Radial excitations in constituent quark model

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Light-cone conference 2021

Contents

Part 1

- Radial excitations of heavy hadrons
- Nonrelativistic quark model - [2S]

  —> Strong decay

Part 2

- Light-front quark model - [1S & 2S]

  —> Mass spectrum & decay constant

  —> Form factor & charge radii

Introduction

- **Recent experimental reports**
  —> $\Lambda_b(6072)$: Observation [1, LHCb]
  —> $\Xi_c(2970)$: J^P determination [2, Belle]
  —> $D_s(2590)$: Observation [3, LHCb]

- **Some analogous states are not identified**
  —> $\Xi(2S)$, $\Xi_b(2S)$, etc.
  —> $B_s(2S)$, etc

  ⇨ Finding the missing siblings

- **Some interesting behaviors**
  —> Similar excitation energy
  —> Large decay width

  ⇨ Internal structure?

[1] LHCb, JHEP 06 (2020) 136
[2] Belle, PRD 103 (2021) L111101
[3] LHCb, PRL 126 (2021) 122002

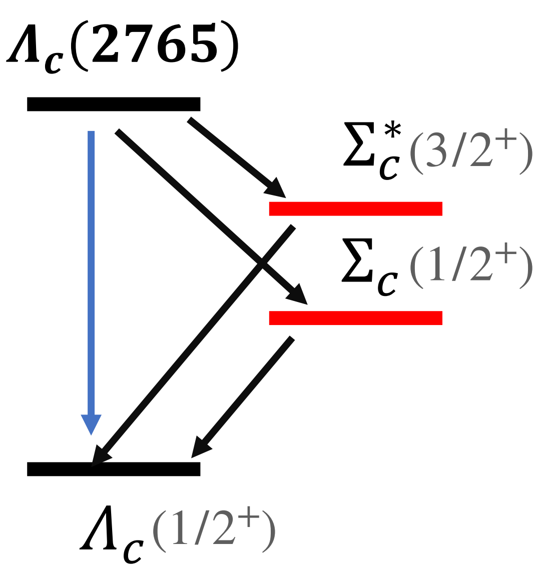
[2S state] Spin-parity (J^P)

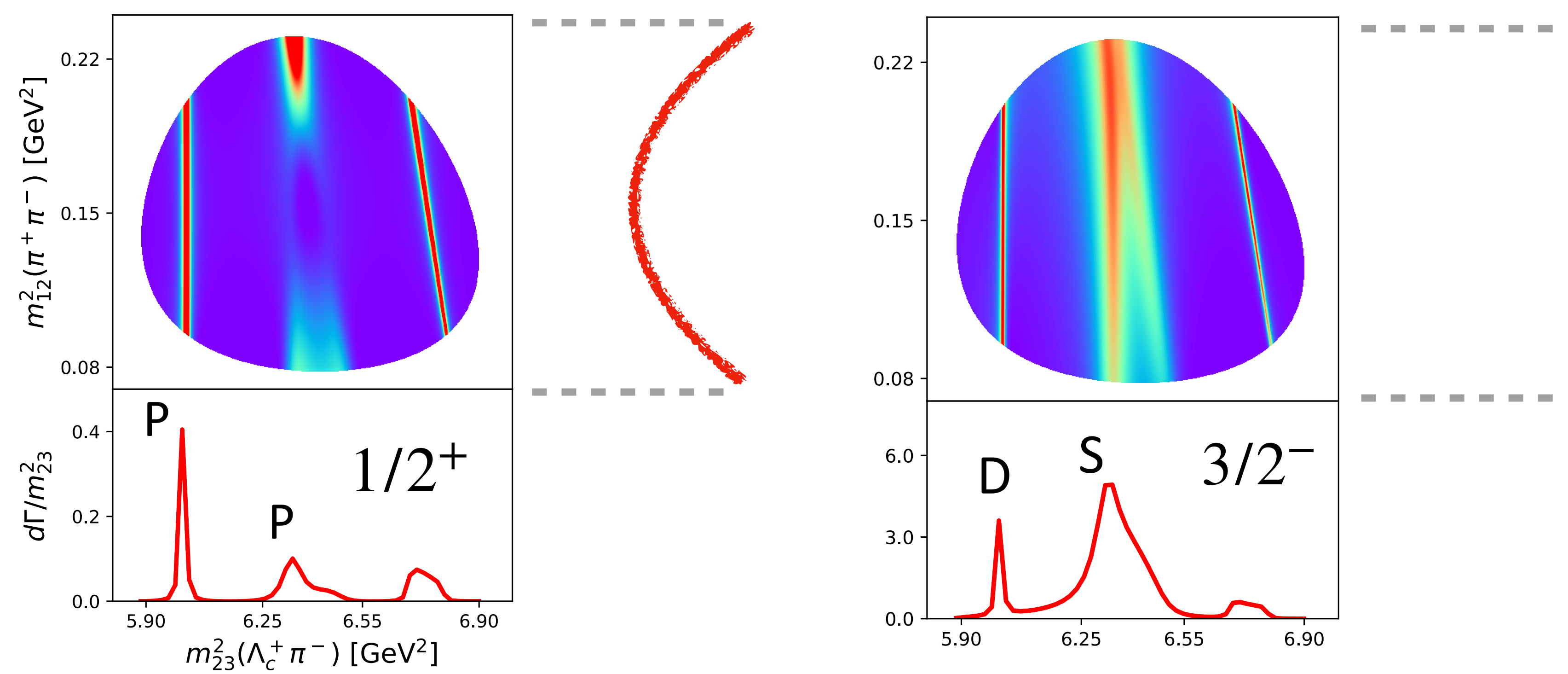
- **Can be determined from Dalitz plot (3-body decay)**

  —> Example: $\Lambda_c(2675)$ state [1]

