Proton image and momentum distributions from light-front dynamics

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Motivation

- The proton light-front wave function gives access to many observables in momentum space.
- For example:
  - Electromagnetic form factors
  - The parton distribution function, $f_1(x_1)$, i.e. probability distribution for a quark having a momentum fraction. Extracted from inclusive deep inelastic scattering.
  - Transverse momentum distribution. Dependence on both momentum fraction x and transverse one $\vec{k}_\perp$. Associated with semi-inclusive deeply inelastic scattering (SIDIS).
- Additionally, in the double parton scattering cross section enters the double parton distribution function (DPDF) [1]:

$$D(x_1, x_2, \vec{\eta}_\perp) = \sum_{n=3}^{\infty} D_n(x_1, x_2, \vec{q}_\perp) = \sum_{n=3}^{\infty} \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{d^2k_{2\perp}}{(2\pi)^2} \left\{ \prod_{i\neq 1,2} \int \frac{d^2k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\}$$
$$\times \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta\left(\sum_{i=1}^{n} \vec{k}_{i\perp}\right) \Psi_n^\dagger(x_1, \vec{k}_{1\perp} + \vec{\eta}_\perp, x_2, \vec{k}_{2\perp} - \vec{\eta}_\perp, ...) \Psi_n(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, ...)\,, \quad (1)$$

- The first of Mellin moments of DPDF has recently been calculated within lattice QCD [2].

[1] B. Blok et al, PRD 83 (2011) 071501 (R).

[2] G. S. Bali, JHEP09 (2021) 106.

Proton model on the light-front

- In this work we consider a dynamical three-body model directly in Minkowski space, allowing to compute observables on the LF, ultimately including the full BS amplitude.
- As a first step, Fock basis truncated to valence order and spin degree-of-freedom not included.
- The quark-quark transition amplitude has a pole representing the s-wave diquark introduced through the zero-range interaction between two of the quarks. In that sense it is an effective low-energy model.

Three-body Faddeev-Bethe-Salpeter equation with zero interaction

- Faddeev-Bethe-Salpeter (FBS) equation with zero-range interaction [1]:

$$v(q,p) = 2\mathrm{i}\mathcal{F}(M_{12}^2) \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\epsilon} \frac{\mathrm{i}}{(p-q-k)^2 - m^2 + i\epsilon} v(k,p) \tag{2}$$

- Currently, bare propagators for the quarks.
- $v(q,p)$ is one of the Faddeev components of the total vertex function.
- Di-quark concept introduced via assuming a pole in $\mathcal{F}(M_{12}^2)$, corresponding either to a two-body bound ($a > 0$) or virtual ($a < 0$) state, where a denotes the scattering length
- $\mathcal{F}(M_{12}^2)$, where $M_{12}^2 = (p-q)^2$, given by

$$\mathcal{F}(M_{12}^2) = \frac{\Theta(-M_{12}^2)}{\frac{1}{16\pi^2 y} \log \frac{1+y}{1-y} - \frac{1}{16\pi ma}} + \frac{\Theta(M_{12}^2)\,\Theta(4m^2 - M_{12}^2)}{\frac{1}{8\pi^2 y'} \arctan y' - \frac{1}{16\pi ma}} + \frac{\Theta(M_{12}^2 - 4m^2)}{\frac{y''}{16\pi^2} \log \frac{1+y''}{1-y''} - \frac{1}{16\pi ma} - \frac{iy''}{16\pi}}, \tag{3}$$

- The FBS equation was recently solved including the infinite number of Fock components in Euclidean [2] and Minkowski [3] space.

