine $\cos \to 1$ $\quad$ "expansion method"

→ keep cosine factor $\quad$ "cosine method"

used in Aicher, Schafer, Vogelsang (2010) analysis

Pion PDFs with resummation

Deriving resummation expressions – MF

Claim: yellow terms give rise to the resummation expressions

$$\frac{C_{q\bar{q}}}{e_q^2} = \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right]$$
$$+\frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ +1-z\right]\right.$$
$$\left.+\frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right)-2(1-z)\right]\right\}$$

Claim: Red terms are power suppressed in $(1-z)$ and don't contribute to the same order as the yellow terms

Generalized threshold resummation

Rewrite the (z, y) coefficients in terms of (z_a, z_b), and for the red term:

$$z_{a,b} = \frac{\sqrt{\tau}e^{\pm Y}}{x_{a,b}}$$

$$\mathrm{d}z\,\mathrm{d}y\,\frac{1}{1-z}\left(\frac{1}{y}+\frac{1}{1-y}\right) = \mathrm{d}z_a\mathrm{d}z_b\,\frac{1}{(1-z_a)(1-z_b)}\left[1+\mathcal{O}(1-z_a, 1-z_b)\right].$$

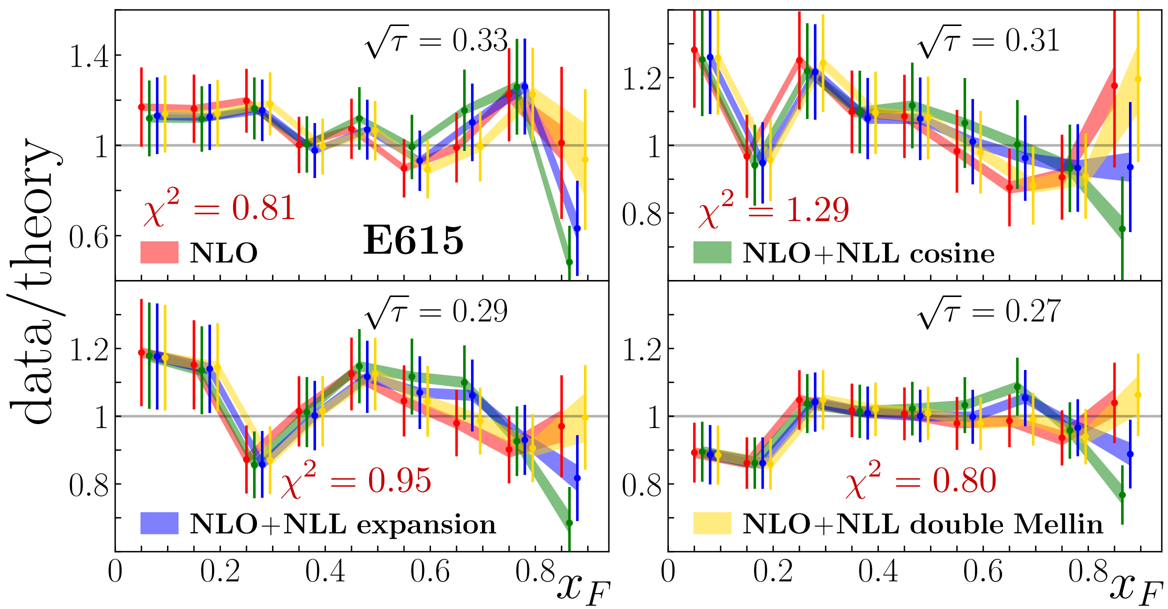
This is *not* power suppressed in $(1-z_a)$ or $(1-z_b)$ - cannot disentangle (z, y)

Double Mellin method, however, includes these terms

Lustermans, Michel, Tackmann (2019)

→ Patrick Barry

Pion PDFs with resummation



→ good fits to data for most resummation methods (slightly worse for "cosine" method)