  <> Invariant mass and ratio,

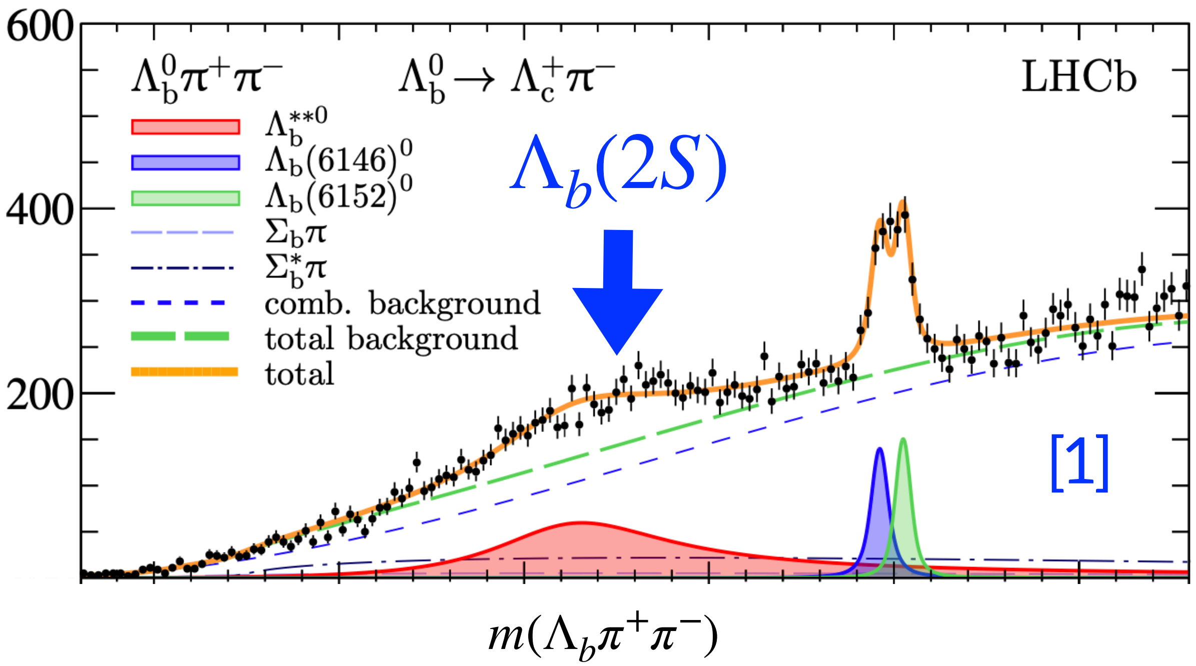
  <> Angular correlation

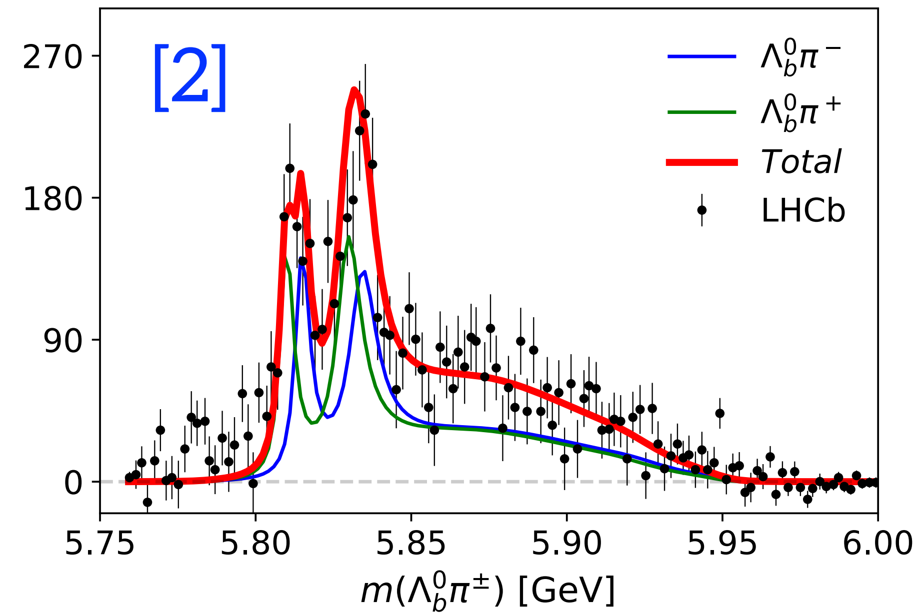




[1] Arifi et al, PRD 101 (2020), 094023

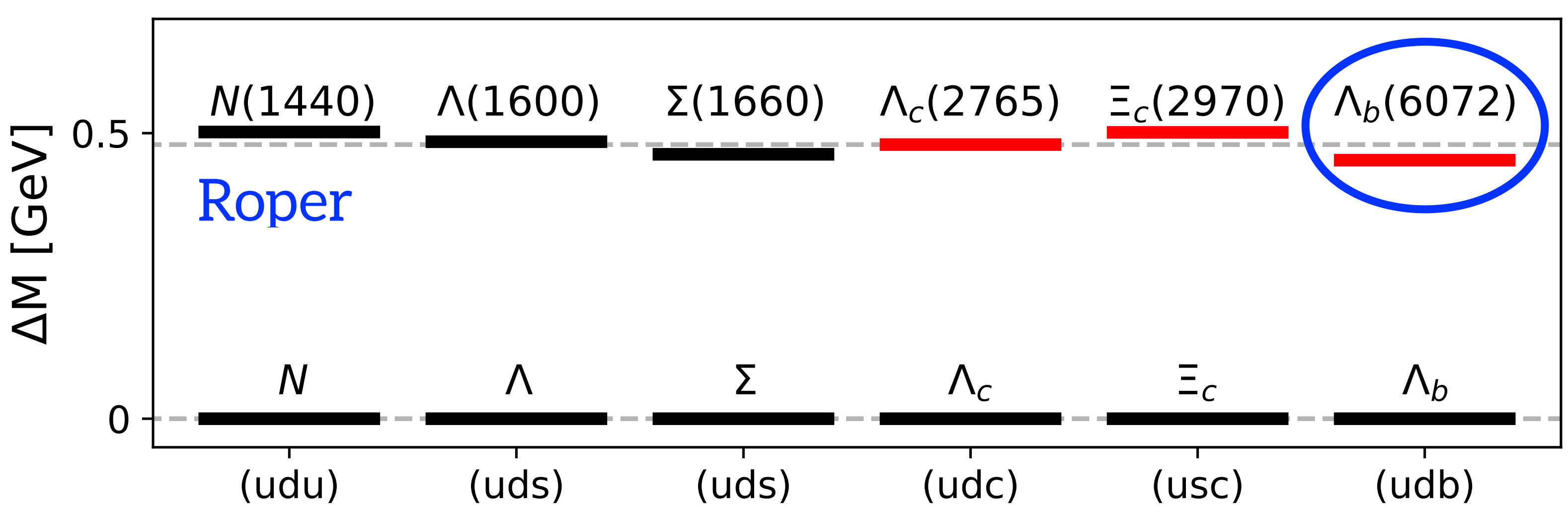
[2S state] Mass & width





<> A broad resonance ($\Gamma = 72$ MeV).

<> Evidence for $1/2^+$.



<> Similar mass gap
ΔM ~ 500 MeV
Universal?
Accidental?

[1] LHCb, JHEP 06 (2020), 136
[2] Arifi et al, PRD 101 (2020), (R) 111502

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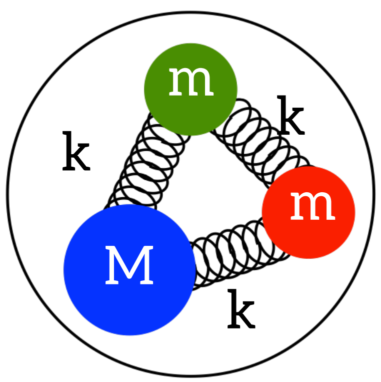
Problem [Decay width]

<> $\Lambda_b(6072)$ —> $\Gamma = 72$ MeV [Experimental data]

- **Non-relativistic quark model (NRQM)**



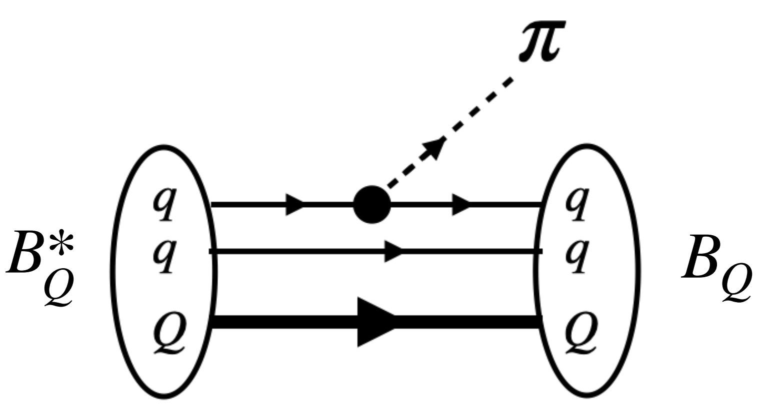
Baryon wave functions [1S and 2S]

—> Gaussian type (Harmonic oscillator)

Quark-pion interaction $\mathcal{L}_{\pi qq} = \frac{g_A^q}{2 f_\pi} \bar{q} \gamma^\mu \gamma_5 \vec{\tau} q \cdot \partial_\mu \vec{\pi}$

—> axial-vector type.