[1] T. Frederico, PLB 282 (1992) 409
[2] E. Ydrefors et al, PLB 770 (2017) 131
[3] E. Ydrefors et al, PLB 791 (2019) 276

Valence LF equation

- The valence three-body LF equation given by [1, 2]:

$$\Gamma(x,k_\perp) = \frac{\mathcal{F}(M_{12}^2)}{(2\pi)^3}\int_0^{1-x}\frac{dx'}{x'(1-x-x')}\int_0^\infty d^2k'_\perp\left[\frac{1}{M_0^2-M_N^2}-\frac{1}{M_0^2+\mu^2}\right]\Gamma(x',k'_\perp) \quad (4)$$

where μ is a cut-off, $k_\perp$ transverse momentum and x momentum fraction of spectator. Furthemore, the squared free three-body mass

$$M_0^2 = (k_\perp'^2+m^2)/x' + (k_\perp^2+m^2)/x + ((k'_\perp+k_\perp)^2+m^2)/(1-x-x') \quad (5)$$

- The three-body valence LF wave function is given by

$$\Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}) = \frac{\Gamma(x_1,\vec{k}_{1\perp})+\Gamma(x_2,\vec{k}_{2\perp})+\Gamma(x_3,\vec{k}_{3\perp})}{\sqrt{x_1x_2x_3}(M_N^2-M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))}, \quad (6)$$

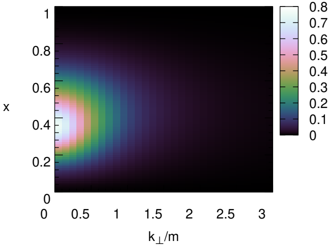
where due to momentum conservation: $x_3 = 1-x_2-x_3$ and $\vec{k}_{3\perp} = -\vec{k}_{1\perp}-\vec{k}_{2\perp}$.

[1] J. Carbonell and V.A. Karmanov, PRC 67 (2003) 037001

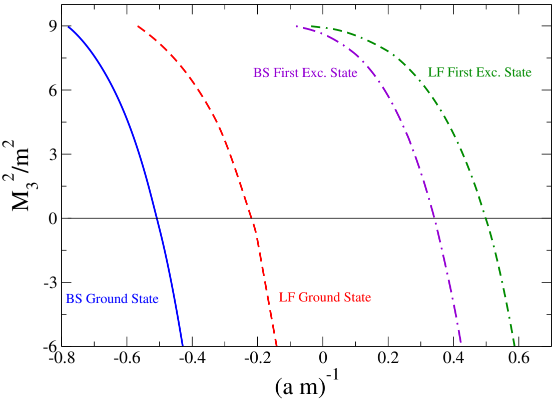
[2] T. Frederico, PLB 282 (1992) 409

Results for the vertex function

m [MeV]	a $[m^{-1}]$	μ/m	μ [MeV]	M_2 [MeV]	M_N/m
343	5.0	3.0	1029	668	6.825

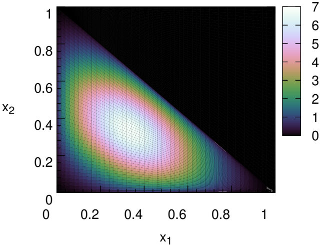


- The two parameters, namely a and μ, fitted to reproduce the experimental Dirac form factor (up to ~ 1 GeV^2).
- The proton structure contained in the vertex function $\Gamma(x, k_\perp)$. Concentrated at small $k_\perp$ and $x \approx 1/3$.



- As studied in PLB 770 (2017) 131, it exists a lower-lying unphysical solution with $M_N^2 < 0$. This is the relativistic analog of the well-known Thomas collapse. But, contrary to the non-relativistic case the unphysical state has a finite energy, due to a short-range repulsion of purely relativistic origin.
- Difference between valence LF result and full BS solution, due to a contribution coming from an infinite number of diagrams involving anti-particles, which can be interpreted as an effective three-body force.

Distribution amplitude



- The distribution amplitude is defined as

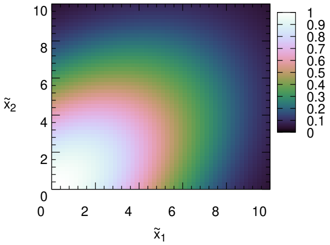
$$\phi(x_1, x_2) = \int d^2k_{1\perp} d^2k_{2\perp} \Psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}). \tag{7}$$

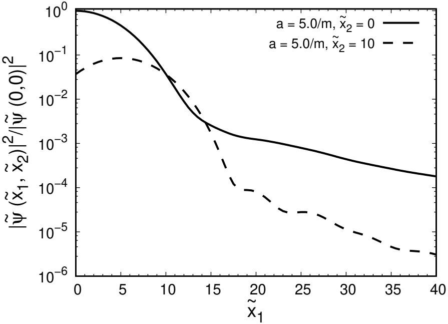
- It shows the dependence of the wave function on the momentum fractions for the case when the quarks share the same position.