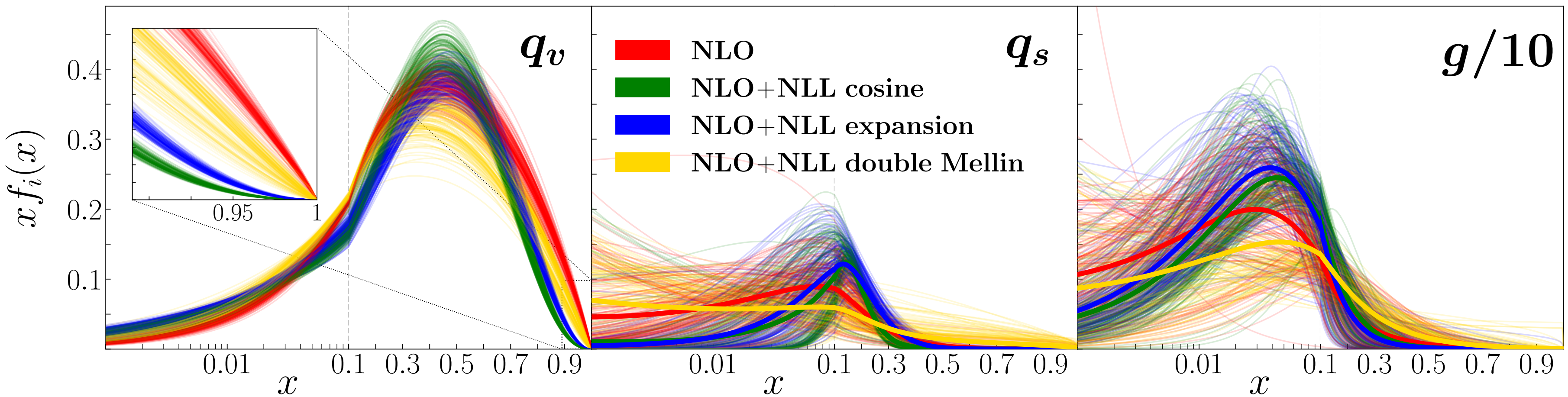
→ valence quarks give ~ 5% momentum fractions to gluons after resummation (for all methods)

resummation method	$\langle x \rangle_v$	$\langle x \rangle_s$	$\langle x \rangle_g$
NLO	0.53(2)	0.14(4)	0.34(6)
NLO+NLL cosine	0.47(2)	0.14(5)	0.39(6)
NLO+NLL expansion	0.46(2)	0.16(5)	0.38(6)
NLO+NLL double Mellin	0.46(3)	0.15(7)	0.40(5)

Barry, Ji, Sato, WM
PRL (2021)
arXiv:2108.05822

Pion PDFs with resummation

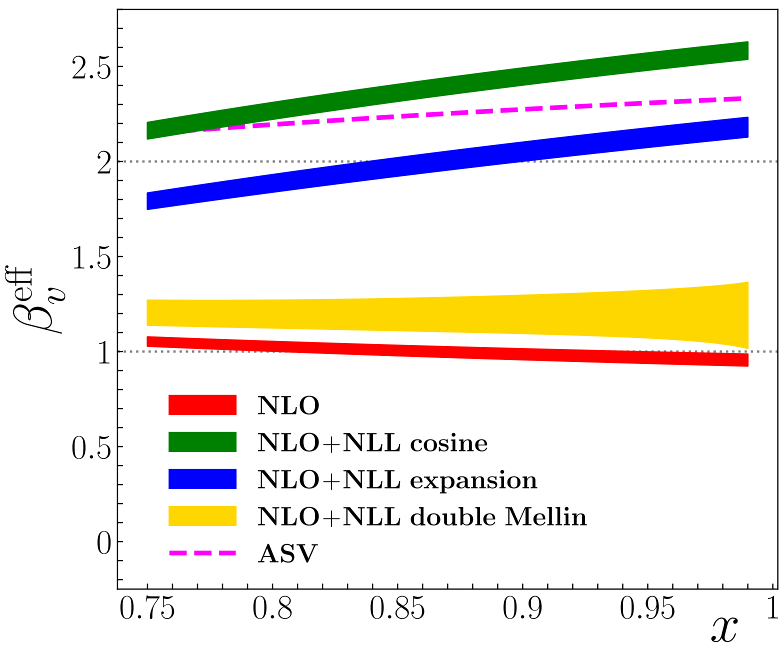
⟶ redistribution of x dependence



⟶ effective exponent

$$\beta_v^{\rm eff}(x,Q) = \frac{\partial \log |q_v(x,Q)|}{\partial \log(1-x)}$$

⟶ double Mellin method similar to fixed-order NLO



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Outlook

- JAM global QCD analysis allows simultaneous description of Drell-Yan (p_T integrated and differential) and leading neutron electroproduction data in terms of universal set of pion PDFs

  ⟶ map out pion structure from low x to high x

- Successful extension to incorporate transverse momentum

  ⟶ more precise data needed to constrain gluon PDF at high x

  ⟶ extraction of pion TMDs

- Global QCD analysis with threshold resummation

  ⟶ suggests $\sim (1-x)$ behavior at large x

- Framework easily extended to kaon structure, when data available