—> Non-relativistic expansion



Decay amplitude $\mathcal{T} = \langle \pi \, \Lambda_c | \mathcal{L}_{\pi qq} | \Lambda_c^* \rangle$

—> A convolution of wave functions

<> $\Lambda_b(6072)$ —> $\Gamma = 5$ MeV (Very narrow) [Quark Model]

Our attempt [Relativistic correction]

Quark-pion interaction

—> Non-relativistic expansion

—> Next-leading order: FWT transformation [1].

$$\mathcal{L}_{\pi qq} = \frac{g_A^q}{2f_\pi}\bar{q}\gamma^\mu\gamma_5\vec{\tau}q\cdot\partial_\mu\vec{\pi}$$

$$H = \underbrace{H(1/m^0) + H(1/m)}_{\text{NR}} + \underbrace{H(1/m^2)}_{\text{RC}} + \ldots$$

Relativistic effect

—> Essential

State	Multiplet	Channel	Γ_{NR}	$\Gamma_{\text{NR+RC}}$	Γ_{Exp}	
$\Sigma_b(5810)^+$	$\Sigma_b(1S, 1/2(1)^+)$	$\Lambda_b\pi$	11.9–12.3	0.62–5.11	4.83 ± 0.31	~ (–)
$\Sigma_b(5830)^+$	$\Sigma_b(1S, 3/2(1)^+)$	$\Lambda_b\pi$	20.4–21.4	1.08–8.80	9.34 ± 0.47	
$\Lambda_b(5912)^0$	$\Lambda_b(1P_\lambda, 1/2(1)^-)$	$\Sigma_b\pi$	0.001–0.003	0.001–0.003	< 0.25	~ (0)
$\Lambda_b(5920)^0$	$\Lambda_b(1P_\lambda, 3/2(1)^-)$	$\Sigma_b^*\pi$	0.004–0.008	0.004–0.009	< 0.19	
$\Lambda_b(6072)^0$	$\Lambda_b(2S_{\lambda\lambda}, 1/2(0)^+)$	$\Sigma_b\pi$	0.72–2.17	4.97–20.8		
		$\Sigma_b^*\pi$	1.08–3.00	7.81–31.5		
		Sum	1.80–5.17	12.8–52.3	72 ± 11	~ (+)

Light-front quark model

—> Fully relativistic model, decay width?

[1] Arifi et al, PRD 103 (2021), 094003

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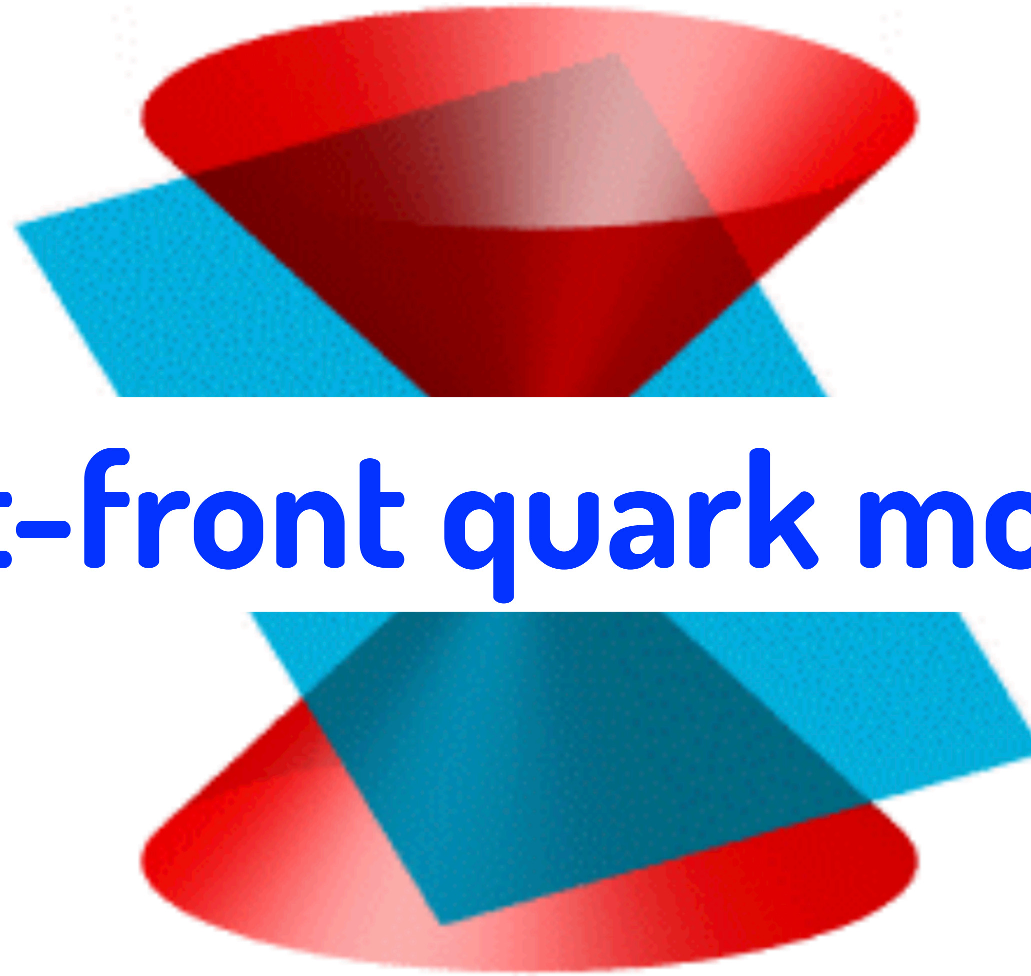


Light-front quark model

Light-front quark model

- Constituent quark model
- Light-front dynamics

⇨ Hadrons: $q\bar{q}$, qqq

Variational principle

<> Trial wave function
—> Gaussian (H.O. basis).

$$\phi_{ns}(x, \mathbf{k}_\perp) = \sqrt{\frac{\partial k_z}{\partial x}} \phi_{ns}(k_z, \mathbf{k}_\perp)$$

$$\phi_{1s} = \frac{1}{\pi^{3/4}\beta^{3/2}} e^{-\mathbf{k}^2/2\beta^2}$$

<> Assume some interactions
—> Linear confinement,
spin-spin int, etc.

$$V_{q\bar{q}} = a + br - \frac{4\alpha_s}{3r} + \frac{2}{3}\frac{\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}}{m_q m_{\bar{q}}} \nabla^2 V_{\text{Coul}}$$

<> Parameters <=> mass spectra
—> β, m, a, b, etc.