Ioffe-time image of the valence state

- Alternatively, the proton can be studied in the on the null-plane, in terms of the transverse position ($\vec{b}_{i\perp}$) and the Ioffe-time $\tilde{x}_i = b_i^- p^+$. The image of the proton is then obtained through the Fourier transform of the proton LF wave function.
- For simplicity, we consider here the case $\vec{b}_{1\perp} = \vec{b}_{2\perp} = \vec{0}_\perp$, and then one has

$$\Phi(\tilde{x}_1, \tilde{x}_2) \equiv \tilde{\Psi}_3(\tilde{x}_1, \vec{0}_\perp, \tilde{x}_2, \vec{0}_\perp) = \int_0^1 dx_1 \, \mathrm{e}^{i\tilde{x}_1 \, x_1} \int_0^{1-x_1} dx_2 \, \mathrm{e}^{i\tilde{x}_2 \, x_2} \, \phi(x_1, x_2) \, , \qquad (8)$$





- For $\tilde{x}_2 = 10$ and $\tilde{x}_1 >= 10$ a rather dramatic decrease of the amplitude is seen.
- An exponential damping is seen with respect to the relative distance in Ioffe-time between the two quarks. We expect this damping to be even more significant if confinement is incorporated, as its more effective at large distances.

- The valence contribution to the Dirac form factor is given by

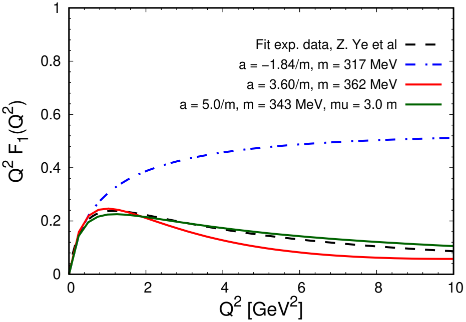
$$F_1(Q^2) = \left\{ \prod_{i=1}^{3} \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\} \delta\left(1 - \sum_{i=1}^{3} x_i\right) \delta\left(\sum_{i=1}^{3} \vec{k}^{\mathrm{f}}_{i\perp}\right) \times \Psi_3^\dagger(x_1, \vec{k}^{\mathrm{f}}_{1\perp}, ...)\Psi_3(x_1, \vec{k}^{\mathrm{i}}_{1\perp}, ...), \quad (9)$$

where $Q^2 = \vec{q}_\perp \cdot \vec{q}_\perp$ and the magnitudes of the momenta read

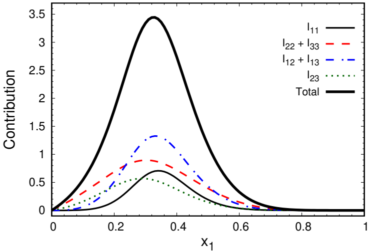
$$\left|\vec{k}^{\mathrm{f(i)}}_{i\perp}\right|^2 = \left|\vec{k}_{i\perp} \pm \frac{\vec{q}_\perp}{2} x_i\right|^2 = \vec{k}^2_{i\perp} + \frac{Q^2}{4} x_i^2 \pm \vec{k}_{i\perp} \cdot \vec{q}_\perp x_i \quad (i = 1, 2), \quad (10)$$

and

$$\left|\vec{k}^{\mathrm{f(i)}}_{3\perp}\right|^2 = \left|\pm \frac{\vec{q}_\perp}{2}(x_3 - 1) - \vec{k}_{1\perp} - \vec{k}_{2\perp}\right|^2 = (1 - x_3)^2 \frac{Q^2}{4} \pm (1 - x_3)\vec{q}_\perp \cdot (\vec{k}_{1\perp} + \vec{k}_{2\perp}) + (\vec{k}_{1\perp} + \vec{k}_{2\perp})^2. \quad (11)$$



- Good agreement with exp. data considering the simplicity of the model, only two parameters.
- However, the scaling laws of the QCD are not built-in, so high-momentum behavior should be viewed with caution.