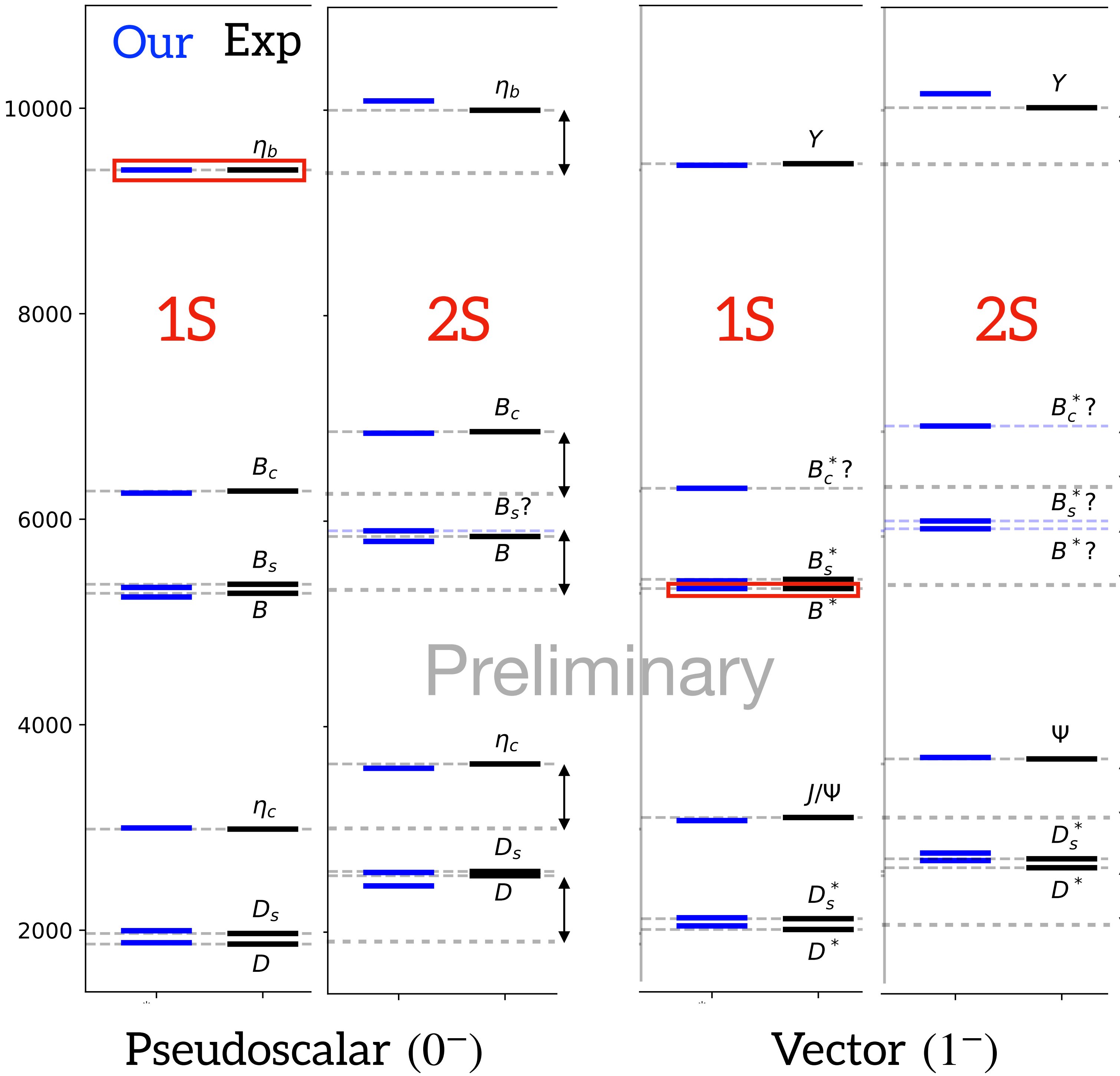
$$M_{q\bar{q}} = \langle\Psi| \left[H_0 + V_{q\bar{q}}\right] |\Psi\rangle$$

$$\frac{\partial \langle\Psi| \left[H_0 + V_0\right] |\Psi\rangle}{\partial\beta} = 0$$

<> Compute various observables
—> decay constant, form factor, decay rate, etc.

[1] HM Choi & CR Ji, PLB 460 (1999) 461.

Mass spectra [heavy mesons]



<> Input:

—> $\eta_b(1S)$ & $B^*(1S)$

<> The same α_s for all.

<> The same β for 2S

—> Orthogonality.

<> Potential parameters:

a = -0.52, b = 0.18, α_s = 0.35.

<> Quark masses:

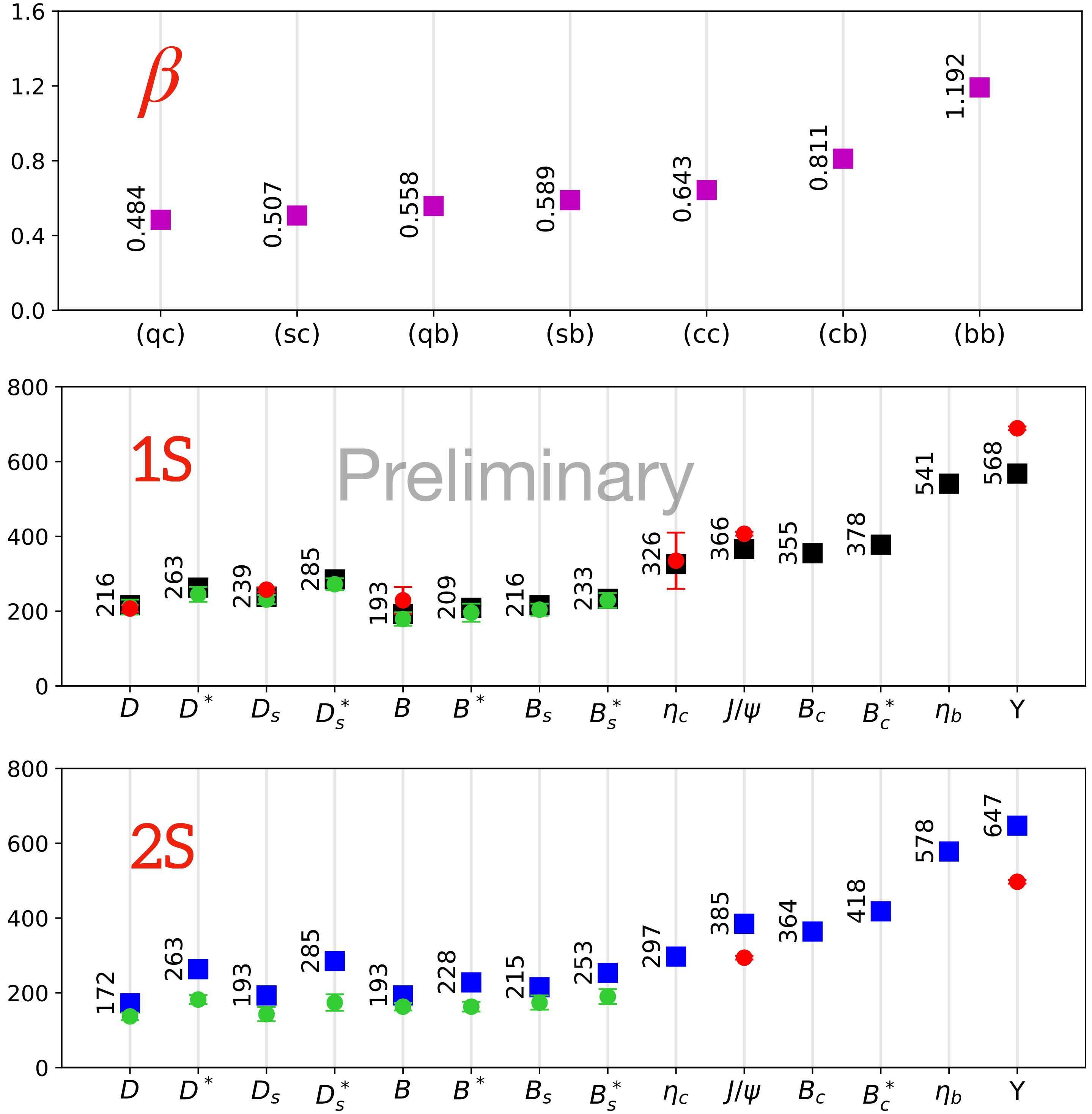
mq = 0.30, ms = 0.45,
mb = 5.0, mc = 1.62

<> β parameters:

—> next slide

$$\phi_{1s} = \frac{1}{\pi^{3/4}\beta^{3/2}} e^{-\mathbf{k}^2/2\beta^2}$$

Decay constants



<> in LFQM

$$\langle 0 | \bar{q}_2 \gamma^\mu \gamma_5 q_1 | M(P) \rangle = i f_P P^\mu$$

$$f_P = \sqrt{6} \int \frac{dx \, d^2\mathbf{k}_\perp}{(2\pi)^3} \phi(x, \mathbf{k}_\perp) \frac{\mathcal{A}}{\sqrt{\mathcal{A}^2 + \mathbf{k}_\perp^2}}$$

<> Nice agreement

—> Lattice [1] & Exp

<> **Problem:** bad agreement

—> Our: increase

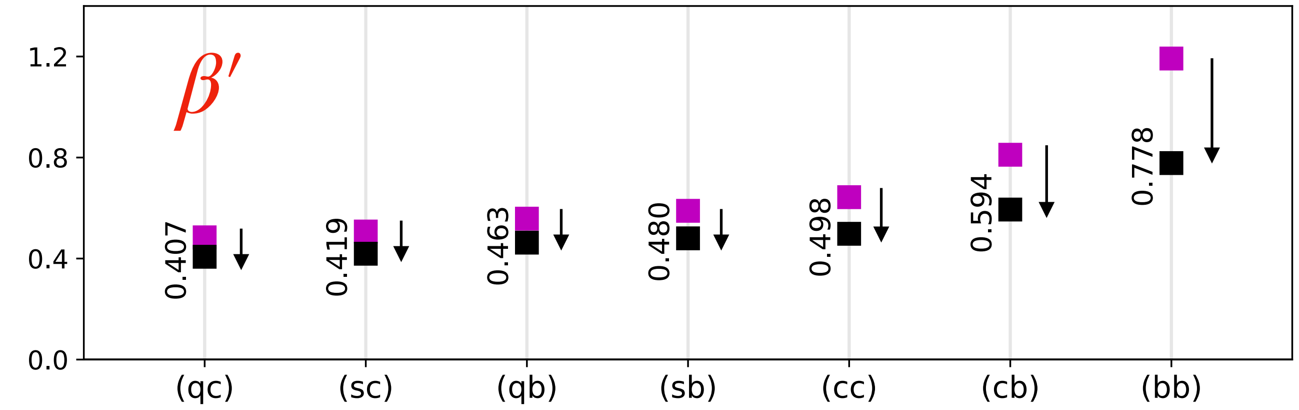
—> Exp/sum-rule [2]: decrease

[1] Becirevic et al, PRD 60 (1999) 074501.
[2] Gellhausen et al, EPJC 74 (2014)2979.

Problem [2S decay constant]

<> Decay constant:

f(2S) << f(1S)

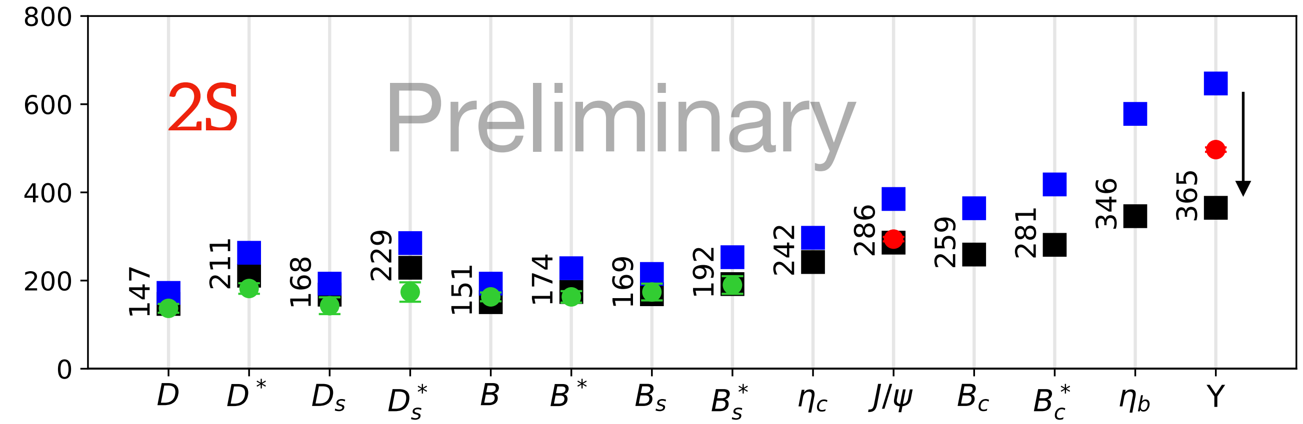


<> If we fit the 2S spectrum,

—> different β

—> reproduce the data

—> orthogonality?



<> 2S - harmonic oscillator basis

—> not a good approximation?

—> hydrogen-like wf [1].

$$\mathrm{Exp}\left(-\frac{k^2}{2\beta}\right) \Longrightarrow \mathrm{Exp}\left(-\frac{n^\delta}{2}\frac{k^2}{\beta}\right)$$

<> Mixing?