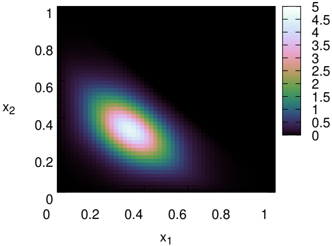


- We define the single parton distribution function (PDF) as

$$f_1(x_1) = \frac{1}{(2\pi)^6}\int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} |\Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp})|^2 =$$
$$I_{11} + I_{22} + I_{33} + I_{12} + I_{13} + I_{23}, \quad \int_0^1 dx_1 f(x_1) = F(0) = 1. \tag{12}$$

with the Faddeev contributions

$$I_{ii} = \frac{1}{(2\pi)^6}\int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma^2(x_i,\vec{k}_{i\perp})}{x_1x_2x_3(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))^2}$$
$$I_{ij} = \frac{2}{(2\pi)^6}\int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma(x_i,\vec{k}_{i\perp})\Gamma(x_j,\vec{k}_{j\perp})}{x_1x_2x_3(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))^2}; \quad i \neq j. \tag{13}$$

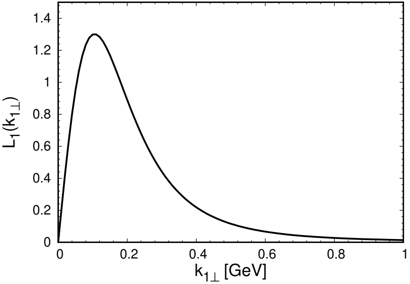


- The valence double parton distribution function (DPDF) is given by

$$D_3(x_1, x_2; \vec{\eta}_\perp) = \frac{1}{(2\pi)^6} \int d^2k_{1\perp} d^2k_{2\perp} \\ \times \Psi_3^\dagger(x_1, \vec{k}_{1\perp} + \vec{\eta}_\perp; x_2, \vec{k}_{2\perp} - \vec{\eta}_\perp; x_3, \vec{k}_{3\perp}) \Psi_3(x_1, \vec{k}_{1\perp}; x_2, \vec{k}_{2\perp}; x_3, \vec{k}_{3\perp}). \tag{14}$$

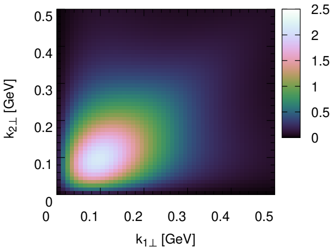
- Fourier transform of $D_3(x_1, x_2, \vec{\eta}_\perp)$ in $\vec{\eta}_\perp$ gives the probability of finding the quarks 1 and 2 with momentum fractions x_1 and x_2 at a relative distance $\vec{y}_\perp$ within the proton.
- $D_3 = 0$ for $x_1 + x_2 > 1$, as it should, due to momentum conservation.

Transverse momentum densities



- The single quark transverse momentum density in the forward limit and integrated in the longitudinal momentum is associated with the probability density to find a quark with momentum $k_\perp$.
- It can be computed as:

$$L_1(k_{1\perp}) = \frac{k_{1\perp}}{(2\pi)^6} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{2\pi} d\theta_1 \int d^2 k_{2\perp} |\psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp})|^2. \tag{15}$$



- The corresponding two-quark one reads

$$L_2(k_{1\perp}, k_{2\perp}) = \frac{k_{1\perp} k_{2\perp}}{(2\pi)^6} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 \times |\psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp})|^2. \tag{16}$$

Conclusions

- We have, in this work, studied the proton in a simple but fully dynamical valence LF model based on a zero-range interaction.
- The model is based on the concept of a strongly bound interacting diquark.
- We have studied the structure of the proton by computing the LF wave function in its Ioffe-time representation and also momentum distributions.
- However, the model is rather crude since e.g. the spin degree of freedom hasn't been included yet. But is a first step towards studying the proton directly in Minkowski space.
- Future plans:
  - Generalization to the infinite set of Fock components (The Faddeev-Bethe-Salpeter equation solved in PLB 791 (2019) 276)
  - Implementation of a more realistic interaction (gluon exchange)
  - Inclusion of spin degree of freedom