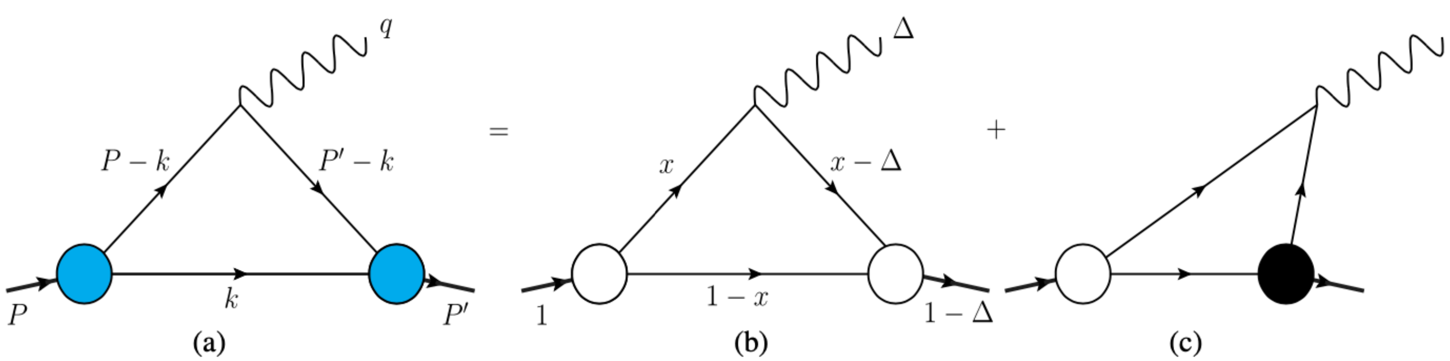
—> as a trial wf, we may mix 1S-2S [2],

$$\psi_{2S} = c_1\phi_{1S} + c_2\phi_{2S}$$

<> Any other idea?

[1] H.W. Ke et al, PRD 82 (2010) 034023.
[2] M. Li et al, Arxiv:2111.07087 [2021].

Form factors & Charge radii [1S]

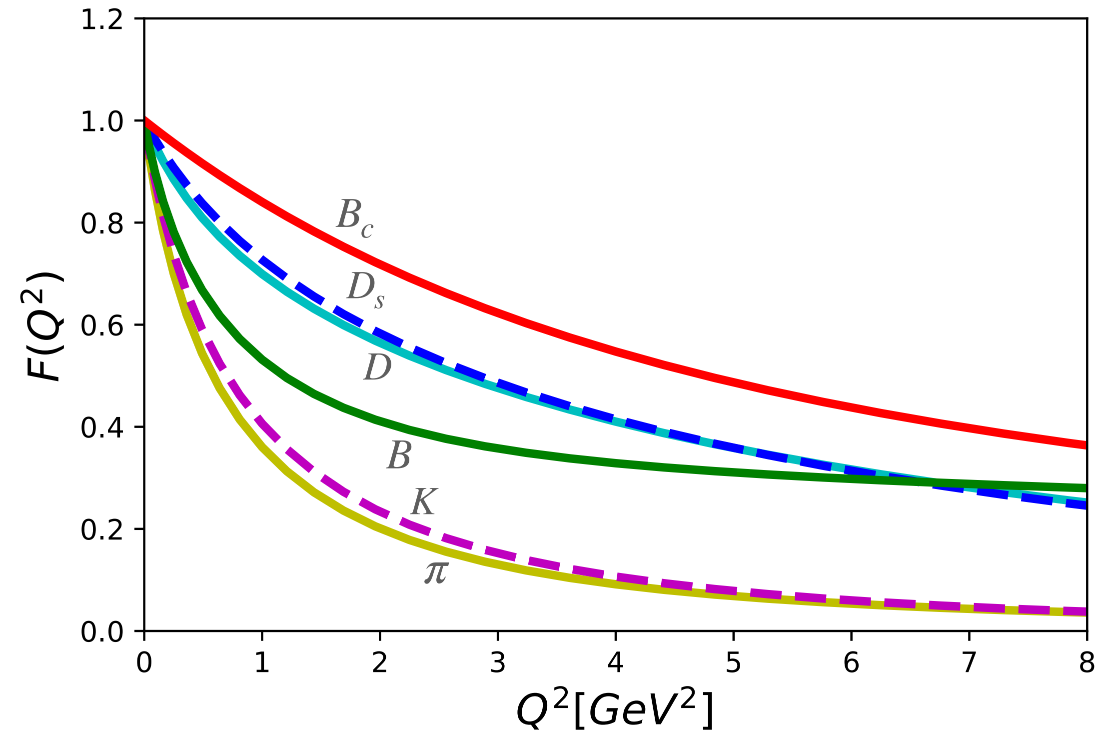


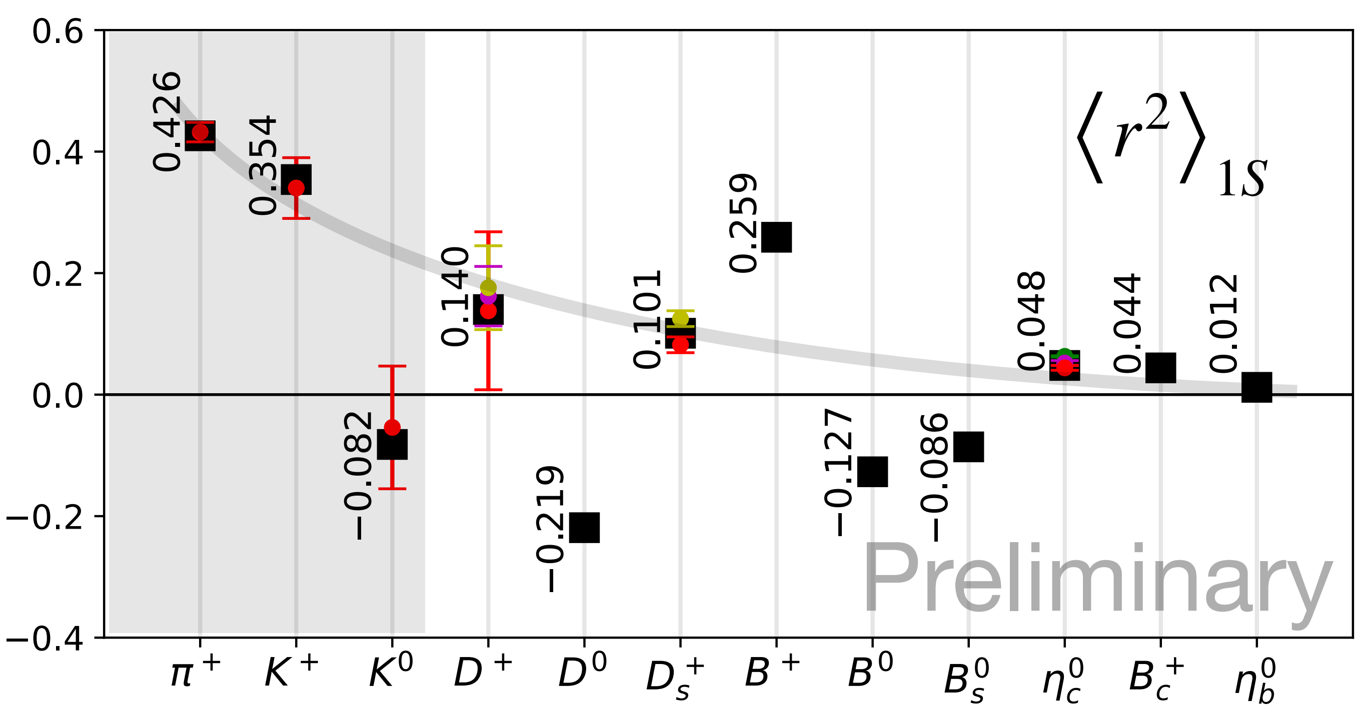
LF valence LF nonvalence

$$F^+(Q^2) = e_q I^+(Q^2, m_q, m_{\bar{q}}) + e_{\bar{q}} I^+(Q^2, m_{\bar{q}}, m_q)$$

Quark Anti-quark

"⊥" current —> give the same result [1].





$$\langle r^2 \rangle = -6 \frac{dF(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

- Radii: Exp & lattice —> good agreement
- B^+ meson radius —> relatively large

[1] HM Choi & CR Ji, PRD 19 (2015) 014018

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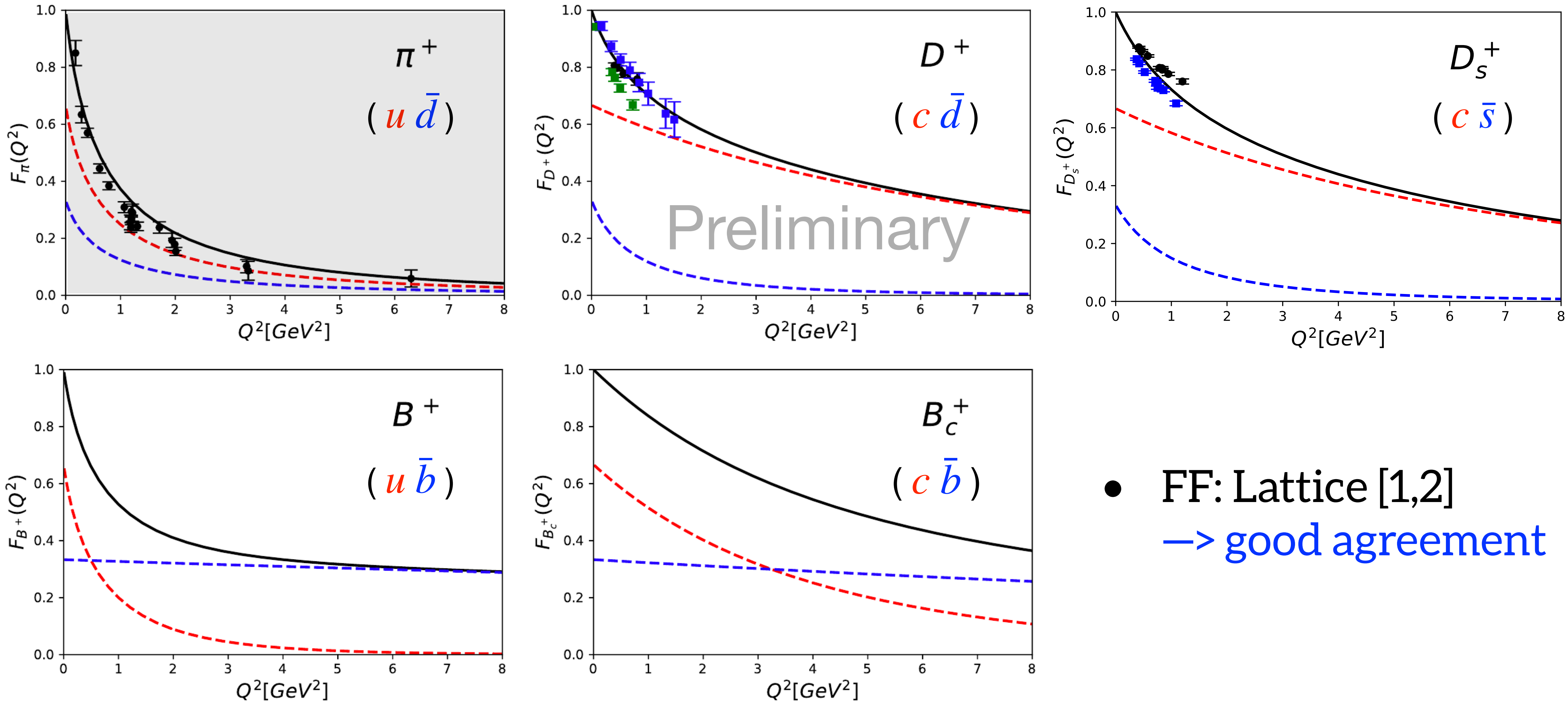
Flavor decomposition of FFs

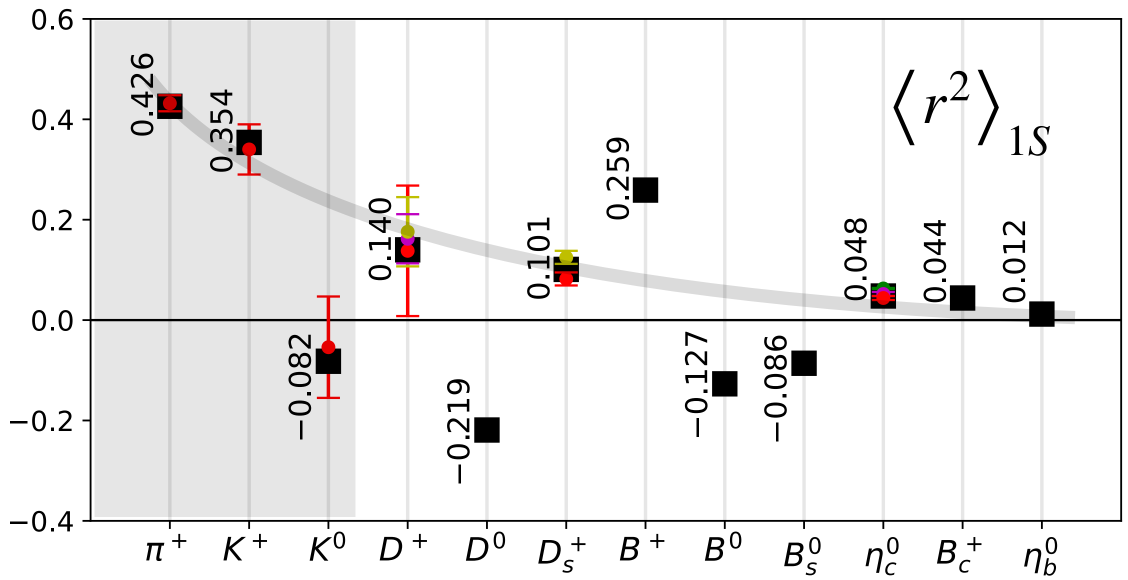


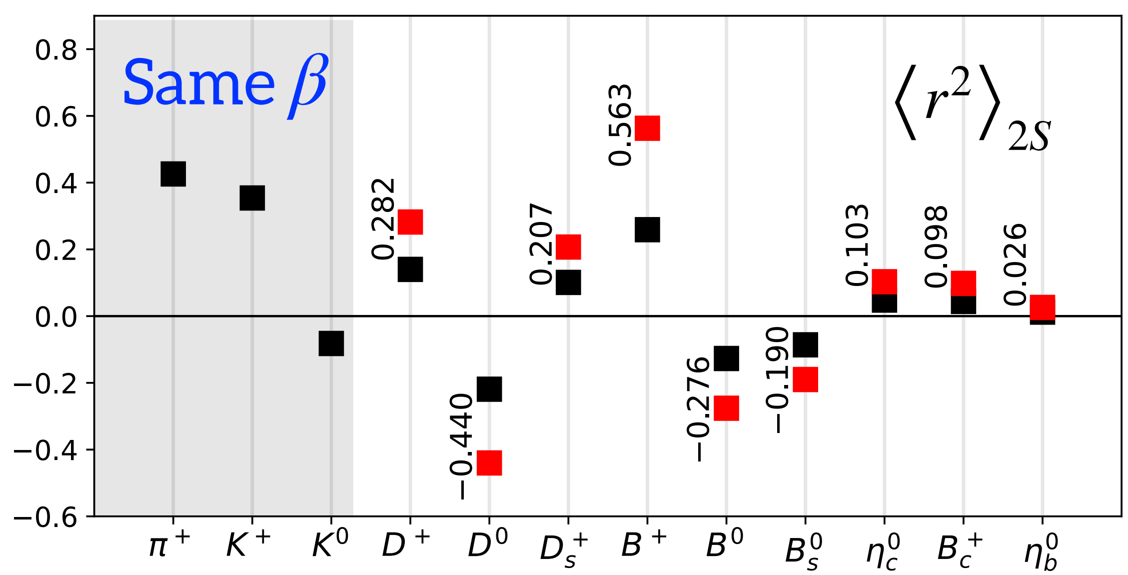
- FF: Lattice [1,2]
  —> good agreement

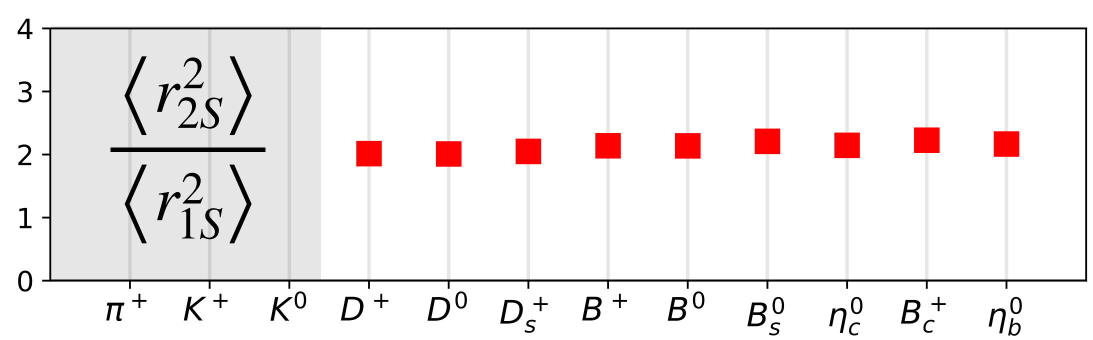
- Charge Radii —> light quark has an important role.

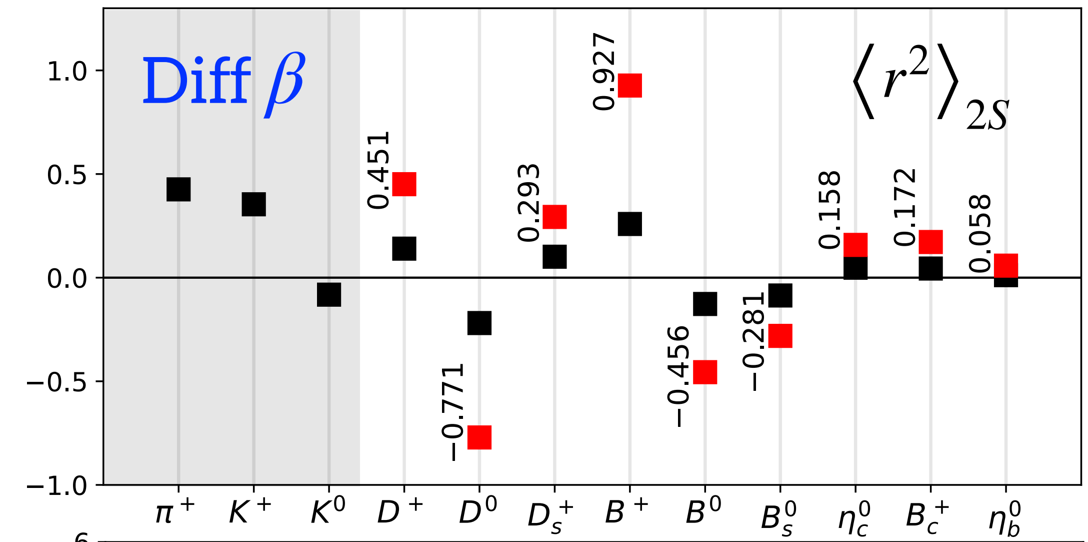
[1] U. Can, PLB 719 (2013) 103.
[2] Li & Wu, EPJA 53 (2017) 56.

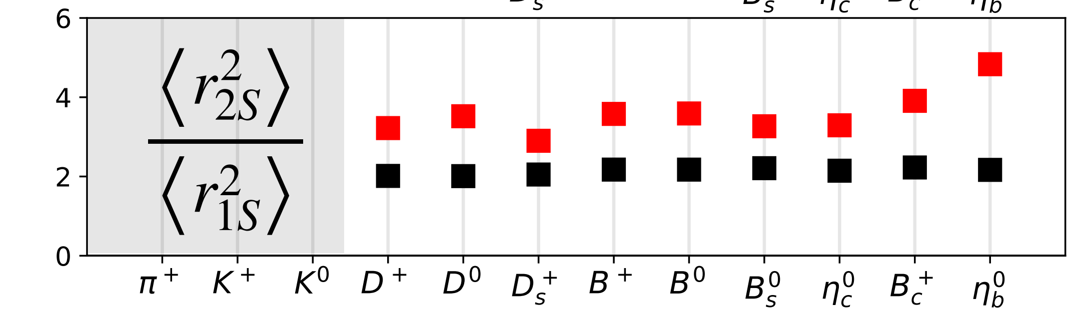
Charge radii [2S]











- Generally, radii of 2S are larger than 1S.
  —> Sensitive to wf.
  —> No lattice data.
- Ratio of 2S/1S radii
  —> Our: ~ 2 (same beta), ~ 3-4 (diff beta)
  —> Other [1]: ~ 4

[1] Y. Li et al. PLB 758 (2016) 118.

Summary

- Several discoveries of **radial excitations** of hadrons.
  —> Further search of missing states.

- They have some similarities and interesting behaviors.
  —> useful to understand hadron resonances.

- We believe they are standard quark model states,
  but we still have problems to explain their properties.
  —> e. g. : decay widths and decay constants.

- Further systematic studies of related observables.
  —> provide hints/constraints to their internal structures.

- Comparative studies of non-rel. and rel. quark model
  —> may clarify the role of relativistic effects

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